

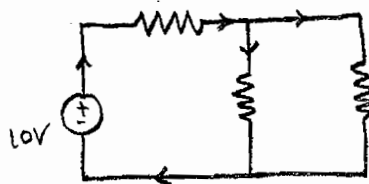
13/8/2013

FUNDAMENTALS

CIRCUITS

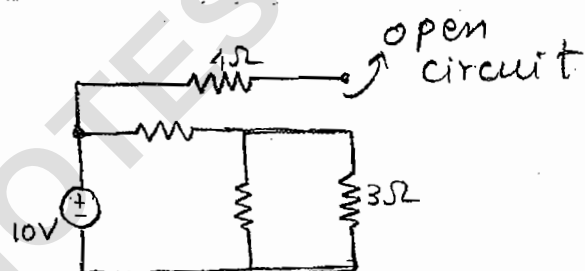
→ Current is intended to flow through all elements.

This closed path concept is circuit.



NETWORKS

→ Current does not necessarily flow through all the elements.



→ Circuits or networks are interconnection of various components to act together.

→ Most of our practical sys. are big interconnector n/ws but we do circuit analysis to some of its parts where electrical energy flows.

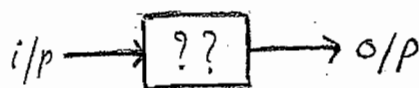
So circuits are building blocks of networks.

• NETWORK COMPONENTS / ELEMENTS

All our applications are our components, but when these components are modelled as a circuit or n/w, we use fundamental n/w component to model them like, V, I, R, L, C , etc.

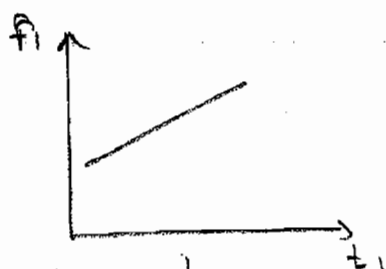


Analysis
(unique solution)

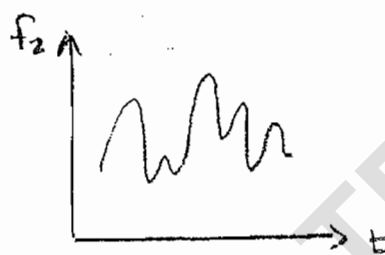


Synthesis
(various procedures can be performed)

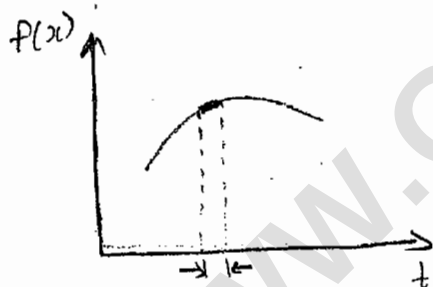
• LINEAR COMPONENT / CIRCUIT



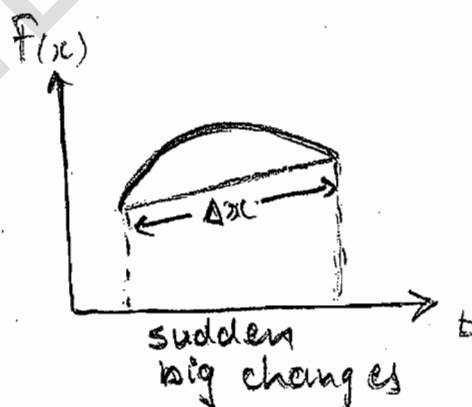
can be modelled mathematically
 $y = mx + c$



can not be modelled mathematically



small incremental changes



sudden big changes

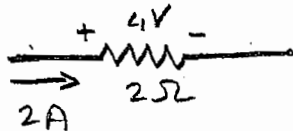
→ Most of our practical components / n/w are non linear in nature, but any non-linear sys. can be linearised for small incremental changes in time.

But the same sys. under sudden big changes undergo non-linear mode of operation.

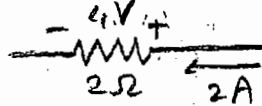
We design all our n/w practically for specified

ratings & as long as they obey Ohm's Law, Kirchhoff's Law, superposition principle, etc. they are said to be linear.

• BILATERAL ELEMENTS :-



$$P_{\text{lost}} = 8 \text{ W}$$

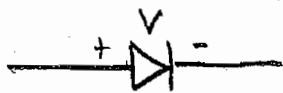


$$P_{\text{lost}} = 8 \text{ W}$$

Property independent to
Voltage $\rightarrow$ Polarity
Current $\rightarrow$ dirⁿ

eg:- R, L, C

• UNILATERAL ELEMENTS :-



Frwd bias
Can conduct
 $R_{\text{on}} = 0 \Omega$

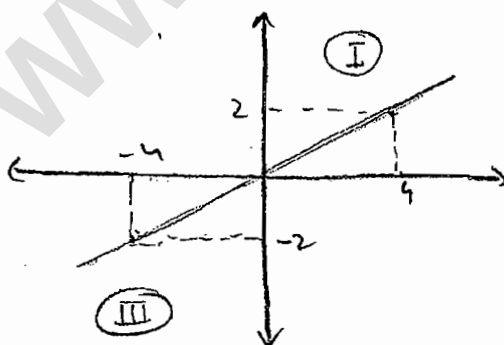


Reverse bias
Cannot conduct
 $R_{\text{on}} = \infty \Omega$

Property depends on
Voltage $\rightarrow$ Polarity
Current $\rightarrow$ dirⁿ

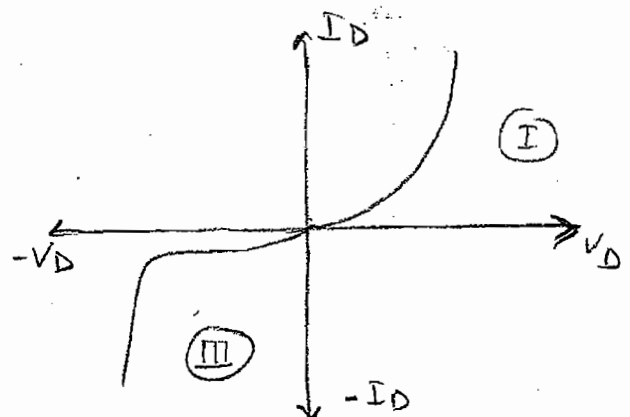
eg:- BJT, FET, diode, etc.

Based on V-I characteristics :-



$$\textcircled{I} = \textcircled{III}$$

Bilateral



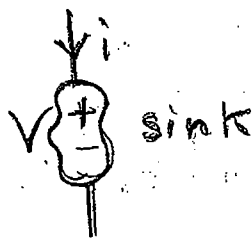
$$\textcircled{I} \neq \textcircled{III}$$

Unilateral

• PASSIVE ELEMENTS (sinks)

- absorb
 - dissipate
 - waste
 - convert
 - store
- electrical energy

eg:- R, L, C.



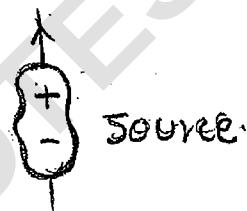
$P_{\text{absorbed}} \Rightarrow +ve$

$P_{\text{delivered}} \Rightarrow -ve$

• ACTIVE ELEMENTS (sources)

- Energize
 - Drive externally
 - deliver
 - give out
- electrical energy

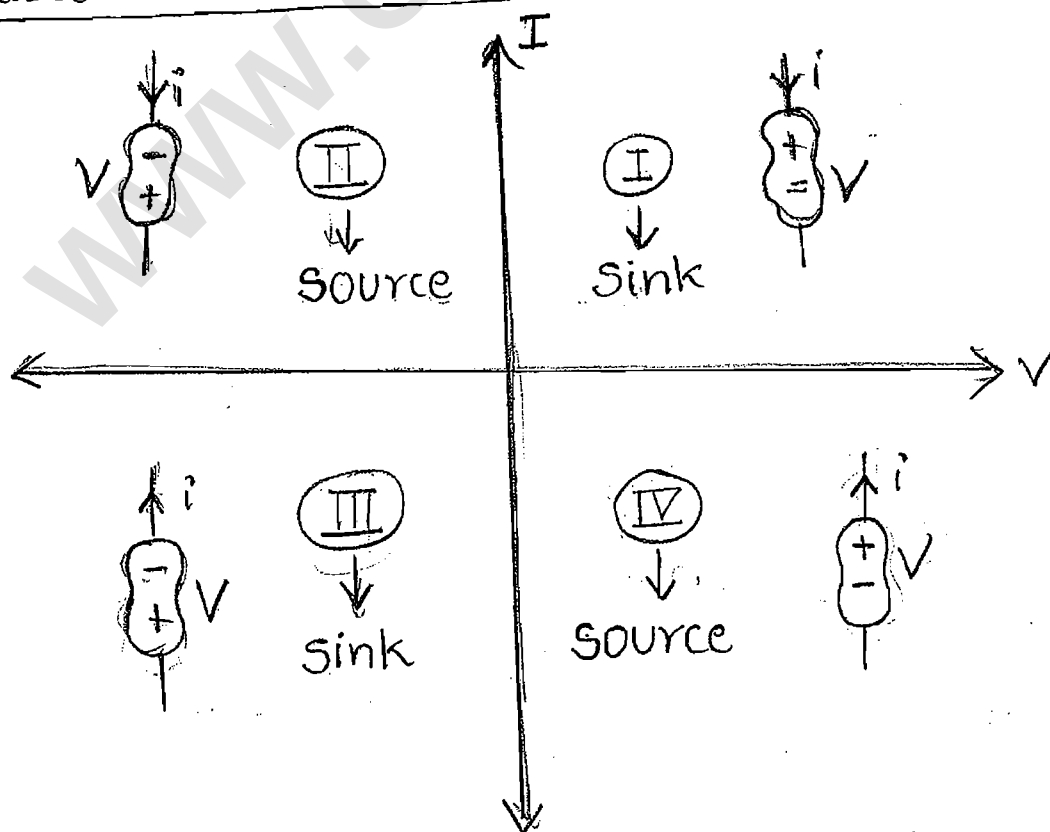
eg:- V, I



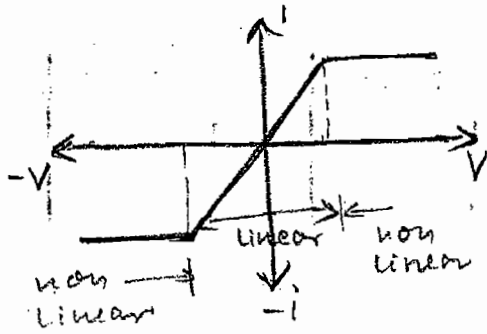
$P_{\text{absorbed}} \Rightarrow -ve$

$P_{\text{delivered}} \Rightarrow +ve$

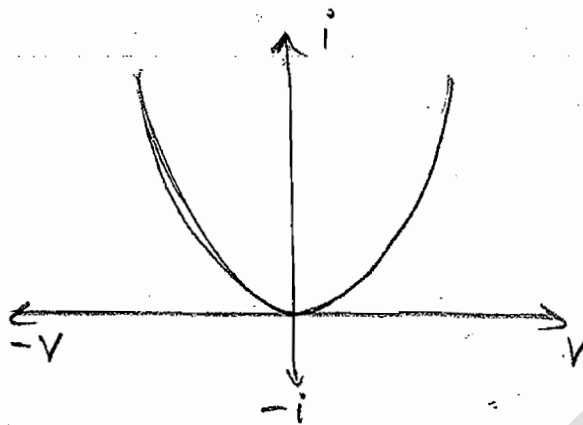
Based on V-I characteristics :-



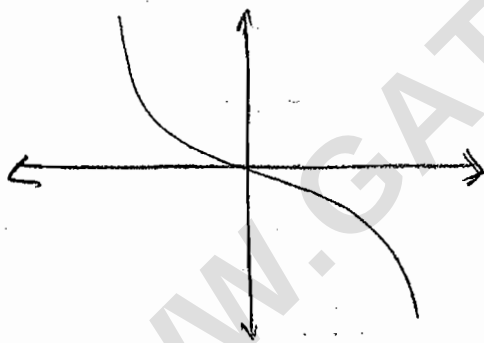
Eg: 1) The static V-I charac. of component is shown below, then component is _____.



- ① Linear, active, bilateral
- ② Linear, passive, bilateral
- ③ Non-linear, active, unilateral
- ✓ ④ Non-linear, passive, bilateral



- Non-linear
- Unilateral
- Both active & passive
- overall → Active

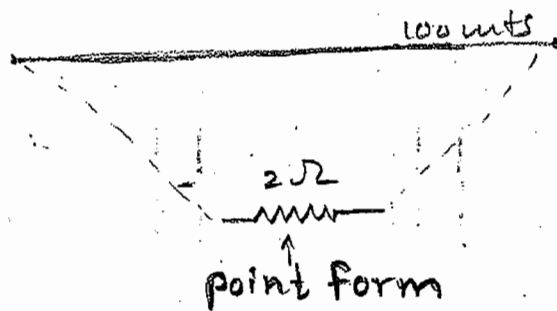


- Non linear
- Active
- Bilateral.

Active elements can act as passive elements, but passive elements can't act as active

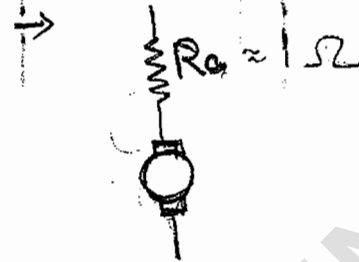
eg:- Capacitor always acts as a sink; either it charges or discharges.

• LUMPED PARAMETERS



Eg :-

→ PCB component



Properties :-

- Simple
- Linear Algebraic equation
- Solutions are fast
- Approximated values.

• DISTRIBUTED PARAMETERS



$$R_{\text{actual}} = \frac{\partial R}{\partial l}$$

Eg :-

- Long Tx line
- Emf concepts
- Antenna (waveguide)

Properties :-

- Complex
- Linear Differential Equations
- Solutions are Tedious
- Very accurate values.
- Exact modelling

2) The relations like $V = n\lambda$ holds good for _____

IES

(a) Lumped

✓ (b) Distributed

(c) Lumped & Dist.

(d) None

- **NODE (n) :**

A node is a point of interconnection or junction b/w 2 or more components.

- **BRANCH (b) :**

A branch is an elemental connection between two nodes.

- **Degree of a Node (S) :**

No. of branches incident or connected at any node represents its degree.

If $S = 2 \rightarrow$ simple node (n_s)

$S > 2 \rightarrow$ principle node (n_p)

NOTE :-

For any ckt or n/w

$$\sum_{i=1}^n S_i = 2 \times b$$

- **MESH (m) :**

Mesh is a closed path of ckt or n/w which should not have further closed path in it.

• Loops (l) :

Loops are all possible closed path of n/w

NOTE:-

→ For any ckt or n/w : $m = b - n + 1$

→ The mini. no. of eqⁿ to solve any ckt or n/w is :

$$m = b - n + 1$$

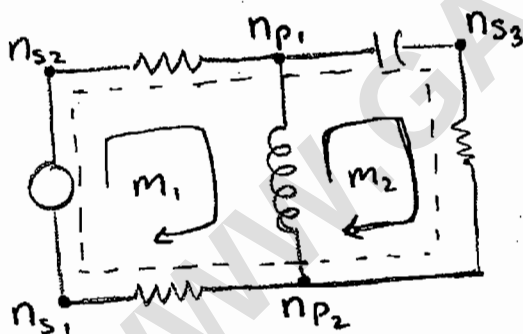
→ Meshes are specifically called as independent loops.

→ All meshes are by default loops but all loops are not meshes.

→ In nodal analysis we may neglect single node & one of the principle node is considered as reference.

Ex:-

3)



$$n = 5$$

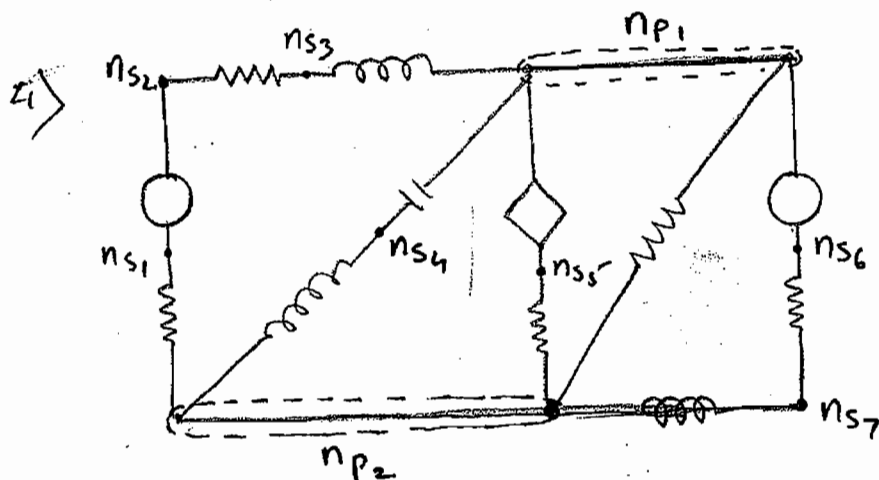
$$b = 6$$

$$m = 2$$

$$l = 2 + 1 = 3$$

$$\sum \delta_i = 2 + 2 + 2 + 3 + 3 = 12$$

$$b \times 2 = 6 \times 2 = 12$$



$$n = 9$$

$$b = 12$$

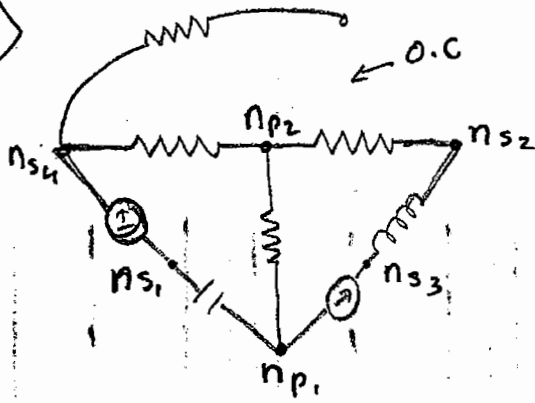
$$m = 4$$

$$l = 4 + 5 = 9$$

$$\sum \delta_i = (7 \times 2) + 5 + 5 = 24$$

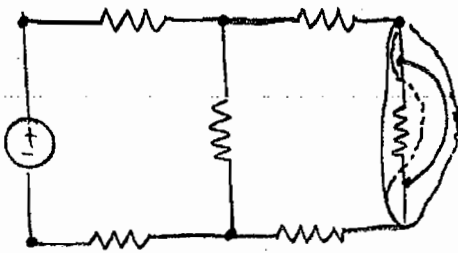
$$= 2 \times 12$$

5)



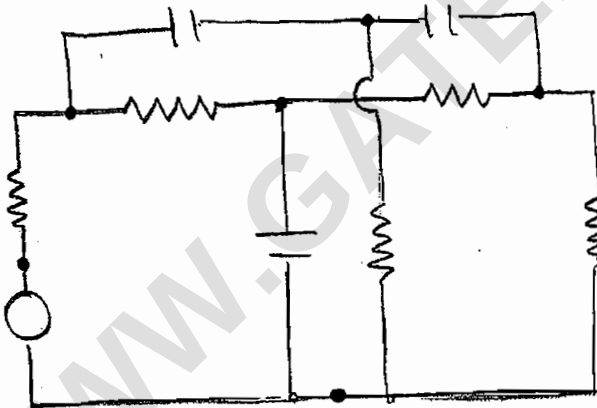
$$\begin{aligned} n &= 6 \\ b &= 7 \\ m &= 2 \end{aligned}$$

6)

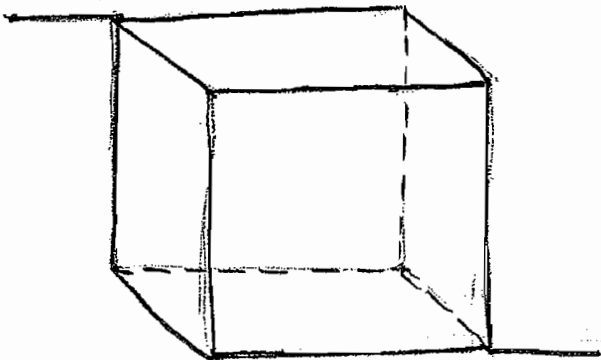


$$\begin{aligned} n &= 5 \\ b &= 6 \\ m &= 2 \end{aligned}$$

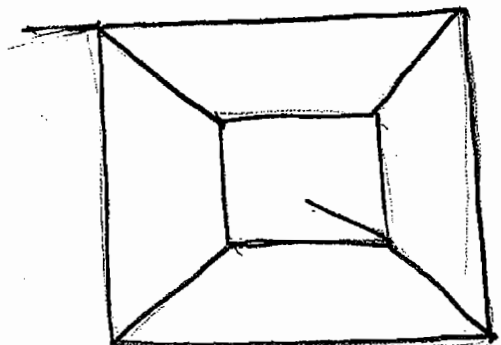
7) The mini. no. of eq^s required to solve the ckt below is 4.



$$\begin{aligned} m &= b - n + 1 \\ &= 9 - 6 + 1 \\ &= 4 \end{aligned}$$



Non-planar



Planar

* OHM'S LAW

→ L.T.I

→ Temperature is const.

→ Uniform cross section of material

$$\boxed{J = \sigma E} \quad (1^{st} \text{ Form})$$

Current density $\swarrow$ $\downarrow$ Conductivity $\searrow$ Electric field intensity

$$\frac{I}{a} = \sigma \frac{V}{l}$$

$$V = \left[\frac{l}{\sigma a} \right] I$$

but $\frac{l}{\sigma} = \rho$ (resistivity)

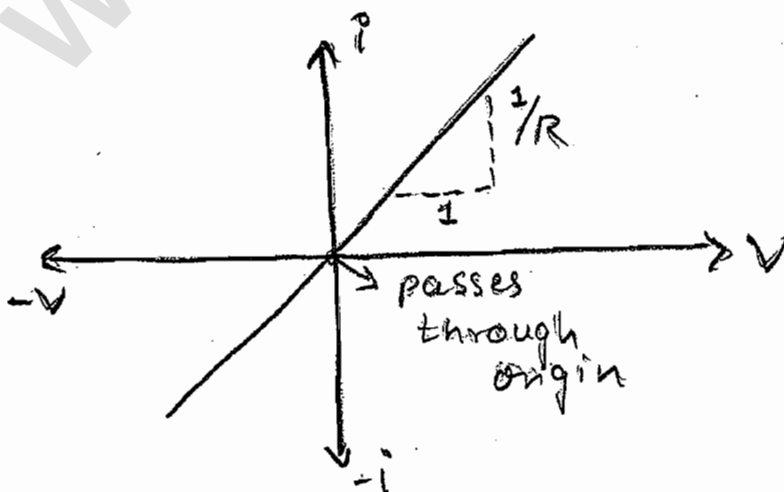
$$V = \left[\frac{\rho l}{a} \right] I$$

$$\boxed{V = IR} \quad (2^{nd} \text{ Form})$$

$\downarrow$
Resistance (Ω)

Circuit form of Ohm's Law in 'R'

Graphically, $I = \left(\frac{1}{R} \right) V$



$[L] \rightarrow \Sigma \text{electromagnetic}$

$$\boxed{\Psi = Li} \quad (3^{\text{rd}} \text{ Form})$$

$$\Psi = N\Phi$$

$\hookrightarrow$ Flux linkage (wb-T)

$$\boxed{N\Phi = Li}$$

$$N \frac{d\Phi}{dt} + \Phi \frac{dN}{dt} = L \frac{di}{dt} + i \frac{dL}{dt}$$

$\therefore \boxed{V = L \frac{di}{dt}}$ (4^{th} Form) Critical form of Ohm's Law in 'L'

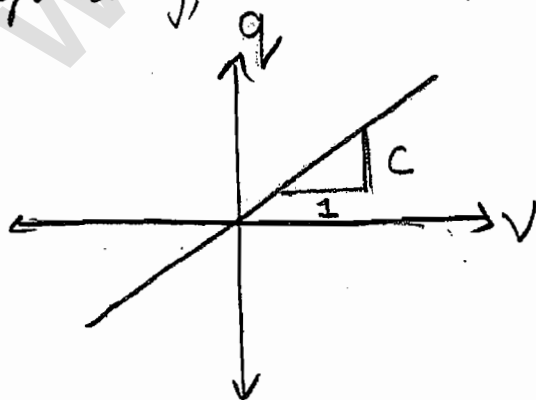
$[C] \rightarrow \Sigma \text{electro-static}$

$$\boxed{q = CV} \quad (5^{\text{th}} \text{ Form})$$

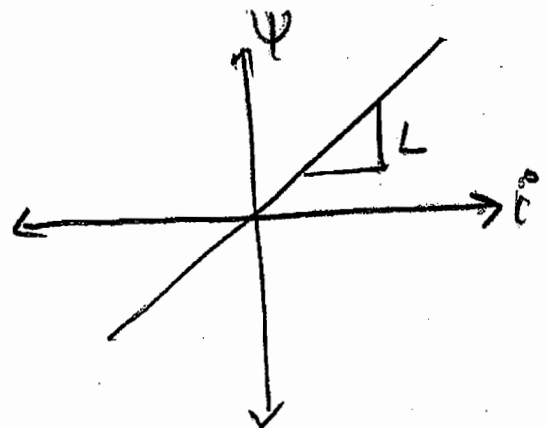
$$\frac{dq}{dt} = C \frac{dV}{dt} + V \frac{dC}{dt} \rightarrow \text{If } \epsilon = 0 \Rightarrow \frac{dC}{dt} = 0$$

$\boxed{i = C \frac{dV}{dt}}$ (6^{th} Form) Critical form of Ohm's Law in 'C'

Graphically,



Slope = C



Slope = L

* DC CIRCUIT ANALYSIS

• Properties of DC Supply :-

- Unipolar
- Unidirectional
- No change in phase / polarity
- Power freq. = 0 Hz.

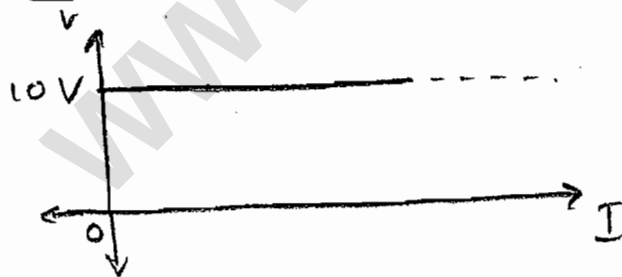
They are used in small, independent isolated power supply systems, where electrical energy can be stored in small capacities.

Eg:-

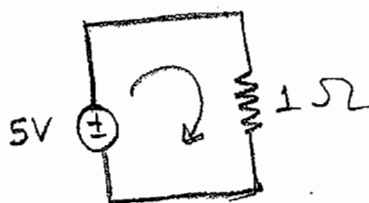
- Machine Tools
- Medical Instruments
- Cell phones, Toy
- Defence Applications

Precision
Accuracy
Automation

• Standard DC Waveform :-



$$T \rightarrow \infty$$
$$f \rightarrow 0$$



* AC CIRCUIT ANALYSIS

• Properties of AC supply :-

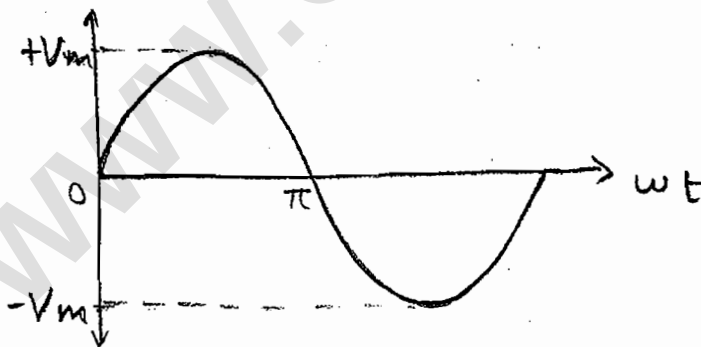
- Bipolar
- Bidirectional
- Definite change in phase / polarity.
- Power freq. exist (India = 50 Hz)

They are used in Large, Bulk, Continuous power supply systems, where electrical energy cannot be stored.

Eg:-

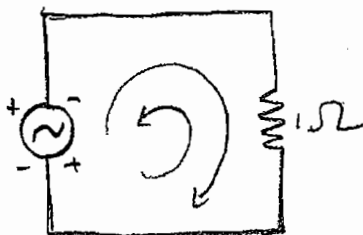
- Domestic
 - Industrial Applications.
- } Robust
Powerful

• Standard AC Waveform :-



Sinusoid $\left\{ \begin{array}{l} \rightarrow \text{sine} \\ \rightarrow \text{cosine} \end{array} \right.$

$$V = V_m \sin \omega t$$



VOLTAGE :

$$V = \frac{dW}{dq}$$

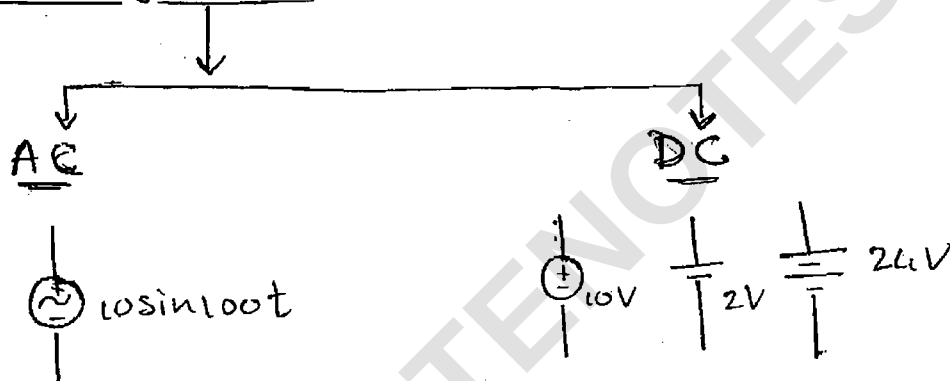
→ It is the force (Emf), which can drive charges.

→ Units : volts or J/C.

→ Range : kV, mV, V, MV, μ V.

→ Symbols : v , $\bar{V}$, $v(t)$

→ Circuit Symbols :



→ Examples :

DC → Cell, Battery, Fuel cells.

P-V solar panels

Rectified power sources

Conventional DC converter

AC → UPS

Inverter

Alternator

CURRENT⁺:

$$i = \frac{dq}{dt} \quad \text{or} \quad I = \frac{\Delta Q}{\Delta t} = \frac{Q}{t}$$

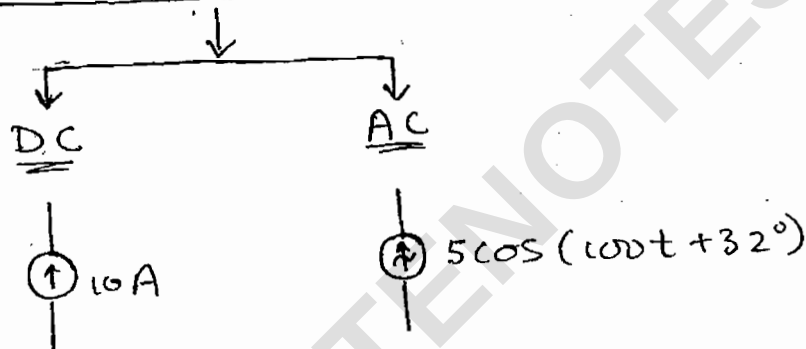
→ It is rate of flow of charge

→ Units: Ampere (A) or C/sec

→ Range: μA , mA , A , kA

→ Symbols: i , I , $\bar{I}$, $i(t)$

→ Circuit Symbols:



⇒ Examples:

DC → A DC series generator can be modeled as DC current source

→ A BJT can be modeled as a DC dependent current source.

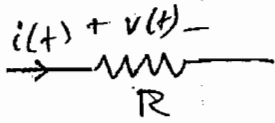
AC → A Feeder can be modeled as a AC current source.

(Feeder → const. current density conductors.)

RESISTANCE :

→ It is electrical property of matter.

→ "Resistor" is a component to model it.



→ it is classified based on the material.

- Carbon
- Tungsten
- Ceramic

→ Units : ohms (Ω),

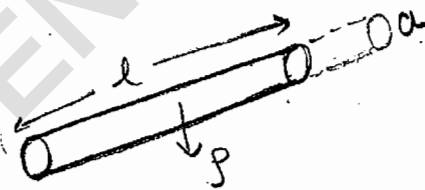
$$\frac{V}{A}$$

→ Range : $\mu\Omega$, $m\Omega$, Ω ,
 $k\Omega$, $M\Omega$, $G\Omega$

$$V = IR \Rightarrow I = \frac{V}{R}$$

Basic Formula :

$$R = \frac{\rho l}{a} \Omega$$



l → length of material

a → cross section area

ρ → specific resistance (or)
resistivity of material

→ Resistance depends upon temperature

$$R_t = R_0 [1 + \alpha t]$$

α → temperature co-eff. of resistance

α is +ve → conductors

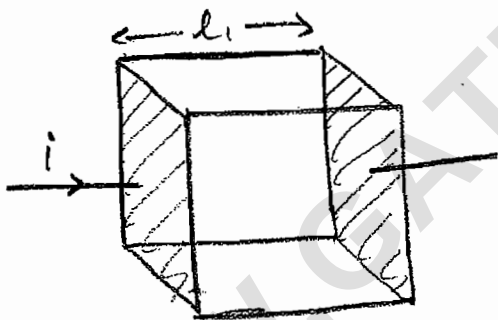
α is -ve → semiconductors.

Examples:-

- All industrial & Domestic wiring.
- Communication & Tx. lines.
- PCB components.

Ex:-

- 1) A cube shaped material has a resistance of 2Ω between any of its opposite faces. Now if this material is stretched in one direction by applying a linear force to double its original length, then the resistance between the two opposite stretched faces is _____.



$$R_1 = \frac{\rho_1 l_1}{a_1} = 2$$

$$\rho_2 = \rho_1$$

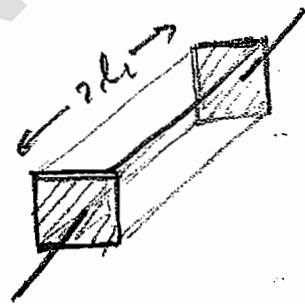
$$l_2 = 2l_1$$

$$V_1 = V_2$$

$$l_1 a_1 = l_2 a_2$$

$$\therefore a_1 = 2a_2$$

$$\therefore a_2 = \frac{a_1}{2}$$



$$R_2 = \frac{\rho_2 l_2}{a_2} = \frac{\rho_1 (2l_1)}{a_1/2}$$

$$= 4 \left[\frac{\rho_1 l_1}{a_1} \right]$$

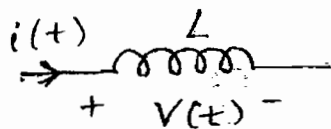
$$= 4(2)$$

$$= 8 \Omega$$

INDUCTANCE

Electromagnetic property matter

→ "Inductor" is a component to model it.



→ It is classified based on CORE material

- Iron
- Ferrite
- air

Units : Henry (H) ,

$$\frac{V \cdot \text{sec}}{A}$$

Range : μH , mH , H

$$V = L \frac{di}{dt} \Rightarrow i = \frac{1}{L} \int v dt$$

$$i = \frac{1}{L} \int_{-\infty}^t v dt$$

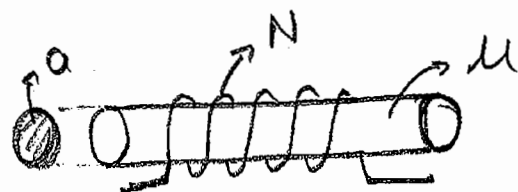
$$= \frac{1}{L} \int_{-\infty}^0 v dt + \frac{1}{L} \int_0^t v dt$$

initial current

$$i = I_0 + \frac{1}{L} \int_0^t v dt$$

Basic Formula :-

$$L = \frac{\mu N^2 a}{l} H$$



$\mu = \mu_0 \mu_r \rightarrow$ permeability of CORE

$$\mu_0 = 4\pi \times 10^{-7} H/m$$

$$\mu_r = 1 \text{ (air)}$$

$$\mu_r > 1000 \text{ (iron)}$$

$N \rightarrow$ No. of turns of coil

$a \rightarrow$ cross-sectional area of core (m^2)

$l \rightarrow$ effective length of magnetic flux path (m)

Examples :-

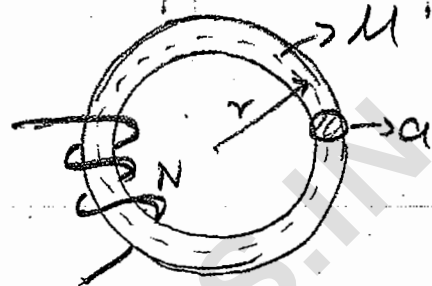
$\rightarrow$ Filter

$\rightarrow$ Choke coils

$\rightarrow$ MIC windings

$\rightarrow$ Tx lines.

mH / μ H / kH



$$l = 2\pi r$$

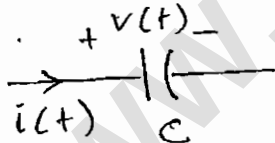
$l \rightarrow$ mean circumference.

CAPACITANCE :

$\rightarrow$ Electrostatic property matter

$\rightarrow$ "Capacitor" is a component to model it.

$\rightarrow$ It is classified based on Dielectric element/material.



Units : Faraday (F),

$$\frac{A \cdot \text{sec}}{V}$$

$\rightarrow$ Electrolytic

$\rightarrow$ Ceramic

$\rightarrow$ Polyester

Range : pF, nF, μ F, mF

$$i = C \frac{dv}{dt}$$

$$V = \frac{1}{C} \int i dt$$

$$V = \frac{1}{C} \int_{-\infty}^t i dt = \frac{1}{C} \int_{-\infty}^0 i dt + \frac{1}{C} \int_0^t i dt$$

initial voltage

$$V = V_0 + \frac{1}{C} \int_0^t i dt$$

Basic Formula :-

$$C = \frac{\Sigma A}{d} F$$

$\Sigma = \Sigma_0 \Sigma_r \rightarrow$ permittivity of dielectric

$$\Sigma_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\Sigma_r = 1 \text{ (air)}$$

$$\Sigma_r = 6 \text{ (polyester)}$$

$$\Sigma_r = 700 \text{ (ceramic)}$$

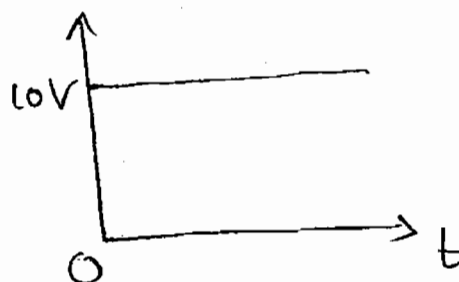
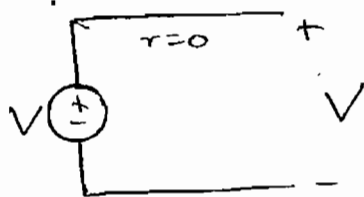
$d \rightarrow$ dist. between electrodes (m)

$A \rightarrow$ common cross-sectional area b/w electrodes (m^2)

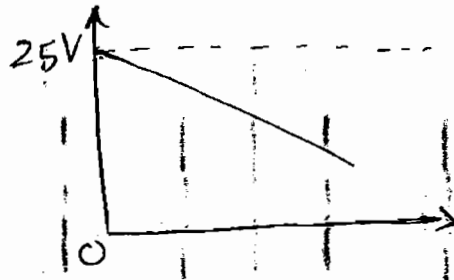
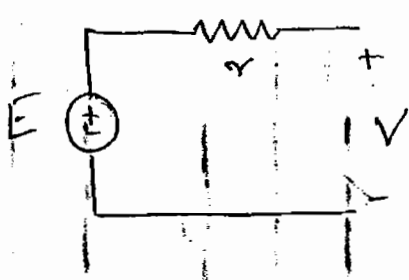
Examples :-

- $\rightarrow$ Filters
- $\rightarrow$ Power system
- $\rightarrow$ PDC circuits
- $\rightarrow$ Tx lines $\mu\text{F/ph/km}$

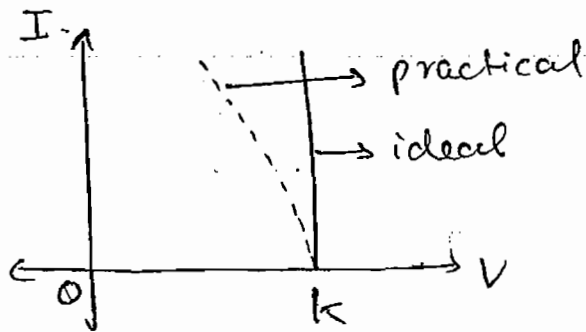
IDEAL VOLTAGE SOURCE :-



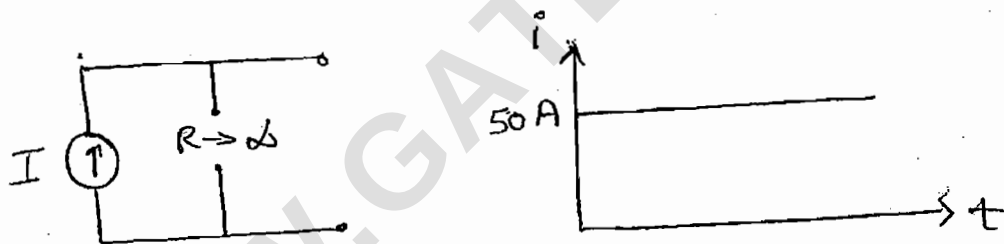
PRACTICAL VOLTAGE SOURCE :-



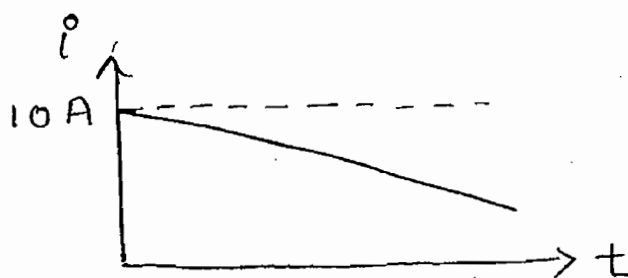
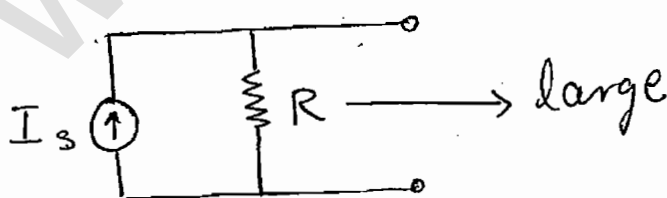
V-I Characteristics :-



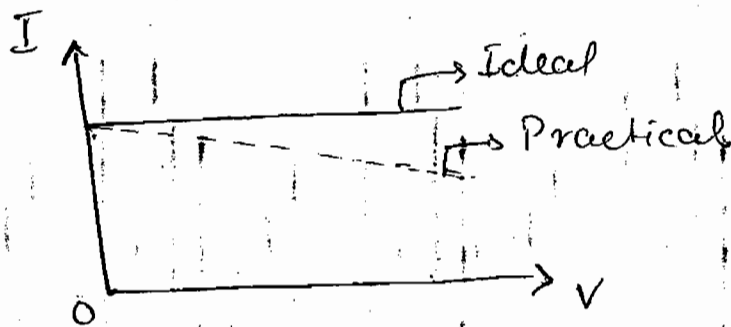
IDEAL CURRENT SOURCE



PRACTICAL CURRENT SOURCE



V-I Characteristics :-



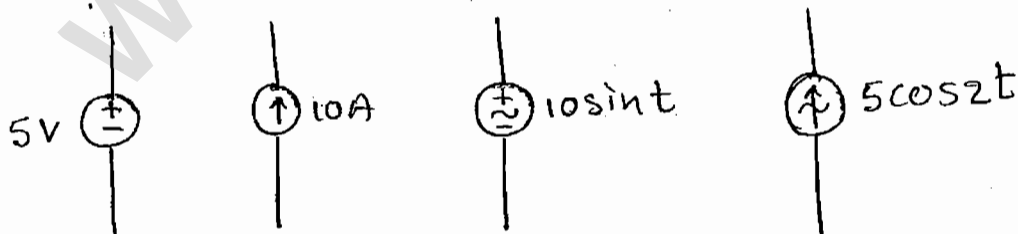
INDEPENDENT SOURCES :-

- Properties
- Characteristics
- Values [Mag, Freq., etc]

These are independent to any other parameter within or outside the circuit.

Examples :-

→ Ideal sources



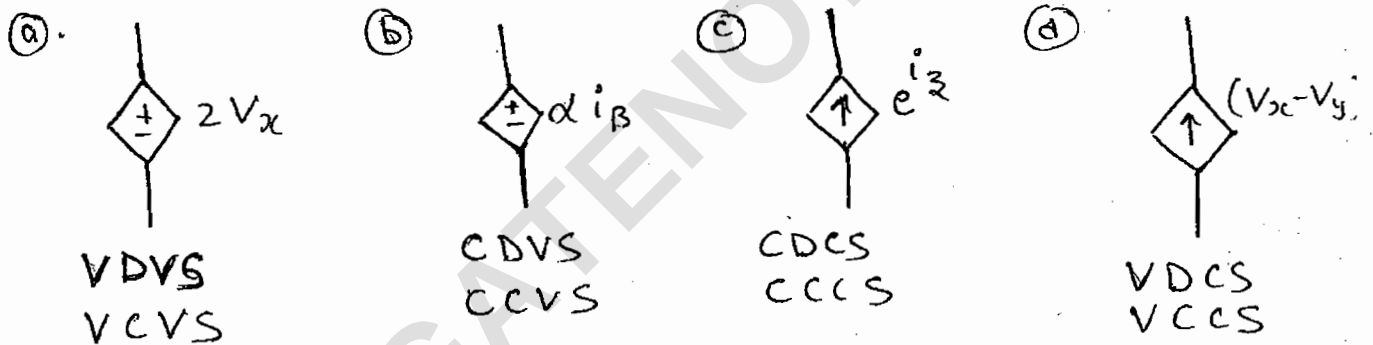
DEPENDENT SOURCE :-

- Properties
 - Characteristics
 - Values [Mag, Freq, etc]
- } depend upon any other parameter within or outside the ckt

Examples :-

- Practical sources
- BJT
- Solar cell

→ Standards. (4) Types :-



→ Unlike independent sources dependent sources cannot be suppressed in terms of resistance; as these models by themselves represent complex circuits

POWER :

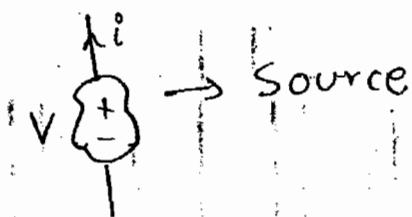
Rate of change in energy.

$$P = \frac{dW}{dt} = \frac{dE}{dt} ; P = \frac{E}{t}$$

Units : Watts or J/sec

Range : mW, W, kW, MW, GW

$$1 \text{ hp} = 746 \text{ W}$$



$$P_{\text{delivered}} = + \underline{V i} \text{ W}$$

$$= \frac{dW}{dq} \cdot \frac{dq}{dt} = \frac{dW}{dt}$$

L.T.I

[R]

$$P_R = V_R i_R = i_R^2 R = \frac{V_R^2}{R}$$

[L]

$$P_{\text{avg}} = 0$$

instantaneous power

$$P_L = V_L \cdot i_L = L i \frac{di}{dt} \text{ W}$$

[C]

$$P_{\text{avg}} = 0$$

instantaneous power

$$P_C = i_C V_C = C V \frac{dV}{dt} \text{ W}$$

ENERGY :-

Capacity to do work (electrical)

$$E = \int P dt$$

$$E = P \times t$$

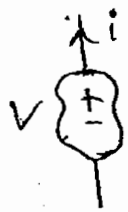
Units : Joules or W-sec

$$1 \text{ unit of E.E.} = 1 \text{ kWh}$$

$$1 \text{ kWh} = 1000 \text{ W} \times 1 \text{ hr}$$

$$= 500 \text{ W} \times 2 \text{ hr}$$

$$= 100 \text{ W} \times 10 \text{ hr} = 2000 \text{ W} \times \frac{1}{2} \text{ hr}$$



$$E_{\text{delivered}} = + \underline{V \cdot i \cdot t} \quad \text{J}$$

$$\boxed{1 \text{ kWh} = 36 \times 10^5 \text{ J}}$$

[R]

$$E_R = \int P_R dt = \int V_R i_R dt = \int i_R^2 R dt = \int \frac{V_R^2}{R} dt$$

→ But for L.T.I.

$$E_R = V_R \cdot i_R \cdot t = i_R^2 R \cdot t = \frac{V_R^2}{R} \cdot t \quad \text{J}$$

[L]

$$E_L = \int P_L dt = \int L i \frac{di}{dt} \cdot dt$$

→ Now for L.T.I.

$$E_L = \int L i \frac{di}{dt} \cdot dt = \frac{1}{2} L i^2$$

$$\text{But } \psi = L i$$

$$\therefore \boxed{E_L = \frac{1}{2} L i^2 = \frac{1}{2} \psi i = \frac{\psi^2}{2L} \quad \text{J}}$$

[C]

$$E_C = \int P_C dt = \int C V \frac{dV}{dt} \cdot dt$$

→ But for L.T.I.

$$E_C = \int C V \frac{dV}{dt} dt = \frac{1}{2} C V^2$$

$$\text{But } q = C V$$

$$\boxed{E_C = \frac{1}{2} C V^2 = \frac{1}{2} q V = \frac{q^2}{2C} \quad \text{J}}$$

ENERGY STORAGE CAPACITY in a Battery:—

→ Ampere-hours (Ahr)

Eg: Pencil cell (AA) → 1.5 V, 500 mAh

Cell phones → 3.7 V (1500–1800) mAh

Cell batteries → 12 V, 40 Ahr

Eg:-

40 Ahr → 40 A × 1 hr

→ 20 A × 2 hr

→ 1 A × 40 hr

→ 80 A × $\frac{1}{2}$ hr

KIRCHOFF'S LAWS:—

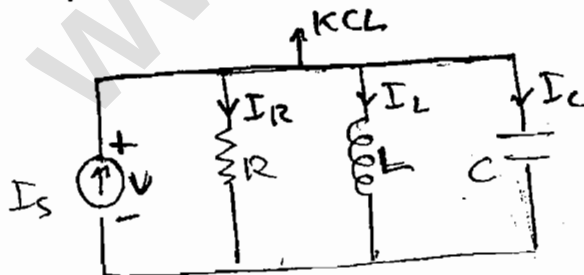
(1) K^{1st} Law → KCL → Node

$$\sum_i I_{\text{node}} = 0$$

↓ $\frac{V}{R}$ ↓ $\frac{1}{L} \int V dt$ ↓ $C \frac{dV}{dt}$

Based on Law of conservation of charge.

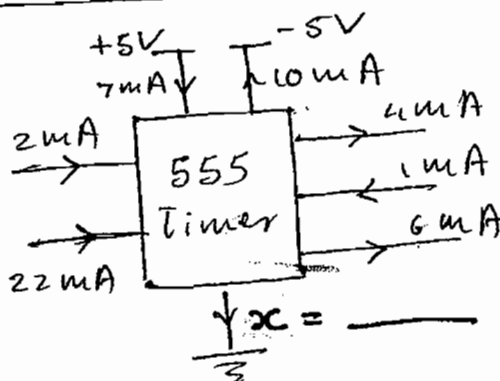
Eg:-



$$-I_s + I_R + I_L + I_C = 0$$

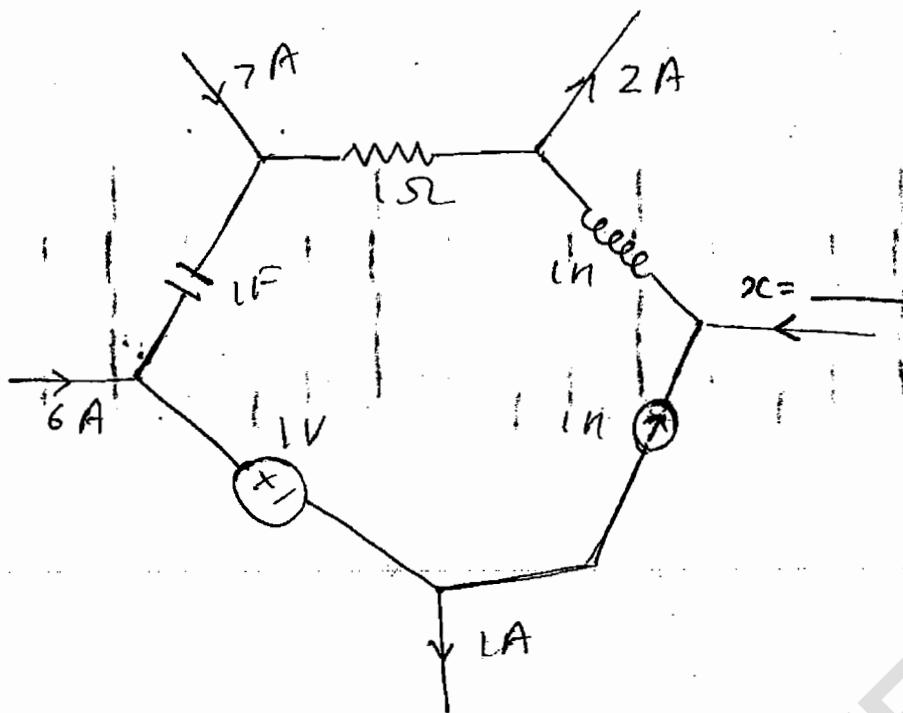
$$I_s = \frac{V}{R} + \frac{1}{L} \int V dt + C \frac{dV}{dt}$$

Eg:-



$$2 + 2 + 7 + 1 = x + 6 + 4 + 10$$

$$\therefore \boxed{x = 12 \text{ mA}}$$



$$7 + 6 + x = 1 + 2$$

$$13 + x = 3$$

$$x = -10$$

Actually the current is coming out of the node.

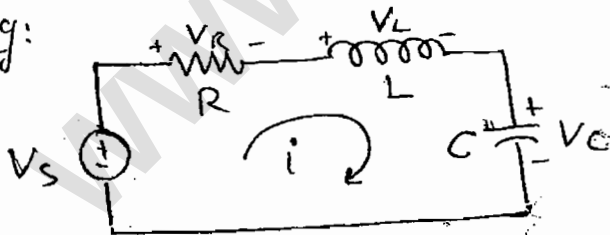
(2) Knd Law → KVL → Mesh

→ Based on law of conservation of energy

$$\sum V_{\text{mesh}} = 0$$

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

Eg:



KVL

$$-V_s + V_R + V_L + V_C = 0$$

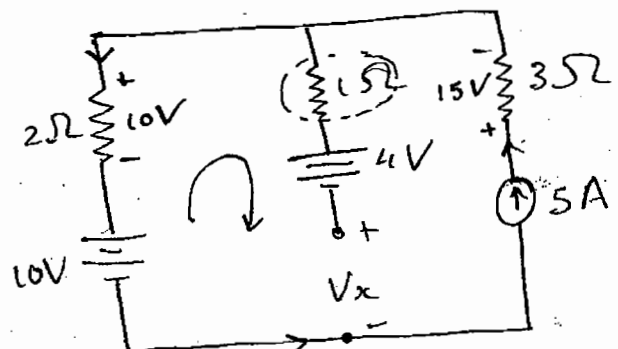
$$V_s = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$

Eg:

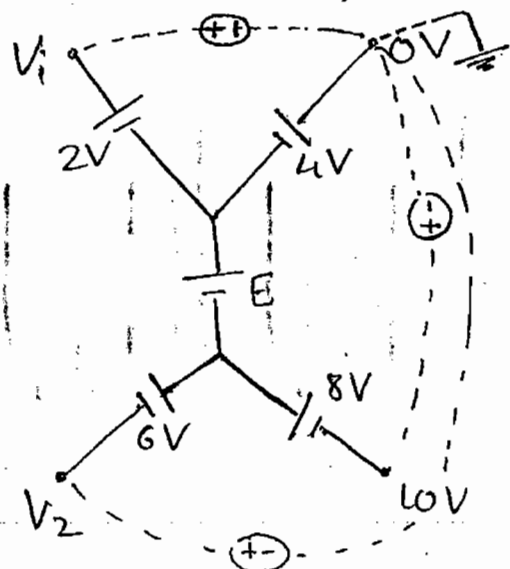
Find $V_x =$

$$+10 + 10 - 4 = V_x$$

$$V_x = 16 \text{ V}$$



Eg: Find E, V_1, V_2 .



$$0 - 4 - E - 8 = 10$$

$$\boxed{E = -22 \text{ V}}$$

$$V_2 + 6 - 8 = 10$$

$$V_2 = 12 \text{ V}$$

$$V_1 - 2 + 4 = 0$$

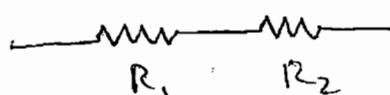
$$\boxed{V_1 = -2 \text{ V}}$$

$$V_2 + 6 + E + 4 = 0$$

$$\boxed{V_2 = 12 \text{ V}}$$

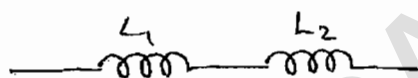
• Series Connection of Elements :-

[R]



$$R_s = R_1 + R_2$$

[L]



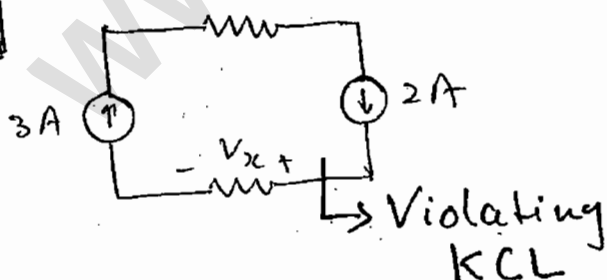
$$L_s = L_1 + L_2$$

[C]

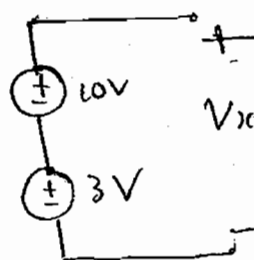


$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

[I]



[V]



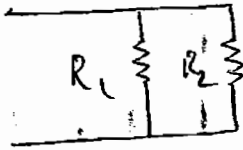
→ Current sources of different values can never exist in series. They violate KCL

→ If 2 current sources are in series, they must be equal both in magnitude & direction.

Voltage sources of any value can be series.

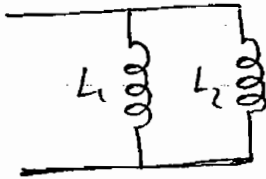
• Parallel Connection of Elements:

[R]



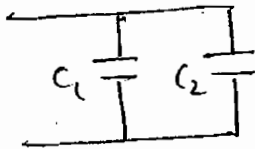
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

[L]



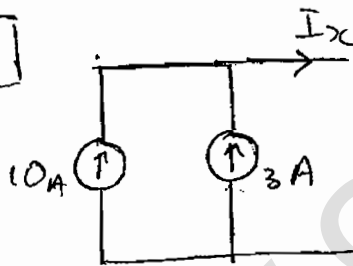
$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2}$$

[C]

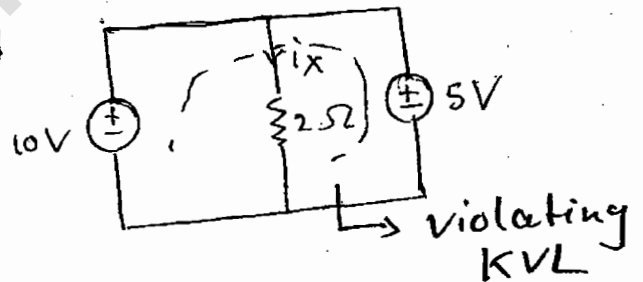


$$C_p = C_1 + C_2$$

[I]



[V]



→ Never 2 ideal voltage sources of diff values can exist in ||. They violate KVL.

→ If 2 vltg sources exist in ||, they must be equal both in magnitude and polarity.

→ Practical vltg sources can always exist in parallel.

→ Current sources of any value can be in parallel.

Open Circuit / o.c

→ In an o.c, $i = 0$ for any voltage

$$R_{o.c} = \frac{V}{0} = \infty \Omega$$

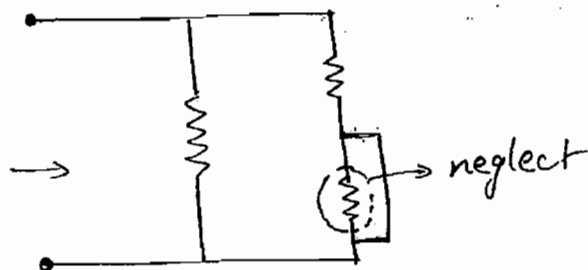
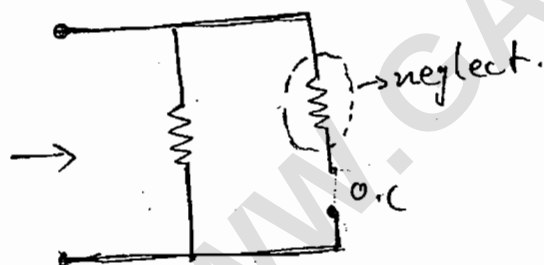
→ Any passive element in series to o.c, can be neglected.

Short Circuit / s.c

→ In a s.c, $V = 0$ for any (i)

$$R_{s.c} = \frac{0}{i} = 0 \Omega$$

→ Any passive element in parallel to s.c, can be neglected.

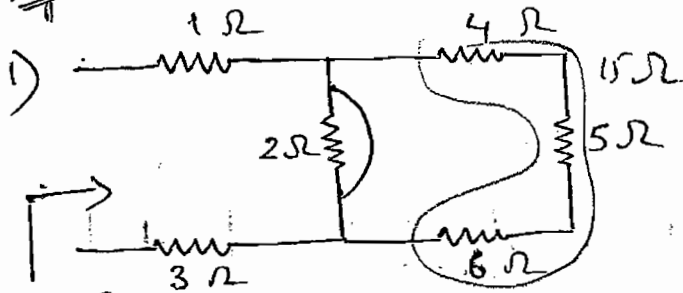


In high vltg engineering o.c. live wire faults are more dangerous than s.c., to human beings.

However s.c always have protection, both at high & low level. vltg by designing some correct rated fuses.

→ Resistance is offered by the path when current can flow as seen from target terminal.

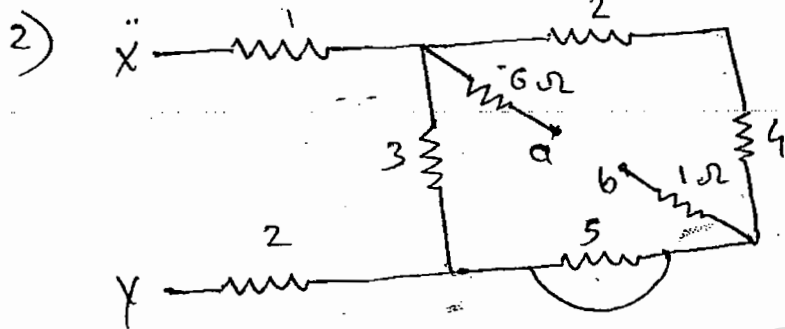
Eg:-



$R_{eq} = 9$

$$1 + 5 + 6 = 12$$

$$R_{eq} = 1 + 3 = 4 \Omega$$



R_{xy}, R_{ab}, R_{xa}

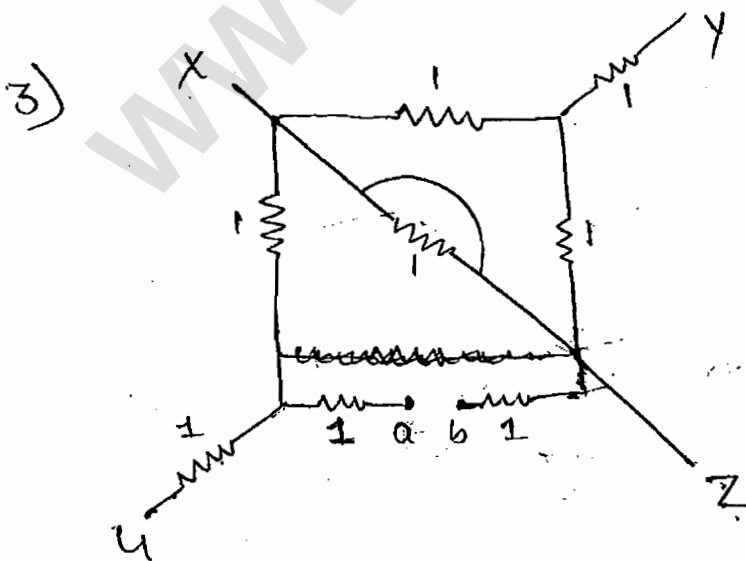
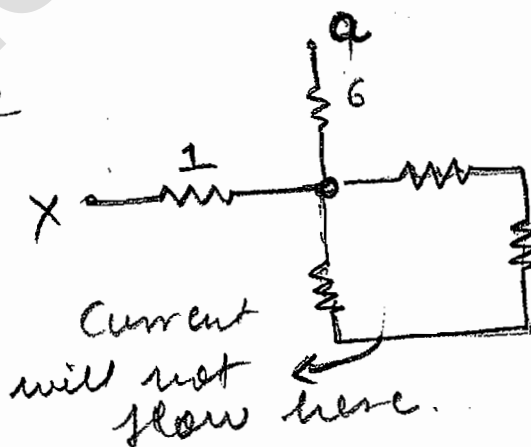
R_{yb}

$$\frac{3 \times 6}{9} = 2 ; R_{xy} = 1 + 2 + 2 = 5 \Omega$$

$$R_{ab} = 1 + 6 + 2 = 9 \Omega$$

$$R_{xa} = 1 + 6 = 7 \Omega$$

$$R_{yb} = 1 + 2 = 3 \Omega$$



$$R_{xy} = \frac{1}{2} + 1 = \frac{3}{2} \Omega$$

$$R_{xz} = 0 \Omega$$

$$R_{xu} = 2 \Omega$$

$$R_{yz} = \frac{1}{2} + 1 = \frac{3}{2} \Omega$$

$$R_{yu} = \frac{1}{2} + 3 = \frac{7}{2} \Omega$$

$$R_{ay} = 3 + \frac{1}{2} = \frac{7}{2} \Omega$$

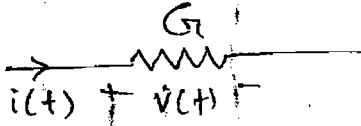
$$R_{zu} = 2 \Omega \quad R_{bx} = 1 \Omega$$

$$R_{ab} = 3 \Omega \quad R_{by} = 2 + \frac{1}{2} = \frac{5}{2} \Omega \quad R_{az} = 2 \Omega$$

CONDUCTANCE :

→ It is the ability to conduct electrically

→ It is used to further classify conductor (metals)



Units :- mho (Ω) or siemens (S) or $\frac{A}{V}$

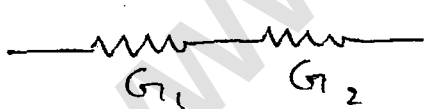
$$G = \frac{1}{R} ; G = \frac{a}{\rho l}$$

$$\boxed{G = \frac{\sigma a}{l}} ; \because \frac{1}{\rho} = \sigma \text{ (conductivity)}$$

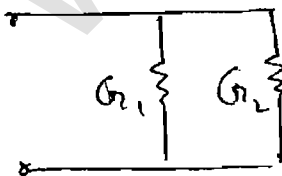
Units for ' σ ' $\rightarrow (\Omega \cdot m)^{-1}$
 $\rightarrow \Omega/m$ or S/m

$$V = \frac{I}{G} ; I = V \cdot G$$

$$P_G = V_G \cdot i_G = \frac{i_G^2}{G} = \frac{V_G^2}{G} \cdot G \quad \underline{W}$$



$$\frac{1}{G_s} = \frac{1}{G_1} + \frac{1}{G_2}$$



$$G_p = G_1 + G_2$$

Based on Conductivity :-

Rank $\rightarrow$ 1 Silver

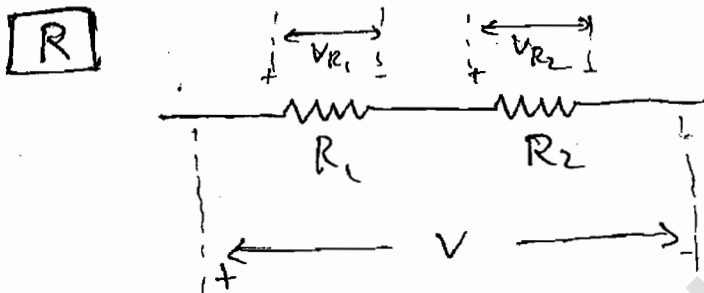
2 Copper $\rightarrow$ used in high current density compact sys.
 $\rightarrow$ Domestic, Industrial, m/c, PCB

3 Gold

4 Aluminium → used in external Tx & distribution lines.
→ cheap
→ light weight

Voltage Division Rule

→ series connected elements only



L

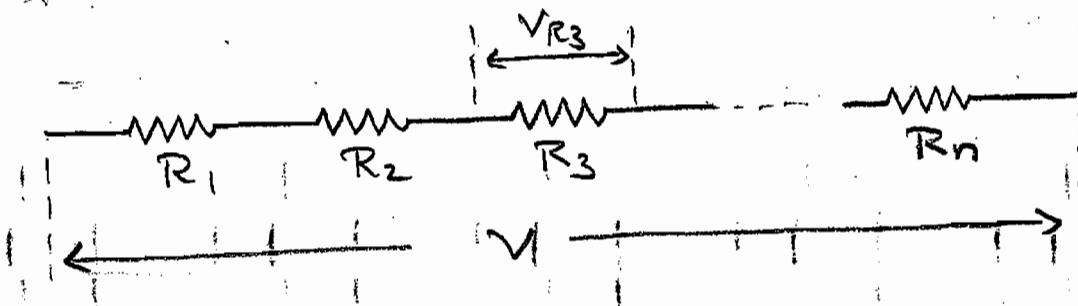
$$V_{L1} = V \left[\frac{L1}{L1 + L2} \right]$$
$$V_{L2} = V \left[\frac{L2}{L1 + L2} \right]$$

C

$$V_{C1} = V \left[\frac{C2}{C1 + C2} \right]$$
$$V_{C2} = V \left[\frac{C1}{C1 + C2} \right]$$

G

$$V_{G1} = V \left[\frac{G2}{G1 + G2} \right]$$
$$V_{G2} = V \left[\frac{G1}{G1 + G2} \right]$$



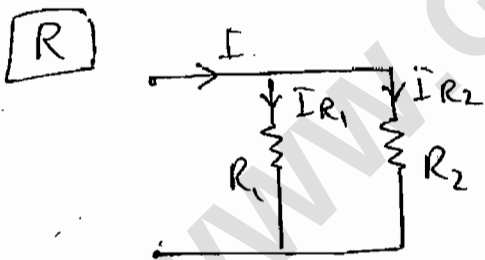
$$V_{R_3} = V \left[\frac{R_3}{\sum_{i=1}^n R_i} \right]$$

But suppose these are capacitors:

$$V_{C_3} = V \left[\frac{1/C_3}{\sum_{i=1}^n 1/C_i} \right]$$

Current Division Rule :-

Parallel connected elements only.



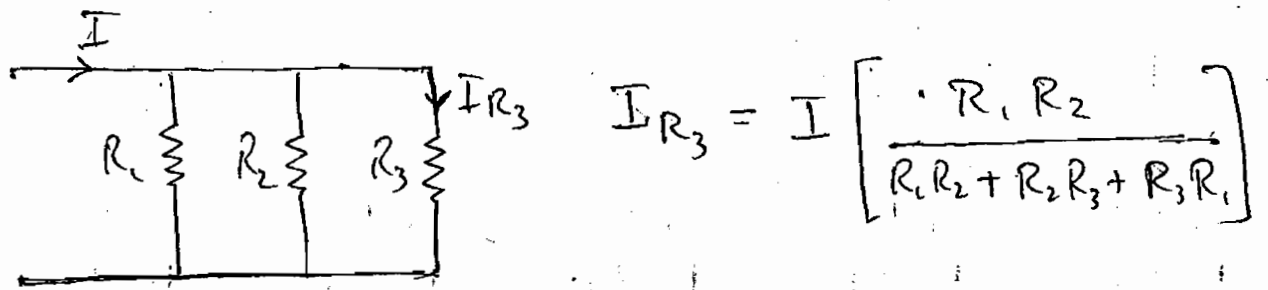
$$I_{R_1} = I \left[\frac{R_2}{R_1 + R_2} \right]$$

$$I_{R_2} = I \left[\frac{R_1}{R_1 + R_2} \right]$$

$$[L] \quad I_{L_1} = I \left[\frac{L_2}{L_1 + L_2} \right] ; \quad I_{L_2} = I \left[\frac{L_1}{L_1 + L_2} \right]$$

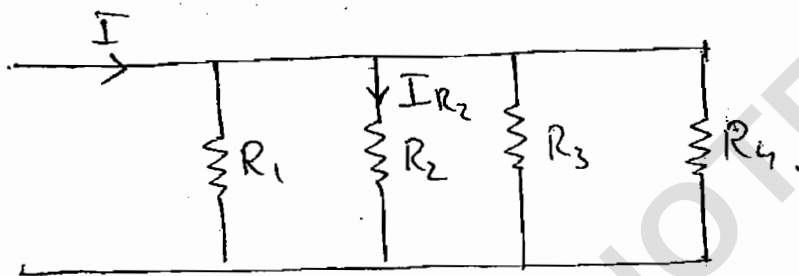
$$[C] \quad I_{C_1} = I \left[\frac{C_1}{C_1 + C_2} \right] ; \quad I_{C_2} = I \left[\frac{C_2}{C_1 + C_2} \right]$$

$$[G] \quad I_{G_1} = I \left[\frac{G_1}{G_1 + G_2} \right] ; \quad I_{G_2} = I \left[\frac{G_2}{G_1 + G_2} \right]$$



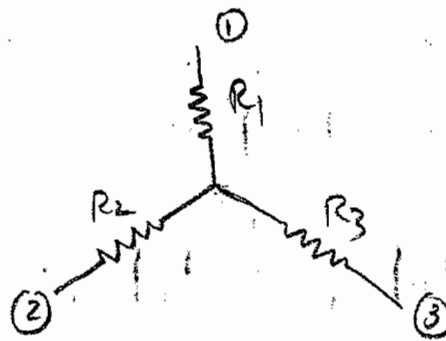
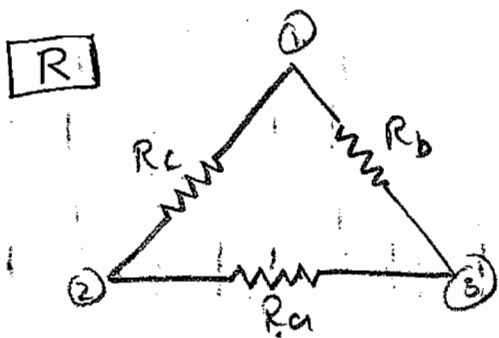
→ But suppose capacitors are there

$$I_{C_3} = I \left[\frac{\frac{1}{C_1} \cdot \frac{1}{C_2}}{\frac{1}{C_1} \cdot \frac{1}{C_2} + \frac{1}{C_2} \cdot \frac{1}{C_3} + \frac{1}{C_3} \cdot \frac{1}{C_1}} \right]$$



$$I_{R_2} = I \left[\frac{R_1 R_3 R_4}{R_1 R_2 R_3 + R_2 R_3 R_4 + R_3 R_4 R_1 + R_4 R_1 R_2} \right]$$

DELTA - TO - STAR



$$R_1 = \frac{R_a R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_c R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_a R_b}{R_a + R_b + R_c}$$

L

$$L_1 = \frac{L_a L_c}{L_a + L_b + L_c}$$

$$L_2 = \frac{L_a L_b}{L_a + L_b + L_c}$$

C

$$\frac{C_1}{C_1} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

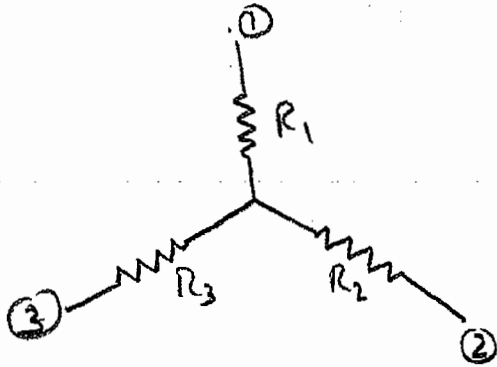
$$\frac{1}{C_2} = \frac{\frac{1}{C_a} \cdot \frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

G

$$\frac{1}{G_1} = \frac{\frac{1}{G_a} \cdot \frac{1}{G_c}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

$$\frac{1}{G_2} = \frac{\frac{1}{G_a} \cdot \frac{1}{G_b}}{\frac{1}{G_a} + \frac{1}{G_b} + \frac{1}{G_c}}$$

STAR - TO - DELTA



[R]

$$R_a = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_b = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_c = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

[L]

$$L_a = L_1 + L_2 + \frac{L_1 L_2}{L_3}$$

[C]

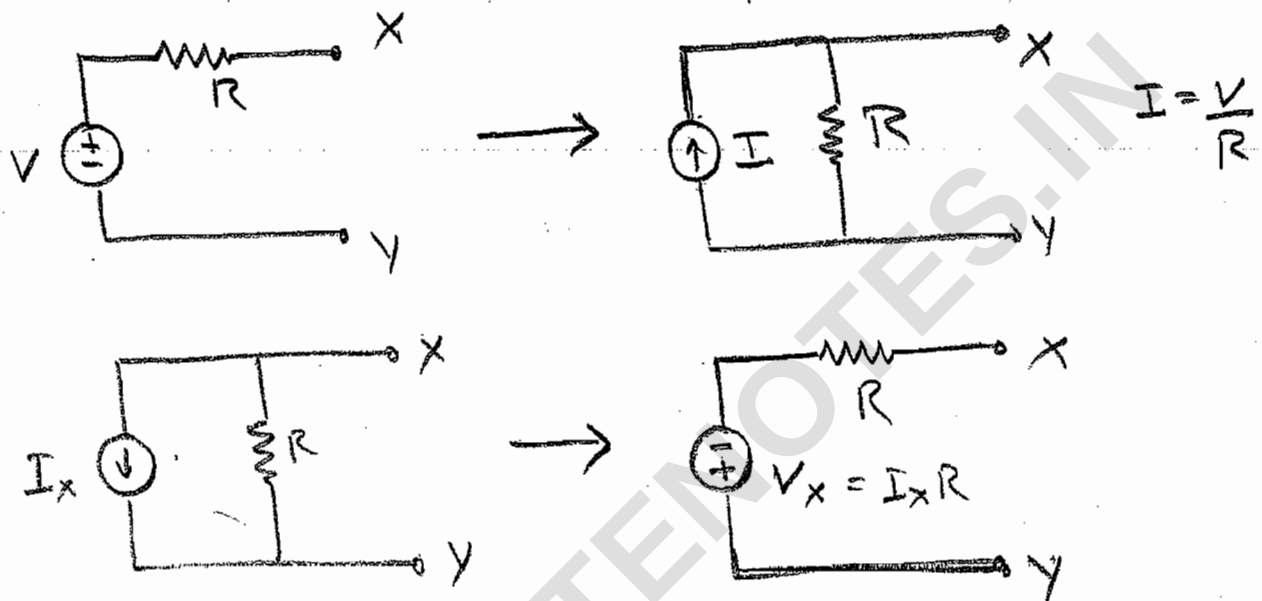
$$\frac{1}{C_a} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{\frac{1}{C_1} \cdot \frac{1}{C_2}}{\frac{1}{C_3}}$$

[G]

$$\frac{1}{G_a} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{\frac{1}{G_1} \cdot \frac{1}{G_2}}{\frac{1}{G_3}}$$

SOURCE TRANSFORMATION

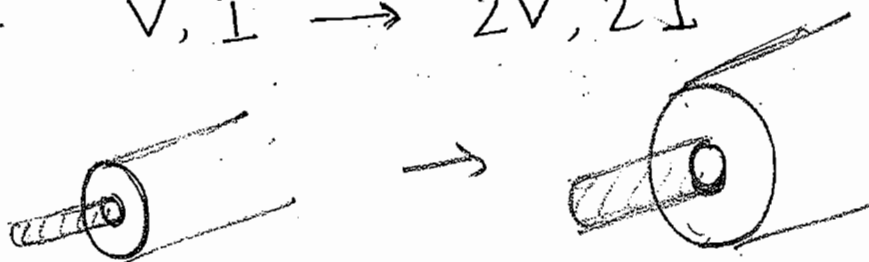
→ An ideal vltg source in series with resis. can be converted into ideal current source in 11^{th} with same resis. across the same terminals & vice-versa.



RATINGS / Specifications :

- They represent the maxi. permissible or allowable safe values for continuous operation of an electrical device.
- Most of our electrical or electronic components or ~~ndws~~ will have vltg, current, power, freq, ratings etc.

$$V, I \rightarrow 2V, 2I$$



$I \uparrow \rightarrow$ conductor cross-sectional area $\uparrow$

$V \uparrow \rightarrow$ insulation withstanding capacity $\uparrow$

$\rightarrow$ Most of our Electrical or Electronic equipment are designed for constant Voltage.

$\rightarrow$ But their current carrying capacity depends upon Loading level.

Low Wattage

$V \uparrow$

$I \downarrow$

$a \downarrow$

$R \uparrow$

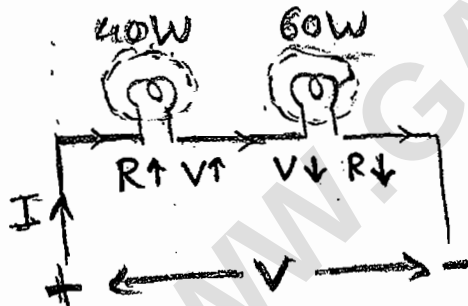
High Wattage

$V \downarrow$

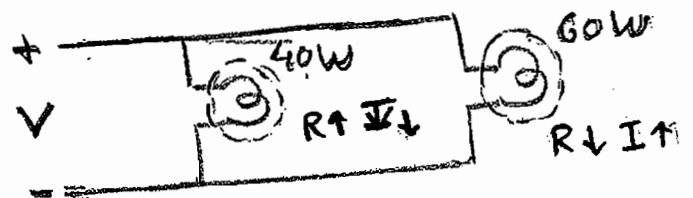
$I \uparrow$

$a \uparrow$

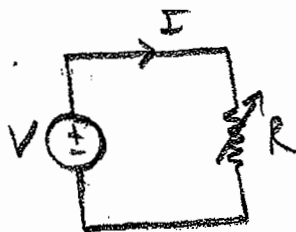
$R \downarrow$



which glows brighter?
= 40W



which glows brighter?
= 60W

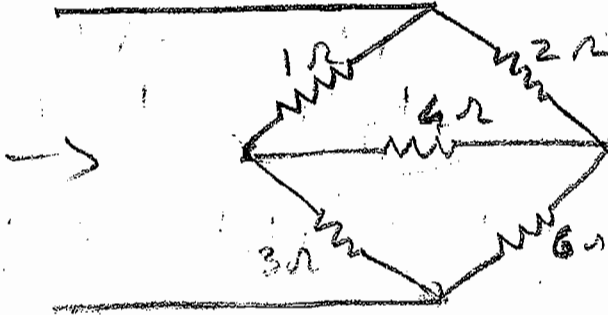


If load $\uparrow \Rightarrow$ more power is drawn
 $\Rightarrow$ more current is drawn
as vltg remain same.

$\rightarrow$ The load always decides the power drawing capability or current capability of the system.

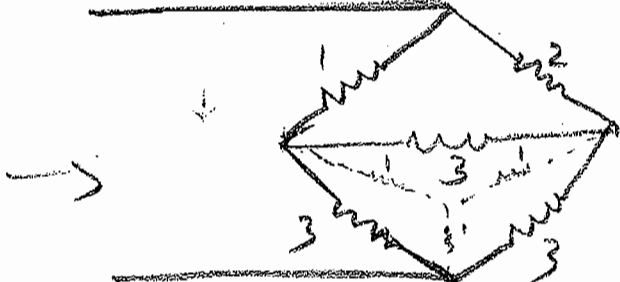
RESISTOR REDUCTION TECHNIQUE:

①



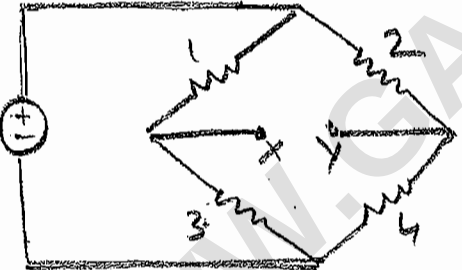
$4 \parallel 8$
 $\therefore \frac{4 \times 8}{12} = \frac{8}{3} \Omega$

②



$1 + [2 \parallel 3]$
 $1 + \frac{6}{5} = \frac{11}{5} \Omega$

③



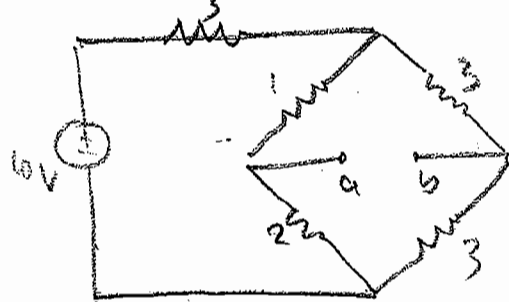
$\Rightarrow$
 $[1 \parallel 3] + [2 \parallel 4]$
 $\frac{3}{4} + \frac{4}{3}$
 $= \frac{25}{12} \Omega$

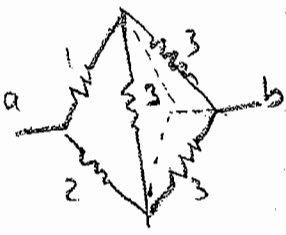
④



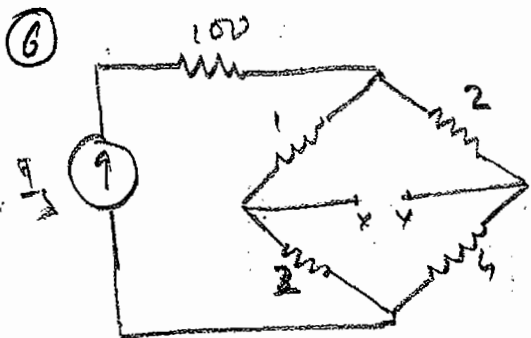
$\frac{3 \times 7}{10} = \frac{21}{10}$

⑤



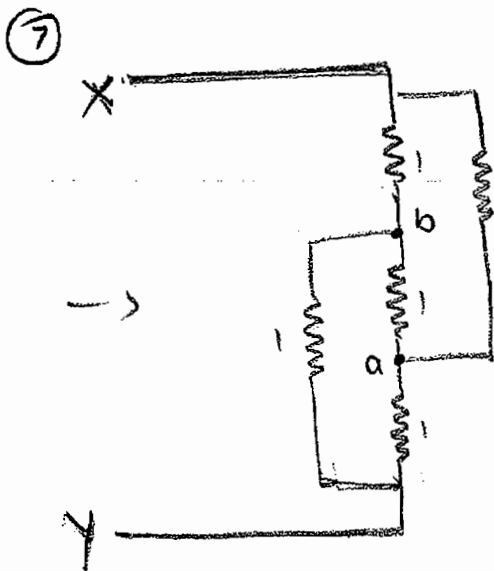
$\Rightarrow$


same as 2nd.

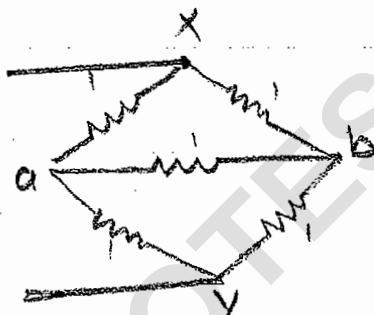


→ Open circuit the current source

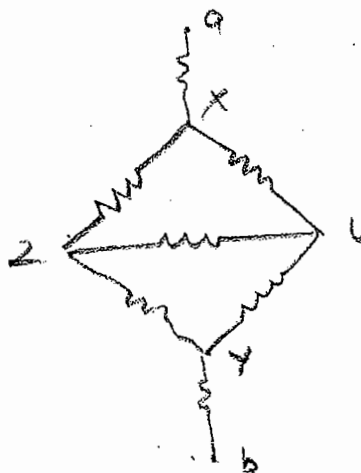
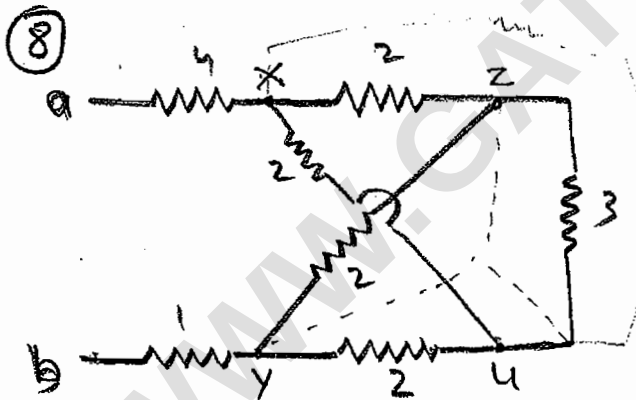
$$R_{xy} = 3 \parallel 6 = \frac{3 \times 6}{9} = 2 \Omega$$



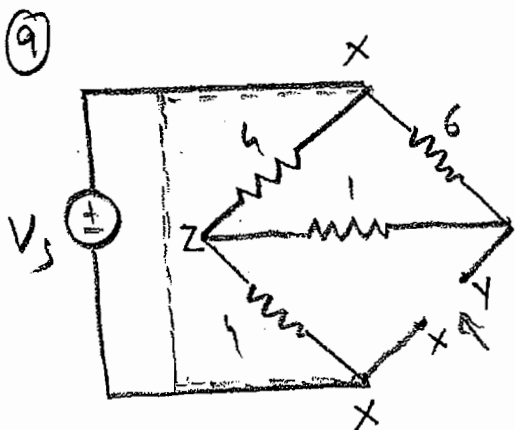
Node shifting tech.



$$\therefore R_{xy} = 2 \parallel 2 = 1$$

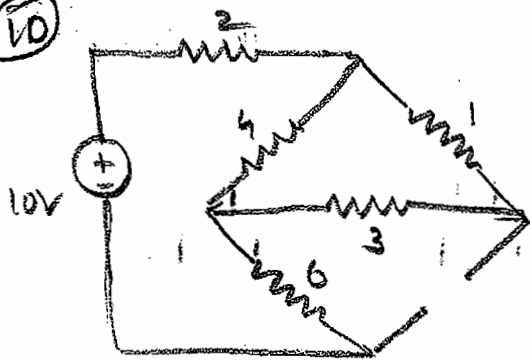


$$7 \Omega$$



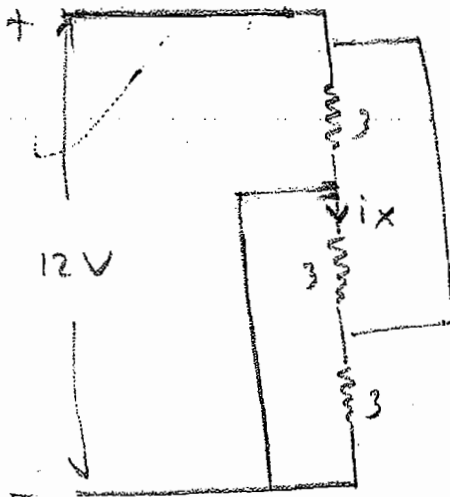
$$2 \Omega$$

10



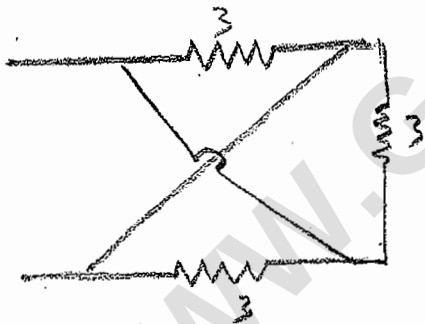
$$9/4 \Omega$$

11

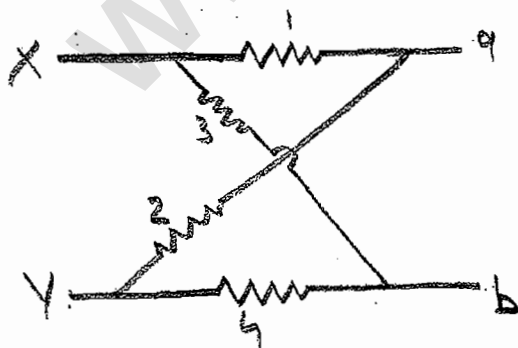


$$-4 A$$

12



13



$$R_{xy}$$

$$R_{ab}$$

$$R_{xa}$$

$$R_{yb}$$

$$R_{xb}$$

$$R_{ya}$$

$$\begin{aligned} R_{xy} &= (1+2) \parallel (3+4) \\ &= \frac{3 \times 7}{10} \\ &= \frac{21}{10} \end{aligned}$$

$$\begin{aligned} R_{ab} &= (1+3) \parallel (2+4) \\ &= \frac{4 \times 6}{10} \\ &= \frac{12}{5} \end{aligned}$$

$$\begin{aligned} R_{xa} &= 1 \parallel (3+2+4) \\ &= \frac{1 \times 9}{10} = \frac{9}{10} \end{aligned}$$

$$R_{Yb} = 4 // (1+3+2)$$

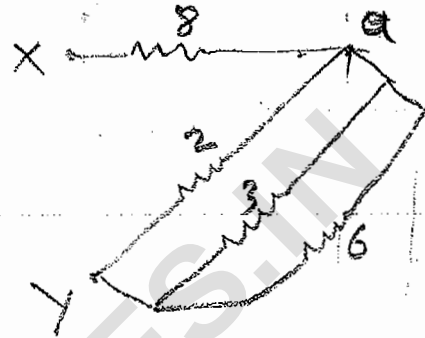
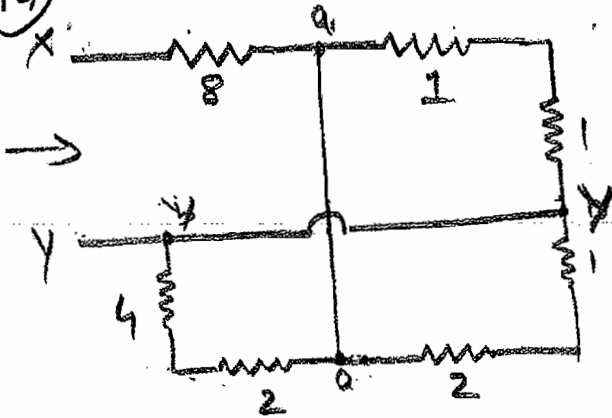
$$= \frac{4 \times 6}{10}$$

$$= \frac{12}{5}$$

$$R_{Xb} = 3 // (1+2+4)$$

$$R_{Ya} = 2 // (1+3+4)$$

(14)

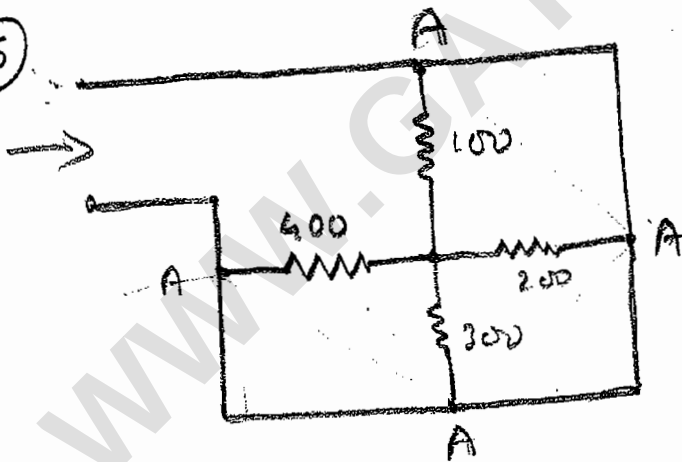


$$R_{XY} = 8 + (2 // 3 // 6)$$

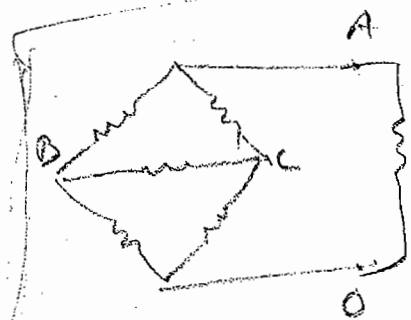
$$= 8 + 1$$

$$= 9$$

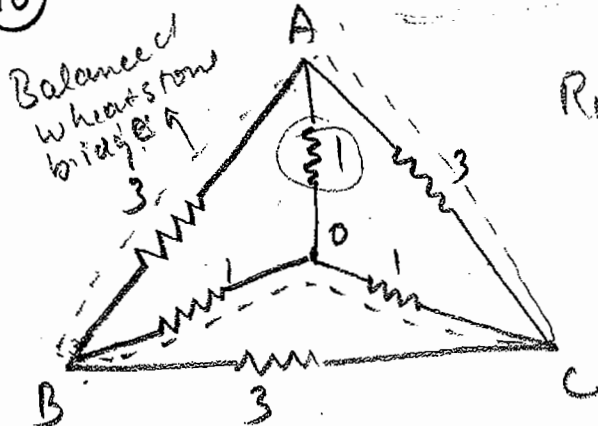
(15)



= 0



(16)



Balanced
Wheatstone
bridge

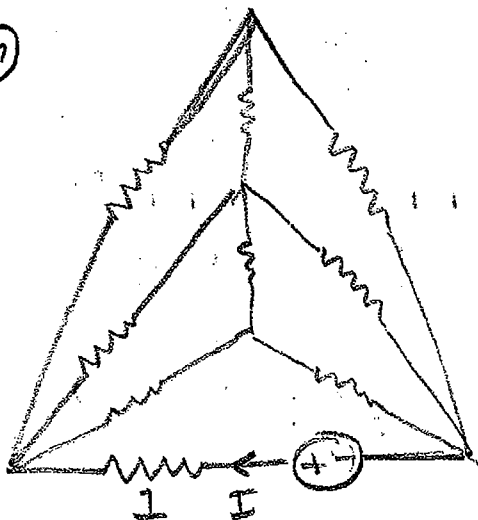
$$R_{Bc} = 3 // 6 // 12 = 1$$

Always convert internal
star $\rightarrow$ delta.

$\therefore$ Converting delta to star
centre point may not
be same.

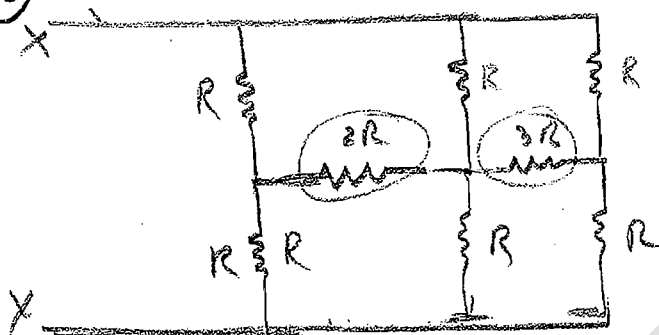
$$R_{AO} = \frac{7}{3} \quad R_{Bc} = 1$$

(17)



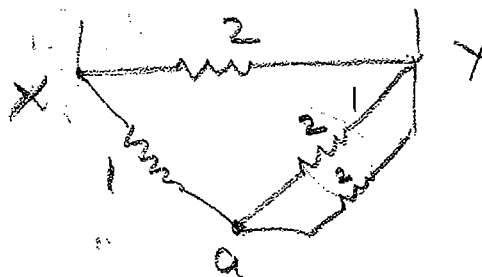
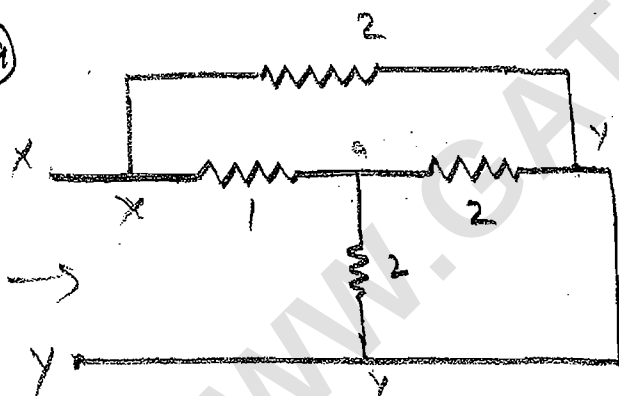
$$I = \frac{10}{2} = 5 \text{ A}$$

(18)



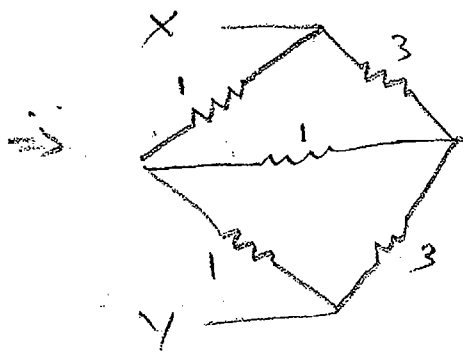
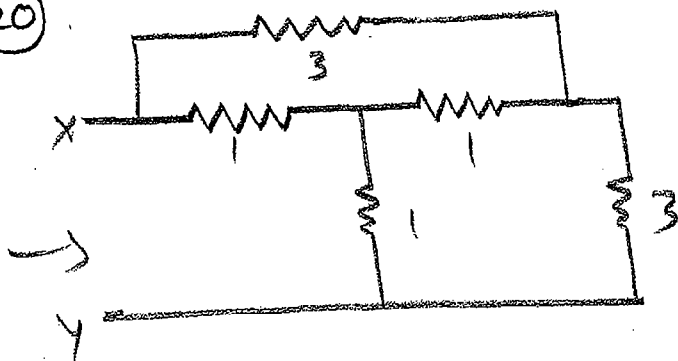
$$\Rightarrow \frac{2}{3} R$$

(19)



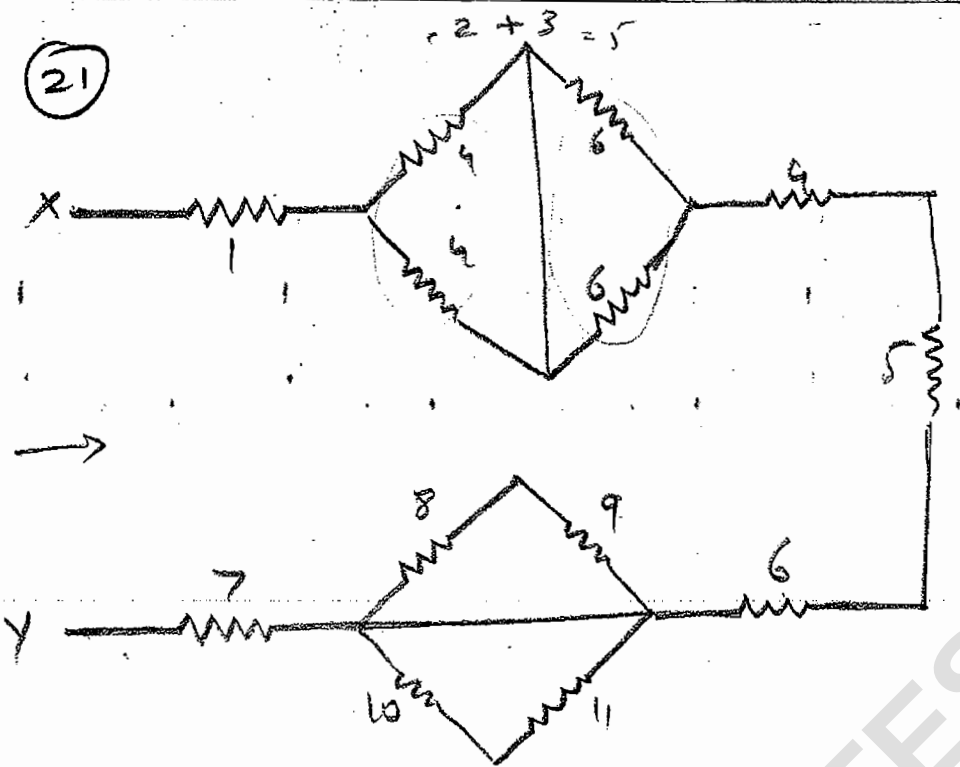
$$R_{xy} = 2 // 2 = 1 \Omega$$

(20)



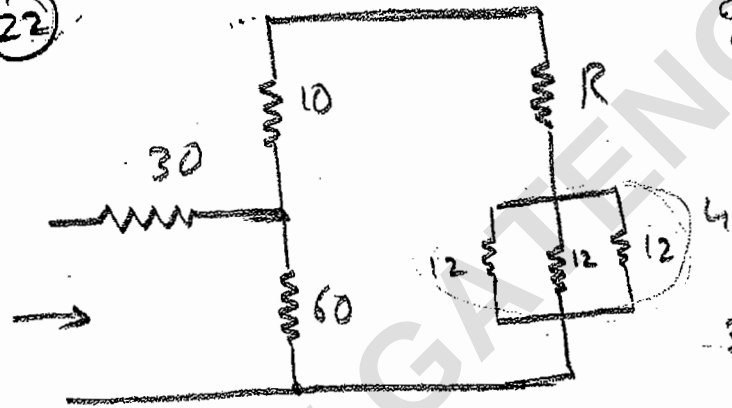
$$R_{xy} = 2 // 6 = \frac{3}{2} \Omega$$

21



$$R_{xy} = 1 + 5 + 4 + 5 + 6 + 7 = 28 \Omega$$

22



Req = 50, then
R = _____

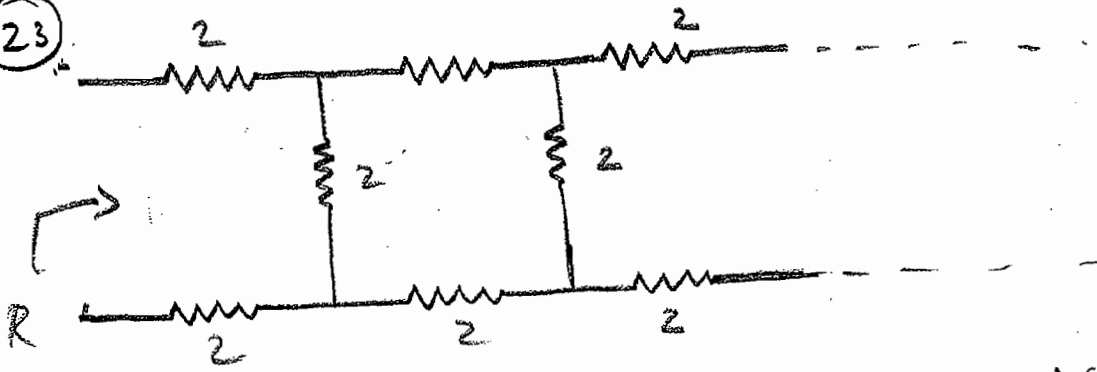
$$30 + [60 \parallel (14 + R)] = 50$$

$$\frac{60 \times (14 + R)}{R + 74} = 20 \Rightarrow 360(14 + R) = 20(R + 74)$$

$$412 + 3R = R + 74 \frac{4}{3}$$

$$R = 16 \Omega$$

23



The v-i relation is a component is $V = i^2$

$$R_{dynamic} = \text{_____} \quad R_{static} \quad \left\{ \begin{array}{l} R_{static} = \frac{V}{I} \end{array} \right.$$

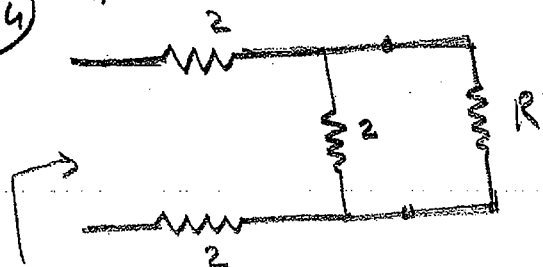
$$V = i^2$$

$$\frac{dV}{dt} = 2 \frac{di}{dt}$$

$$R_{\text{dynamic}} = \left[\frac{\frac{dV}{dt}}{\frac{di}{dt}} \right] = 2$$

$$\therefore R_{\text{dynamic}} = 2 R_{\text{static}}$$

(24)



$$R = 4 + [2 || R]$$

$$= 4 + \frac{2R}{2+R}$$

$$\therefore 2R + R^2 = 8 + 2R + 4R$$

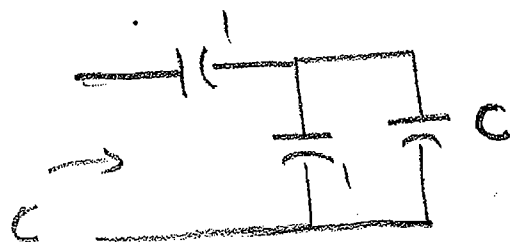
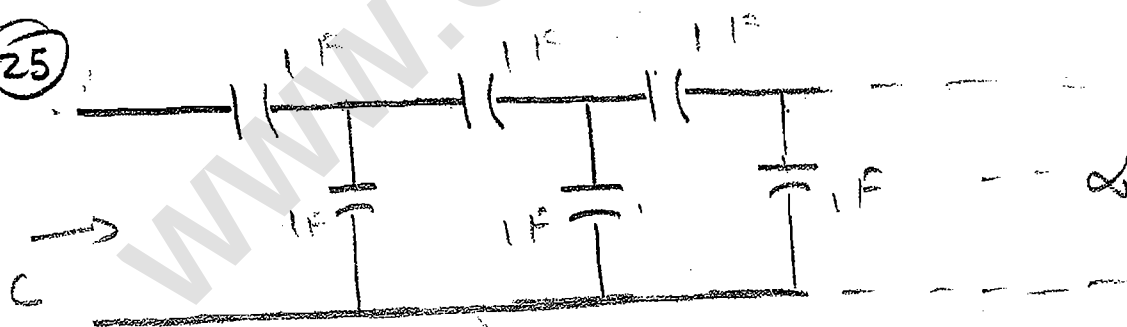
$$\therefore R^2 - 4R - 8 = 0$$

$$R = \frac{4 \pm \sqrt{16 - 4(-8)}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$$R = (2 \pm 2\sqrt{3})$$

$$\therefore R = (2 + 2\sqrt{3}) \Omega$$

(25)



$$C = 1 \text{ } (\textcircled{S}) (1+C)$$

$$\therefore C = \frac{1+C}{2+C}$$

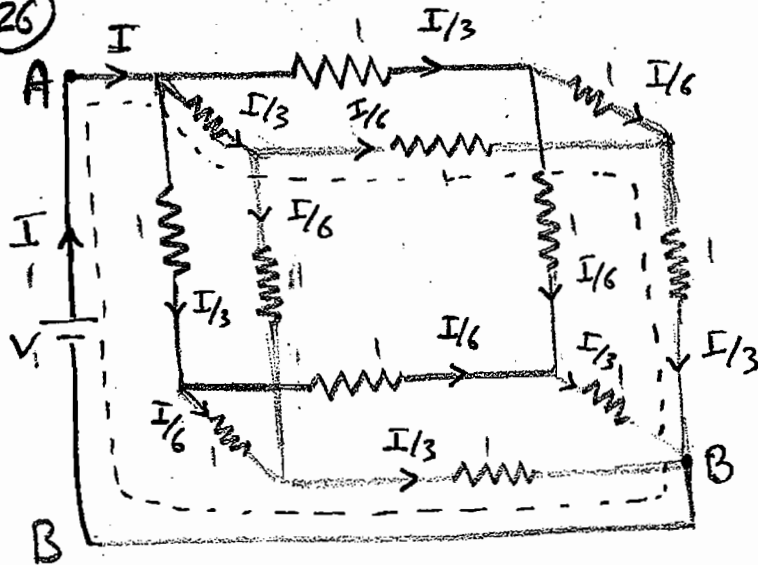
$$2C + C^2 = 1 + C$$

$$C^2 + C - 1 = 0$$

$$C = \frac{-1 \pm \sqrt{5}}{2}$$

$$\therefore C = \frac{\sqrt{5}-1}{2} \text{ F}$$

26



Symmetrical

- Resis. are equal value
- Travelling from source 'to dest'n' resis. remains same

Applying KVL in the above loop.

$$-V + \frac{I}{3}(1) + \frac{I}{6}(1) + \frac{I}{3}(1) = 0$$

$$V = I \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right] = I \left[\frac{5}{6} \right]$$

$$R_{AB} = \frac{V}{I} = \frac{5}{6} \Omega$$

$$R_{AB} = \frac{5}{6} r \Omega$$

$$L_{AB} = \frac{5}{6} l$$

$$C_{AB} = \frac{6}{5} C$$

$$G_{AB} = \frac{6}{5} g$$

KVL → for capacitors :-

$$-V + \frac{1}{C} \int \frac{I}{3} dt + \frac{1}{C} \int \frac{I}{6} dt + \frac{1}{C} \int \frac{I}{3} dt = 0$$

$$\therefore V = \frac{1}{C} \left[\frac{1}{3} + \frac{1}{6} + \frac{1}{3} \right] \int I dt$$

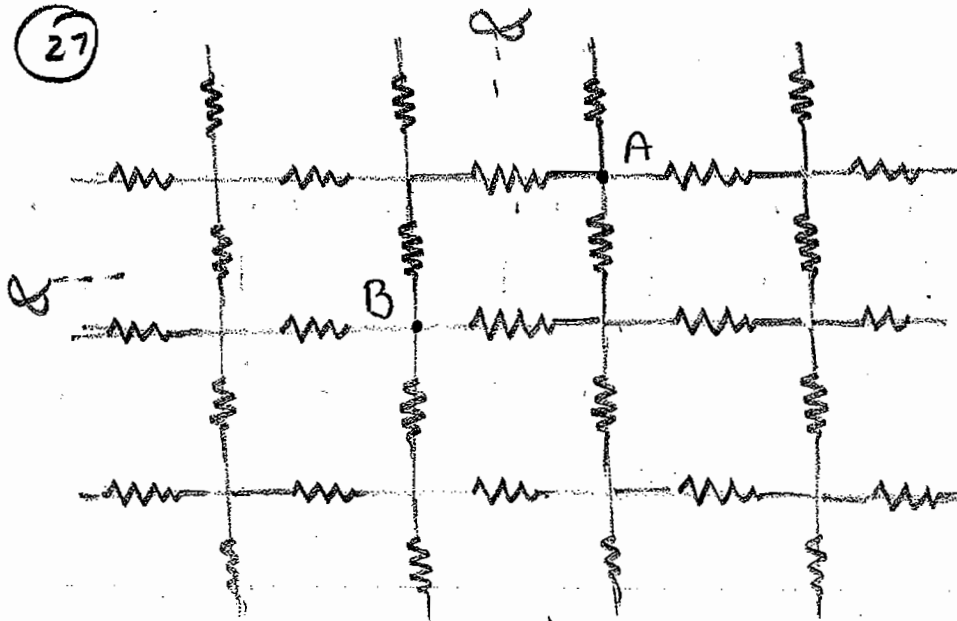
$$= \left[\frac{1}{C} \left[\frac{5}{6} \right] \right] \int I dt$$

$$\frac{1}{C_{AB}} = \frac{1}{C} \left[\frac{5}{6} \right]$$

$$\therefore C_{AB} = \frac{6C}{5}$$

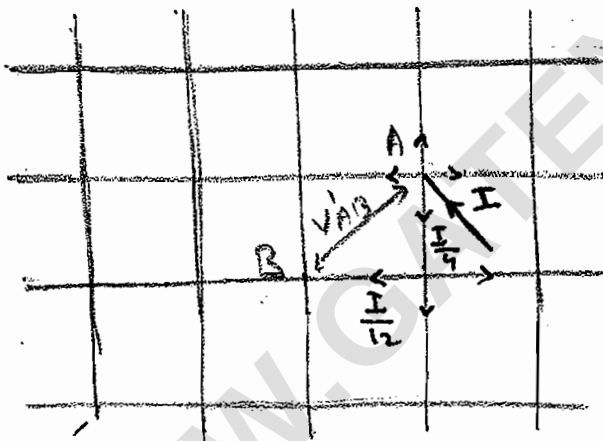
27

Each $r = 1 \Omega$



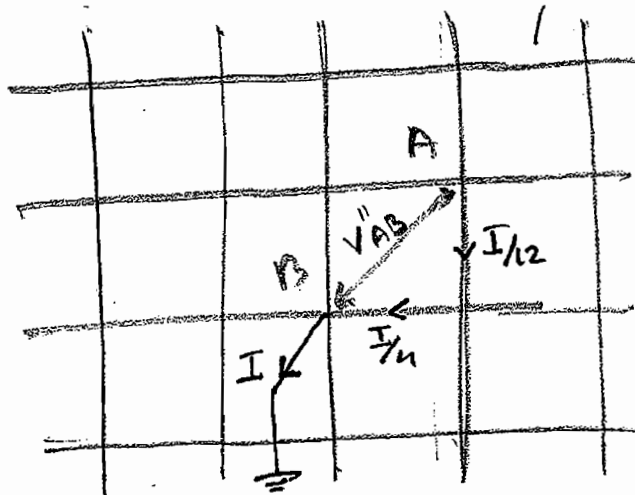
Use Super-position principle :

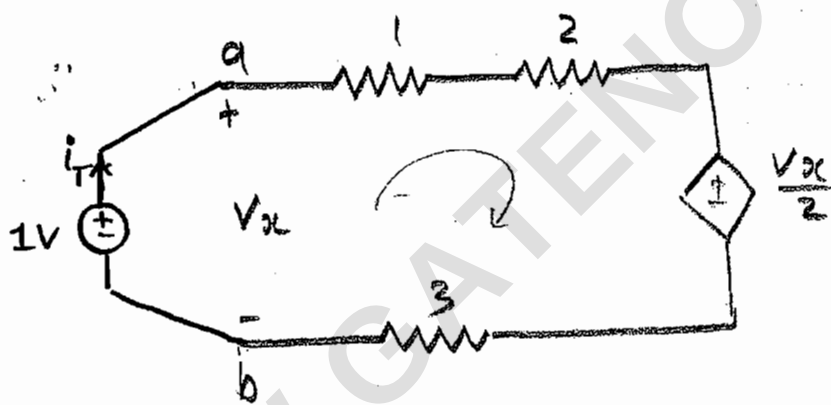
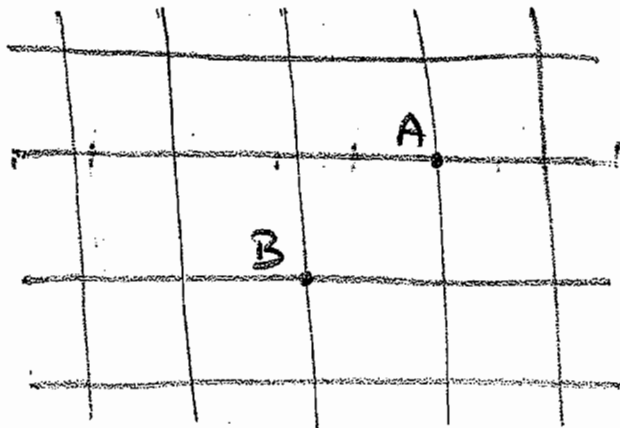
Step ①



~~Step ②~~

(∞) $\swarrow I$ (∞)





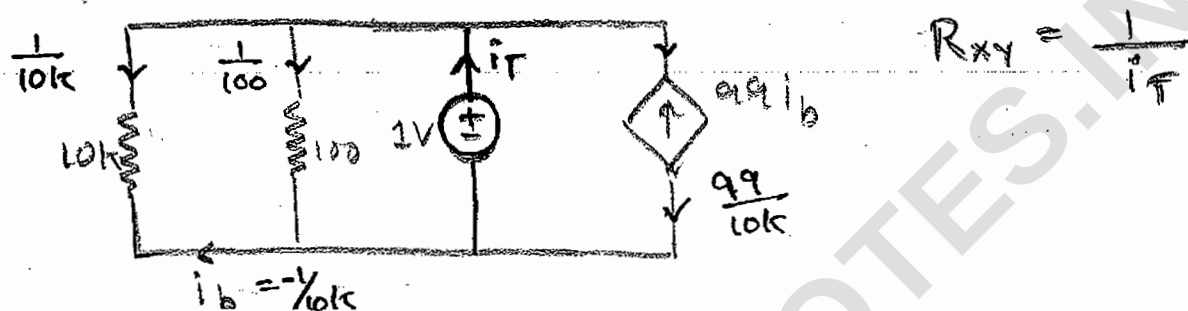
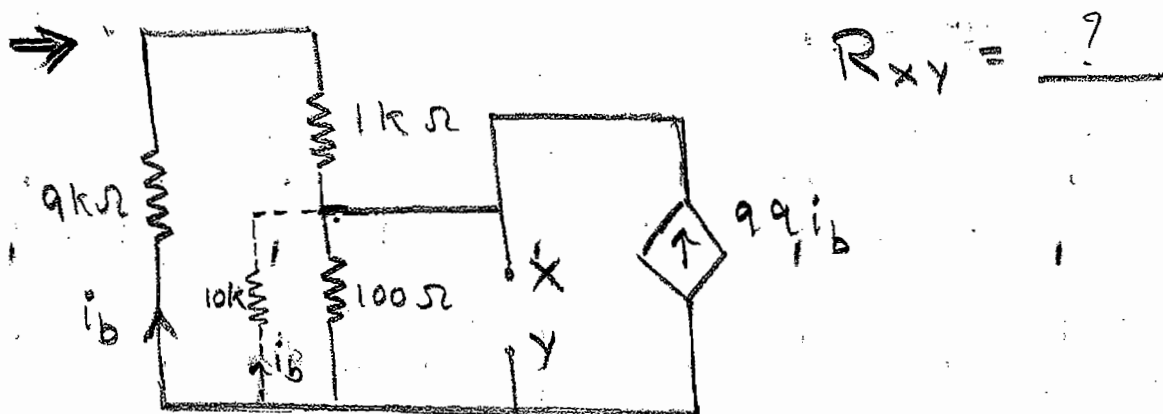
$$R_{ab} = \frac{1}{i_T}$$

KVL

$$-1 + i_T + 2i_T + \frac{1}{2} + 3i_T = 0$$

$$6i_T = \frac{1}{2} \Rightarrow i_T = \frac{1}{12}$$

$$R_{ab} = \frac{1}{i_T} = \frac{1}{1/12} = \boxed{12 \Omega}$$

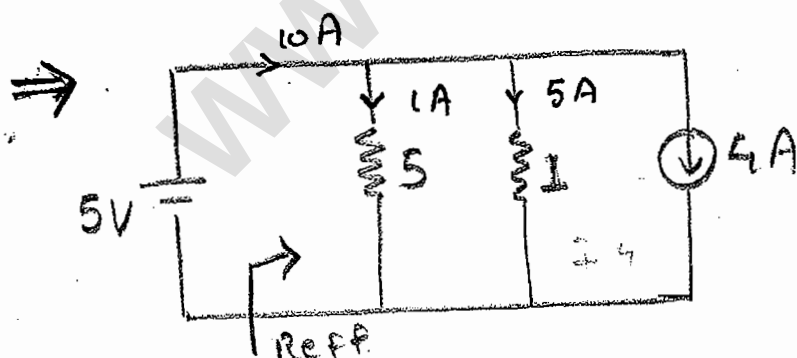


$$i_T = \frac{1}{100} + \frac{1}{10k} + \frac{99}{10k}$$

$$i_T = \frac{1}{100} + \frac{1}{10k} [100]$$

$$= \frac{1}{100} + \frac{1}{100} = \frac{2}{100} = \frac{1}{50}$$

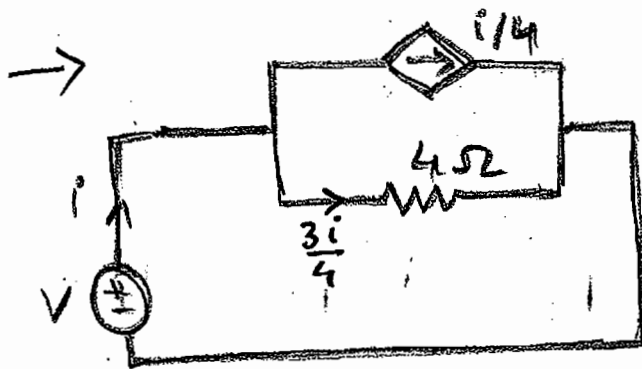
$$\therefore R_{xy} = 50 \Omega$$



What is the effective resis. seen by the vltg source?

→ Effective resistance is the resistance offered by the circuit under working condition.

$$\therefore R_{eff} = \frac{5}{i_T} = \frac{5}{10} = 0.5 \Omega$$



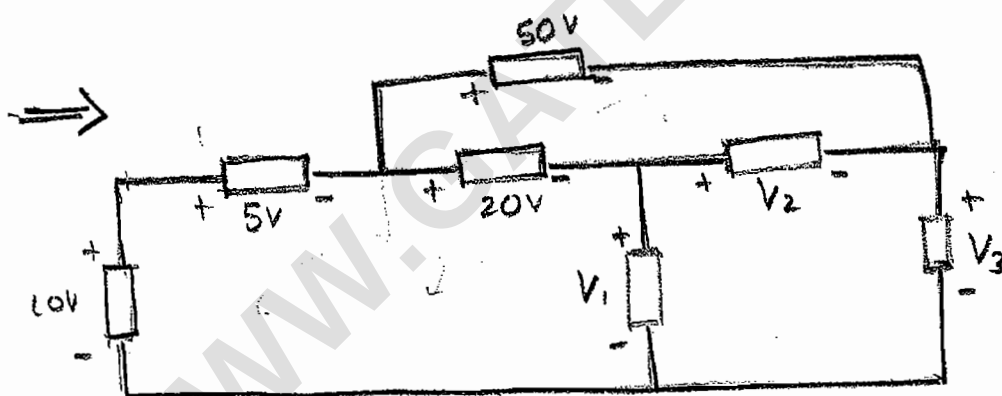
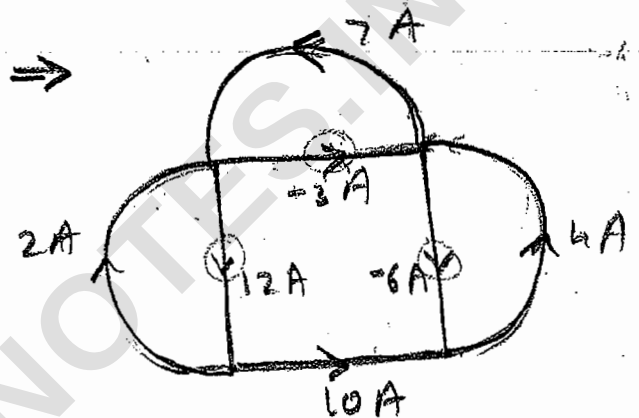
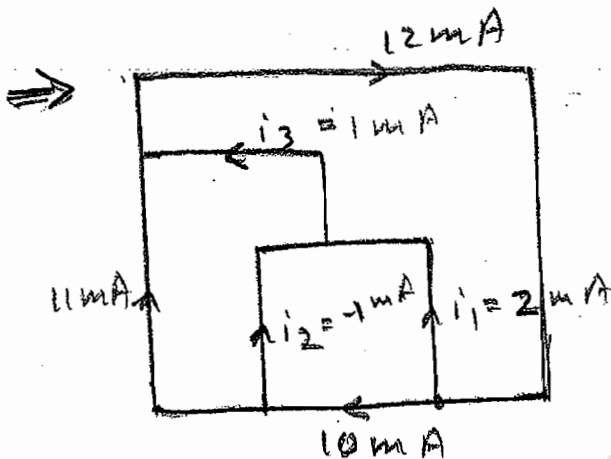
$$R_{eff} = \frac{V}{i} = 3 \Omega$$

What is R_{eff} seen by the vltg source?

KVL

$$-V + \frac{3i}{4} \times 4 = 0$$

$$\therefore V = 3i$$



$$10 - 25 = V_1$$

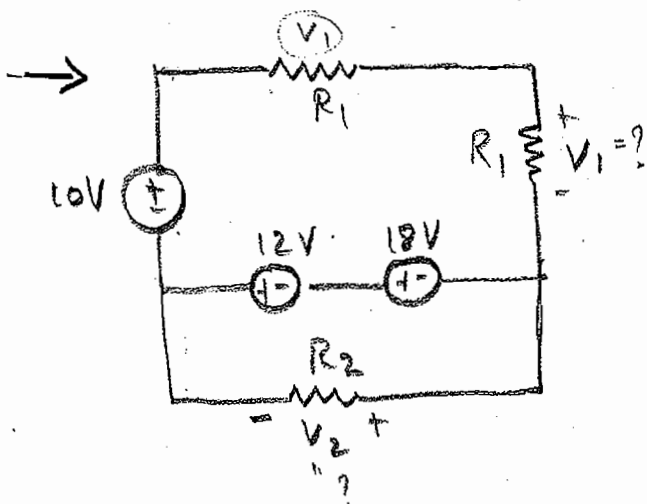
$$V_1 = -15 \text{ V}$$

$$20 - 50 + V_2 = 0$$

$$V_2 = 30 \text{ V}$$

$$-15 - 30 = V_3$$

$$V_3 = -45 \text{ V}$$

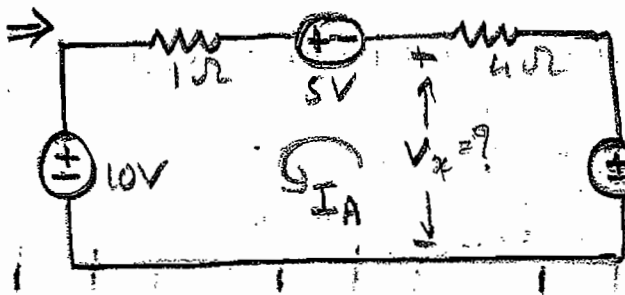


$$V_2 = -30 \text{ V}$$

$$10 + 2V_1 - 30 = 0$$

$$2V_1 = 20$$

$$V_1 = 10$$

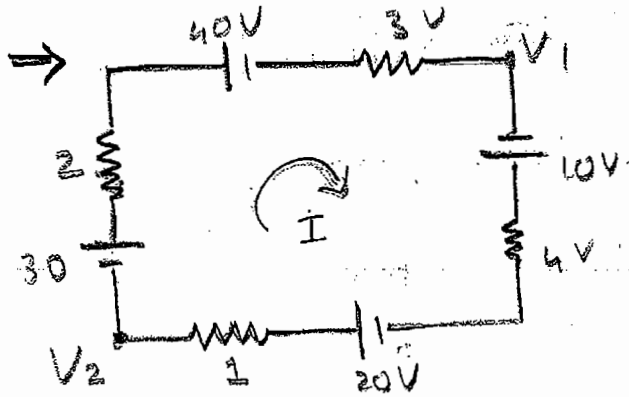


$$I_A = \frac{20}{5} = 4 \text{ A}$$

$$V_x = 25 - 4 I_A$$

$$= 25 - 16$$

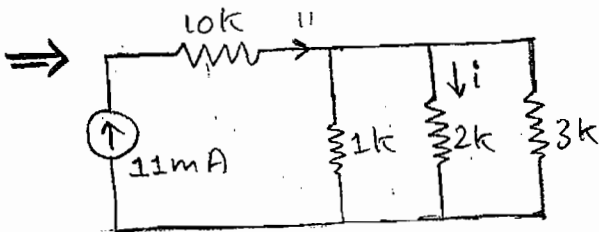
$$= 9 \text{ V}$$



$$I = \frac{20}{10} = 2 \text{ A}$$

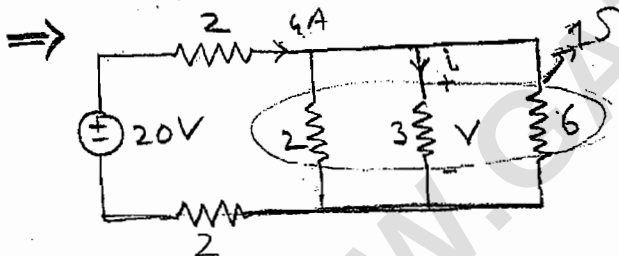
$$V_2 = -30 + 5I + 40 + V_1$$

$$\therefore V_2 = V_1 + 20$$



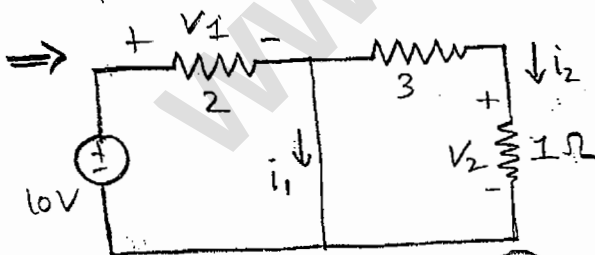
$$i = \frac{\frac{3}{4} \times 11}{\frac{3}{4} + 2}$$

$$= \frac{33}{11} = 3 \text{ A}$$



$$V = \frac{1 \times 20}{4 + 1} = \frac{20}{5}$$

$$= 4 \text{ V}$$

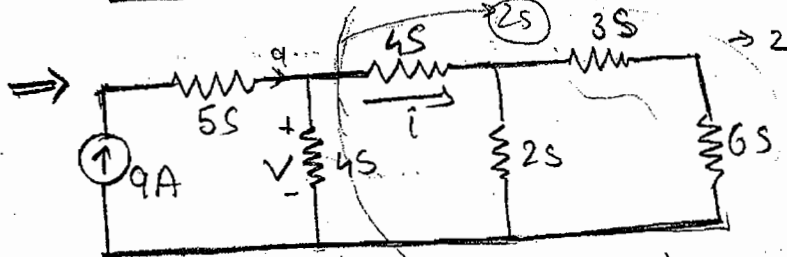


$$i_1 = \frac{10}{2} = 5 \text{ A}$$

$$i_2 = 0$$

$$V_1 = 2 \times 5 = 10 \text{ V}$$

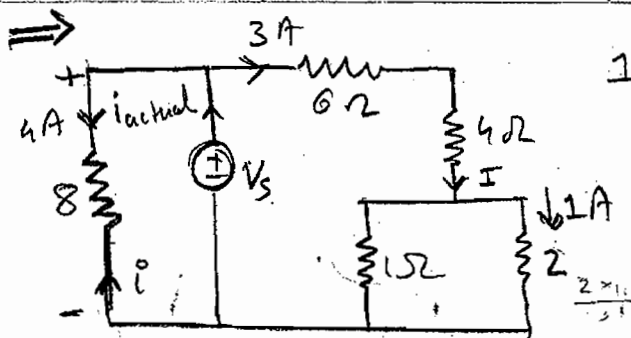
$$V_2 = 0$$



$$i = \frac{9 \times 4}{4 + 2} = \frac{3 \times 4^2}{8 \text{ S}}$$

$$= 6 \text{ A}$$

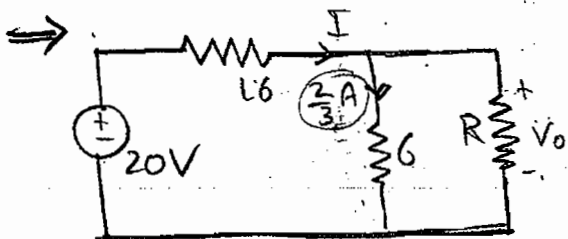
S $\rightarrow$ Siemens (i.e. conductance)



$$1 = \frac{1}{3} \times I \Rightarrow I = 3 \text{ A}$$

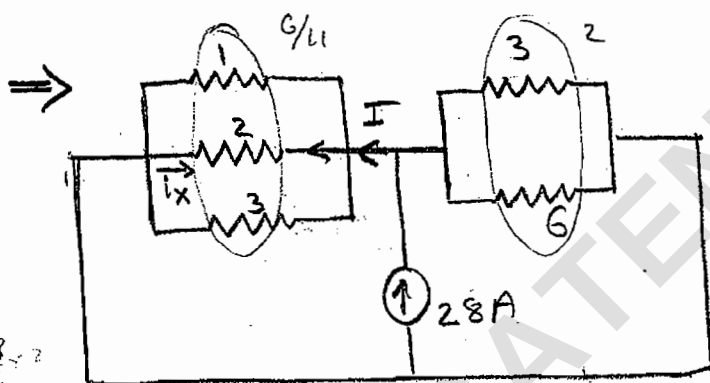
$$V_s = 3 \times \left(10 + \frac{2}{3}\right) = 32 \text{ V}$$

$$i = -\frac{32}{8} = -4 \text{ A}$$



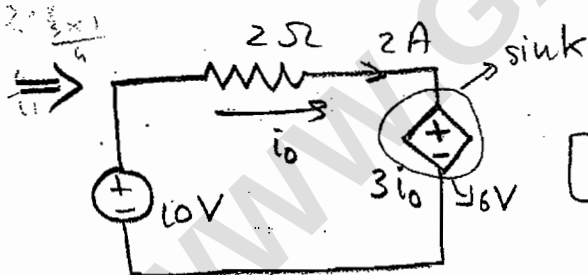
$$i) V_o = 4 \text{ V}, R = \frac{9}{12}$$

$$I = \frac{16}{16} = 1 \text{ A}$$



$$I = \frac{2 \times 28}{[2 + 6/11]}$$

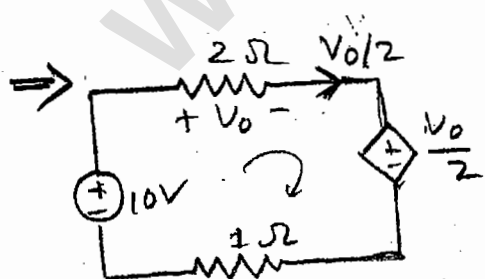
$$-i_x = \frac{2 \times 28 \times 4 \times \frac{3}{4}}{\frac{2 \times 8}{4(2) + \frac{3}{4}}} = 6$$



$$\frac{10 - 3i_o}{2} = i_o \Rightarrow 5i_o = 10$$

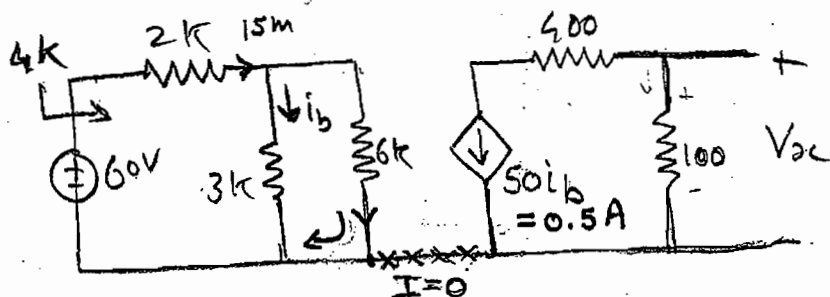
$$i_o = 2$$

$$P = -6 \times 2 = -12 \text{ W}$$



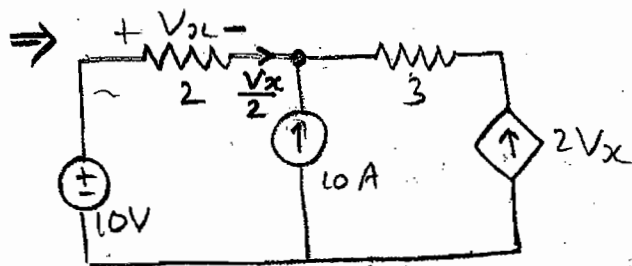
$$+10 - V_o - \frac{V_o}{2} - \frac{V_o}{2} = 0$$

$$\therefore V_o = \frac{10}{2} = 5 \text{ V}$$



$$i_b = \frac{28 \times 15}{98} = 10 \text{ mA}$$

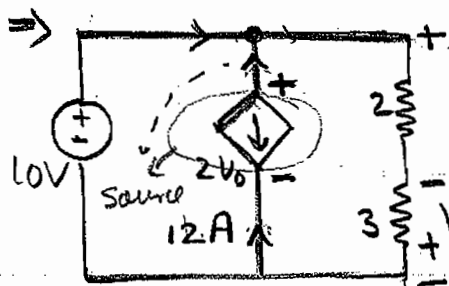
$$V_o = -400(0.5) = -50 \text{ V}$$



$$\frac{V_x}{2} + 10 + 2V_x = 0$$

$$\therefore 5V_x = -20$$

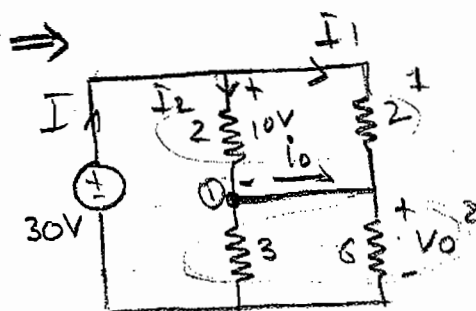
$$\therefore V_x = -4V$$



Power delivered by dependent source?

$$V_o = -\left(\frac{3 \times 10}{3+2}\right) = -6V$$

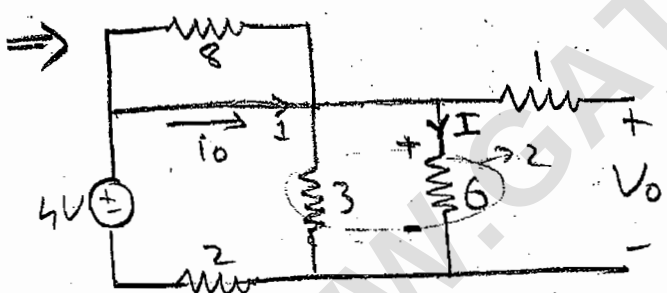
$$P = +10 \times 12 = 120W$$



$$V_o = \frac{30 \times 2}{3} = 20V$$

$$\text{KCL} \quad \frac{V_o}{2} = i_o + \frac{20}{3} \Rightarrow i_o = \frac{30-40}{6}$$

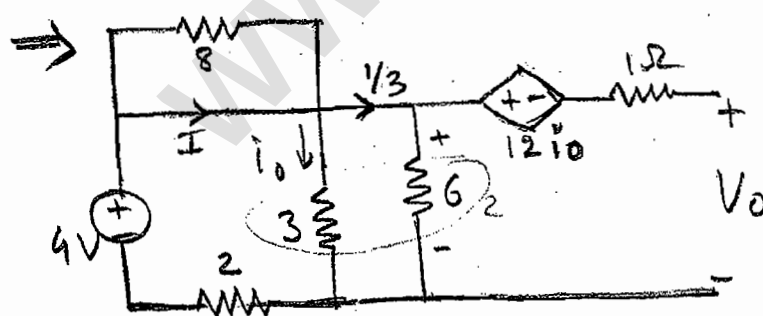
$$i_o = -\frac{10}{6} = -\frac{5}{3}A$$



$$i_o = \frac{4}{4} = 1A$$

$$I = \frac{3}{9} = \frac{1}{3}$$

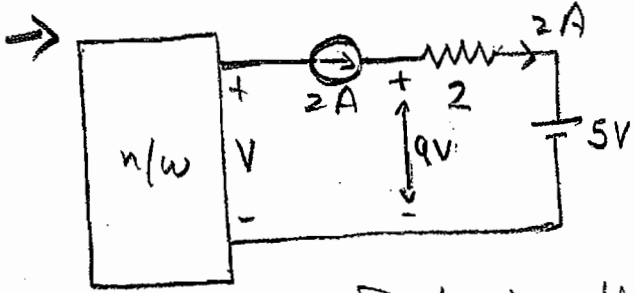
$$V_o = 6 \times \frac{1}{3} = 2V$$



$$I = \frac{4}{4} = 1A$$

$$i_o = \frac{6}{9} = \frac{2}{3}A$$

$$V_o = -12\left(\frac{2}{3}\right) + 6\left(\frac{1}{3}\right) = -6V$$

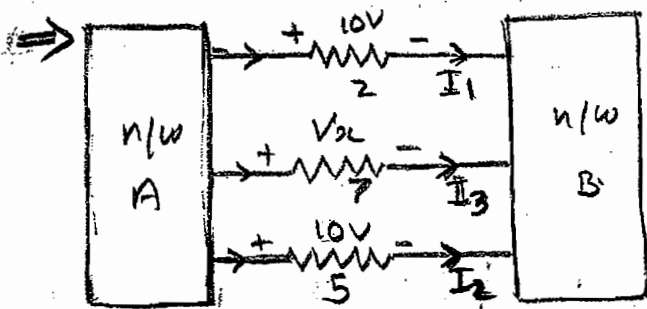


$$\frac{V-5}{2} = 2$$

$$V = 9V$$

∵ We missed the vltg across the current source

∴ Data Insufficient ✓



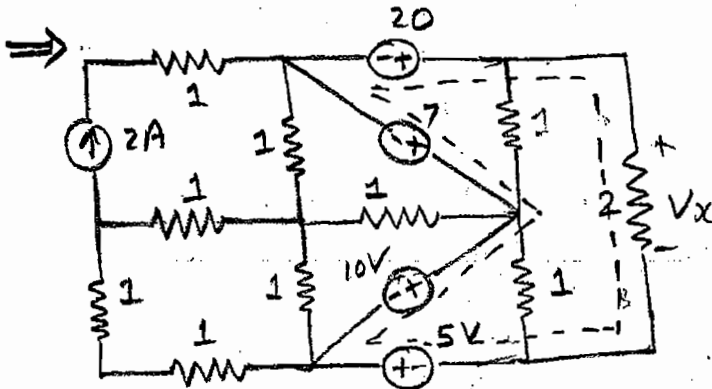
$$V_{2c} = ?$$

$$I_1 = 5A \quad I_2 = 2A$$

$$\therefore I_3 = I_1 + I_2 \quad (\text{KCL})$$

$$= -7A$$

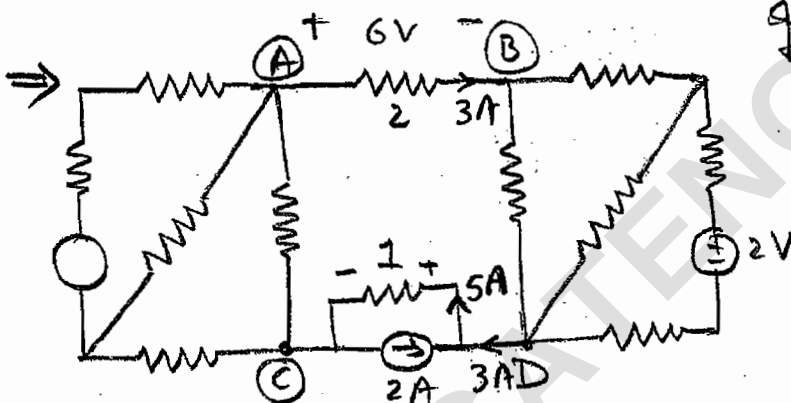
$$V_{2c} = -49V$$



$$-5 - 10 + 7 - 20 + V_x = 0$$

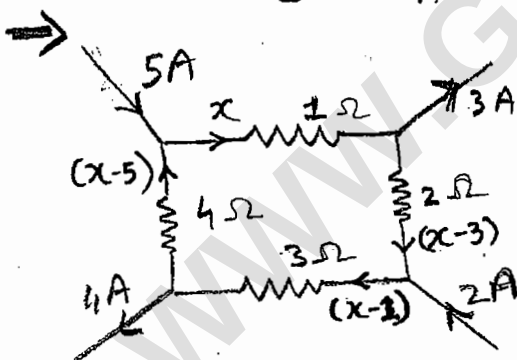
$$V_x = 35 - 7$$

$$V_{2c} = 28V$$



$$V_A - V_B = 6V, \quad V_C - V_D = ?$$

$$V_C - V_D = -5V$$



Find power lost in n/w

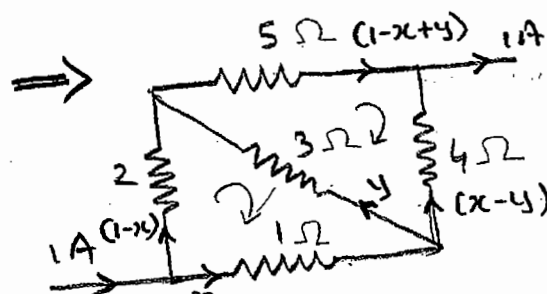
KVL

$$x + 2(x-3) + 3(x-1) + 4(x-5) = 0$$

$$\therefore x = 2.9A$$

$$P_T = (2.9)^2 + (0.1)^2(2) + (1.9)^2(3) + (2.1)^2(4)$$

$$= 36.9W$$



Find all Branch current

$$\text{KVL} \quad 2(1-x) - 3y - x = 0$$

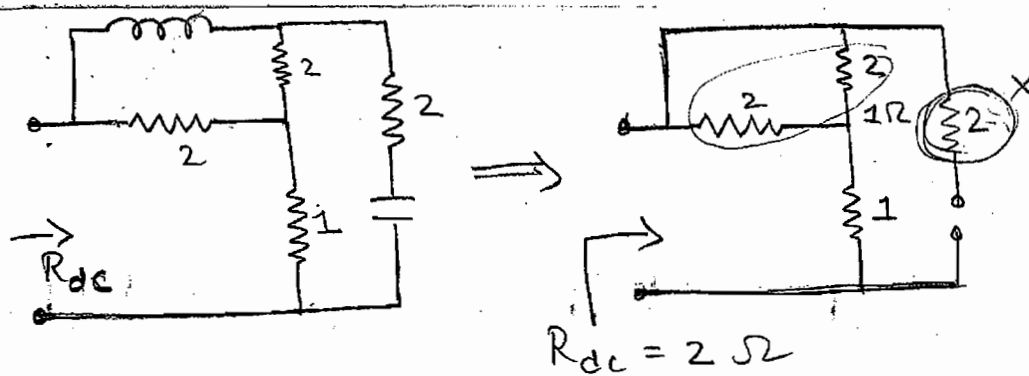
$$3x + 3y = 2 \quad \text{--- (1)}$$

$$3y + 5(1-x+y) - 4(x-y) = 0$$

$$9x - 12y = 5 \quad \text{--- (2)}$$

From (1) & (2)

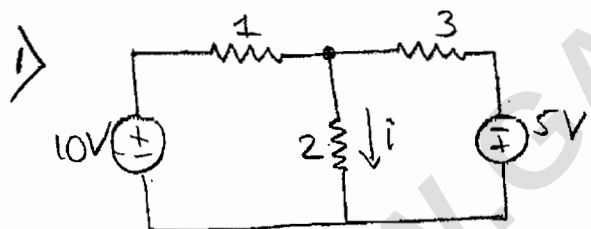
$$x = 0.619 \quad y = 0.047$$



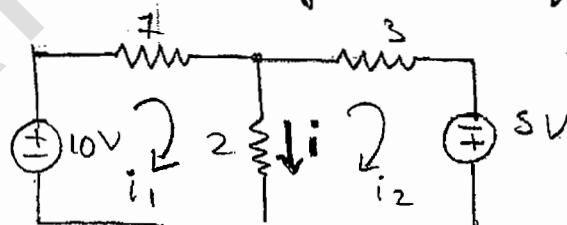
Methods of Analysis :

- Mesh Analysis = KVL + Ohms' Law
[I ↑ V ↓]
- Nodal Analysis = KCL + Ohms' Law
[V ↑ I ↓]

In nodal analysis, we can eliminate the use of simple nodes, if not required.



Find 'i' using mesh & nodal analysis



Mesh Analysis

$$\textcircled{1} \quad 10 - 3i_1 + 2i_2 = 0$$

$$\therefore 3i_1 - 2i_2 = 10$$

$$\textcircled{2} \quad 5 - 5i_2 + 2i_1 = 0$$

$$\& \quad 2i_1 - 5i_2 = -5$$

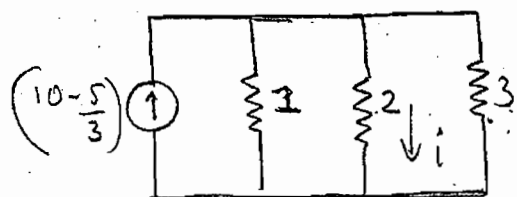
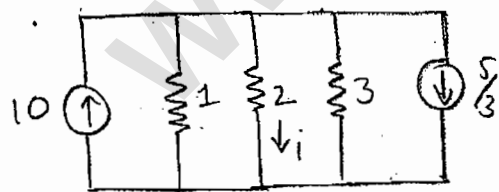
From ① & ② -

$$i_1 = \frac{60}{11} \quad i_2 = \frac{35}{11}$$

Now,

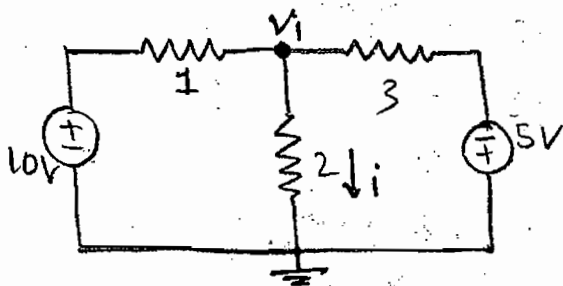
$$i = i_1 - i_2$$

$$= \frac{25}{11} \text{ A}$$



$$i = \left(10 - \frac{5}{3}\right) \left[\frac{3}{2+3+6}\right]$$

$$= \frac{25}{3} \times \frac{3}{11} = \frac{25}{11} \text{ A}$$



Nodal Analysis

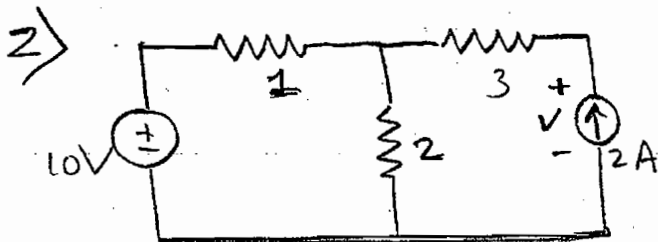
$$\frac{V_1 - 10}{1} + \frac{V_1}{2} + \frac{V_1 - 5}{3} = 0$$

$$\therefore \frac{V_1}{2} \quad 6V_1 - 60 + 3V_1 + 2V_1 - 10 = 0$$

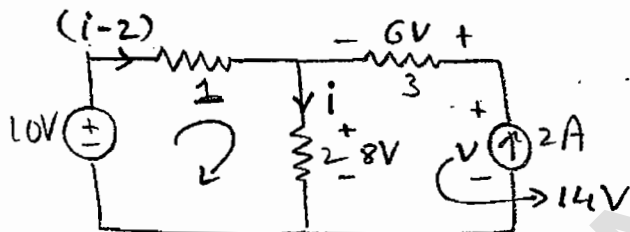
$$V_1 = \frac{50}{11} \text{ V}$$

Now,

$$i = \frac{V_1}{2} = \frac{25}{11} \text{ A}$$



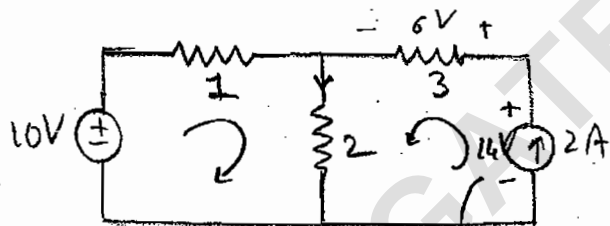
Find power delivered by current source using mesh & nodal.



$$-10 + i - 2 + 2i = 0$$

$$i = 4 \text{ A}$$

$$\therefore P_{\text{deliv.}} = + 2 \times 14 = 28 \text{ W}$$

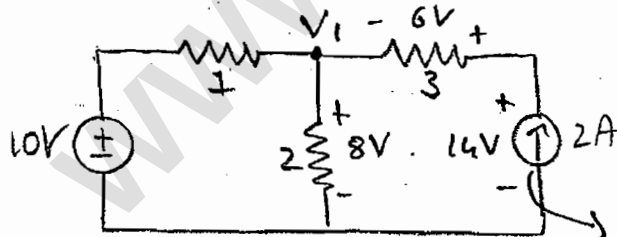


$$-10 + i_1 + 2(i_1 - i_2) = 0$$

$$3i_1 - 2i_2 = 10 \quad \text{--- (1)}$$

By comparison: $i_2 = -2 \quad \text{--- (2)}$

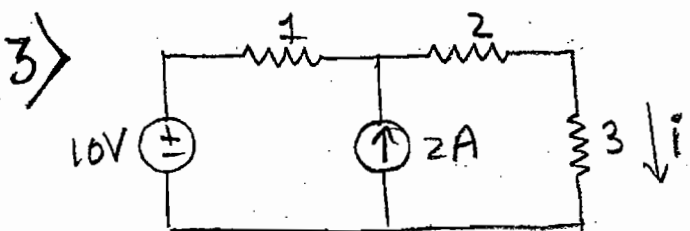
$$P_{\text{del}} = 28 \text{ W} \quad \therefore 3i_1 = 6 \Rightarrow i_1 = 2$$



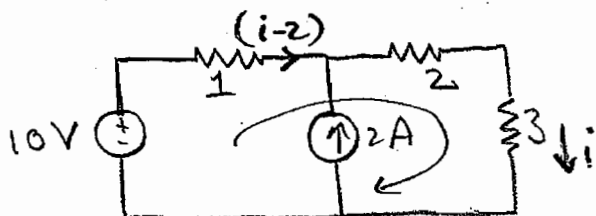
$$\frac{V_1 - 10}{1} + \frac{V_1}{2} - 2 = 0$$

$$\therefore V_1 = 8 \text{ V}$$

$$P_{\text{del}} = 28 \text{ W}$$

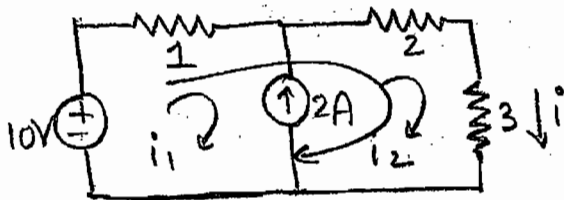


Find i using mesh & nodal analysis



$$-10 - i - 2 + 5i = 0$$

$$i = 2$$



Here $i_2 = i$

$$10 - i_1 - 5i = 0$$

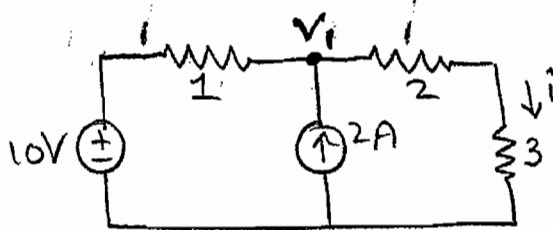
$$i_1 + 5i = 10$$

$$\therefore 6i = 12 \Rightarrow \boxed{i = 2A}$$

Now,

$$i - i_1 = 2$$

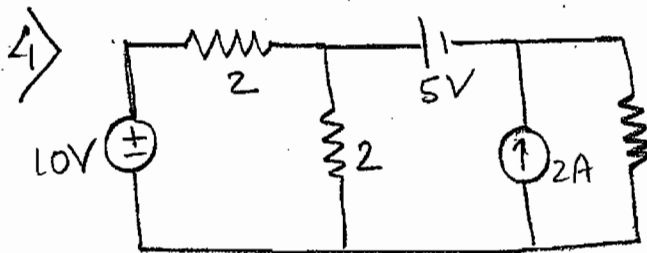
$$i_1 = i - 2$$



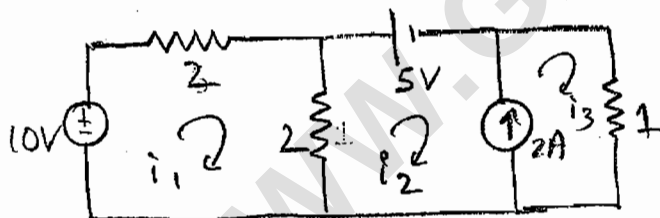
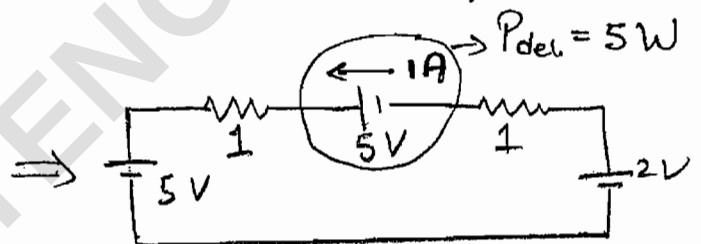
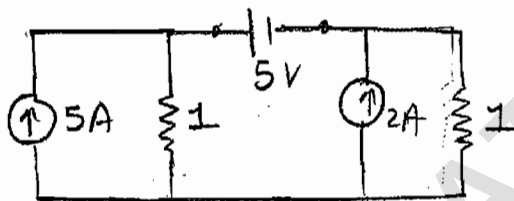
$$\frac{V_1 - 10}{1} = 2 + \frac{V_1}{5} = 0$$

$$6V_1 = 60 \Rightarrow V_1 = 10V$$

$$i = \frac{V_1}{5} = \frac{10}{5} = 2A$$



Find the power delivered by 5V source using mesh and nodal analysis.



$$i_2 + i_3 = 0$$

$$2i_2 = -2$$

$$i_2 = -1$$

$$-10 + 2i_1 + 2(i_1 - i_2) = 0$$

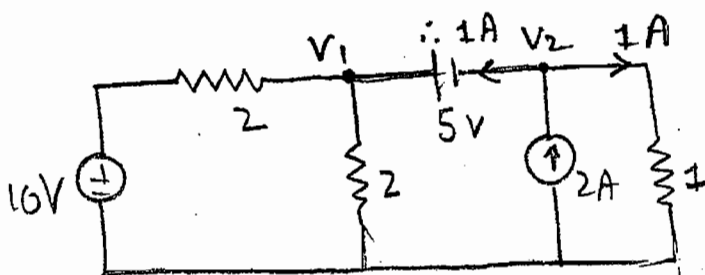
$$2i_1 - i_2 = 5 \quad \text{--- (1)}$$

$$2(i_2 - i_1) + 5 + i_3 = 0$$

$$-2i_1 + 2i_2 + i_3 = 5 \quad \text{--- (2)}$$

$$-i_2 + i_3 = 2 \quad \text{--- (3)}$$

$$\Rightarrow \boxed{P_{del.} = 5W}$$



$$\therefore P_{deliv.} = 5 \times 1 = 5W.$$

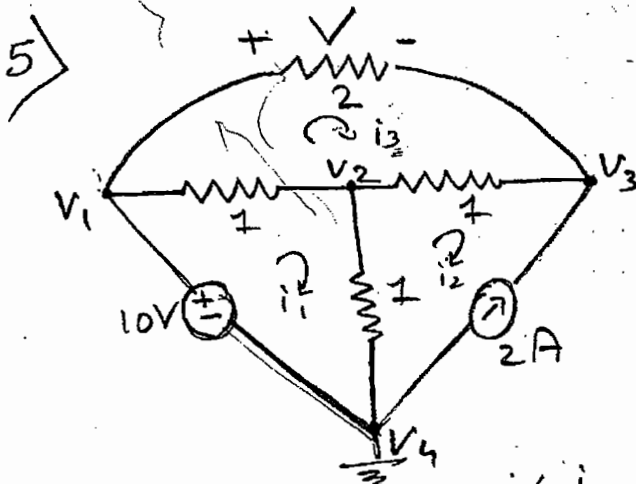
$$\frac{V_1 - 10}{2} + \frac{V_1}{2} - 2 + \frac{V_2}{1} = 0$$

$$V_1 + V_2 = 7 \quad \text{--- (1)}$$

$$V_1 - V_2 = 5 \quad \text{--- (2)}$$

$$\therefore V_1 = 6V$$

$$V_2 = 1V$$



Find V using mesh & nodal analysis.

MESH :-

$$10 - 2i_1 + i_3 + i_2 = 0 \quad \text{--- (1)}$$

$$i_2 = -2 \quad \text{--- (2)}$$

$$4i_3 - i_1 - i_2 = 0 \quad \text{--- (3)}$$

$$-2i_1 + i_3 + 8 = 0$$

$$-i_1 + 4i_3 + 2 = 0$$

$$\Rightarrow 4i_3 - 2i_1 = -8$$

$$-8i_3 + 2i_1 = +24$$

$$\hline -7i_3 - 7 = -20 \quad 4$$

$$i_3 = \frac{14}{7} = 2$$

$$\therefore V = 2 \left(\frac{14}{7} \right) = \boxed{\frac{8}{7} V}$$

NODAL :-

$$V_1 = 10 V \quad \text{--- (1)}$$

$$\frac{V_2 - 10}{1} + \frac{V_2 - V_3}{1} + \frac{V_2}{1} = 0$$

$$\therefore 3V_2 - V_3 = 10 \quad \text{--- (2)}$$

$$\frac{V_3 - V_2}{1} + \frac{V_3 - 10}{2} - 2 = 0$$

$$\therefore 3V_3 - 2V_2 = 14 \quad \text{--- (3)}$$

From (2) & (3), ;

$$6V_2 - 2V_3 = 20$$

$$-6V_2 + 9V_3 = 42$$

$$\hline 7V_3 = 62$$

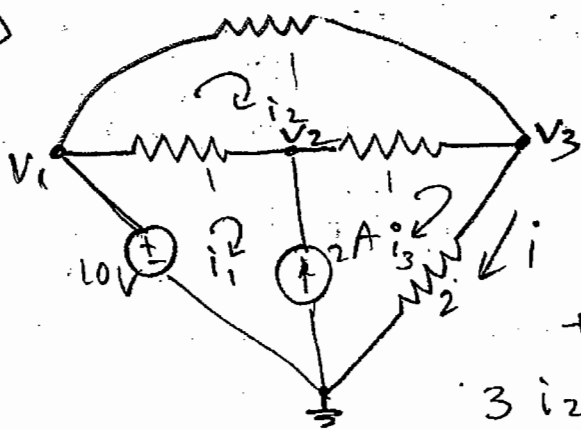
$$V_3 = \frac{62}{7}$$

$$\therefore V = V_1 - V_3$$

$$= 10 - \frac{62}{7}$$

$$\boxed{V = \frac{8}{7} V}$$

6)



Find 'i' using mesh & nodal.

Mesh :-

$$+10 - i_1 - i_3 - i_1 = 2 \quad \text{--- (1)}$$

$$3i_2 - i_1 - i_3 = 0 \quad \text{--- (2)}$$

$$10 - i_1 - 3i_3 + 2i_2 = 0 \quad \text{--- (3)}$$

Now, from (2) & (1); $3i_2 - i_3 + 2 - i_3 = 0$

$$3i_2 - 2i_3 = -2 \quad \text{--- (4)}$$

Now, from (1) & (3); $-i_3 + 2 - 3i_3 + 2i_2 = 10$

$$\therefore 2i_2 - 4i_3 = +8 \quad \text{--- (5)}$$

$$6i_2 - 2i_3 = -4$$

$$-6i_2 + 12i_3 = -24$$

$$\hline 8i_3 = -28$$

NODAL :- $V_1 = 10 \text{ V} \quad \text{--- (1)}$

$$\frac{V_2 + 10}{1} + \frac{V_2 - V_3}{1} - 2 = 0$$

$$2V_2 - V_3 = 12 \quad \text{--- (2)}$$

$$\frac{V_3 - 10}{1} + \frac{V_3 - V_2}{1} + \frac{V_3}{2} = 0$$

$$\therefore -2V_2 + 5V_3 = 20 \quad \text{--- (3)}$$

$$\therefore 4V_3 = 32$$

$$V_3 = 8 \text{ V}$$

$$\Rightarrow i = \frac{8}{2} = \boxed{4 \text{ A}}$$



What is the power delivered by the vltg source?

Mesh :-

$$i_2 - i_1 = 10 \quad \text{--- (1)}$$

$$3i_1 + 2i_2 - 2i_3 = -5$$

$$\therefore 3(i_2 - 10) + 2i_2 - 2i_3 = -5$$

$$3i_2 - 30 + 2i_2 - 2i_3 = -5$$

$$\therefore 5i_2 - 2i_3 = 25 \quad \text{--- (2)}$$

$$3i_3 - 2i_1 = 5$$

$$3i_3 - 2(i_2 - 10) = 5$$

$$\therefore -2i_2 + 3i_3 = -25 \quad \text{--- (3)}$$

$$10i_2 - 6i_3 = 75$$

$$-4i_2 + 6i_3 = -30$$

$$\frac{18i_2}{8} = 45$$

$$\therefore i_2 = \frac{45}{8}$$

$$i_3 = \frac{4\left(\frac{45}{8}\right) - 25}{2}$$

$$= \frac{45 - 50}{4} = -\frac{5}{4}$$

$$P_{del.} = -5 \times \left(\frac{45}{8} + \frac{5}{4} \right)$$

$$= \boxed{-\frac{275}{8} \text{ W}}$$

NODAL :-

$$\frac{V_1}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{1} = 0$$

$$5V_1 - V_2 - 2V_3 = 0 \quad \text{--- (1)}$$

$$(V_2 - V_3) = 5 \quad \text{--- (2)}$$

$$\frac{V_2 - V_1}{2} - 10 + \frac{V_3}{1} + \frac{V_3 - V_1}{1} = 0$$

$$\therefore -3V_1 + V_2 + 4V_3 = 20 \quad \text{--- (3)}$$

$$\text{From (1) \& (2); } \begin{matrix} -21 & +3 & -600 \\ 7V_1 & -V_2 & = 20 \end{matrix}$$

$$\text{From (2) \& (3); } \begin{matrix} 5V_1 & -3V_2 & = -10 \\ \hline 16V_1 & & = 70 \end{matrix}$$

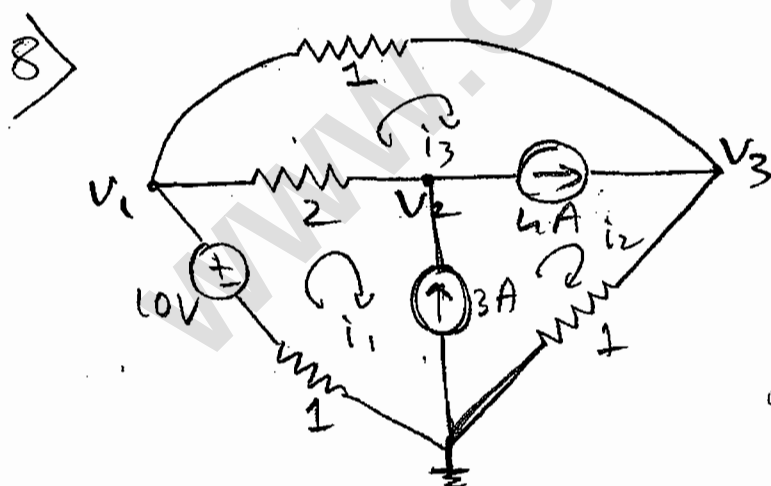
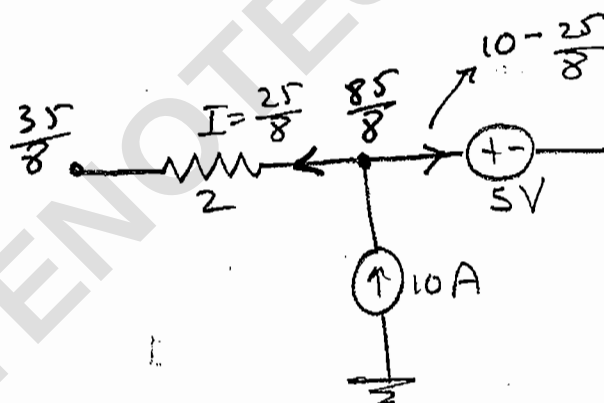
$$V_2 = 7\left(\frac{35}{8}\right) - 20$$

$$\therefore V_2 = \frac{85}{8}$$

$$V_1 = \frac{35}{8}$$

$$\therefore P_{\text{deliv}} = -5\left(10 - \frac{25}{8}\right)$$

$$= -\frac{275}{8} \text{ W}$$



Find power delivered by abtg source using mesh & nodal.

MESH :-

$$i_1 + i_2 + i_3 = 10 \quad \text{--- (1)}$$

$$i_2 - i_1 = 3 \quad \text{--- (2)}$$

$$i_2 = 3 + i_1$$

$$i_2 - i_3 = 4 \quad \text{--- (3)}$$

$$3 + i_1 - i_3 = 4$$

$$\therefore i_3 = i_1 - 1$$

$$\therefore i_1 + 3 + i_1 + i_1 - 1 = 10$$

$$\therefore 3i_1 = 8 \Rightarrow i_1 = \frac{8}{3}$$

$$\therefore P_{\text{deliv.}} = 10 \times \frac{8}{3} = \boxed{\frac{80}{3} \text{ W}}$$

NODAL : —

$$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{1} = 0$$

$$\therefore 5V_1 - V_2 - 2V_3 = 20 \quad \text{--- (1)}$$

$$\frac{V_2 - V_1}{2} + 4 - 3 = 0$$

$$\therefore V_2 - V_1 = -2 \quad \text{--- (2)}$$

$$\frac{V_3}{1} - 4 + \frac{V_3 - V_1}{1} = 0$$

$$\therefore -V_1 + 2V_3 = 4 \quad \text{--- (3)}$$

From (1) & (2); $5V_1 - V_1 + 2 - 2V_3 = 20$

$$\therefore 4V_1 - 2V_3 = 18 \quad \text{--- (4)}$$

$$\therefore V_3 = \frac{4 + \frac{22}{3}}{2}$$

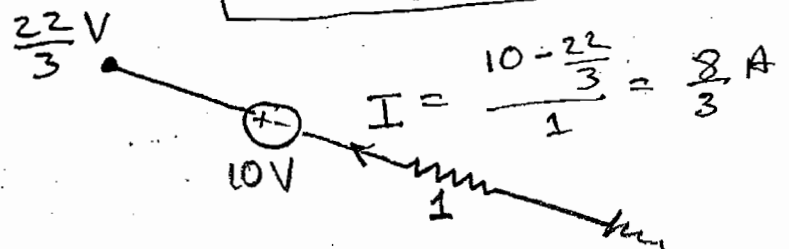
$$\therefore \frac{V_1 + 2V_3 = 4}{3V_1} = 22$$

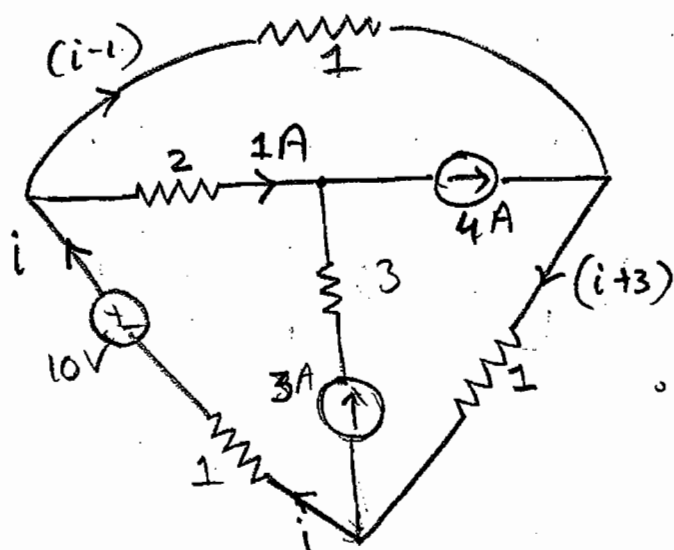
$$\therefore V_3 = \frac{34}{6} \text{ V}$$

$$\therefore \boxed{V_1 = \frac{22}{3} \text{ V}}$$

Now,

$$P_{\text{deliv.}} = 10 \times \frac{8}{3} = \frac{80}{3} \text{ W}$$



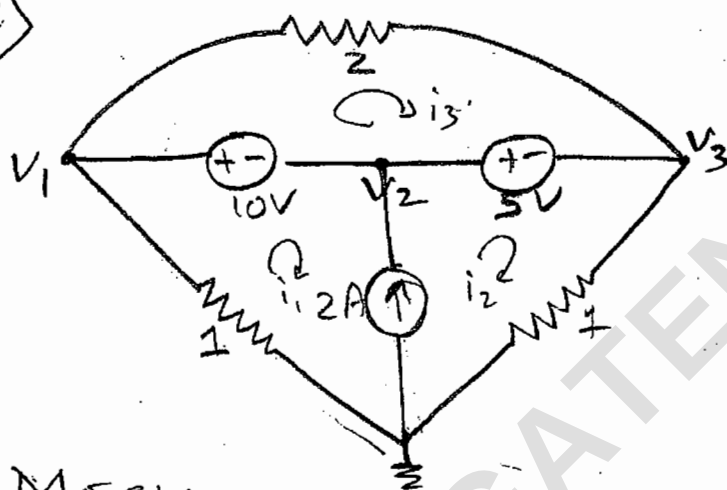


$$i - 10 + i - 1 + i + 3 = 0$$

$$3i = 8$$

$$i = \frac{8}{3}$$

$$\therefore P_{\text{deliv.}} = 10 \times \frac{8}{3} = \frac{80}{3} \text{ W}$$



Find the power delivered by the current source using mesh & nodal.

MESH :-

$$i_1 + 1i_2 + 2i_3 = 0 \quad \text{--- (1)}$$

$$2i_3 = 15 \Rightarrow i_3 = \frac{15}{2} \quad \text{--- (2)}$$

$$i_1 + i_2 = -15 \quad \text{--- (3)}$$

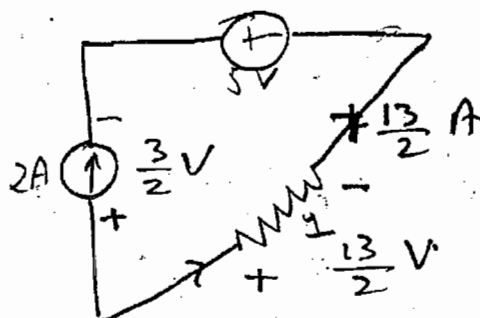
$$-i_1 + i_2 = 2 \quad \text{--- (4)}$$

$$i_2 = -\frac{13}{2}$$

$$i_1 = -\frac{17}{2}$$

$$\therefore P_{\text{delivered}} = -2 \times \frac{3}{2}$$

$$= -3 \text{ W}$$



NODAL :-

$$\frac{V_1}{1} + \frac{V_1 - V_3}{2} - 2 + \frac{V_3}{1} + \frac{V_3 - V_1}{2} = 0$$

$$\therefore 3V_1 - V_3 = 4$$

$$\therefore V_1 + V_3 = 2 \quad \text{--- (1)}$$

$$V_1 - V_2 = 10 \quad \text{--- (2)}$$

$$V_2 - V_3 = 5 \quad \text{--- (3)}$$

$$\Rightarrow V_2 + V_1 = 7$$

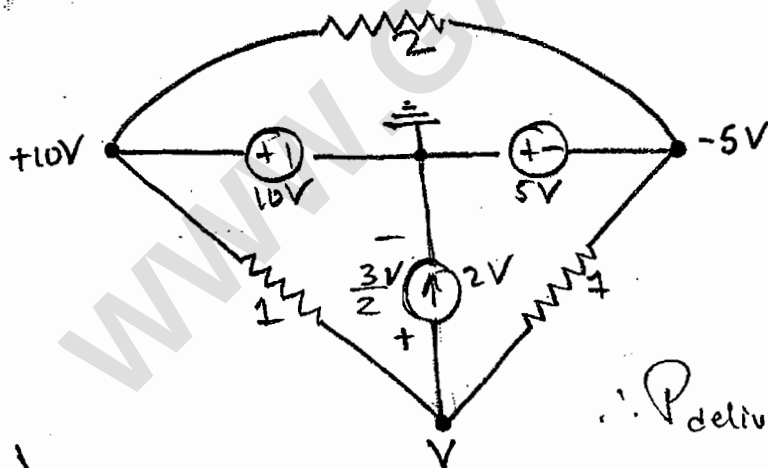
$$\therefore 2V_1 = 17$$

$$V_1 = \frac{17}{2}$$

$$V_2 = \frac{17}{2} - 10$$

$$V_2 = -\frac{3}{2} \text{ V}$$

$$P_{\text{delivered}} = 2 \times \left(-\frac{3}{2}\right) = -3 \text{ W}$$



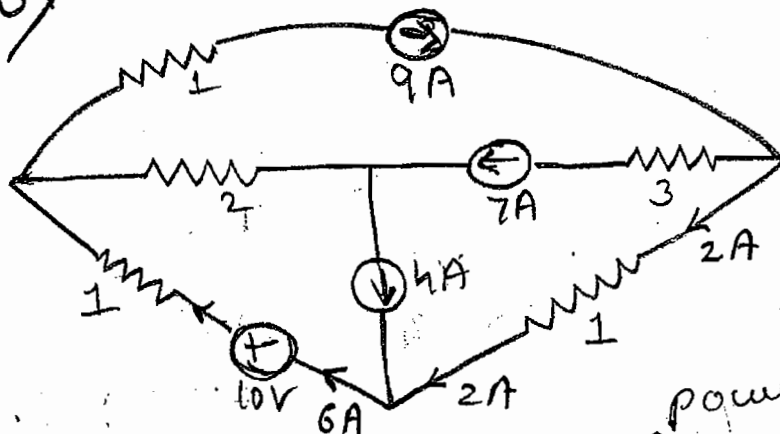
$$\frac{V-10}{1} + \frac{V+5}{1} + 2 = 0$$

$$\therefore 2V = 3$$

$$V = \frac{3}{2} \text{ V}$$

$$\therefore P_{\text{deliv.}} = -\frac{3}{2} \times 2 = -3 \text{ W}$$

10/



Just write mesh & nodal eqⁿ governing the circuit & determine power delivered by vltg source by inspection.

MESH :-

$$i_1 - i_2 = 4 \quad \text{--- (1)} \quad i_3 = 9 \text{ A} \quad \text{--- (2)}$$

$$i_3 - i_2 = 7 \quad \text{--- (3)} \Rightarrow i_2 = 2 \text{ A}$$

$$\therefore i_1 = 6 \text{ A}$$

NODAL :-

$$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{2} + 9 = 0 \quad \text{--- (1)}$$

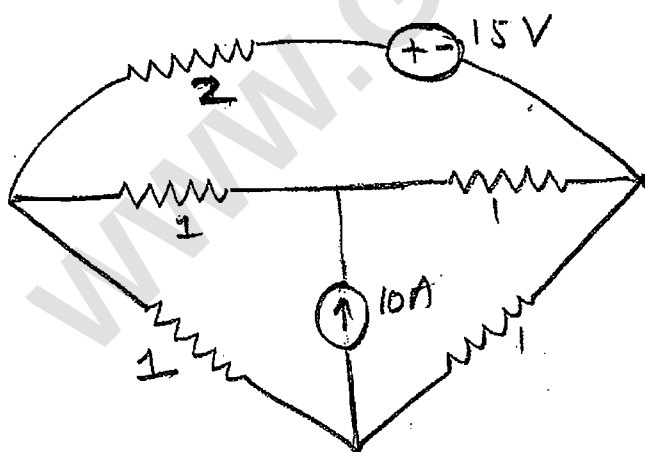
$$\therefore \frac{V_2 - V_1}{2} + 4 - 7 = 0 \quad \text{--- (2)}$$

$$\frac{V_3}{1} + 7 - 9 = 0 \quad \text{--- (3)}$$

Now,

$$P_{\text{delivered}} = 10 \times 6 = \boxed{60 \text{ W}}$$

11)



What is the power delivered by vltgy source using mesh & nodal.

MESH :-

$$i_2 - i_1 = 10 \quad \text{--- (1)}$$

$$4i_3 - i_1 - i_2 = -15 \quad \text{--- (2)} \Rightarrow i_1 + i_2 = 4i_3 + 15$$

$$i_1 + i_2 + 2i_3 = -15 \quad \text{--- (3)} \Rightarrow 4i_3 + 15 + 2i_3 = -15$$

$$P_{\text{delivered}} = +15 \times \frac{10}{2} = +\underline{\underline{75 \text{ W}}} \quad \therefore i_3 = \underline{\underline{-5 \text{ A}}}$$

NODAL:-

$$\frac{V_1}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - 15 - V_3}{2} = 0$$

$$\therefore 5V_1 - 2V_2 - V_3 = 15 \quad \text{--- (1)}$$

$$\frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{1} - 10 = 0$$

$$\therefore -V_1 + 2V_2 - V_3 = 10 \quad \text{--- (2)}$$

$$\frac{V_3}{1} + \frac{V_3 - V_2}{1} + \frac{V_3 + 15 - V_1}{2} = 0$$

$$\therefore -V_1 - 2V_2 + 5V_3 = -15 \quad \text{--- (3)}$$

$$\text{From (1) \& (2); } 4V_1 - 2V_3 = 25$$

$$\text{From (2) \& (3); } \frac{-2V_1 + 4V_3}{4} = \frac{-5}{10}$$

$$6V_3 = 15$$

$$V_1 = \frac{25 + \frac{15}{3}}{4}$$

$$= \frac{90}{12} = \frac{15}{2} \text{ V}$$

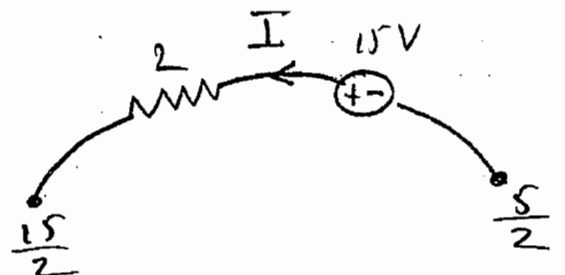
$$V_3 = \frac{15}{6} \text{ V} = \frac{5}{2} \text{ V}$$

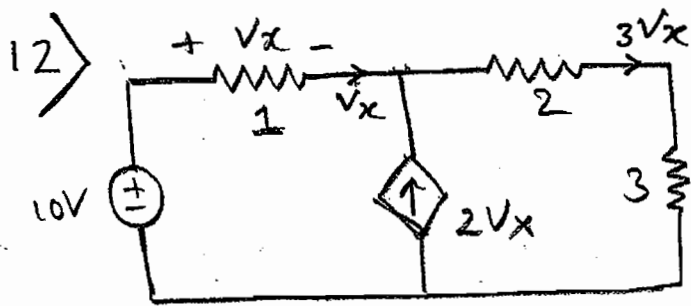
$$\therefore I = \frac{V_3 + 15 - V_1}{2} = \frac{\frac{5}{2} + 15 - \frac{15}{2}}{2}$$

$$= \frac{20}{4} = 5 \text{ A}$$

$$\therefore P_{\text{delivered}} = 15 \times 5$$

$= 75 \text{ W}$





Find V_x using mesh & nodal

KVL

$$-10 + V_x + 15V_x = 0$$

$$\therefore \boxed{V_x = \frac{5}{8} V}$$

MESH :—

$$i_1 + 5i_2 = 10 \quad \text{--- (1)}$$

$$-i_1 + i_2 = 2V_x \quad \text{--- (2)}$$

$$\therefore -3i_1 + i_2 = 0 \quad \text{--- (4)}$$

From (1) & (4);

$$\therefore i_1 = 10 - 5i_2$$

$$i_1 = \frac{5}{8}$$

$$3i_1 + 15i_2 = 30$$

$$-3i_1 + i_2 = 0$$

$$\hline 16i_2 = 30$$

$$i_2 = 2 \text{ A}$$

$$\therefore V_x = \frac{5}{8} V$$

NODAL :—

$$\frac{V_1 - 10}{1} - 2V_x + \frac{V_1}{5} = 0$$

$$5V_1 - 50 - 10V_x + V_1 = 0$$

$$6V_1 - 10V_x = 50$$

$$3V_1 - 5V_x = 25 \quad \text{--- (1)}$$

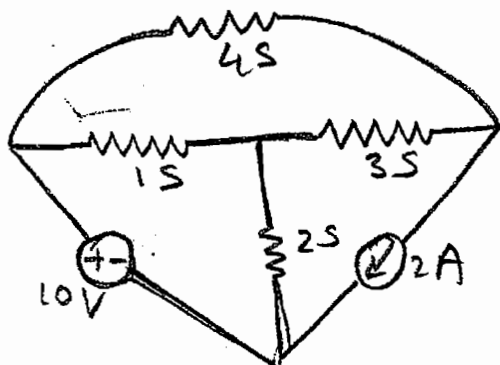
$$V_x = 10 - V_1 \quad \text{--- (2)}$$

$$3(10 - V_x) - 5V_x = 25$$

$$8V_x = 5$$

$$V_x = \frac{5}{8} V$$

13

MESH

$$-10 + \frac{(i_1 - i_3)}{1} + \frac{(i_1 - i_2)}{2} = 0 \quad \text{--- (1)}$$

$$\therefore i_2 = 2 \quad \text{--- (2)}$$

$$\frac{i_3}{4} + \frac{(i_3 - i_2)}{2} + \frac{(i_3 - i_1)}{1} = 0 \quad \text{--- (3)}$$

$$P_{3S} = \frac{|i_2 - i_3|^2}{3} = \underline{\hspace{2cm}} \text{ W}$$

NODAL :-

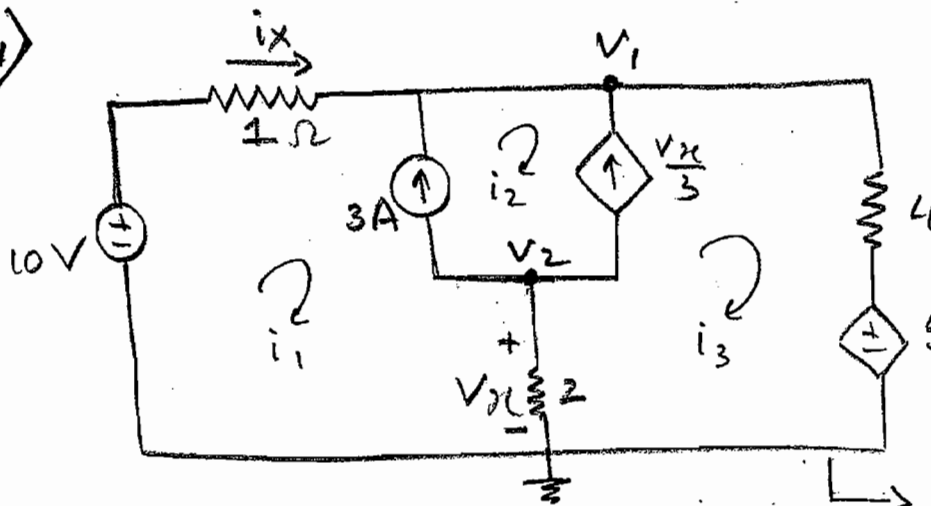
$$V_1 = 10 \quad \text{--- (1)}$$

$$1[V_2 - V_1] + 2V_2 + 3[V_2 - V_3] = 0 \quad \text{--- (2)}$$

$$+2 + 3(V_3 - V_2) + 4(V_3 - V_1) = 0 \quad \text{--- (3)}$$

$$P_{3S} = [V_2 - V_3]^2 \cdot 3 = \underline{\hspace{2cm}} \text{ W}$$

14)

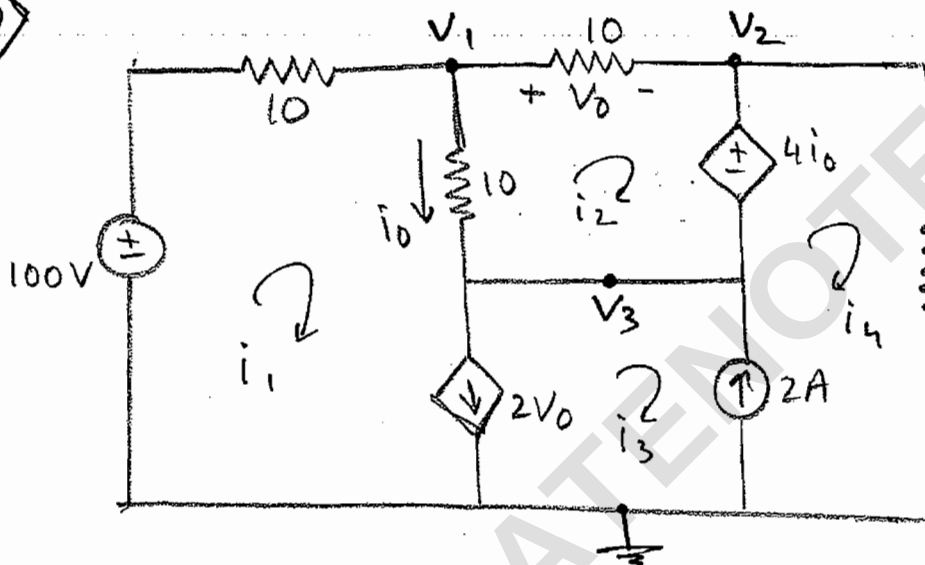


Write mesh
& nodal
equation
governing
the circuit.

$$\text{Mesh} = 3 + 2 = 5$$

$$\text{Nodal} = 2 + 2 = 4$$

15)



$$\text{Mesh} = 4 + 2 = 6$$

$$\text{Nodal} = 3 + 2 = 5$$

14)

MESH :-

$$10 - i_1 - 4i_3 - 5i_x = 0 \quad \text{--- (1)}$$

$$-i_1 + i_2 = 3 \quad \text{--- (2)}$$

$$-i_2 + i_3 = \frac{V_x}{2} \quad \text{--- (3)}$$

$$i_x = i_1 \quad \text{--- (4)}$$

$$V_x = (i_1 - i_3) 2 \quad \text{--- (5)}$$

NODAL :-

$$\frac{V_1 - 10}{1} + \frac{V_1 - 5i_x}{1} - 3 - \frac{V_x}{3} = 0 \quad \text{--- (1)}$$

$$\frac{V_2}{2} + 3 + \frac{V_x}{3} = 0 \quad \text{--- (2)}$$

$$V_x = V_2 \quad \text{--- (3)}$$

$$i_x = 10 - V_1 \quad \text{--- (4)}$$

15) MESH :-

$$100 - 20i_1 + 10i_2 + 4i_0 - 5i_4 = 0 \quad \text{--- (1)}$$

$$-10i_1 + 20i_2 + 4i_0 = 0 \quad \text{--- (2)}$$

$$i_4 - i_3 = 2 \quad \text{--- (3)} \quad i_1 - i_2 = i_0 \quad \text{--- (5)}$$

$$i_1 - i_3 = 2V_0 \quad \text{--- (4)} \quad V_0 = 10i_2 \quad \text{--- (6)}$$

NODAL :-

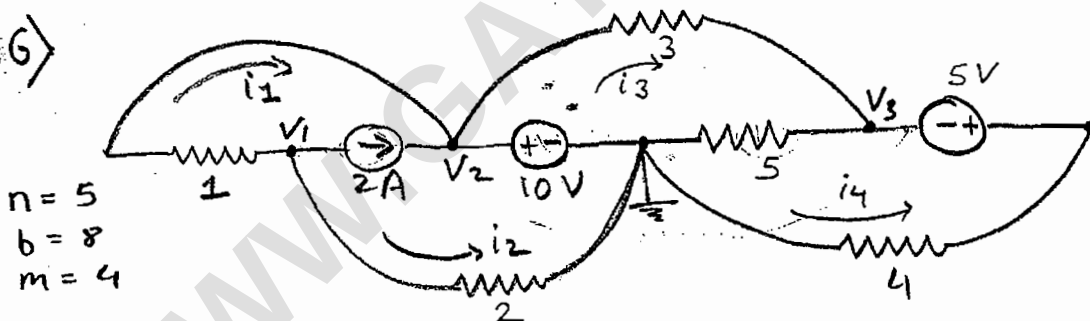
$$\frac{V_1 - 100}{10} + \frac{V_1 - V_2}{10} + \frac{V_1 - V_3}{10} = 0 \quad \text{--- (1)}$$

$$\frac{V_2 - V_1}{10} + \frac{V_2}{5} + 2V_0 - 2 + \frac{V_3 - V_1}{10} \quad \text{--- (2)}$$

$$V_2 - V_3 = 4i_0 \quad \text{--- (3)}$$

$$V_0 = V_1 - V_2 \quad \text{--- (4)} \quad i_0 = \frac{V_1 - V_3}{10} \quad \text{--- (5)}$$

16)



MESH :-

$$i_1 + 10 - 2i_2 = 0 \quad \text{--- (1)}$$

$$+i_1 + i_2 = -2 \quad \text{--- (2)}$$

$$8i_3 + 5i_4 = 10 \quad \text{--- (3)}$$

$$5i_3 + 4i_4 + 5 = 0 \quad \text{--- (4)}$$

NODAL :-

$$\frac{V_1 - V_2}{1} + 2 + \frac{V_1}{2} = 0 \quad \text{--- (1)}$$

$$V_2 = 10 \quad \text{--- (2)}$$

$$\frac{V_3 - V_2}{3} + \frac{V_3}{5} + \frac{V_3 + 5}{4} = 0 \quad \text{--- (3)}$$

MESH:-

$$i_1 - 2i_2 = -10$$

$$-i_1 + i_2 = +2$$

$$-3i_2 = -8$$

$$i_2 = \frac{8}{3} \text{ A}$$

$$i_1 = -10 + 2\left(\frac{8}{3}\right)$$

$$i_1 = \frac{-14}{3} \text{ A}$$

$$40i_3 + 25i_4 = 50$$

$$40i_3 + 72i_4 = -40$$

$$-48i_4 = 90$$

$$i_4 = \frac{-90}{48} \text{ A}$$

$$i_3 = \frac{15}{48} \text{ A}$$

NODAL:-

$$V_1 - 10 + 2 + \frac{V_1}{2} = 0$$

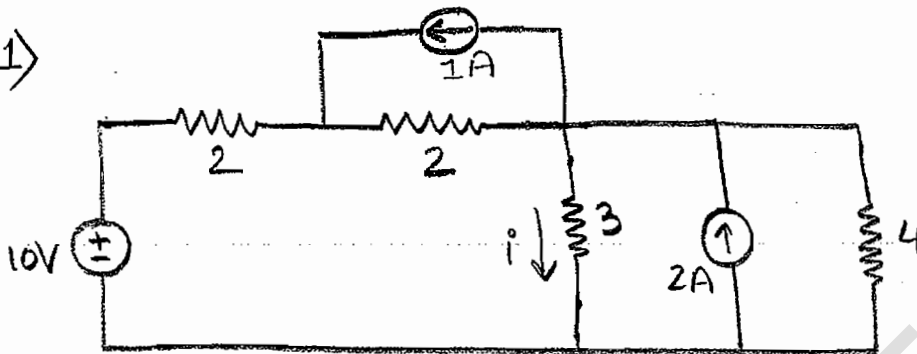
WWW.GATENOTES.IN

NETWORK THEOREMS IN DC

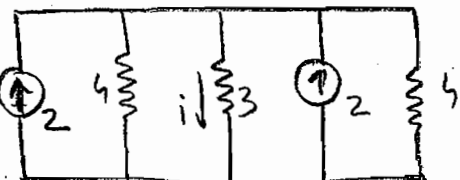
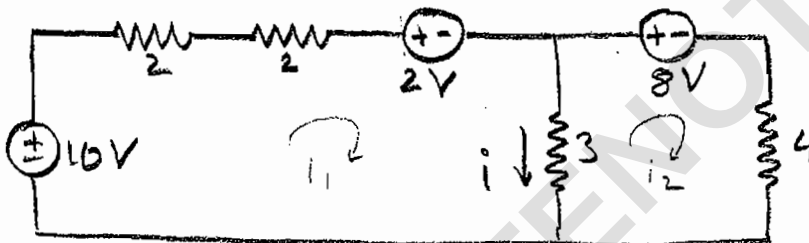
Theorem 1:-

Source Transformation Technique:-

1)

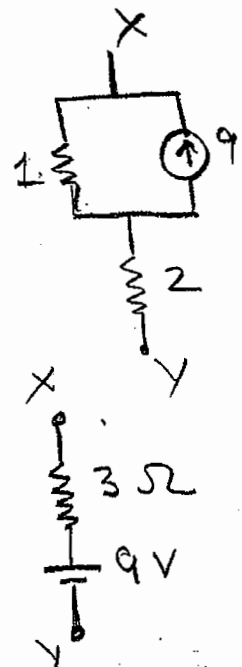
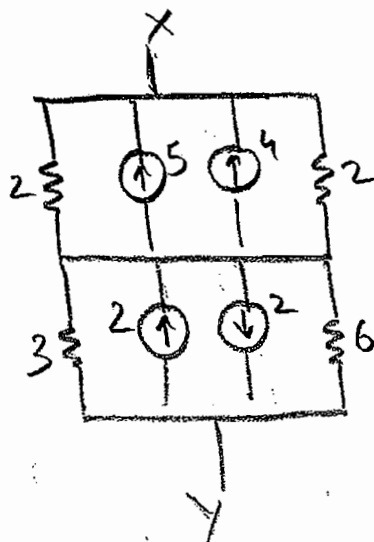
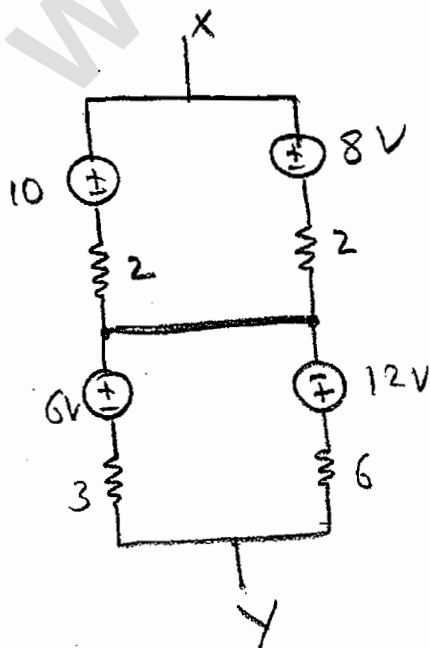


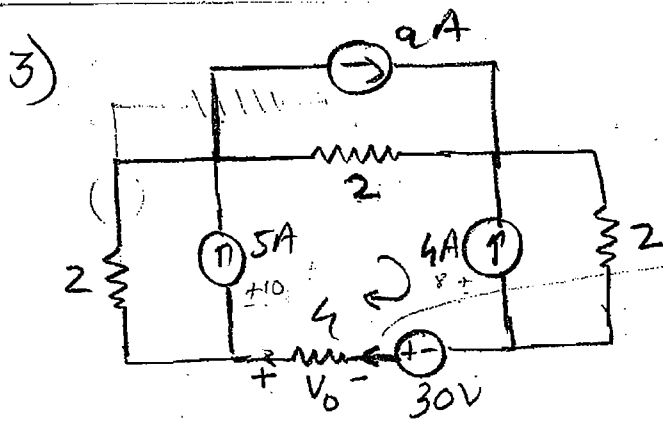
Find 'i'
using S.T.T



$$\therefore i = 4 \left(\frac{18}{18 + 12 + 16} \right) = 4 \left(\frac{18}{46} \right) = \frac{8}{5} \text{ A}$$

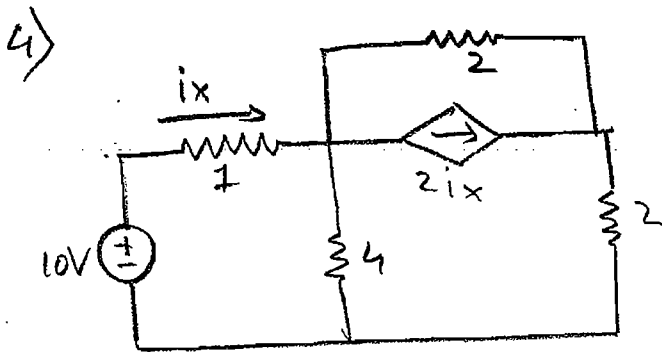
2)



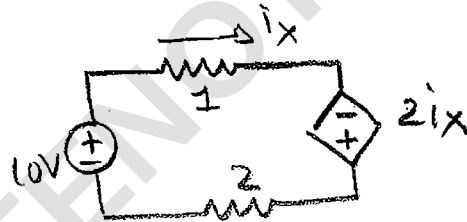
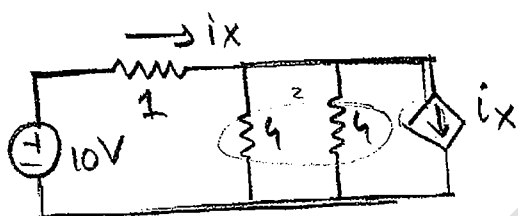
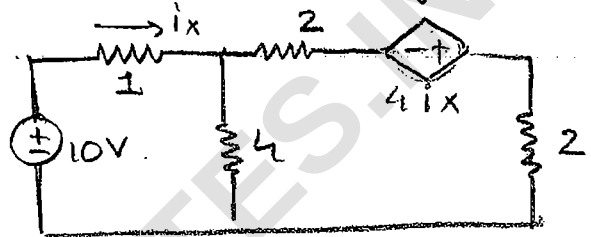


Find V_o in one step

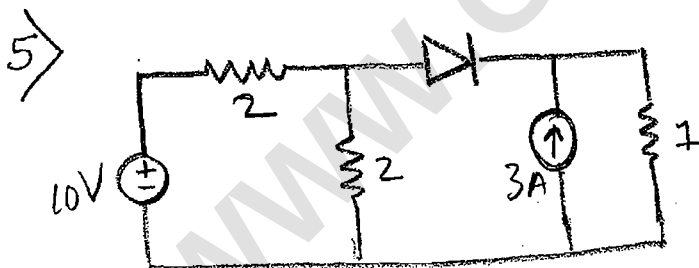
$$V_o = -50 \left(\frac{4}{4+6} \right) = -20 \text{ V}$$



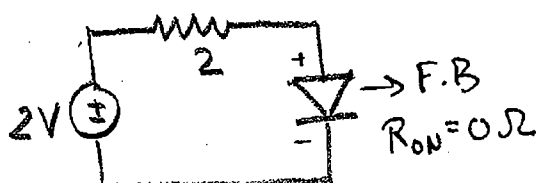
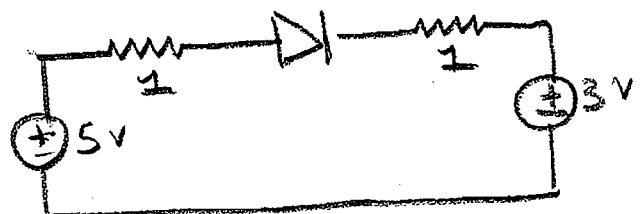
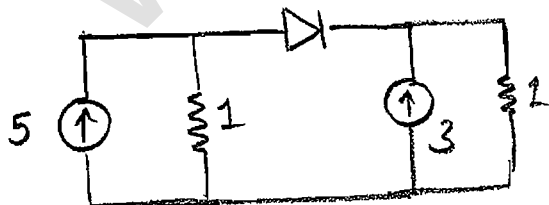
Find i_x using S.T.T.



$$i_x = \frac{10 - 2i_x}{3} \Rightarrow i_x = 10 \text{ A}$$



If diode is ideal one, find current through it.



$$\therefore I_D = \frac{2}{2} = 1 \text{ A}$$

● Theorem 2 :—

● Superposition Theorem :—

● In any linear, active, bilateral n/w, consisting of no. of energy sources, resistances, etc. ; the effect produced in any element when all sources act at a time is equal to sum of effect in same element when each source is considered independently.

● 1) The no. of sub-circuits to be solved by applying S.P.T. is ———

● $\Rightarrow$ Sum of Independent sources only.

● 2) Which of the foll. electrical parameter cannot be directly evaluated by using S.P.T. ?

● (a) Voltage (b) Current

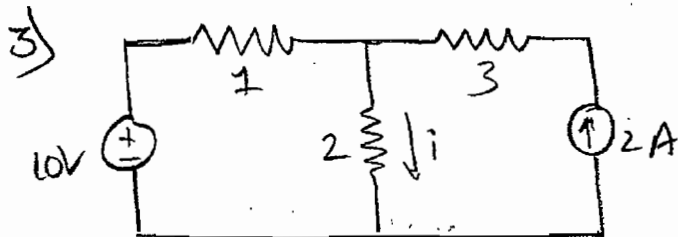
● ☒ (c) Power (d) Charge.

● $\hookrightarrow$ non-linear electrical parameter

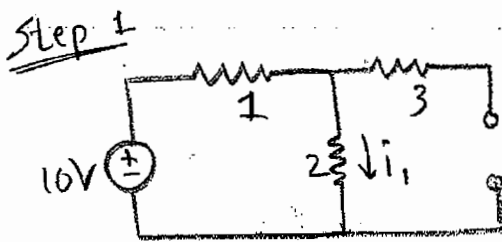
● $\rightarrow$ While applying S.P.T., we consider only 1 independent source in every sub-circuit where other ulty sources are replaced by short circuit & ideal current sources are replaced by open circuit..

● However dependent sources cannot be suppressed.

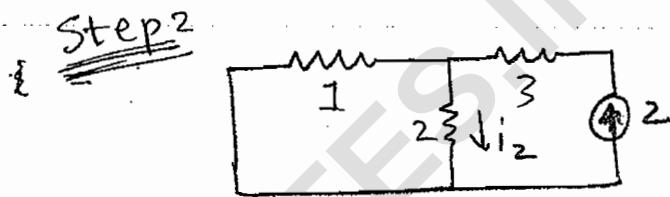
→ Voltages have unique polarities & Current have unique dirⁿ & they must be ~~not~~ respected by while applying S.P.T.



Find 'i' using S.P.T.

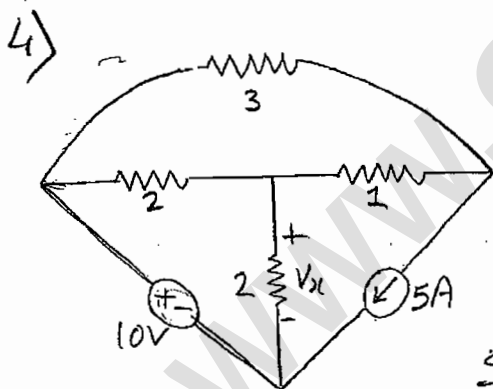


$$i_1 = \frac{10}{3} \text{ A}$$



$$i_2 = 2 \left(\frac{1}{3} \right) = \frac{2}{3} \text{ A}$$

By SPT :- $i = i_1 + i_2 = \frac{10}{3} + \frac{2}{3} = \frac{12}{3} = 4 \text{ A}$



Find V_x with help of S.P.T.

Step 1 (10V only)

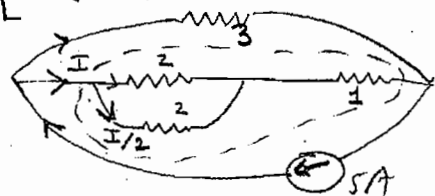
$$V_x' = 10 \left(\frac{2}{2 + 4/3} \right) = 6 \text{ V}$$

Step 2 (5A only)

$$V_x'' = -2 \left[5 \left(\frac{3}{3+2} \right) \times \frac{1}{2} \right] = -3 \text{ V}$$

Using SPT :-

$$V_x = V_x' + V_x'' = 6 - 3 = 3 \text{ V}$$



Check (Nodal) :-

$$V_1 = 10 \text{ --- (1)}$$

$$\frac{V_2 - 10}{2} + \frac{V_2}{2} + \frac{V_2 - V_3}{1} = 0$$

$$4V_2 - 2V_3 = 10 \text{ --- (2)}$$

$$5 + \frac{V_3 - V_2}{1} + \frac{V_3 - 10}{3} = 0$$

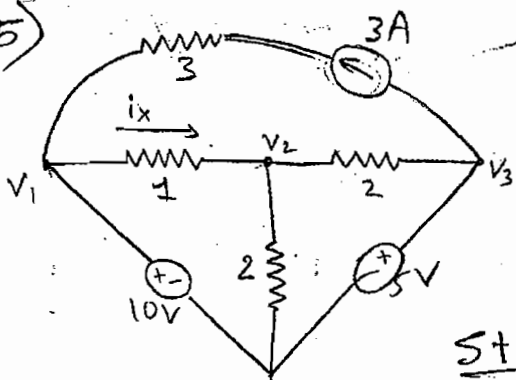
$$-3V_2 + 4V_3 = -5 \text{ --- (3)}$$

$$5V_2 = 15$$

$$V_2 = 3$$

so $V_x = 3$

5)



Find i_x using S.P.T.

Step 1 (10V only)

$$i_x' = 10 \times \frac{1}{1+1} = 5 \text{ A}$$

Step 2 (5V only)

$$i_x'' = -5 \left(\frac{2/3}{2/3+2} \right) = -5 \left(\frac{2}{8} \right) = -\frac{5}{4} \text{ A}$$

Step 3 (3A only)

$$i_x''' = 0 \text{ A}$$

~~Check (Not By SPT)~~

$$i_x = 5 + 0 - \frac{5}{4} = \boxed{\frac{15}{4} \text{ A}}$$

Check (Nodal)

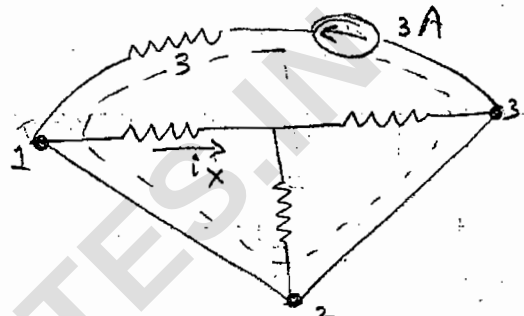
$$\frac{V_2 - 10}{1} + \frac{V_2}{2} + \frac{V_2 - 5}{2} = 0$$

$$\Rightarrow 4V_2 = 25$$

$$V_2 = \frac{25}{4}$$

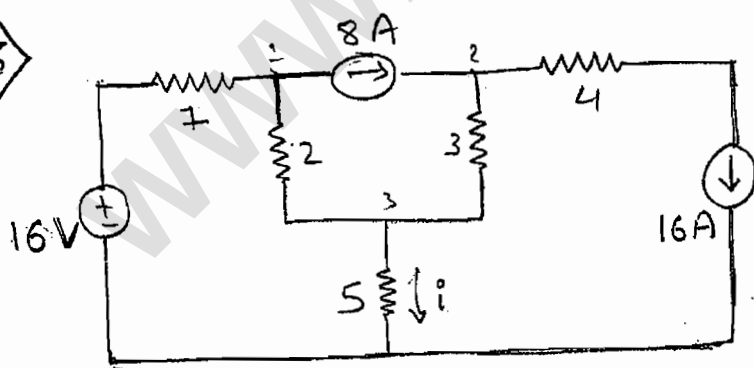
Now,

$$i_x = \frac{V_1 - V_2}{1} = 10 - \frac{25}{4} = \frac{15}{4} \text{ A}$$



1, 2, 3 are at same potential.

6)



What is the power lost in 5Ω resistor using S.P.T.

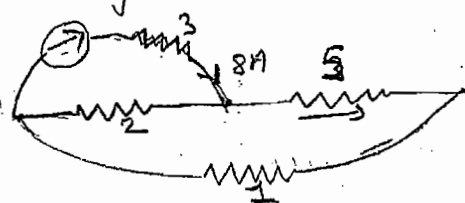
Step 1 :- (16V only)

$$i' = \frac{16}{8} = \frac{2}{1} \text{ A} = 2 \text{ A}$$

Step 2 :- (8A only)

$$i'' = 8 \times \frac{2}{8} = 2 \text{ A}$$

Power is non-linear parameter & hence cannot be calculated directly using S.P.T.



Step 3 :- (16A only)

$$i''' = -16 \times \frac{3}{8} = -6A$$

Using S.P.T

$$i = i' + i'' + i''' = 2 + 2 - 6 = -2A \Rightarrow 5\Omega \uparrow 2A$$

$$\begin{aligned} \text{So, } P_{\text{lost}} &= I_{\text{net}}^2 \times R \\ &= 4 \times 5 = \boxed{20W} \end{aligned}$$

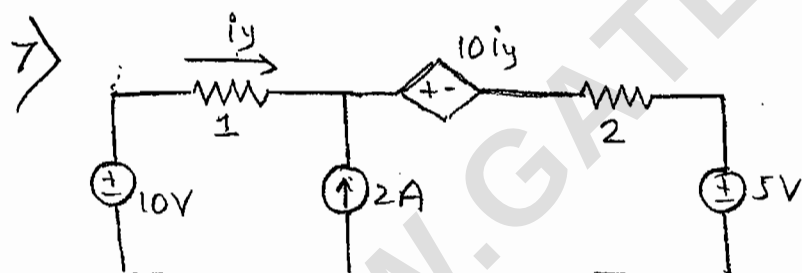
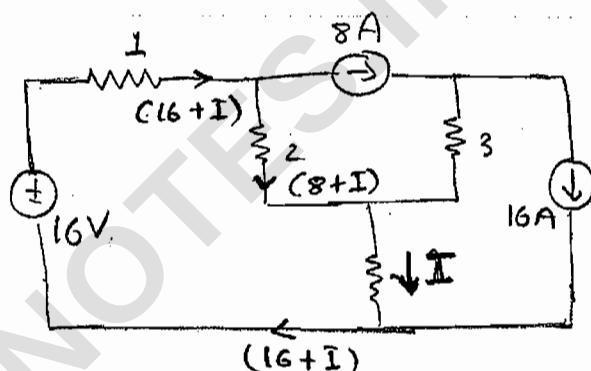
Check

$$-16 + 1(16+I) + 2(8+I) + 5I = 0$$

$$\therefore 8I = -16$$

$$\therefore I = -2A$$

$$P_{\text{lost}} = 4 \times 5 = 20W$$



$$10 - i_y - 10i_y - 2i_y = 0$$

$$\therefore 13i_y = 10 \Rightarrow i_y = \frac{10}{13}$$

Step 2 :- (2A only)

$$i_y'' + 10i_y'' + 2(i_y'' + 2) = 0$$

$$\therefore 13i_y'' = -4$$

$$\therefore i_y'' = -\frac{4}{13}A$$

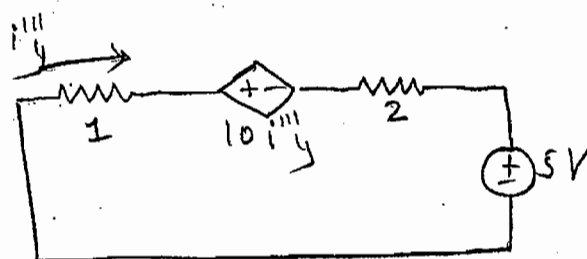
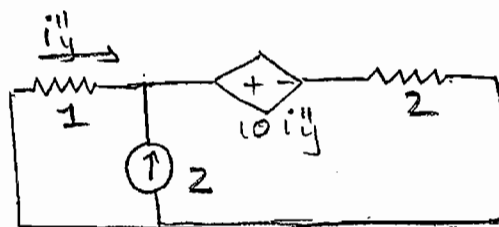
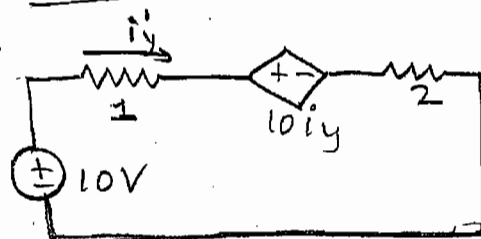
Step 3 :- (5V only)

$$3i_y''' + 10i_y''' + 5 = 0$$

$$\therefore i_y''' = -\frac{5}{13}A$$

Find i_y using S.P.T.

Step 1 (10V only)

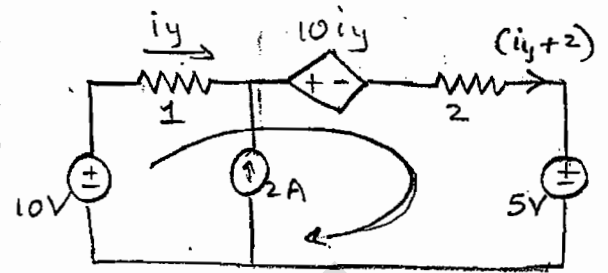


By SPT

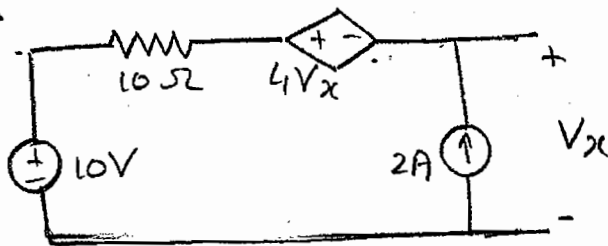
$$i_y = \frac{10}{13} - \frac{4}{13} - \frac{5}{13} = \boxed{\frac{1}{13} \text{ A}}$$

Check

$$\begin{aligned} -10 + i_y + 10i_y + 2(2 + i_y) + 5 &= 0 \\ \therefore 13i_y &= 1 \\ \therefore i_y &= \frac{1}{13} \text{ A} \end{aligned}$$



8)



Find V_x using S.P.T.

Step 1 (10V only)

$$V_x^I = 10 - 4V_x^I$$

$$\therefore V_x^I = 2 \text{ V}$$

Step 2 :- (2A only)

$$4V_x'' - 10(2) + V_x'' = 0$$

$$\therefore V_x'' = 4 \text{ V}$$

check

$$V_x + 4V_x - 10 - 20 = 0$$

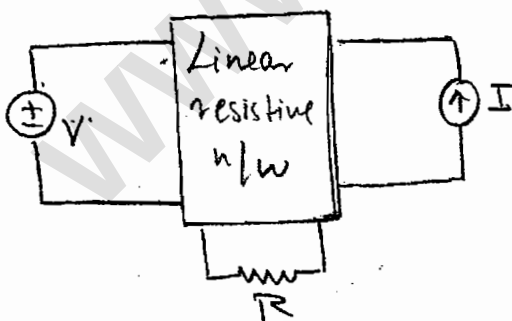
$$5V_x = 30$$

$$V_x = 6 \text{ V}$$

By SPT

$$V_x = 4 + 2 = \boxed{6 \text{ V}}$$

9)



The power lost in resistor R when vltg source alone act is 9W &

when current source alone act is 4W. What is total

power lost in resistor R when both sources act simultaneously. ~~(a) 4W~~ (b) 5W

(c) 13W

~~(d) 25W~~

V alone act :-

$$P_1 = I_1^2 R = 9 \Rightarrow |I_1| = \frac{3}{\sqrt{R}}$$

I alone act :-

$$P_2 = I_2^2 R = 4 \rightarrow |I_2| = \frac{2}{\sqrt{R}}$$

Now,

$$I_{\text{net}} = \pm I_1 \pm I_2$$

$$P_T = [I_{\text{net}}]^2 R = [\pm I_1 \pm I_2]^2 R$$

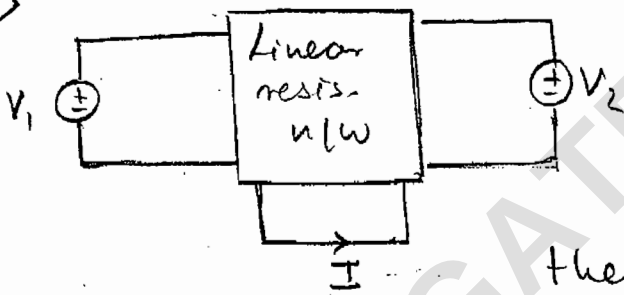
$$= \left[\pm \frac{3}{\sqrt{R}} \pm \frac{2}{\sqrt{R}} \right]^2 R = [\pm 3 \pm 2]^2 W$$

$$\therefore P_T = 1 W \quad (I_1 \& I_2 \text{ diff. dir.})$$

or

$$25 W \quad (I_1 \& I_2 \text{ same dir.})$$

10)



V_1	V_2	I
10V	0V	5A
0V	-5V	1A

then if

$$V_1 = V_2 = 15 V$$

$$I = \underline{\hspace{2cm}}$$

V_1 alone act :-

$$10 \rightarrow 5A$$

$$\therefore 15 \rightarrow \frac{15 \times 5}{10} = 7.5 A \quad (\text{Homogeneity})$$

V_2 alone act

$$-5 \rightarrow 1A$$

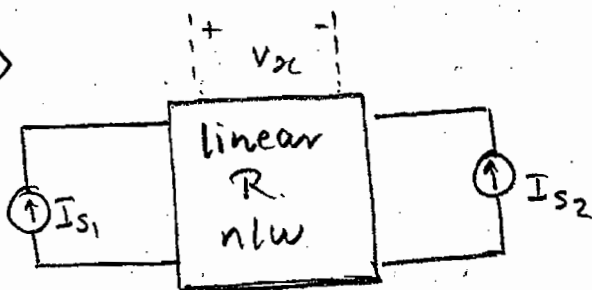
$$15 \rightarrow \frac{15}{-5} = -3 A \quad (\text{Homogeneity})$$

$\therefore$ By SPT

$$I = 7.5 - 3$$

$$= 4.5 A$$

11)



$$\text{If } I_{s1} = 10 \text{ A } I_{s2} = 5 \text{ A.}$$

$$\text{then } V_{RL} = 20 \text{ V}$$

$$\text{If } I_{s1} = 20 \text{ A } , I_{s2} = -5 \text{ A}$$

$$\text{then } V_{RL} = 10 \text{ V}$$

Now if $I_{s1} = I_{s2} = 15 \text{ A}$ then $V_{RL} = \underline{\hspace{2cm}}$

$$V_{RL} = f(I_{s1}, I_{s2})$$

$$\therefore V_{RL} = \alpha I_{s1} + \beta I_{s2}$$

$$20 = \alpha(10) + \beta(5)$$

$$10 = \alpha(20) + \beta(-5)$$

$$30 = \alpha(30)$$

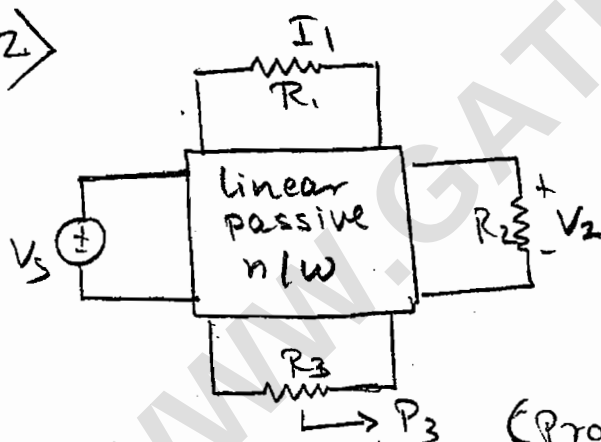
$$\therefore \alpha = 1 \Rightarrow \beta = 2$$

$$V_{RL} = 1 I_{s1} + 2 I_{s2}$$

$$= 1 \times 15 + 2 \times 15$$

$$V_{RL} = 45 \text{ V}$$

12)



$$\text{If } V_s = 30, \text{ then}$$

$$I_1 = 10 \text{ A } , V_2 = 20 \text{ V } , P_3 = 30 \text{ W}$$

$$\text{If } V_s = 50 \text{ V } , \text{ then}$$

$$I_1 = ? , V_2 = ? , P_3 = ?$$

(Proportionality, homogeneity, Ohm's Law).

(I₁)

$$30 \rightarrow 10 \text{ A}$$

$$50 \rightarrow I_{1, \text{new}} = \frac{50 \times 10}{30} = 16.66 \text{ A}$$

(V₂)

$$30 \rightarrow 20 \text{ V}$$

$$50 \rightarrow V_{2, \text{new}} = \frac{50 \times 20}{30} = 33.33 \text{ V}$$

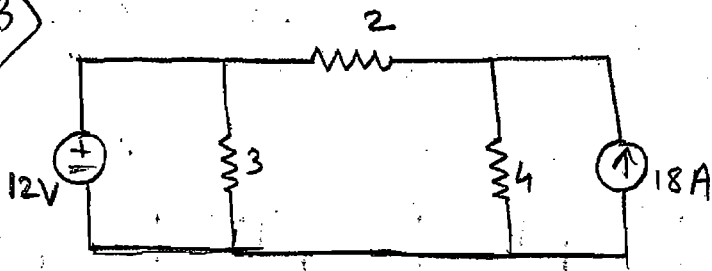
(P₃)

$$P \propto V^2$$

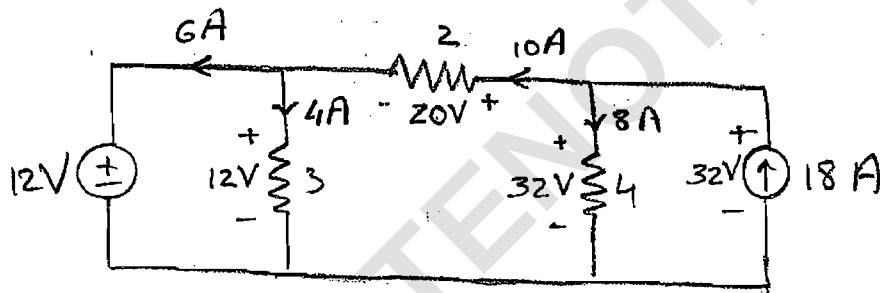
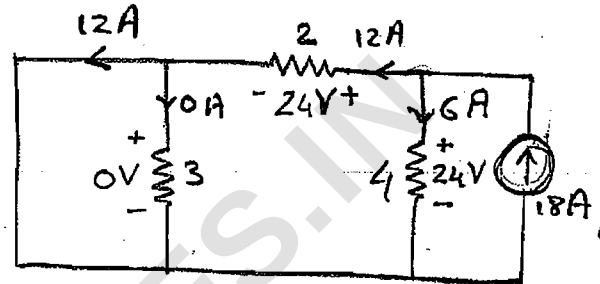
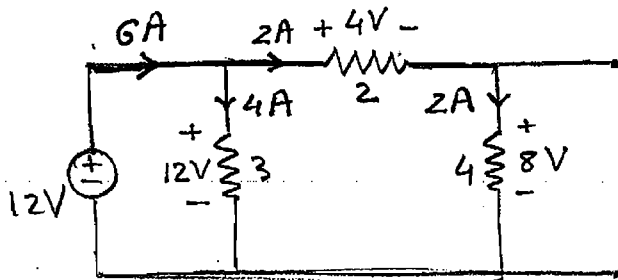
$$(30)^2 \rightarrow 30$$

$$(50)^2 \rightarrow P_{3, \text{new}} = \frac{(50)^2 (30)}{(30)^2} = \frac{250}{3} \text{ W}$$

13)



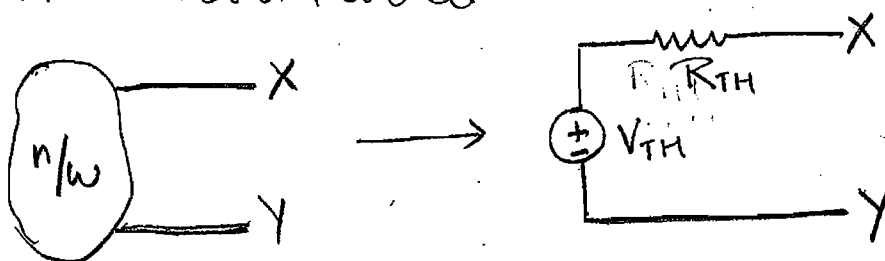
Determine vltg & current across all elements by using SPT.



Theorem 3 :-

Thevenin's Theorem :-

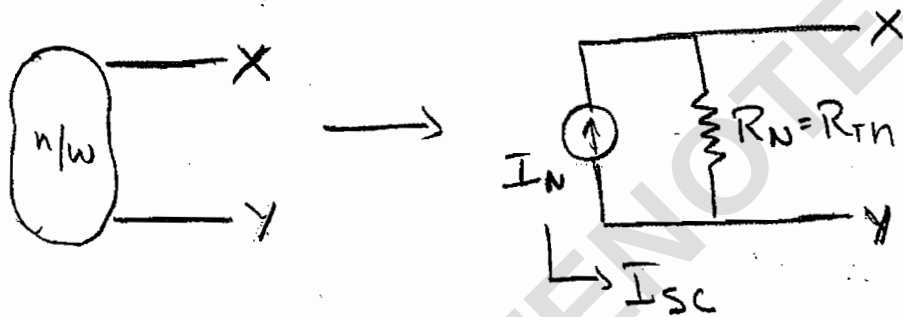
In any linear bilateral (active) n/w consisting of energy sources, resistors, etc with open o/p target terminal defined can be converted into a simple n/w consisting of vltg source in series with resistance.



Theorem 4:-

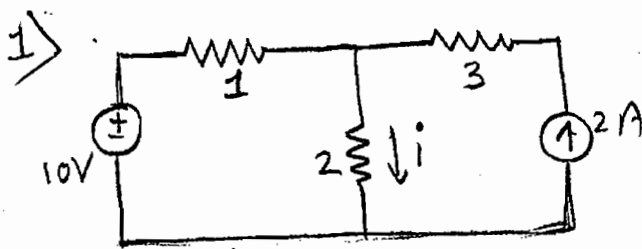
Norton's Theorem:-

In any linear active bilateral n/w consisting of no. of energy sources, resist., etc with open o/p target terminal defined can be converted into a simple n/w consisting of current source in parallel with resistance.



Thevenin & Norton's equivalent are duals of each other.
i.e. They are source transformable.

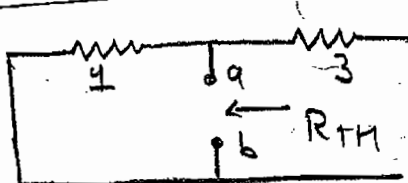
Category 1:- Problems with only independent sources



Determine current i using
① Thevenin th.
② Norton th.

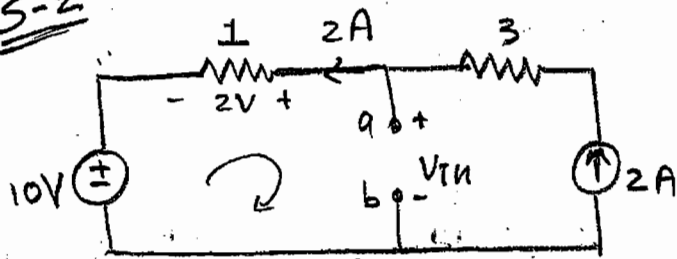
① Thevenin th.

5-1



$$R_{TH} = 1 \Omega$$

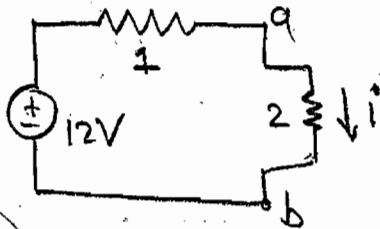
S-2



KVL

$$-10 - 2 + V_{th} = 0$$

$$V_{th} = 12V$$



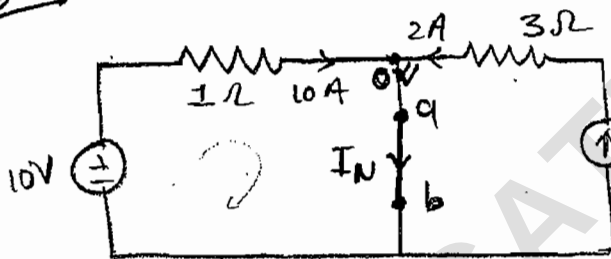
$$i = \frac{12}{3} = 4A$$

② Norton's th.

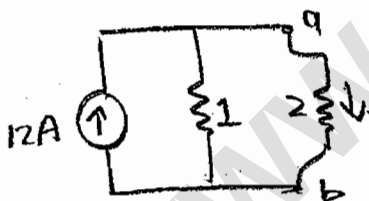
S-1

$$R_N = R_{th} = 1\Omega$$

S-2

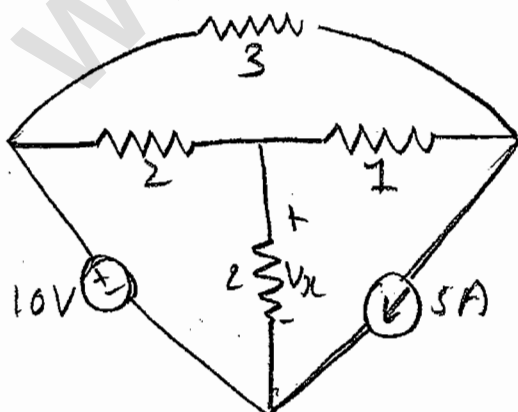


$$I_N = 10 + 2 = 12A$$



$$i = 12 \left(\frac{1}{1+2} \right) = 4A$$

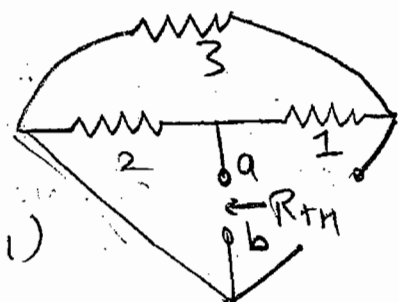
2



Find V_x

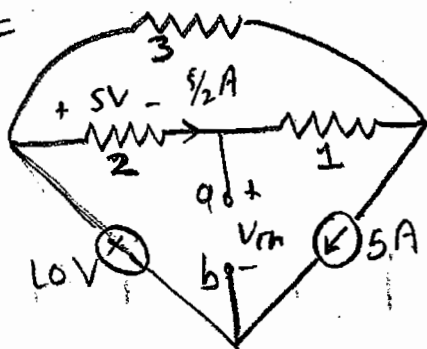
① Thevenin's th.

S-1



$$R_{th} = (2 \parallel 4) = \frac{4}{3}\Omega$$

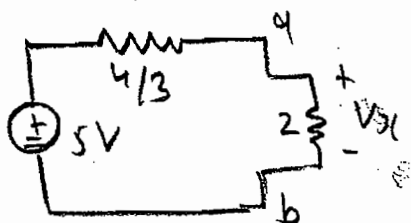
S-2



KVL

$$-10 + 5 + V_{th} = 0$$

$$\Rightarrow V_{th} = 5V$$

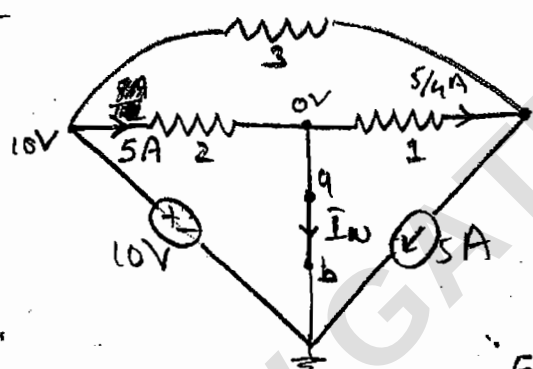


$$V_{th} = 5 \left(\frac{2}{2 + 4/3} \right) = 3V$$

② Norton's the.

S-1 $R_N = R_{th} = 4/3 \Omega$

S-2



KCL KCL

$$5 + \frac{V}{1} + \frac{V-10}{3} = 0$$

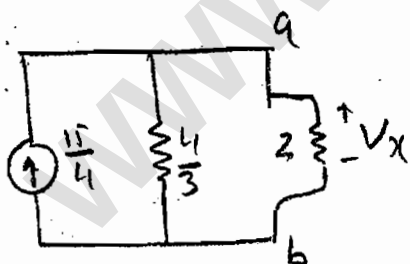
$$4V = -5$$

$$\therefore V = -\frac{5}{4}V$$

KCL

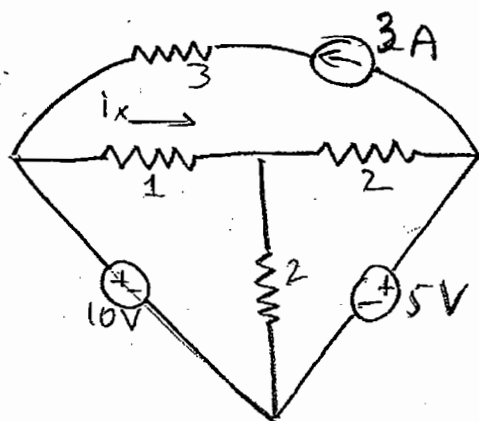
$$5 = I_N + \frac{5}{4}$$

$$I_N = \frac{15}{4}A$$



$$V_{th} = 2 \left[\frac{15/4}{4} \left(\frac{4/3}{2 + 4/3} \right) \right] = 3V$$

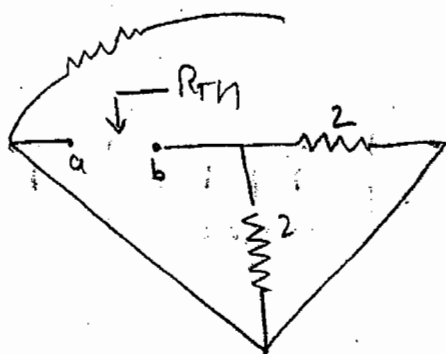
3)



Find i_x using
Thevenin & Norton

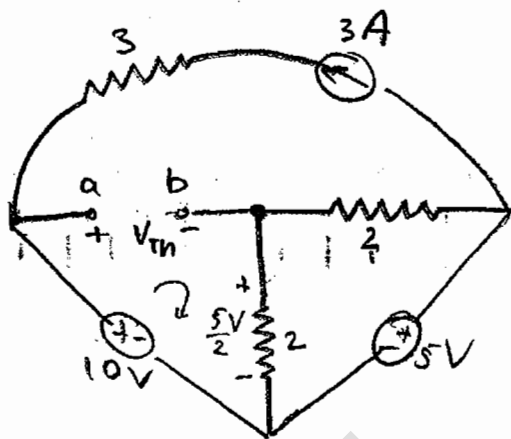
① Thevenin's th.

S-1



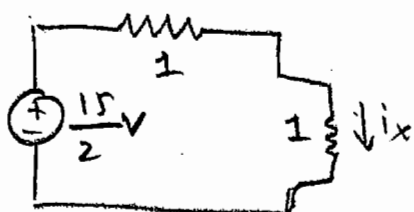
$$R_{TH} = 2 \parallel 2 \\ = 1 \Omega$$

S-2



KVL

$$10 - V_{th} - \frac{5}{2} V = 0 \\ \therefore V_{th} = \frac{15}{2} V$$



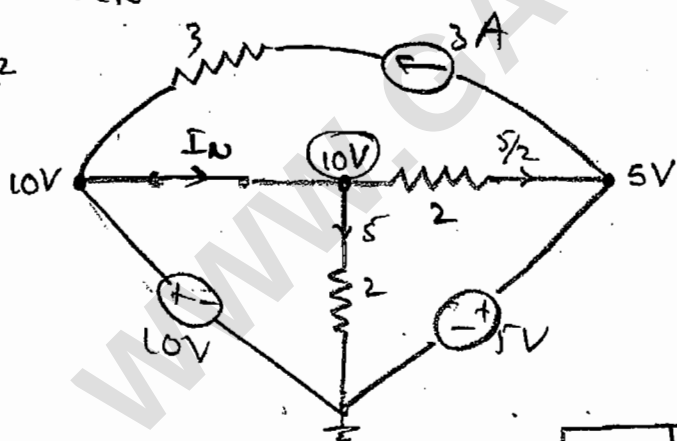
$$i_x = \frac{15}{4} A$$

② Norton's th.

S-1

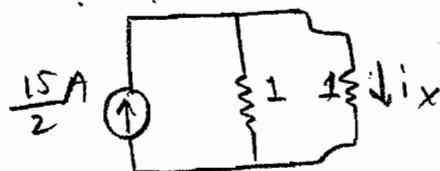
$$R_N = R_{th} = 1 \Omega$$

S-2



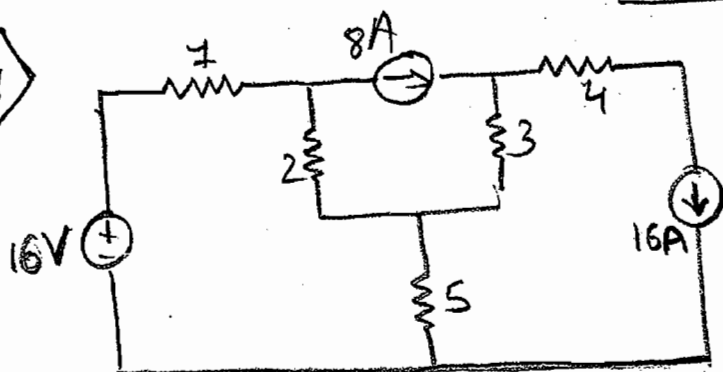
KCL

$$I_N = 5 + \frac{5}{2} \\ = \frac{15}{2} A$$



$$i_x = \frac{15}{2} \left(\frac{1}{2} \right) \\ = \frac{15}{4} A$$

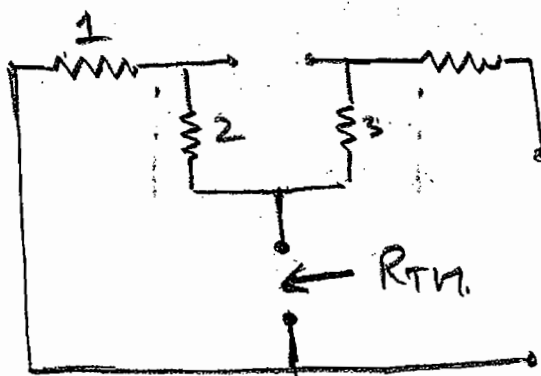
4)



Find power lost
in 5 ohm res.
using thevenin
& Norton

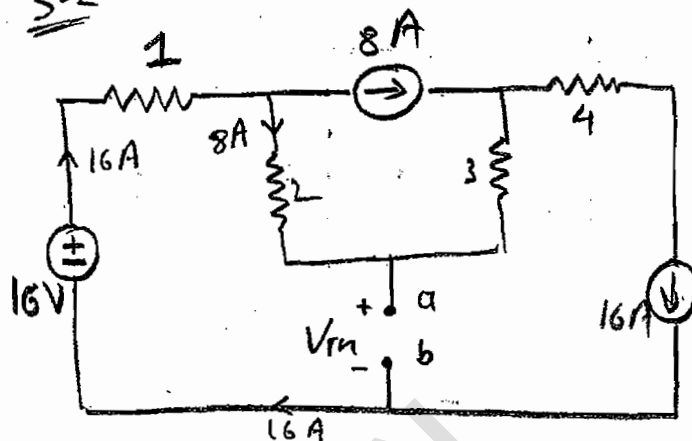
① Thevenius

S-1

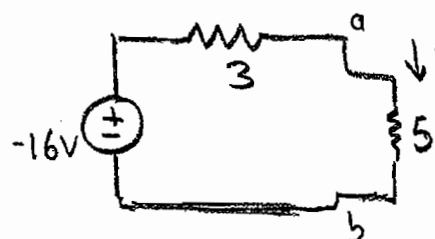


$$R_{TH} = 2 + 1 = 3 \Omega$$

S-2



$$V_{TH} = 16 - 16(1) - 2(8) = 16 - 32 = -16 V$$



$$I_{ab} = \frac{-16}{8} = -2 A$$

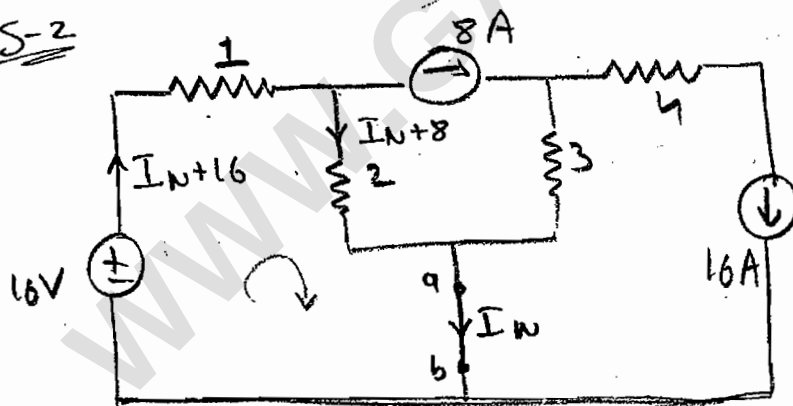
$$\therefore P_{lost} = (-2)^2 \times 5 = 20 W$$

② Nortons

S-1

$$R_N = R_{TH} = 3$$

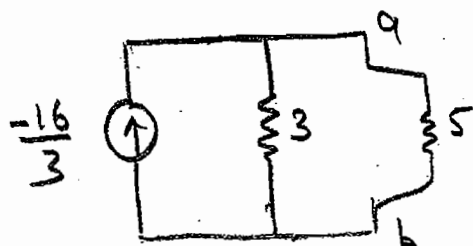
S-2



$$16 - (I_N + 16) - 2(I_N + 8) = 0$$

$$\therefore 3I_N = 16 - 32$$

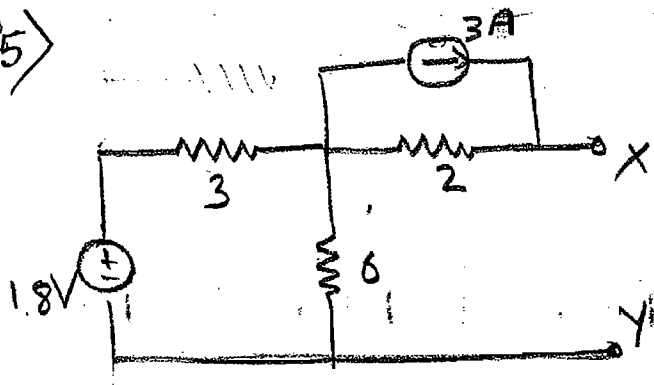
$$\therefore I_N = \frac{-16}{3} A$$



$$I_{ab} = \frac{-16}{3} \left(\frac{3}{8} \right) = -2 A$$

$$\therefore P_{lost} = (-2)^2 \times 5 = 20 W$$

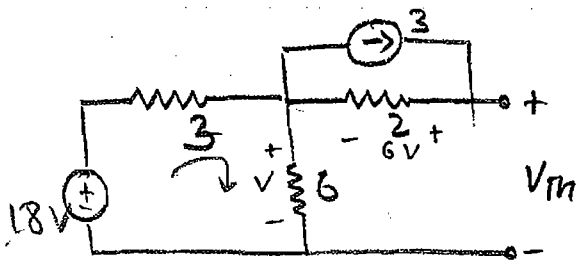
5)



Determine Thevenin
& Norton's
equivalent b/w
X Y

① Thevenin's

$$R_{TH} = 2 + (6 || 3) = 2 + 2 = 4 \Omega$$



$$\begin{aligned} V_{TH} &= 6 + 18 \left(\frac{6}{9} \right) \\ &= 6 + 12 \\ &= 18 \text{ V} \end{aligned}$$

② Norton's

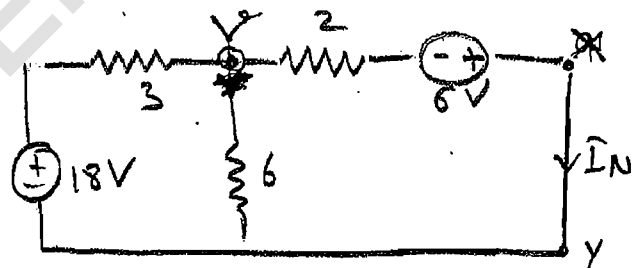
$$R_N = R_{TH} = 4 \Omega$$

KCL

$$\frac{V-18}{3} + \frac{V}{6} + \frac{V+6}{2} = 0$$

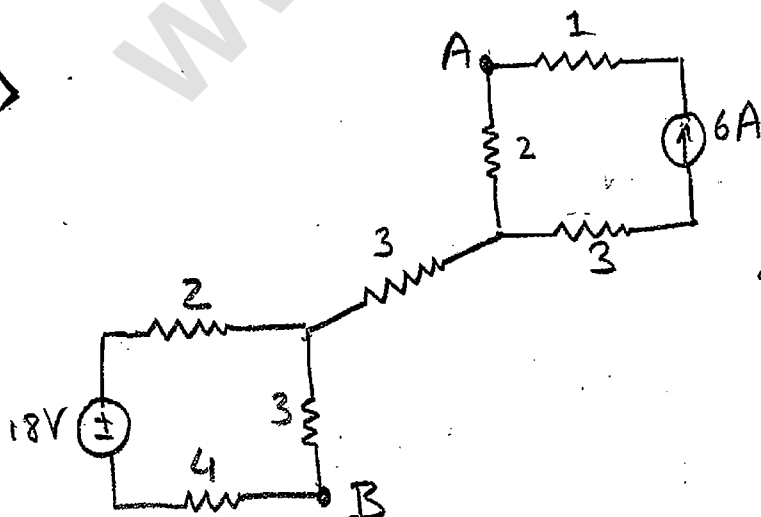
$$\therefore 2V - 36 + V + 3V + 18 = 0$$

$$\therefore V = 3$$



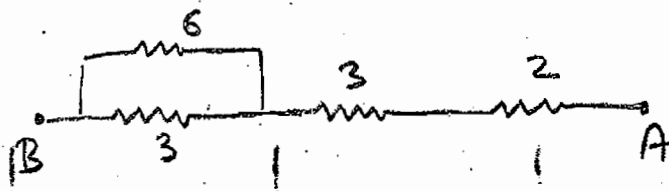
$$\begin{aligned} I_N &= \frac{3+6}{6} \\ &= \frac{9}{2} \text{ A} \end{aligned}$$

6)



Determine
Norton & Thevenin
equivalent b/w
A B

① Thevenin's



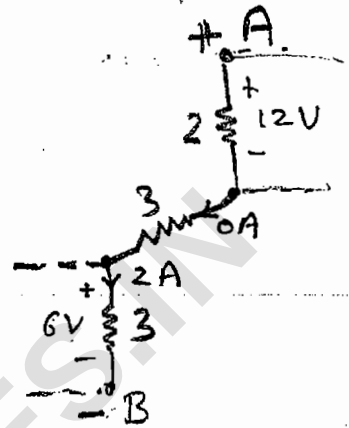
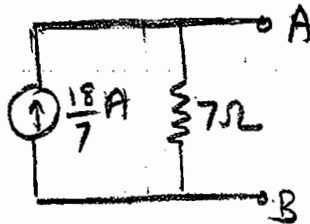
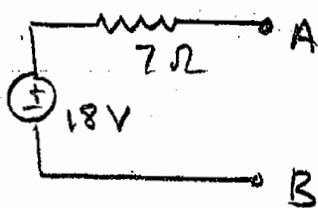
$$R_{TH} = 2 + 3 + (3 || 6)$$

$$= 2 + 3 + 2$$

$$= 7 \Omega$$

$$V_{TH} = 12 + 0 + 6$$

$$= 18 \text{ V}$$



② Norton's

$$R_N = R_{TH} = 7 \Omega$$

Now

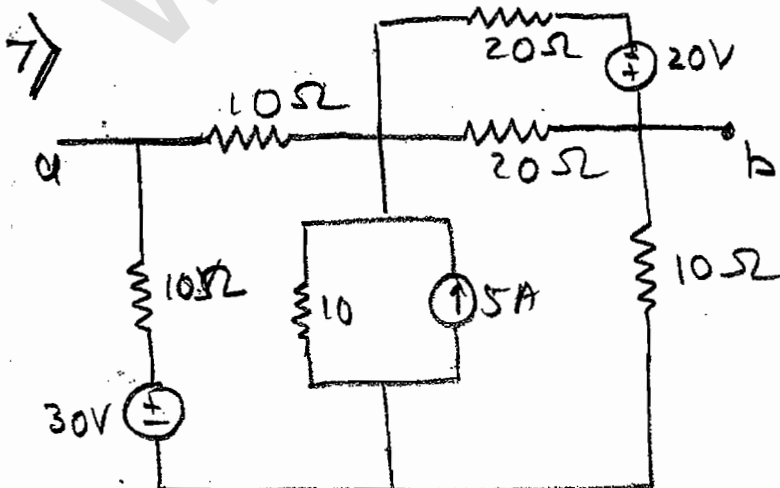
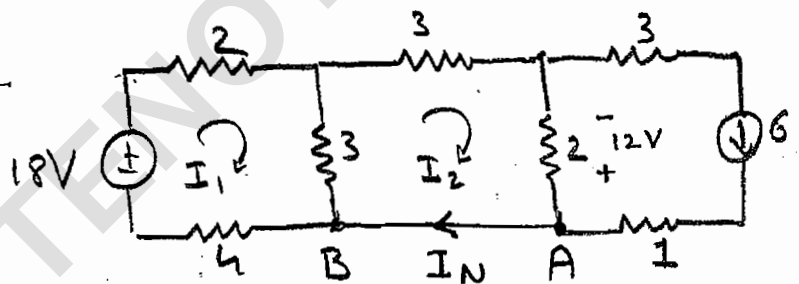
$$18 - 9I_1 + 3I_2 = 0$$

$$\therefore 3I_1 - I_2 = 6 \quad \text{--- (1)}$$

$$3I_1 - 8I_2 = -12 \quad \text{--- (2)}$$

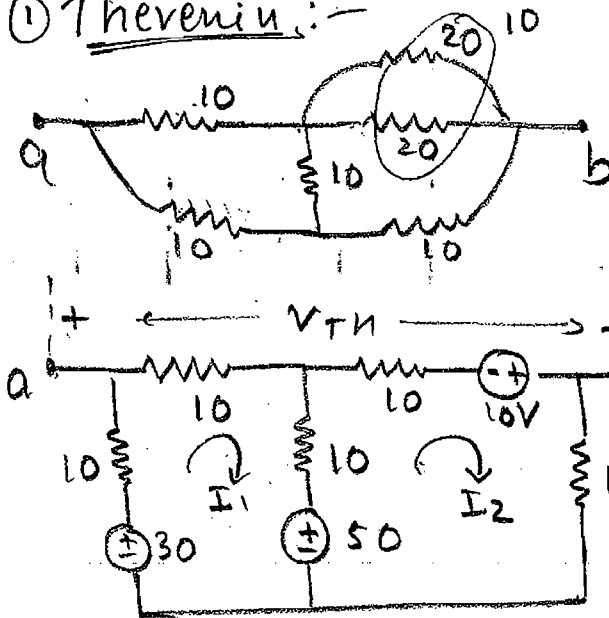
From (1) & (2) : $7I_2 = 18 \Rightarrow I_2 = \frac{18}{7}$

But $I_N = I_2 \Rightarrow I_N = \frac{18}{7} \text{ A}$



Determine
Thevenin &
Norton's equiv.
across ab.

① Theremin :-



$$R_{TH} = 20 \parallel 20$$

$$= 10 \Omega$$

$$30 - 30 I_1 + 10 I_2 = 50$$

$$\therefore -3 I_1 + I_2 = 2 \quad \text{--- (1)}$$

$$50 - 30 I_2 + 10 I_1 + 10 = 0$$

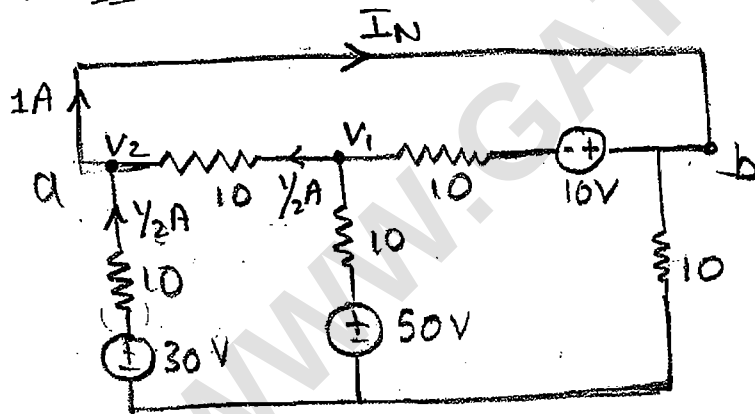
$$I_1 - 3 I_2 = -6 \quad \text{--- (2)}$$

From (1) & (2); $-8 I_2 = -16 \Rightarrow I_1 = 0 \text{ A}$

$$I_2 = \frac{16}{8} = 2 \text{ A}$$

$$\therefore V_{TH} = 0 + 10(2) - 10 = \boxed{10 \text{ V}}$$

② Norton's :-



Nodal

$$\frac{V_1 - V_2}{10} + \frac{V_1 - 50}{10} + \frac{V_1 - V_2 + 10}{10} = 0$$

$$3V_1 - 2V_2 = 40 \quad \text{--- (1)}$$

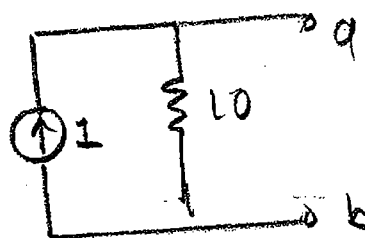
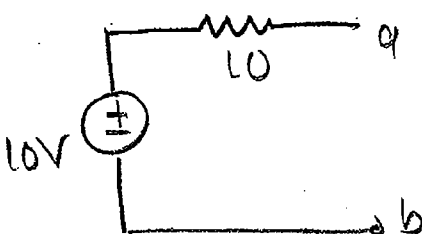
$$\frac{V_2 - V_1}{10} + \frac{V_2 - 30}{10} + \frac{V_2}{10} + \frac{V_2 - V_1 - 10}{10} = 0$$

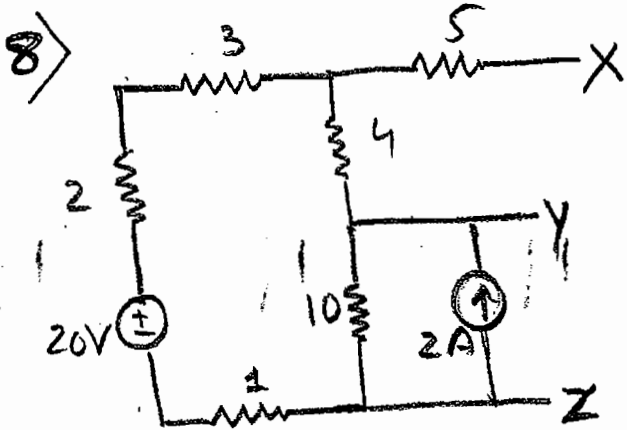
From (1) & (2);

$$4V_1 = 120 \Rightarrow \boxed{V_1 = 30 \text{ V}}$$

$$\boxed{V_2 = 25 \text{ V}}$$

$$\therefore -2V_1 + 4V_2 = 40 \quad \text{--- (2)}$$





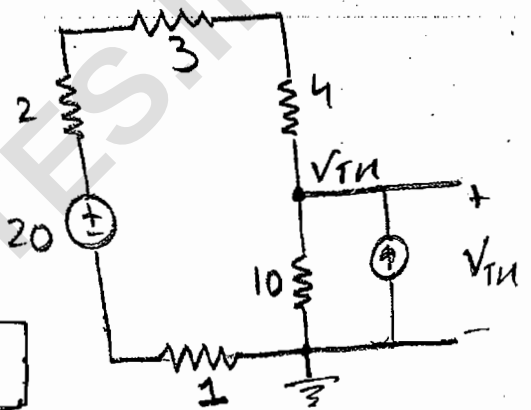
Determine Thevenin equivalent b/w Y & Z.

$$R_{TH} = 10 \parallel (3 + 2 + 1 + 4) = 10 \parallel 10 = 5 \Omega$$

$$V_{TH} \equiv \text{KCL}$$

$$\frac{V_{TH} - 20}{10} + \frac{V_{TH}}{10} - 2 = 0$$

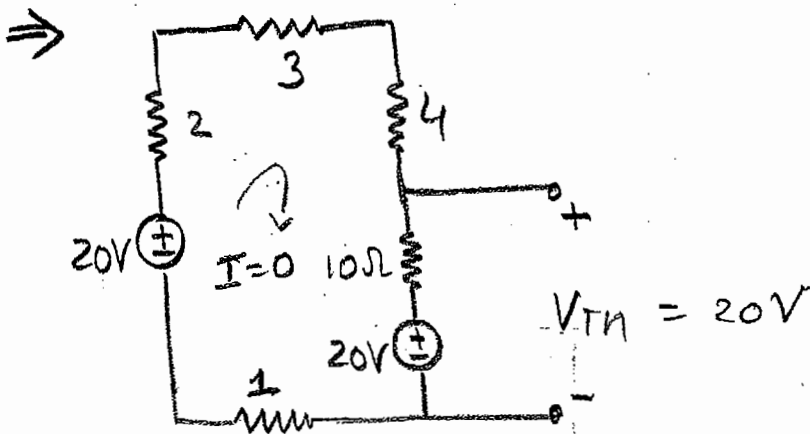
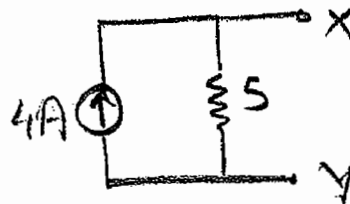
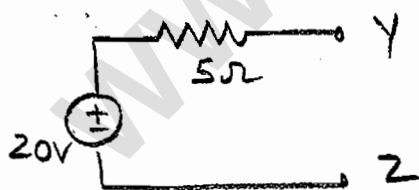
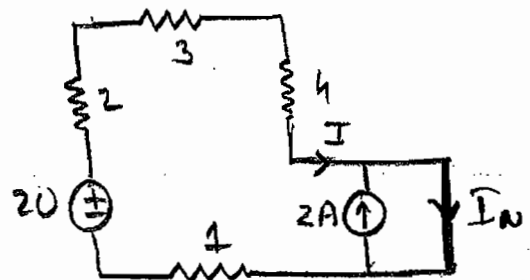
$$\therefore 2V_{TH} = 40 \Rightarrow V_{TH} = 20V$$



Now,

$$R_N = R_{TH} = 5 \Omega$$

$$I_N = 2 + \frac{20}{10} = 4A$$



Category 2 :- Problems with both independent & dependent sources.

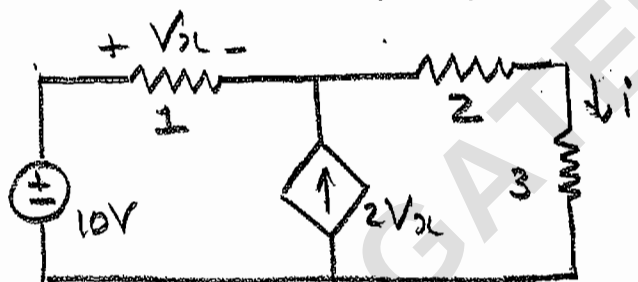
Dependent sources cannot be suppressed directly in terms of their resistances.

So here, finding R_{TH} or R_N is not possible directly.

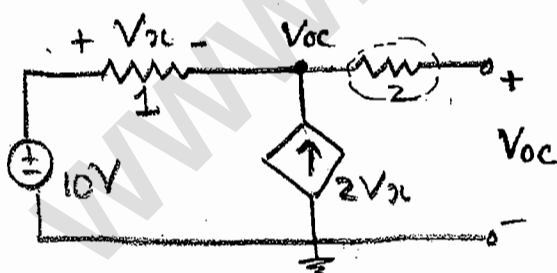
Hence we use Ohm's law where,

$$R_{TH} = R_N = \frac{V_{oc}}{I_{sc}} \text{ at target terminal.}$$

1)



Find current i using Thevenin & Norton's th.

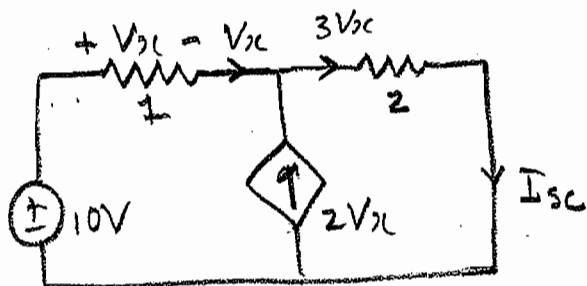


$$\frac{V_{oc} - 10}{1} - 2V_x = 0$$

$$V_x = 10 - V_{oc} \quad \text{--- (1)}$$

$$V_{oc} - 10 - 20 + 2V_{oc} = 0$$

$$V_{oc} = 10 \text{ V}$$



KVL

$$= 10 + V_x + 6V_x = 0$$

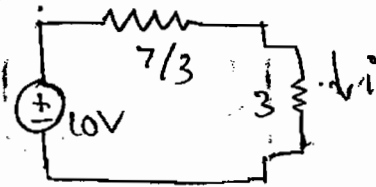
$$V_x = \frac{10}{7}$$

But $I_{sc} = 3V_x$
 $= 3 \times \frac{10}{7} \Rightarrow$

$$I_{sc} = \frac{30}{7} \text{ A}$$

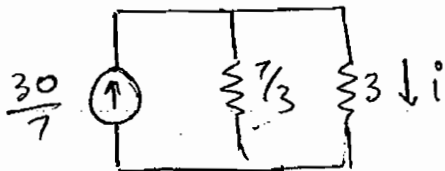
$$R_{TH} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{10 \times 7}{30} = \frac{7}{3} \Omega$$

T.E



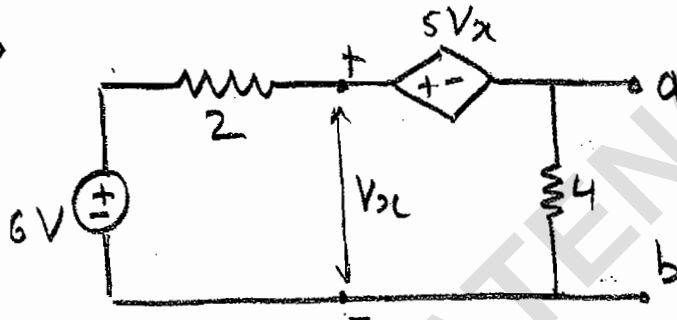
$$i = \frac{10}{3 + \frac{7}{3}} = \frac{15}{8} \text{ A}$$

N.E

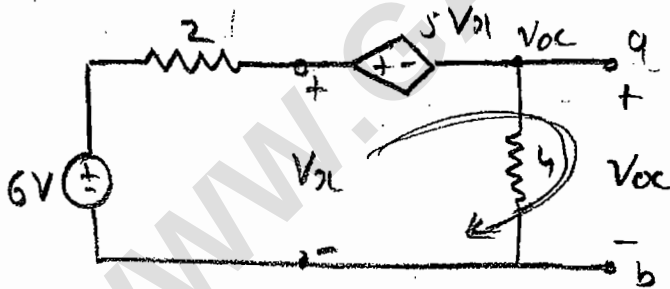


$$i = \frac{30}{7} \left[\frac{\frac{7}{3}}{3 + \frac{7}{3}} \right] = \frac{15}{8} \text{ A}$$

2)



Find Thevenin & Norton's equiv. b/w a & b.



Nodal

$$\frac{V_{oc} - 6 + 5V_x}{2} + \frac{V_{oc}}{4} = 0$$

$$2V_{oc} - 12 + 10V_x + V_{oc} = 0$$

$$\therefore 3V_{oc} + 10V_x = 12 \quad (1)$$

KVL

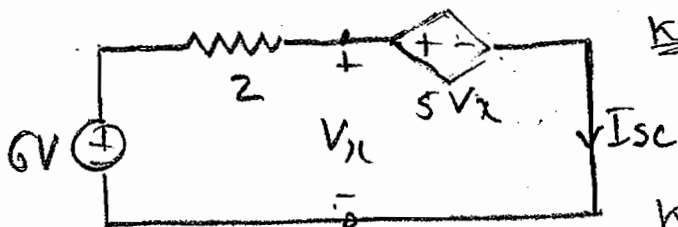
$$-V_x + 5V_x + V_{oc} = 0$$

$$\therefore V_{oc} = -4V_x \quad (2)$$

From (1) & (2); $-12V_x + 10V_x = 12$

$$\therefore V_x = -6 \Rightarrow$$

$$V_{oc} = 24 \text{ V}$$



KVL

$$6 - 2I_s - 5V_x = 0$$

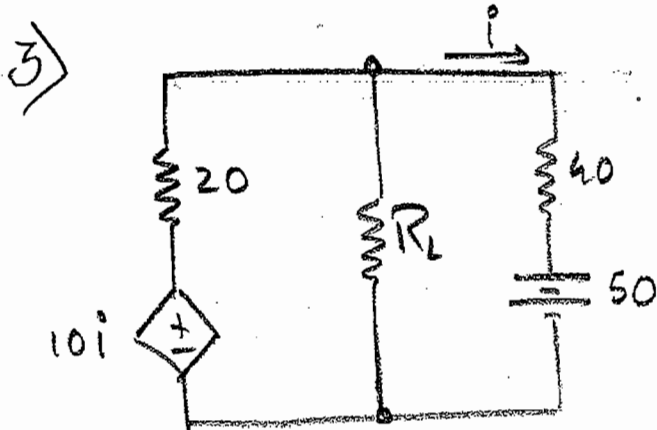
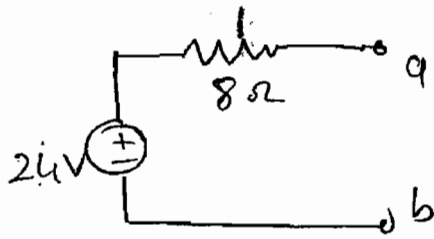
$$2I_s = 6 - 5V_x \quad (1)$$

KVL

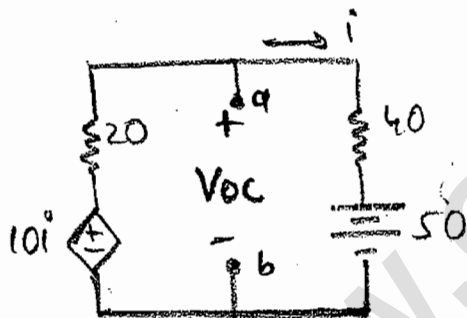
$$-V_x + 5V_x = 0 \Rightarrow V_x = 0$$

From ① & ②; $2 I_{sc} = 6 \Rightarrow \boxed{I_{sc} = 3 A}$

$$R_{TH} = R_N = \frac{V_{OS}}{I_{sc}} = \frac{24}{3} = 8 \Omega$$



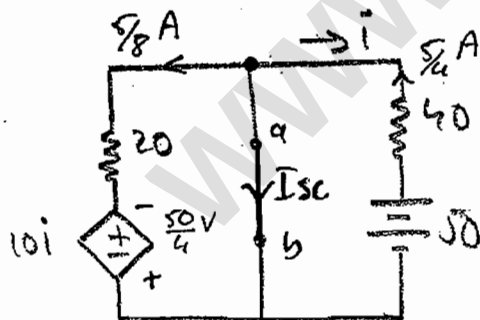
Determine Thevenin
& Norton's equiv.
across the load.



$$10i = 50i + 50$$

$$\therefore i = -1 A$$

$$\therefore V_{OC} = 50 + 40(-1) = 10 V$$

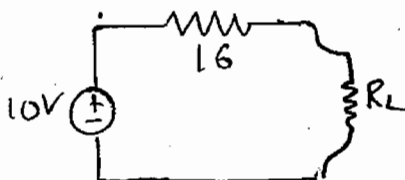


KCL

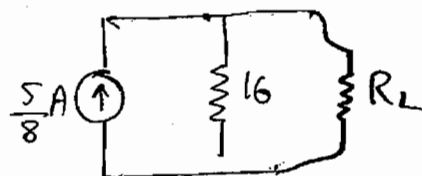
$$\frac{5}{4} = I_{sc} + \frac{5}{8}$$

$$\therefore I_{sc} = \frac{5}{8} A$$

$$R_{TH} = R_N = \frac{V_{OC}}{I_{sc}} = \frac{10 \times 8}{5} = 16 \Omega$$

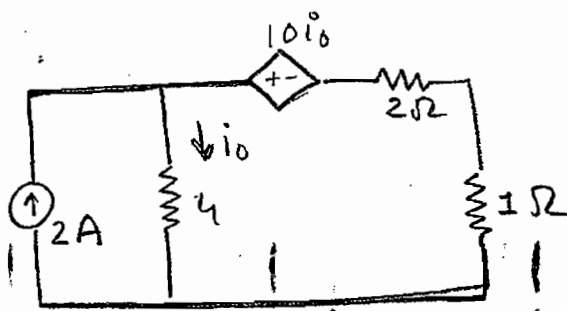


T.E

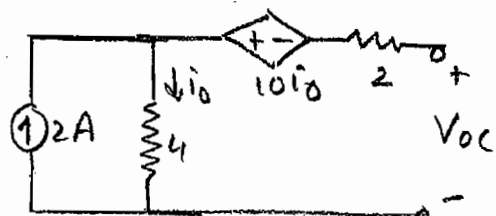


N.E

4)



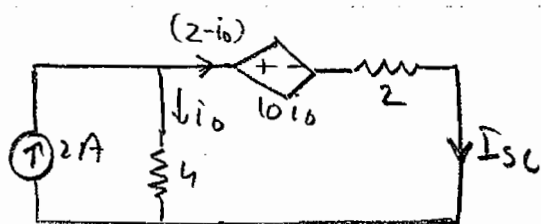
Find current through 1Ω resis. using Norton's theorem & verify directly.



$$i_o = 2\text{ A}$$

$$V_{oc} = 4(2) - 10(2)$$

$$= -12\text{ V}$$



$$-10i_o - 2(2 - i_o) + 4i_o = 0$$

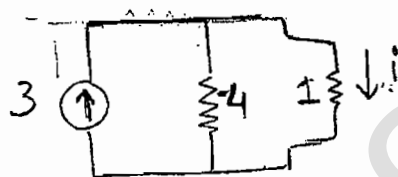
$$\therefore -4i_o = 4$$

$$\therefore i_o = -1\text{ A}$$

$$I_{sc} = 2 - i_o = 2 + 1 = 3\text{ A}$$

$$\therefore R_{TN} = \frac{+V_{oc}}{I_{sc}} = -4\Omega$$

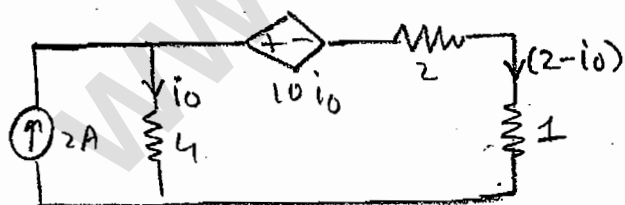
$$\Rightarrow \boxed{R_N = -4\Omega}$$



$$i \neq 3 \times \frac{1}{1+4} = \frac{17}{5}\text{ A}$$

$$\boxed{i = 3 \times \left(\frac{-4}{-4+1} \right) = 4\text{ A}}$$

Direct approach:



$$-4i_o + 10i_o + 3(2 - i_o) = 0$$

$$\therefore 3i_o = -6$$

$$i_o = -2$$

$$I_{1\Omega} = 2 - (-2) = 4\text{ A}$$

→ In above problem R_{TH} or R_N is -ve

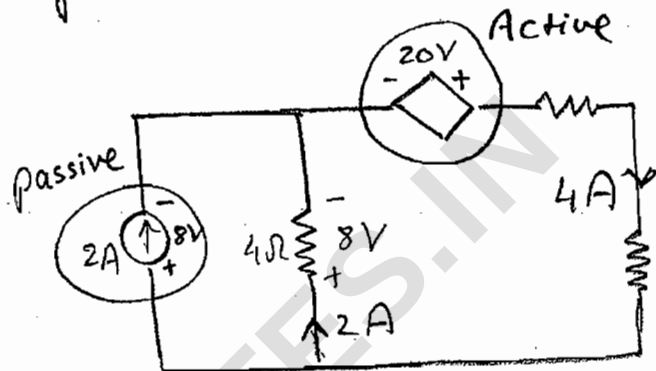
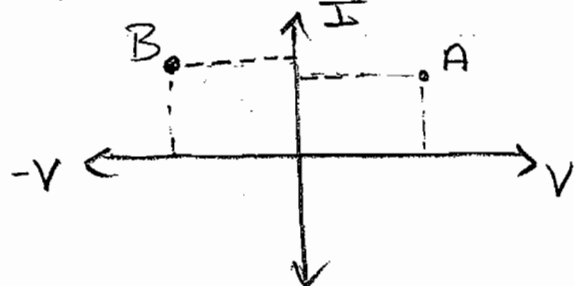
→ -ve resistance is a very powerful way of modelling active elements or active n/w's in circuit analysis.

eg: transistor as an amplifier is active,

thyristor is considered as very high current gain device, optocouplers, etc.

The V-I charac. of this n/w is in quadrant 2.

Hence it is active.



Category 3:- Problems with only dependent sources :-

Such n/w cannot function on their own as there is no independent active element to drive it.

In Thevenin's equivalent $V_{TH} = 0$

In Norton's equivalent $I_N = 0$

but they have only resistance.

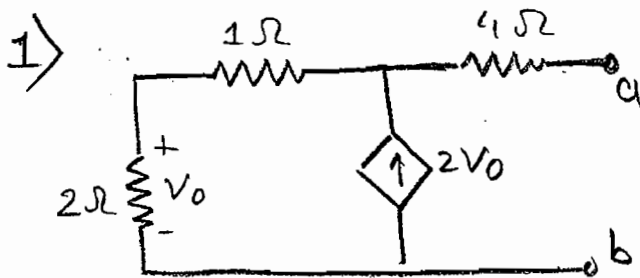
This resist. can be determined indirectly by Ohm's Law by externally exciting them where

$$R_{TH} = R_N = \frac{1 \text{ V}}{1 \text{ A}} = \frac{V_T}{1 \text{ A}}$$

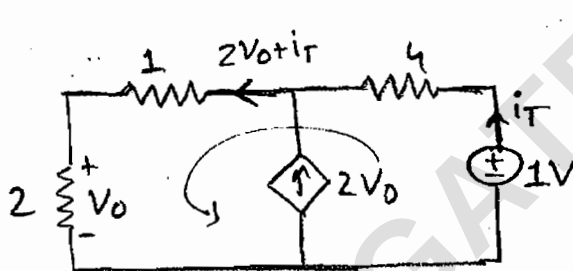
But such models physically represent our n/w & electric devices.

Ex:- ① H-parameter equivalent of BJT in common emitter amp^r.

② Piece-wise PSPICE & MATLAB models of electronics devices, etc.



Determine Thevenin & Norton's equiv. b/w a & b.



KVL

$$-1 + 4i_T + 3(2V_0 + i_T) = 0$$

$$7i_T + 6V_0 = 1 \quad \text{--- (1)}$$

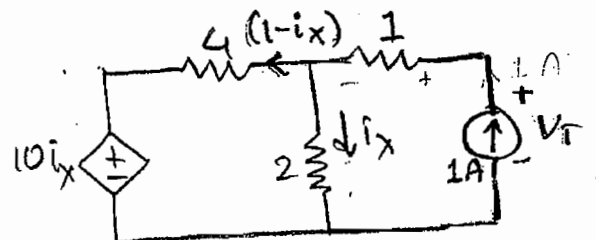
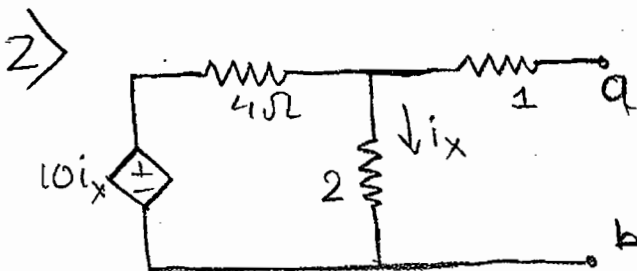
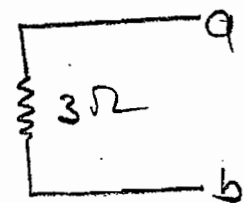
$$V_0 = 2(V_0 + i_T)$$

$$3V_0 + 2i_T = 0 \quad \text{--- (2)}$$

$$\therefore 7i_T + 6\left(\frac{-2}{3}\right)i_T = 1$$

$$3i_T = 1 \Rightarrow i_T = \frac{1}{3}$$

$$R_{Th} = R_N = \frac{1}{1/3} = 3 \Omega$$

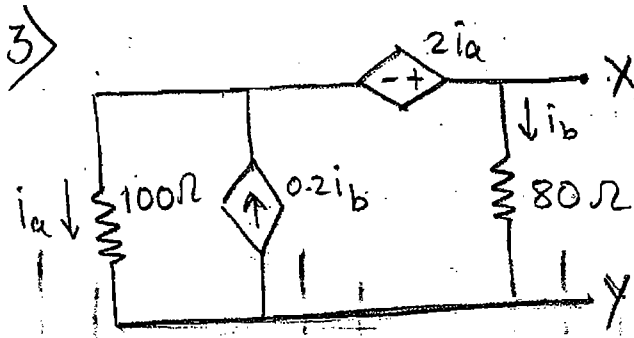


$$10i_x + 4 - 4i_x - 2i_x = 0$$

$$\therefore i_x = -1 \text{ A}$$

$$V_T = 1 + 2(-1) = -1 \text{ V}$$

$$R_N = \frac{-1}{1} = -1 \Omega$$



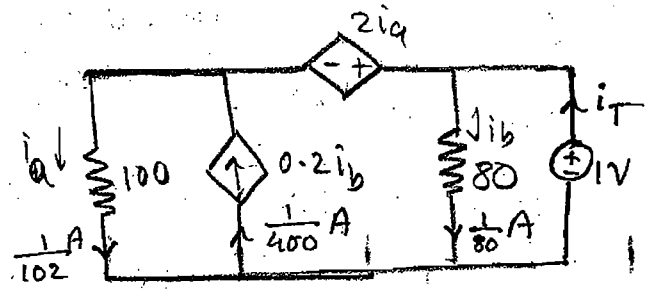
KCL

$$\frac{1}{102} + \frac{1}{80} = \frac{1}{400} + i_T$$

$$i_T = \frac{1}{80} + \frac{1}{102} - \frac{1}{400}$$

$$= 0.0198$$

$$R_{Th} = R_N = \frac{1}{i_T} \approx 50 \Omega$$

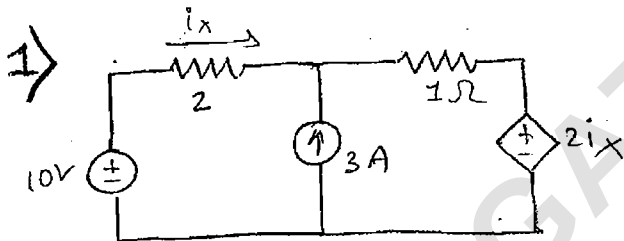


KVL

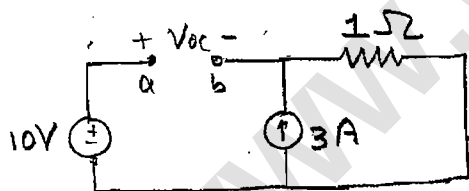
$$-1 + 2i_a + 100i_a = 0$$

$$\therefore i_a = \frac{1}{102}$$

Special Models :-

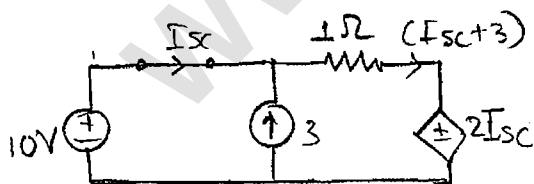


Find i_x using Thevenin & Norton's theorem.



$$-10 + V_{OC} + 3 = 0$$

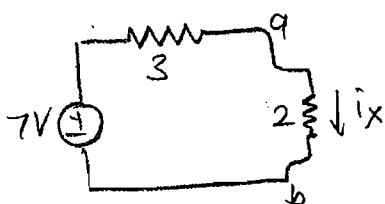
$$V_{OC} = 7 \text{ V}$$



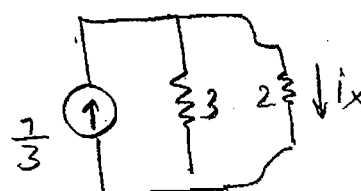
$$-10 + (I_{SC} + 3) + 2I_{SC} = 0$$

$$3I_{SC} = 7 \Rightarrow I_{SC} = \frac{7}{3} \text{ A}$$

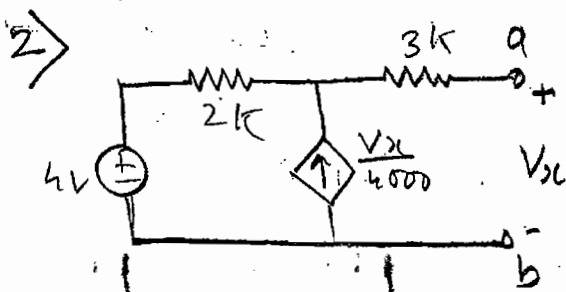
$$R_{Th} = R_N = \frac{7}{\frac{7}{3}} \times 3 = 3 \Omega$$



$$i_x = \frac{7}{5} \text{ A}$$

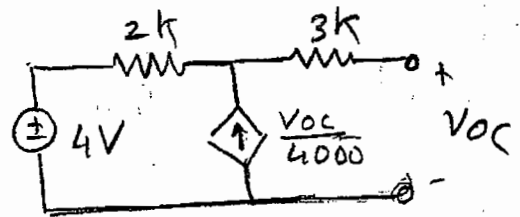


$$i_x = \frac{7}{3} \left(\frac{3}{5} \right) = \frac{7}{5} \text{ A}$$

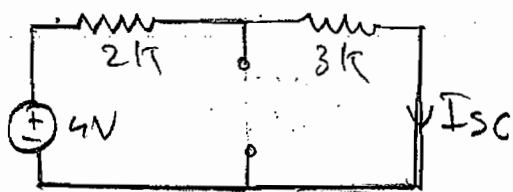


Determine Thevenin equivalent b/w a & b

$$V_{oc} = 2k \left(\frac{V_{oc}}{4000} \right) + 4$$



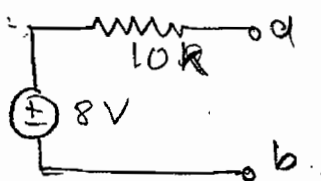
$$2V_{oc} = V_{oc} + 8 \Rightarrow V_{oc} = 8V$$



$$4 - 5I_{sc} = 0$$

$$\therefore I_{sc} = \frac{4}{5} \text{ mA}$$

$$\therefore R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8}{\frac{4}{5}} = 10k\Omega$$



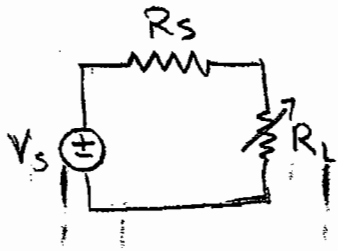
Theorem 5 :-

Maximum Power Transfer Theorem :-

In any linear active bilateral network consisting of no. of energy sources with their internal resistances, maximum power is transferred to the load when load resistance is equal to its equivalent resistance as seen by the load into the supply circuit.

It is indirectly application of Thevenin's theorem in designing the electrical loads

to extract maximum power from source.



$$I_L = \frac{V_s}{R_s + R_L}$$

$$P_L = I_L^2 \times R_L$$

$$P_L = \frac{V_s^2}{(R_s + R_L)^2} \cdot R_L$$

$$\frac{dP_L}{dR_L} = 0 \Rightarrow V_s^2 \left[\frac{(R_s + R_L)^2 \cdot 1 - R_L \cdot 2 \cdot (R_s + R_L)}{(R_s + R_L)^4} \right] = 0$$

$$\therefore (R_s + R_L)^2 = R_L \cdot 2 \cdot (R_s + R_L)$$

$$\therefore \boxed{R_L = R_s}$$

$$\rightarrow P_{\max} = \frac{V_s^2}{4R_s} \text{ W}$$

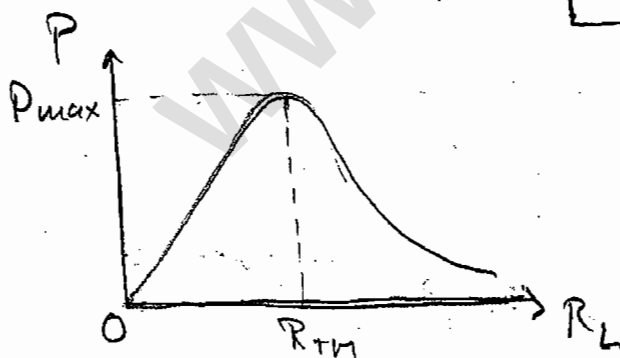
In general:-



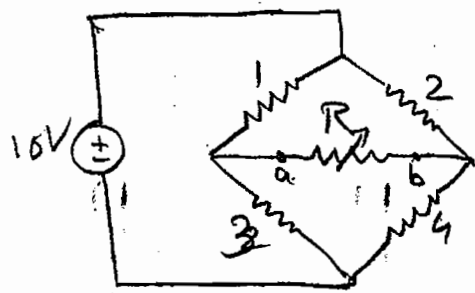
$\rightarrow P_{\max}$ occurs in load when $R_L = R_{TH}$ &

$$\boxed{P_{\max} = \frac{|V_{TH}|^2}{4R_{TH}}} \text{ W}$$

Reduce it to T.E.



During $P_{\max}$ transfer to the load
The o/p efficiency is 50%.

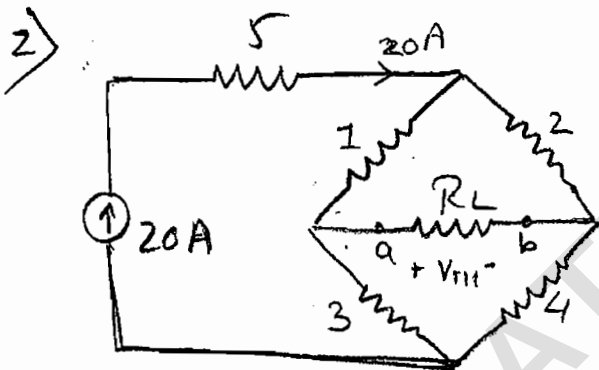
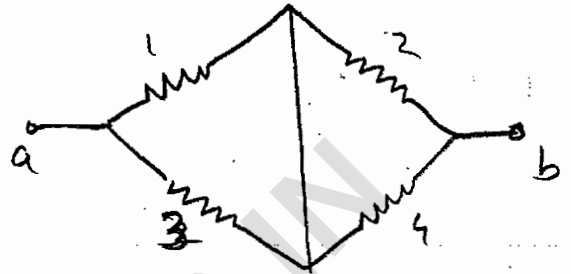


What is the vltg for which maximum power is transferred to load R_L .

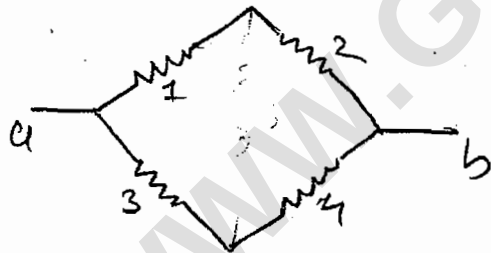
$$R_{TH} = (1 \parallel 3) + (2 \parallel 4)$$

$$= \frac{3}{4} +$$

$$= \frac{25}{12} \Omega$$



What is the max. power transferred to the load.



$$R_{TH} = 3 \parallel 7$$

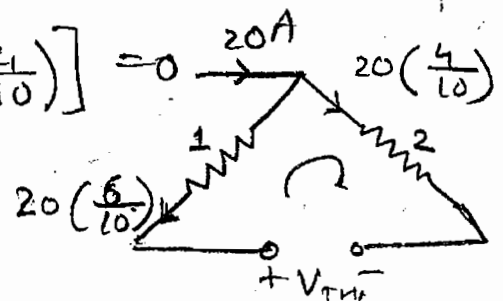
$$= \frac{21}{10} = 2.1 \Omega$$

KVL

$$-V_{TH} - 1 \left[20 \left(\frac{6}{10} \right) \right] + 2 \left[20 \left(\frac{4}{10} \right) \right] = 0$$

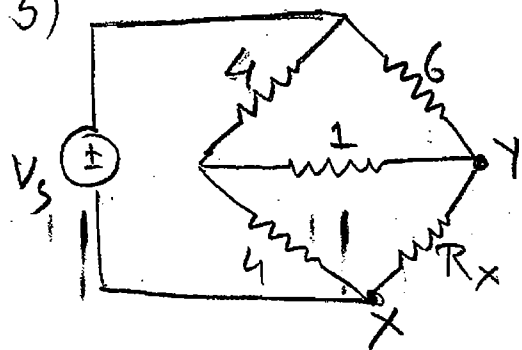
$$V_{TH} = -12 + 16$$

$$V_{TH} = 4 \text{ V}$$



$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{16 \times 10}{4 \times 21} = \frac{40}{21} \text{ W}$$

3)

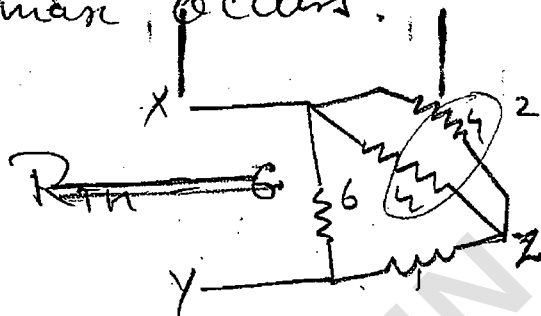


What is the value of R_x for which P_{max} occurs.

$$R_{Th} = (4 \parallel 4) + (6 \parallel R_x)$$

$$= 2 + \frac{6R_x}{6+R_x}$$

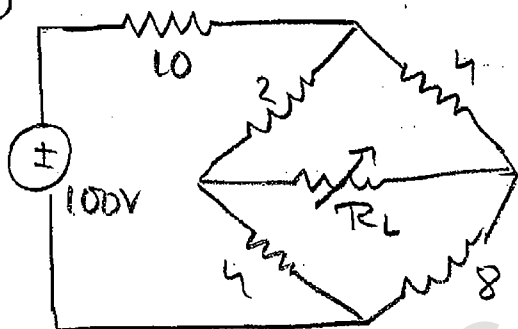
$\therefore R_x = 2 \Omega$ for P_{max}



$$R_{Th} = (2+1) \parallel 6$$

$$= 2 \Omega$$

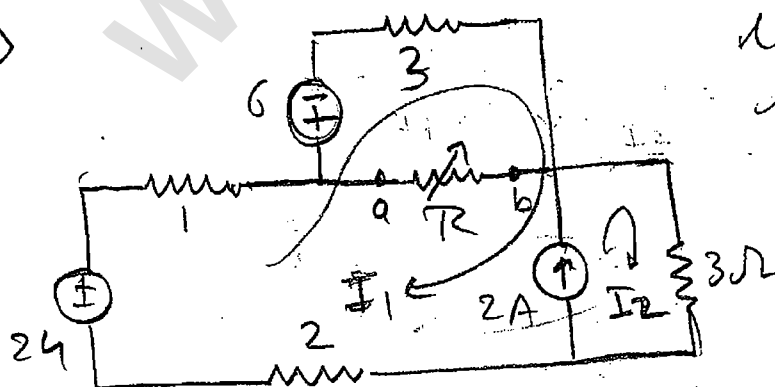
4)



Find the max. Power transferred across R_L .

Since the bridge is balanced so current can not flow through R_L .
Hence V_{Th} across R_L will be zero

5)



What is the max. power transferred to the R .

$$R_{Th} = 3 \parallel 6$$

$$= 2 \Omega$$

V_{Th} (Mesh)

$$24 - 6I_1 + 6 - 3I_2 = 0$$

$$2I_1 + I_2 = 16 \quad \text{--- (1)}$$

$$2I_1 + I_2 = 6 \quad \text{--- (1)}$$

$$-I_1 + I_2 = -2 \quad \text{--- (2)}$$

$$\therefore 3I_1 = 4 \Rightarrow I_1 = \frac{4}{3} \text{ A}$$

Now

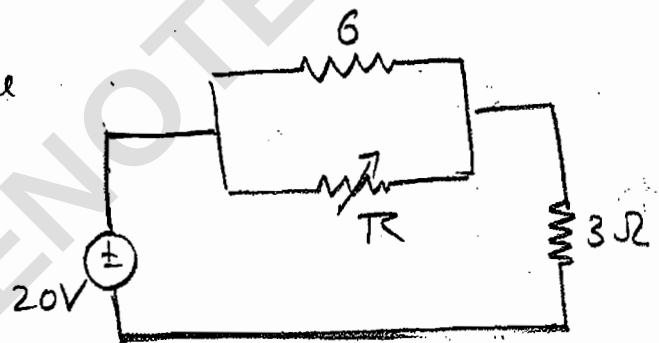
KVL

$$-V_{TH} + 6 + 3I_1 = 0$$

$$\therefore V_{TH} = 6 + 3\left(\frac{4}{3}\right) = 6 + 4 = 10 \text{ V}$$

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{10 \times 10}{4 \times 2} = \frac{25}{2} \text{ W}$$

8) For what value of R , max power is transferred to 3Ω res.



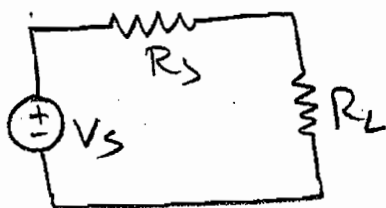
- (a) 6 (b) 3 ☒ (c) 2 (d) infinity

Acc. to concept

$$R_{TH} \text{ across } 3\Omega = 3\Omega$$

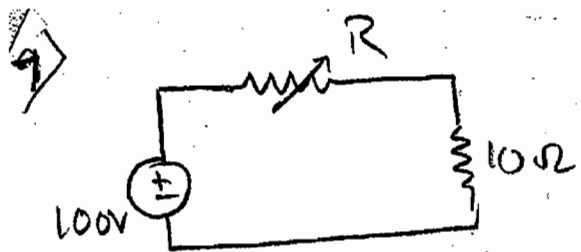
$$6 \parallel R = 3$$

$$\Rightarrow \boxed{R = 3\Omega} \quad \times$$



The value of R_L for which P_{max} occurs in $R_L = R_S \Omega$

The value of R_S for which P_{max} occurs in $R_L = 0\Omega$

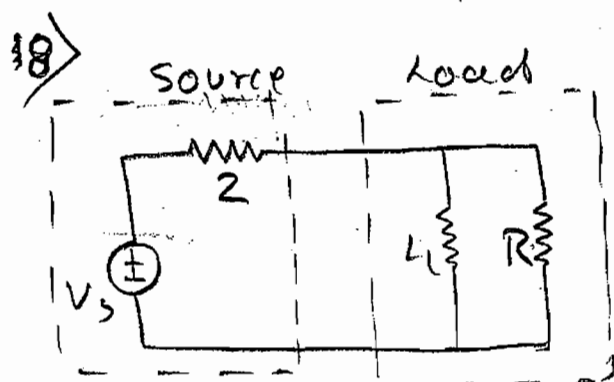


What is the max. power transferred to 10Ω . If $1\Omega \leq R \leq 100\Omega$

Here $R = 1\Omega$

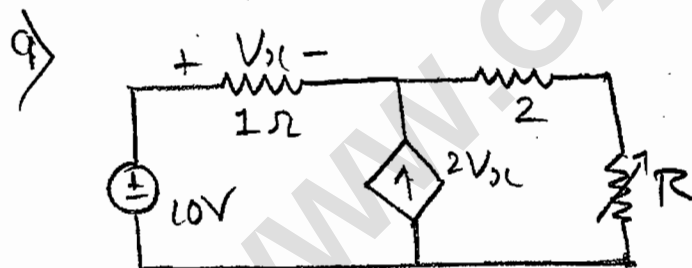
$$\therefore I_{10\Omega} = \frac{100}{10+1} = \frac{100}{11} \text{ A}$$

$$P_{\max} = \left(\frac{100}{11}\right)^2 \times 10 = 826.4 \text{ W}$$

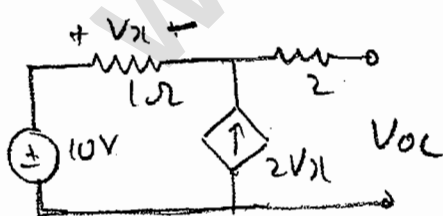


What is the value of R for which max. power is transferred from source to load.

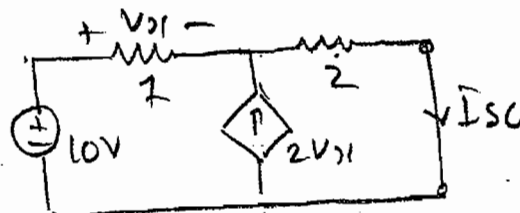
$$R = 4\Omega$$



What is the max. power transferred to R



$$V_{OC} = 10\text{V}$$

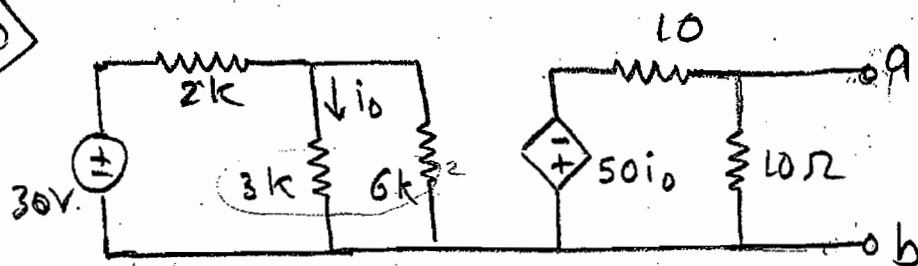


$$I_{SC} = \frac{30}{7} \text{ A}$$

$$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{7}{3} \Omega$$

$$P_{\max} = \frac{10 \times 10}{4 \times \frac{7}{3}} = \frac{75}{7} \text{ W}$$

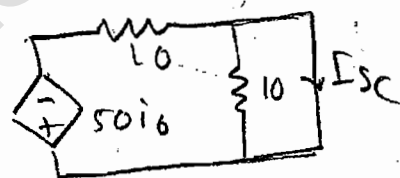
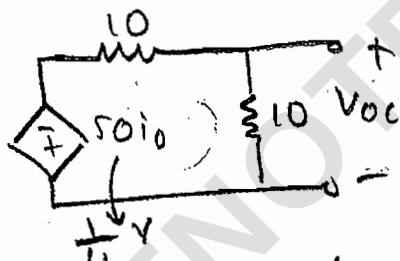
10)



What resist. connected b/w a & b will draw max. power from n/w. & also find this max. power.

$$I = \frac{30}{4k} \text{ A} \Rightarrow i_o = \frac{30}{2k} \left(\frac{6k}{3k+6k} \right) = 5 \text{ mA}$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}}$$



KVL

$$50i_o - 20I = 0$$

$$\therefore 20I = 50 \times 5$$

$$\therefore I = \frac{25}{2} \text{ mA}$$

$$\therefore V_{OC} = -10 \times \frac{25}{2}$$

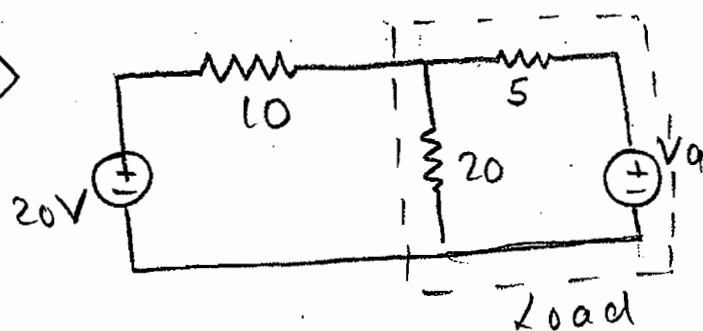
$$V_{OC} = -\frac{1}{4} \left[\frac{10}{20} \right] = -\frac{1}{8} \text{ V}$$

$$I_{SC} = \frac{-1/4}{10} = -\frac{1}{40} \text{ A}$$

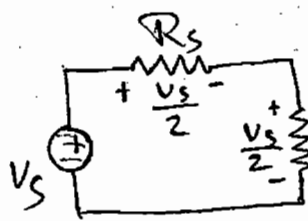
$$R_{TH} = \frac{-1/8}{-1/40} = 5 \Omega$$

$$P_{max} = \frac{V_{TH}^2}{4 R_{TH}} = \frac{(-1/8)^2}{4 \times 5} = 0.781 \text{ W}$$

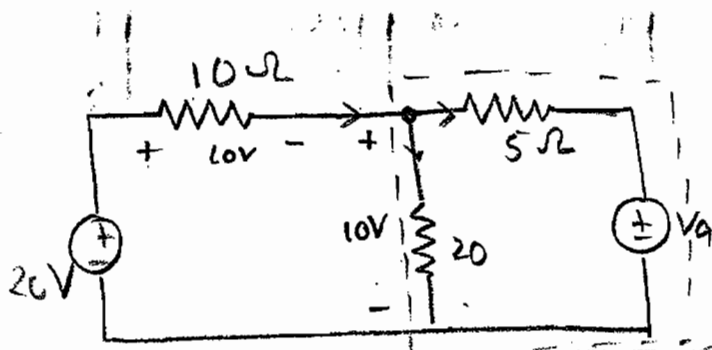
11)



Circuit show a typical battery charging circuit. What is value of V_a for which P_{max} is transferred to load.



During P_{max} transfer to load
voltage across load is 50% of
supply (V_{TH})



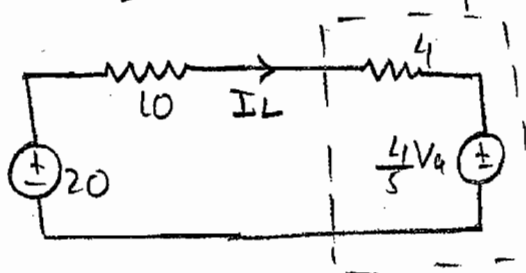
KCL

$$\frac{20-10}{10} = \frac{10}{20} + \frac{10-V_a}{5}$$

$$1 = \frac{1}{2} + \frac{10-V_a}{5}$$

$$\therefore V_a = 10 - \frac{5}{2} = 7.5 \text{ V}$$

Exact Method

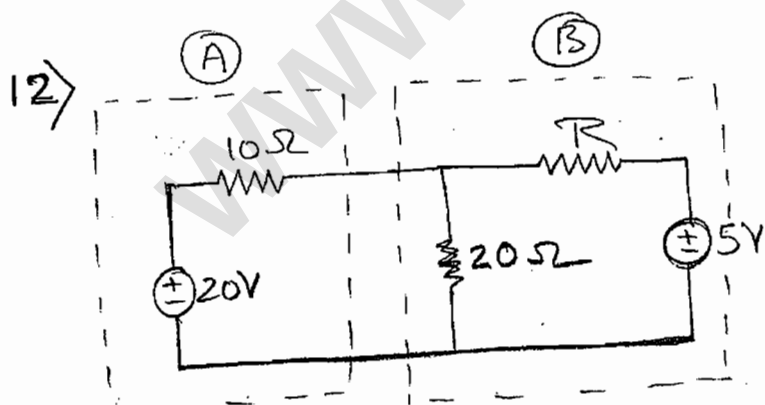


T.E. of load ckt as
seen by source

$$I_L = \frac{(20 - \frac{4}{5}V_a)}{14} = \frac{100 - 4V_a}{14 \times 5}$$

$$= \frac{50 - 2V_a}{35}$$

$$P_L = (I_L)^2 4 + \left(\frac{4}{5}V_a\right) I_L$$



For what value
of R , max. power
is transferred
from ckt. A to ckt B

Shortcut: $\frac{20-10}{10} = \frac{10}{20} + \frac{10-5}{R} \Rightarrow R = 10 \Omega$

$\Rightarrow$ Vltg is divided equally across source and
load.

13)

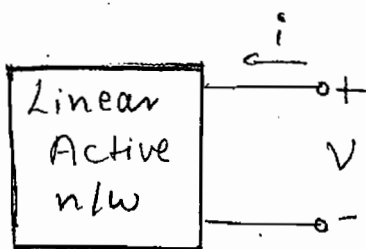


Fig (a)

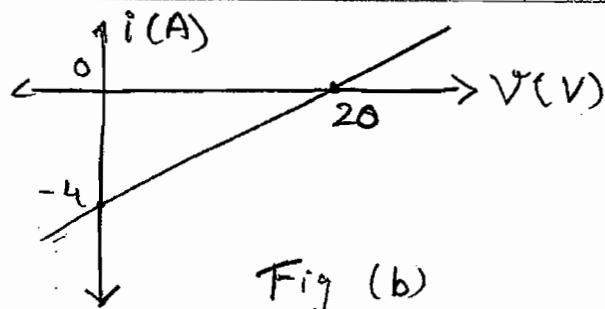
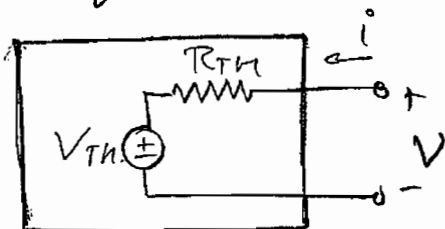


Fig (b)

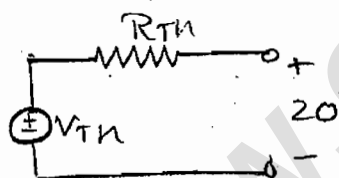
The static VI characteristics of n/w shown in fig (a) is plotted in fig (b) - What is the max. power that can be drawn from the n/w.

→ 4th quadrant $\Rightarrow$ active elements are present in n/w.



1st operating point

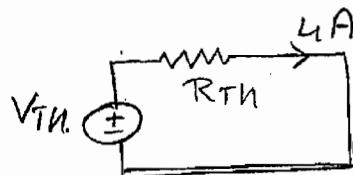
$$I = 0, V = 20V$$



$$\Rightarrow V_{TH} = 20V$$

2nd operating point

$$I = -4A, V = 0$$



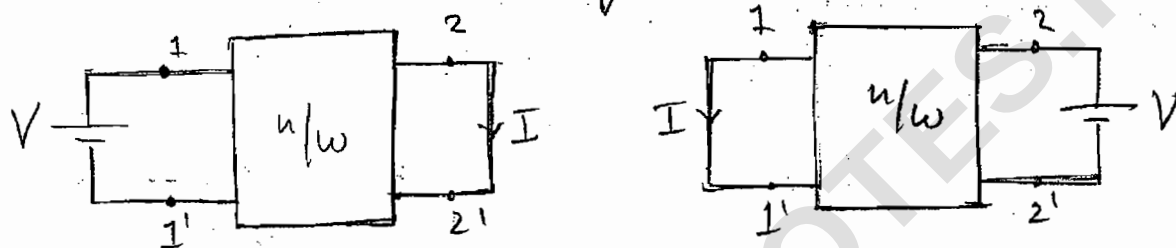
$$V_{TH} = 4R_{TH} \Rightarrow R_{TH} = 5$$

$$\therefore P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{20 \times 20}{4 \times 5} = 20W$$

Theorem 6 :-

Reciprocity Theorem :-

In any linear passive bilateral n/w excited with only a single source, the ratio of response to excitation remains const. even if the positions of source & load are interchanged.



$$\boxed{\frac{I}{V} = \text{const.}} \xrightarrow[\text{Homogeneity principle}]{\text{By applying}} \boxed{\frac{I_1}{V_1} = \frac{I_2}{V_2}}$$

NOTE :-

This theorem is valid for n/w excited with single source only.

This theorem can not be applied for n/w's with dependent source. Since dependent sources can make n/w active.

✓ While writing the Reciprocity n/w of given n/w, ideal independent voltage sources are connected in series to the target branch and ideal independent current sources are connected in parallel to target branch.

Eg:- Communication lines
Electrical power Tx n/w.

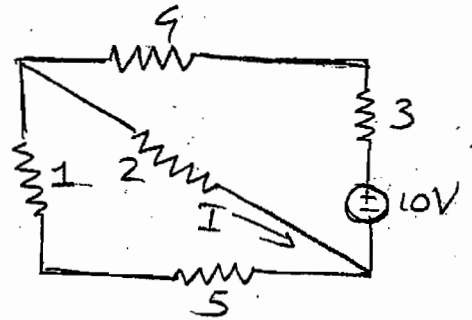
1) Verify Reciprocity theorem for a ckt. shown below in finding current I .

Step 1

Find I $I = \left[\frac{10}{7 + 6 \parallel 2} \right] \left(\frac{6}{2 + 6} \right)$

Total current $= \frac{10}{7 + \frac{12}{8}} \times \frac{6}{8}$

$\left(\frac{V}{R_{eq}} \right) = \frac{15}{17} \text{ A}$

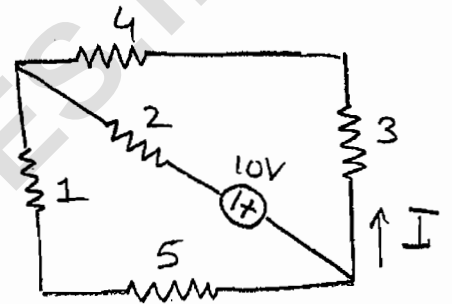


Step 2

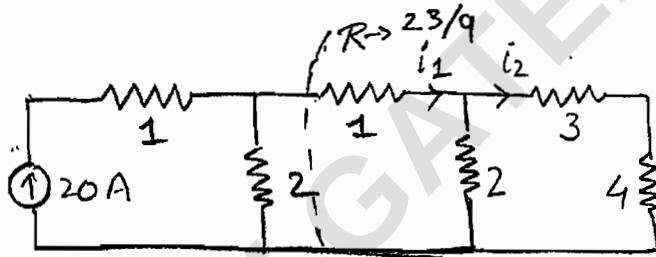
Reciprocal n/w

$I = \left[\frac{10}{2 + 7 \parallel 6} \right] \left(\frac{6}{6 + 7} \right)$

$= \frac{10}{2 + \frac{42}{13}} \left(\frac{6}{13} \right) = \frac{15}{17} \text{ A}$



2)



Verify R.T. & find V_0

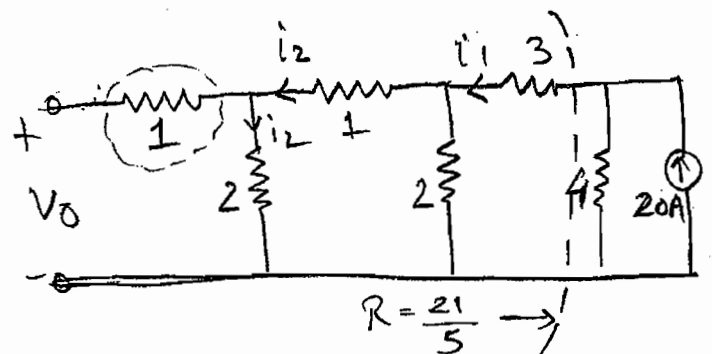
$(7 \parallel 2) + 1 = \frac{23}{9} ; i_1 = 20 \left[\frac{2}{2 + \frac{23}{9}} \right]$

$V_0 = 4 i_2$

$= 4 \left[20 \left(\frac{2}{2 + \frac{23}{9}} \right) \times \frac{2}{9} \right]$

$= \frac{320}{11} \text{ V}$

$i_2 = \left[20 \times \frac{2}{2 + \frac{23}{9}} \right] \times \frac{2}{9}$



reciprocal n/w

$3 + [2 \parallel 3] = 3 + \frac{6}{5} = \frac{21}{5}$

$i_1 = 20 \left(\frac{4}{\frac{21}{5} + 4} \right) ; i_2 = 20 \left[\frac{4}{4 + \frac{21}{5}} \right] \left[\frac{2}{2 + 3} \right]$

$$V_0 = i_2 R = 20 \left[\frac{4}{4 + 21/5} \right] \left[\frac{2}{5} \right] \times 2$$

$$= \frac{20 \times 2 \times 4 \text{ K}\Omega}{\frac{41}{5} \times 5} = \frac{320}{41} \text{ V}$$

3) Use the data given in fig A to find current I in fig. B.

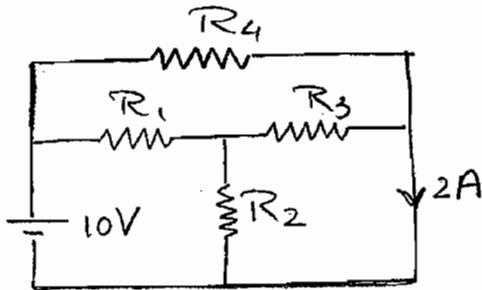


fig (A)

Reciprocal
n/w

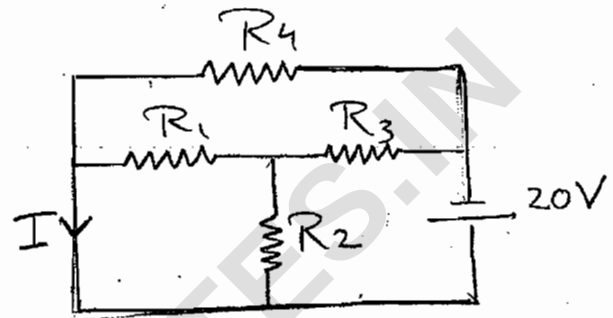
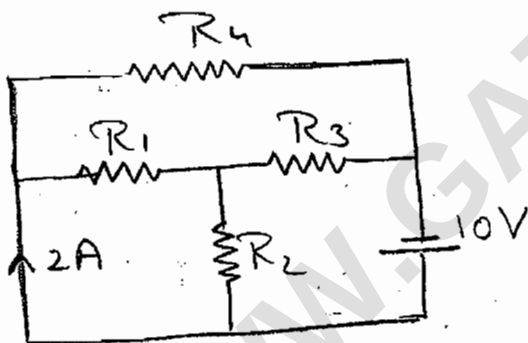


fig (B)

Applying
homogeneity
principle

$$\therefore I = -4 \text{ A}$$

4) Use the data given in fig (A) to find current i in fig. (B)

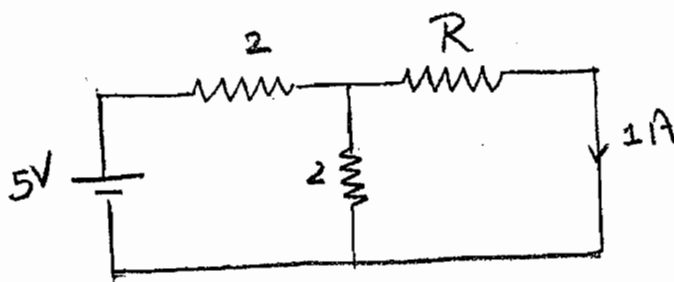


fig (A)

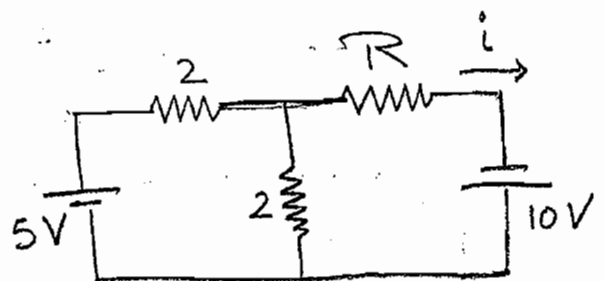
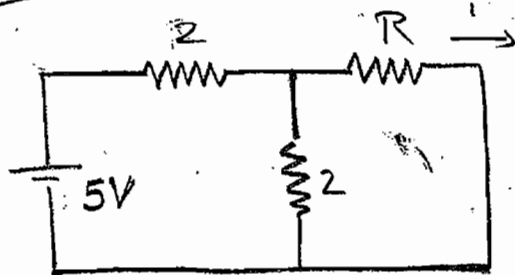


fig (B)

To solve Fig. B use superposition theorem.

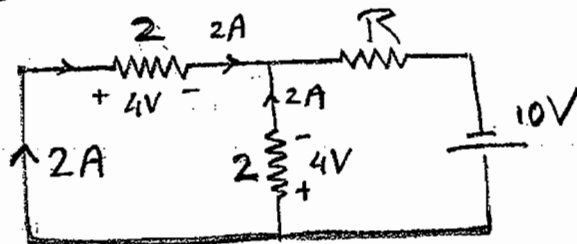
Step 1



(only 5 V)

$$\Rightarrow i' = 1 \text{ A} \quad (\text{from fig A})$$

Step 2



(only 10 V)

2A $\rightarrow$ from reciprocity theorem and homogeneity principle applied in fig A.

$$\therefore i'' = 4 \text{ A}$$

$$\therefore i = i' + i'' = 1 + 4 = 5 \text{ A}$$

5) Use the data in fig A to find current 'i' in fig. B.

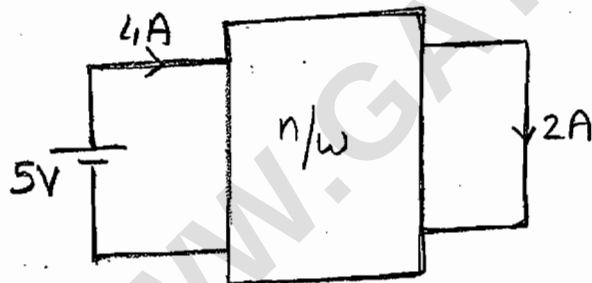


fig A.

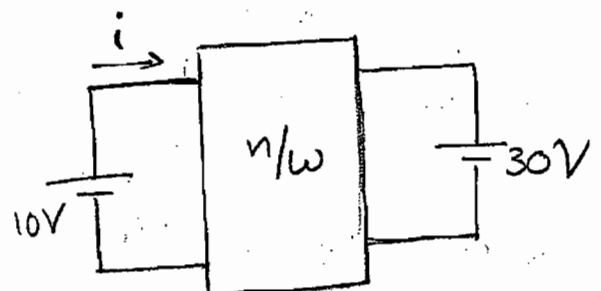
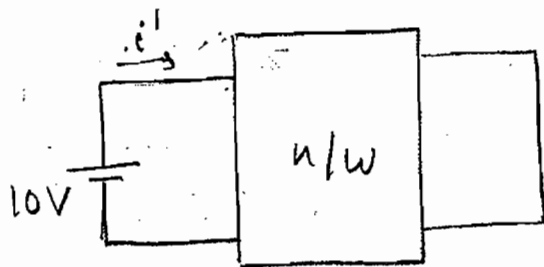


fig. B.

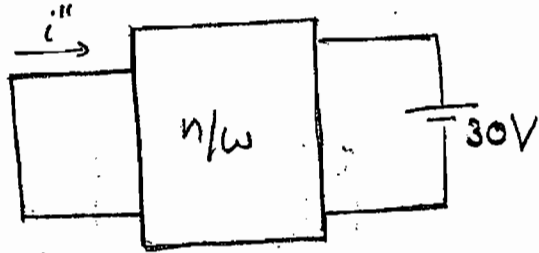
Apply superposition

Step 1



From fig. A
By homogeneity principle
 $i' = 8 \text{ A}$

Step 2:-



From reciprocal n/w of fig. A and applying Homogeneity principle -

$$i'' = -12A$$

$$i = i' + i'' = 8 - 12 = -4A$$

Theorem 7:-

Tellegen's Theorem :-

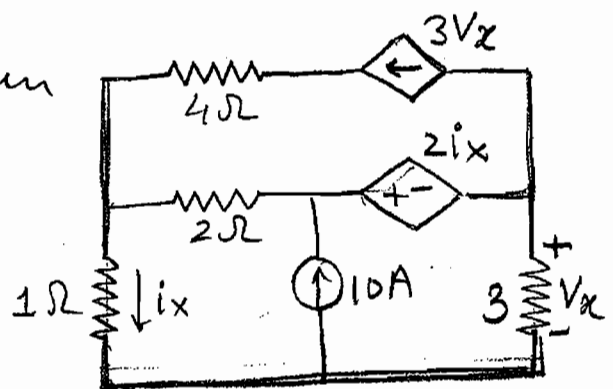
This theorem is verification of law of conservation of energy. However in any linear time invariant system, power at an instant is like an energy over a period. So in this theorem we need to verify.

$$\sum_{k=1}^n V_k \cdot I_k = 0$$

where, $n \rightarrow$ no. of elements in n/w

$$\text{i.e. } \sum VI |_{\text{source}} = \sum VI |_{\text{sinks}}$$

1) Verify Tellegen's theorem for the circuit shown below.



Mesh :

$$\textcircled{1} \quad i_1 + 2(i_1 - i_3) + 2i_x + 3i_2 = 0$$

$$\textcircled{2} \quad -i_1 + i_2 = 0$$

$$\textcircled{3} \quad i_3 = -3V_x$$

$$\textcircled{4} \quad i_x = -i_1$$

$$\textcircled{5} \quad V_x = 3i_2$$

Nodal :

$$\textcircled{1} \quad \frac{V_1}{1} + \frac{V_1 - V_2}{2} - 3V_x = 0$$

$$\textcircled{2} \quad \frac{V_2 - V_1}{2} + \frac{V_3}{3} - 10 + 3V_x = 0$$

$$\textcircled{3} \quad V_2 - V_3 = 2i_x$$

$$\textcircled{4} \quad i_x = \frac{V_1}{1}$$

$$\textcircled{5} \quad V_x = V_3$$

$$i_1 = -105/11 \text{ A}$$

$$i_2 = 5/11 \text{ A}$$

$$i_3 = -45/11 \text{ A}$$

$$V_x = 15/11 \text{ V}$$

$$i_x = 105/11 \text{ A}$$

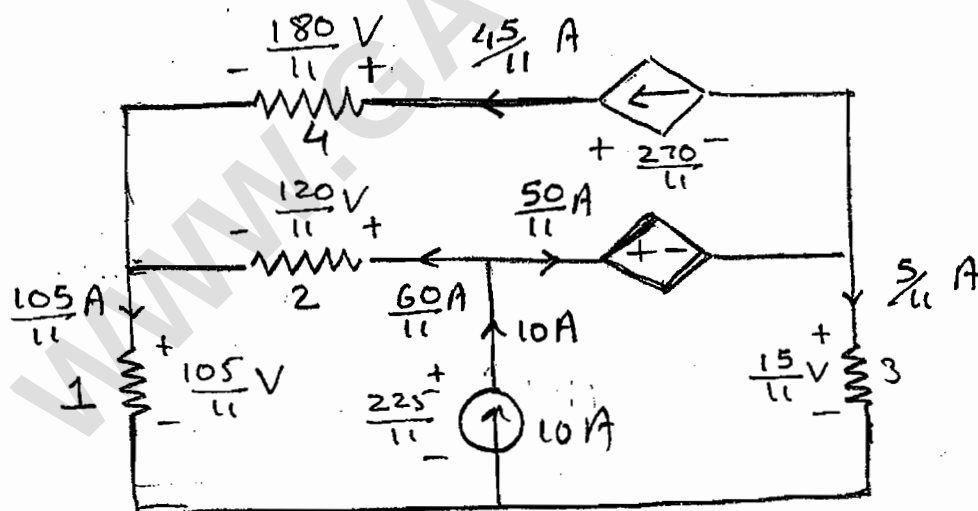
$$V_1 = 105/11 \text{ V}$$

$$V_2 = 225/11 \text{ V}$$

$$V_3 = 15/11 \text{ V}$$

$$i_x = 105/11 \text{ A}$$

$$V_x = 15/11 \text{ V}$$



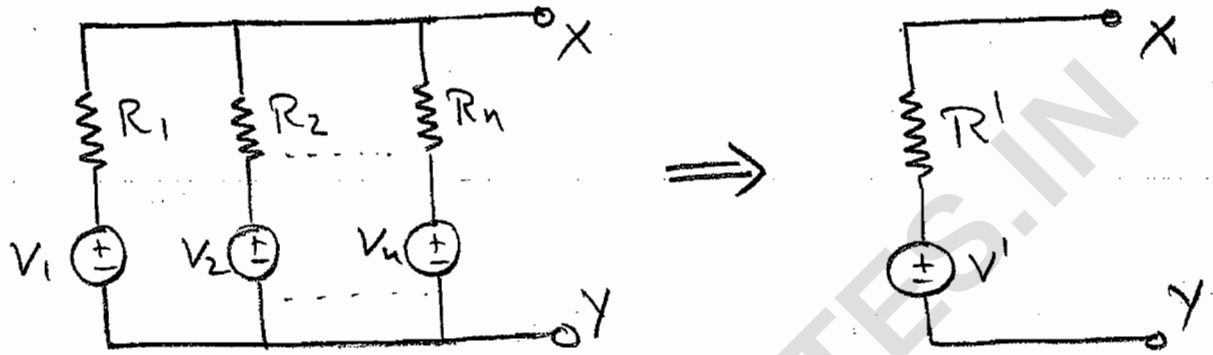
$$\sum P_{\text{source}} = \left(\frac{225}{11} \times 10 \right) + \left(\frac{270}{11} \times \frac{45}{11} \right) = 304.95 \text{ W}$$

$$\begin{aligned} \sum P_{\text{sink}} &= \left(\frac{105}{11} \times \frac{105}{11} \right) + \left(\frac{120}{11} \times \frac{60}{11} \right) + \left(\frac{15}{11} \times \frac{5}{11} \right) + \left(\frac{180}{11} \times \frac{45}{11} \right) \\ &\quad + \left(\frac{210}{11} \times \frac{50}{11} \right) \\ &= 304.95 \text{ W} \end{aligned}$$

Theorem 8:-

Milliman's Theorem:-

[Parallel Generator Theorem]



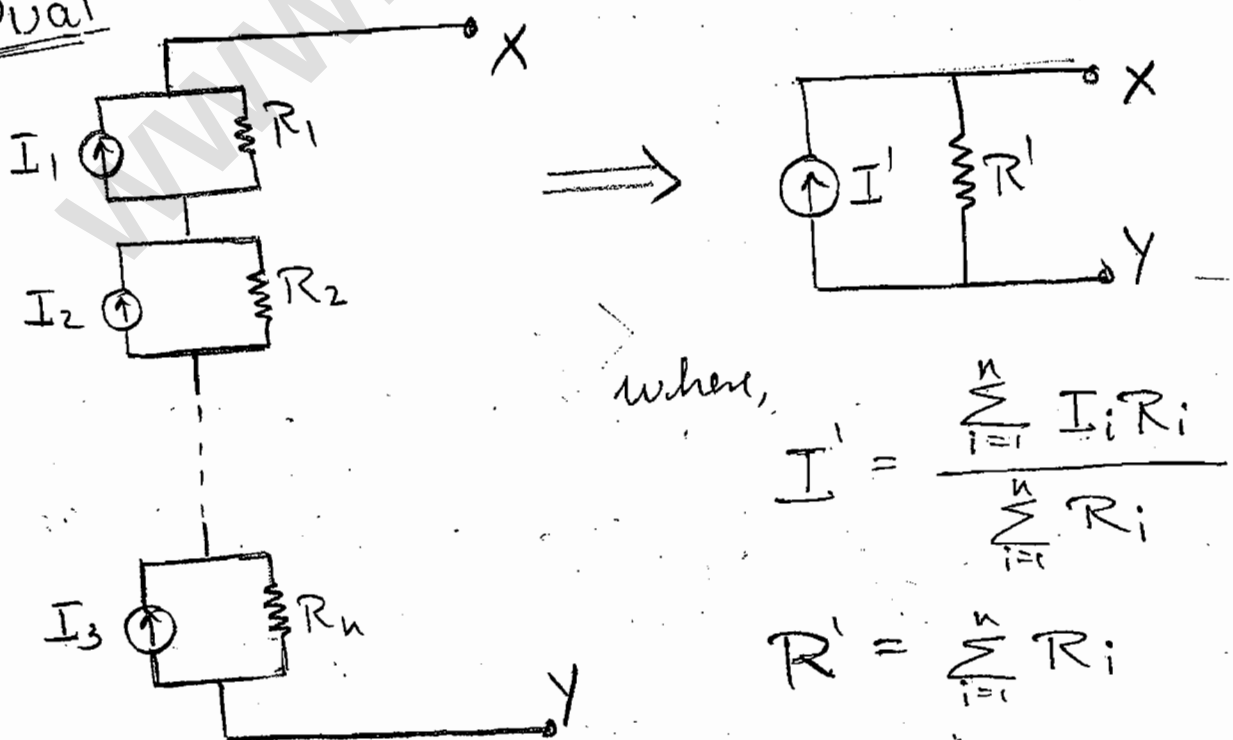
where

$$V' = \frac{\sum_{i=1}^n \frac{V_i}{R_i}}{\sum_{i=1}^n \frac{1}{R_i}} = \frac{\sum_{i=1}^n V_i G_i}{\sum_{i=1}^n G_i}$$

$$R' = \frac{1}{\sum_{i=1}^n \frac{1}{R_i}} = \frac{1}{\sum_{i=1}^n G_i}$$

Dual of Milliman's theorem

Dual



where,

$$I' = \frac{\sum_{i=1}^n I_i R_i}{\sum_{i=1}^n R_i}$$

$$R' = \sum_{i=1}^n R_i$$



ACE
Engineering Academy

ACE ENGINEERING ACADEMY

SUBJECT NAME

ELECTRIC CIRCUITS / ELECTRIC CIRCUITS & FIELDS (E Engineering)
NETWORK THEORY / NETWORK ANALYSIS & TRANSMISSION LINES (E & T Engineering)

IMPORTANCE of SUBJECT

Unique Order of covering IES Syllabus

- 6.9% 1/2 2
- 1M 1/2 4
- 1M
- (1) Fundamentals - Definitions, Notations, Symbols, Units, Formulas, Examples and Applications
 - (2) DC Circuit Analysis - Resistor as Fundamental Component (MESH and NODAL Analysis)
 - (3) DC Network Theorems and Applications
 - (4) Inductors and Capacitors
 - (5) AC Fundamentals - PHASOR, j-Operator, RMS and Average values of Time-Varying Waveforms
 - (6) Concept of POWER in AC, AC Circuit Analysis (MESH and NODAL Analysis)
 - (7) AC Network Theorems and Applications
 - (8) Locus Diagrams, Duals and Duality in Electrical Networks
 - (9) Resonance

(10) Magnetic Circuits

- 11) Network Topology / Graph Theory
- (12) Transient Circuit Analysis (Time-Domain)
- (13) Solution of Network Equations using Laplace Transform
- 14) Network Functions and Filters Concepts
- 15) Two-Port Networks
- 16) Network Synthesis (for IES Only)*
- 17) State Equations for Networks (for IES Only)*
- ~~18) Three Phase Circuits (for EE Only)**~~

L₁ M

Reference Books

- IES/ GATE Rank-1: Engineering Circuit Analysis, by William Hyat and Kemmerly, TMH Publications
GATE Rank-2: Fundamentals of Electric Circuits, by Alexander and Sadiku, TMH Publications
IES Rank-3: Network Analysis, by M E Van Valkenburg, PHI Publications
IES Rank-4: Networks and Systems, by D Roy Choudhury, New Age International Publications
Rank-5: Linear Circuit Analysis, by DeCarlo and Lin, OXFORD University Press

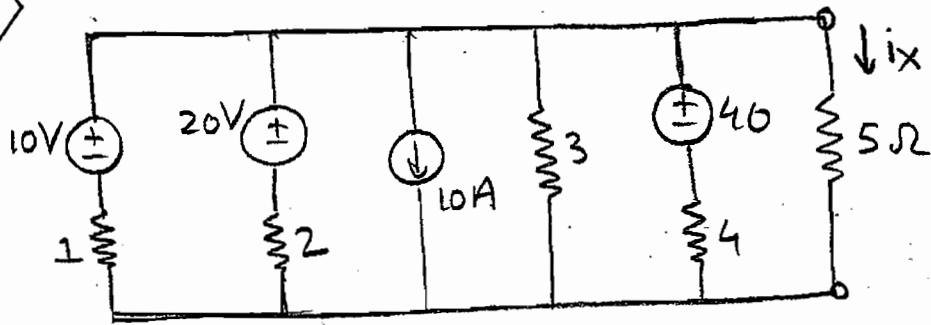
"If I HEAR, I will FORGET"

"If I SEE, I will REMEMBER"

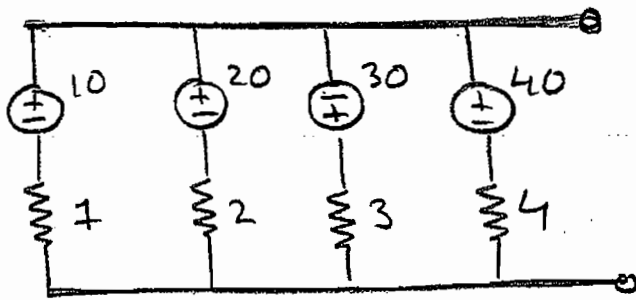
"If I DO, I will UNDERSTAND"

IES = FUNDAMENTALS + CONFIDENCE

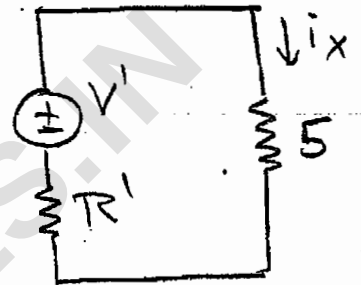
1)



Find i_x



$\Rightarrow$



$$V' = \frac{\frac{10}{1} + \frac{20}{2} - \frac{30}{3} + \frac{40}{4}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

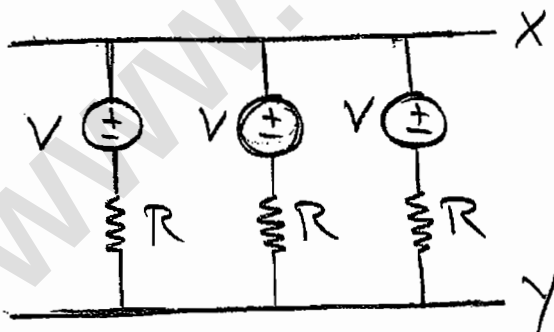
$$= 9.61 \text{ V}$$

$$R' = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}$$

$$= 0.48 \Omega$$

$$\therefore i_x = \frac{9.61}{5.48} = 1.75 \text{ A}$$

2)



(a)



(b)



(c)



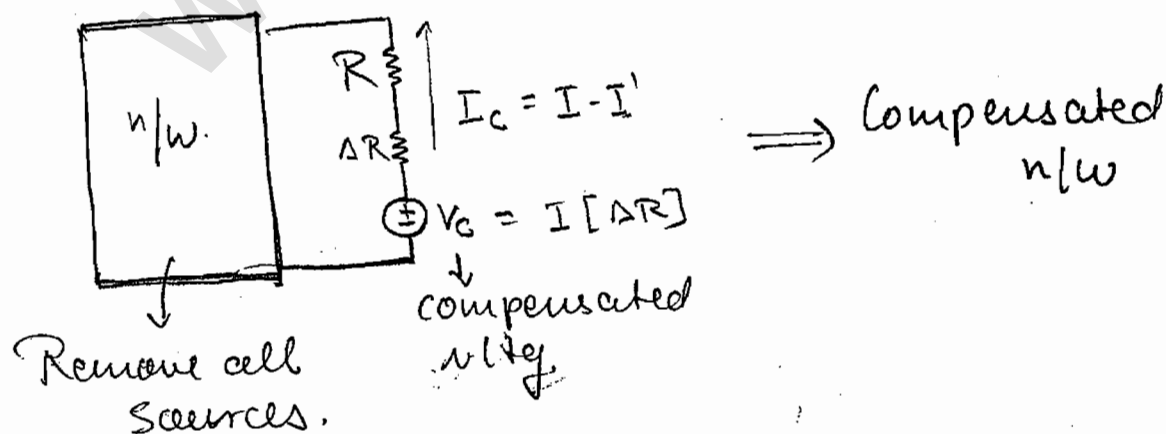
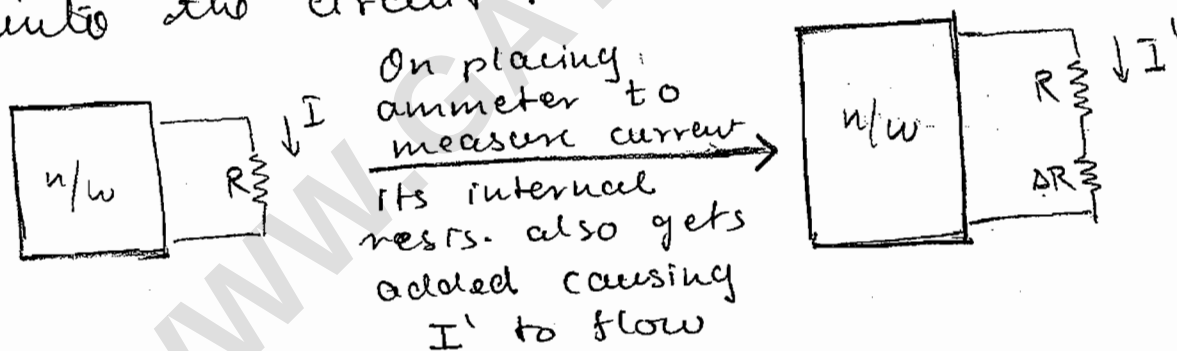
(d)



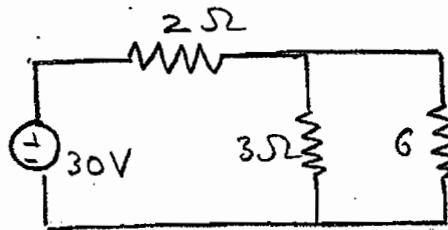
Theorem 9:-

Compensation theorem:-

- This theorem allows us to calculate the correct value of electrical parameters such as voltages & current when they are subjected to parametric variations within the circuit.
- This theorem is exclusively used to determine steady state error in measuring instruments, as practical meters with their internal resistances will alter the ideal values when they are connected into the circuit.



- 1) Find the change in current introduced by ammeter with an internal resist. of 0.15Ω while measuring the current in 6Ω resist. branch. Also determine steady state error introduced by the meter.



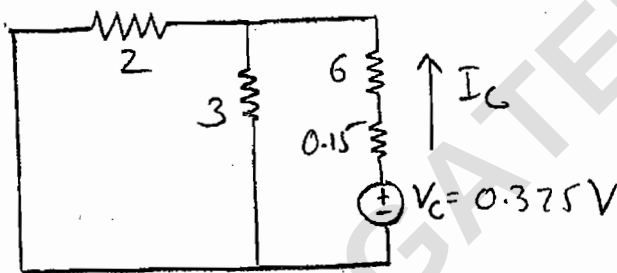
Step 1 :- Find 'I' $\rightarrow$ theoretically

$$I = \frac{30}{2+2} \times \frac{3}{9} = \frac{5}{2} \text{ A}$$

Step 2 :- Calculate compensated vltg

$$V_c = I [\Delta R] = \frac{5}{2} (0.15) = 0.375 \text{ V}$$

Step 3 :- Compensated n/w :-



$$\begin{aligned} I_c &= \frac{(0.375)}{6.15 + (2||3)} \\ &= \frac{0.375}{7.35} \\ &= 0.051 \text{ A} \end{aligned}$$

So by connecting an ammeter with internal resist. of 0.15Ω the current in that 6Ω branch is reduced by 0.051 A .

Now,

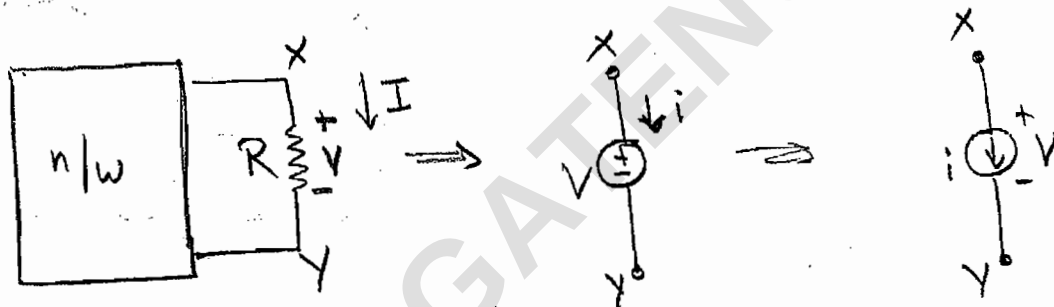
$$\% \text{ Error} = \frac{I - I'}{I} \times 100\% = \frac{I_c}{I} \times 100\%$$

$$= \frac{0.051}{2.5} \times 100\% = \frac{2.05\%}{\text{Acceptable}}$$

Theorem 10:—

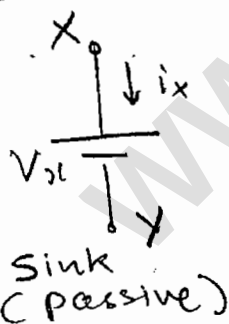
Substitution theorem:—

In any linear active bilateral n/w consisting of no. of energy sources, passive elements, etc. any passive element can be substituted in terms of its equivalent vltg and current for further analysis of n/w w/o disturbing the rest of the n/w provided the power absorbed by this passive element & its equivalently substituted source is same.



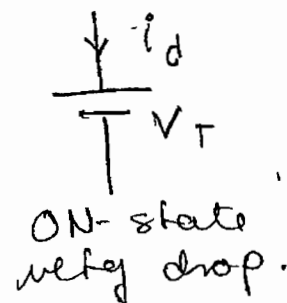
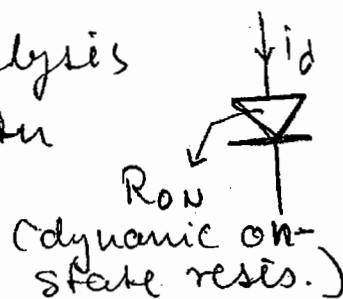
$$P_{\text{absorbed}} = \frac{V^2}{R} = I^2 R = V \cdot I$$

Vice-versa

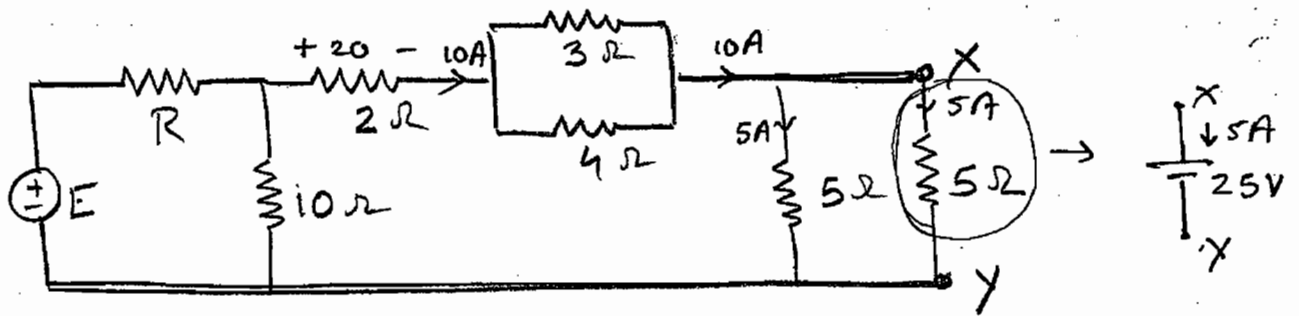


$$R = \frac{V_{xt}}{i_x} \Omega$$

We can model a semiconductor device as a passive element while conducting, in terms of its on-state vltg drop for further analysis using substitution theorem.



1)

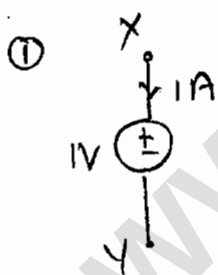
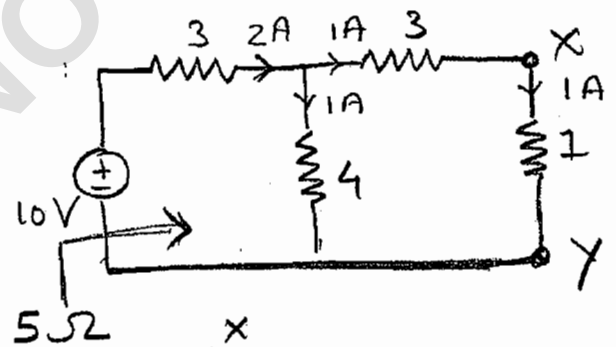


If volty drop across 2Ω resis is $20V$, the 5Ω resis. branch b/w X & Y can be substituted by an equivalent volty of _____?

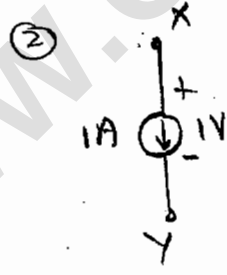
$$P_{\text{absorbed}} = (5)^2 \times 5 = 125W$$

$$P_{\text{abs.}} = 25 \times 5 = 125W$$

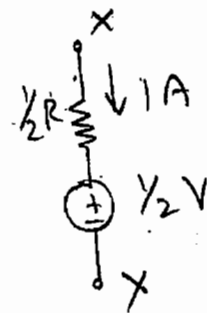
2) Use substitution theorem to substitute 1Ω branch in 5 diff. ways, (atleast)



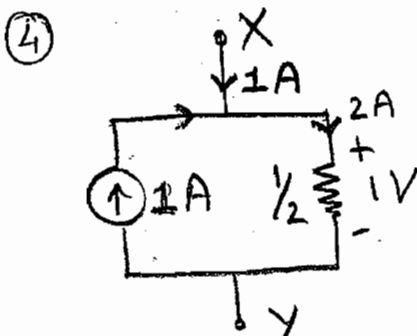
$$P_{\text{abs}} = 1W$$



$$P_{\text{abs}} = 1W$$



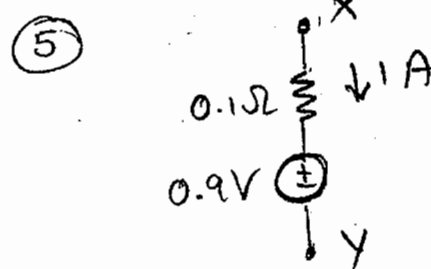
$$P_{\text{abs}} = (1)^2 \times \frac{1}{2} + \frac{1}{2} \times 1 = 1W$$



$$P_{\text{abs}} = (2)^2 \times \frac{1}{2} = 2W$$

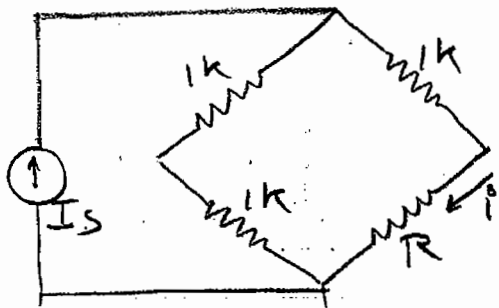
$$P_{\text{del}} = 4 \times 1 = 4W$$

$$P_{\text{abs}} = 1W$$



$$P_{\text{abs}} = (0.1)^2 (0.1) + 0.9 = 1W$$

3) For the balanced bridge determine the value of I_s .



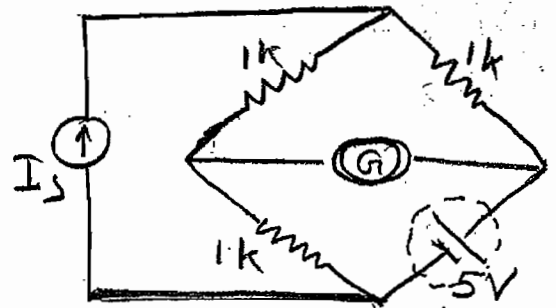
$$5 = iR$$

$\therefore$ Balanced bridge

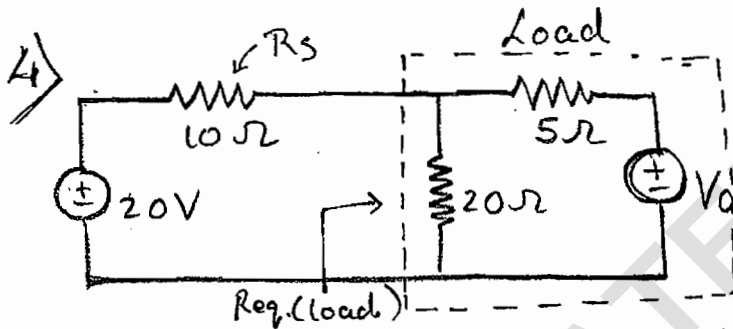
$$\therefore R = 1k\Omega$$

$$i = \frac{5}{R} = \frac{5}{1k} = 5mA$$

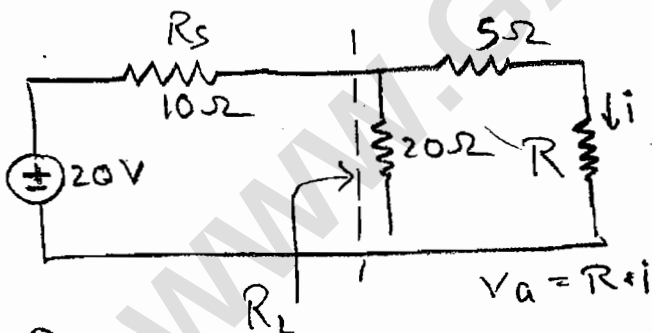
$$\text{So, } I_s = 2 \times i = 2 \times 5mA = 10mA$$



using substitution



Use substitution theo. to find the value of vltg V_a for which max. power is transferred to the load.



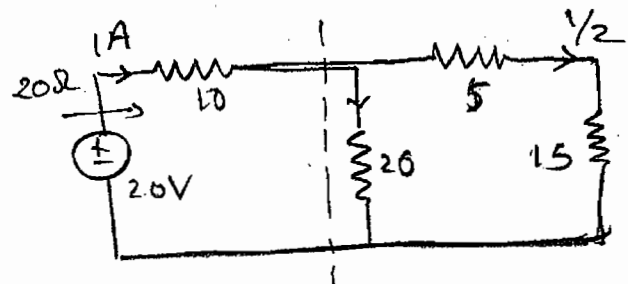
P_{max}

$$R_s = R_L$$

$$\therefore \frac{20(5+R)}{25+R}$$

$$25 + R = 10 + 2R$$

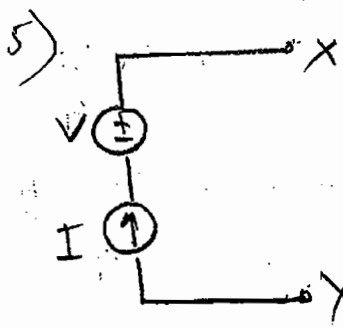
$$\therefore R = 15\Omega$$



$$R_a = 15\Omega$$

$$i = \frac{1}{2}A$$

$$\text{So, } V_a = i \cdot R = \frac{1}{2} \times 15 = 7.5V$$



What is applicable?

(a) T.E only

☒ (b) N.E only

(c) Both

(d) None.

Thevenin theo.

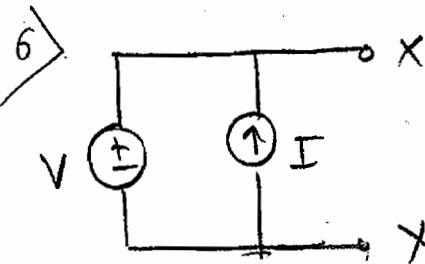
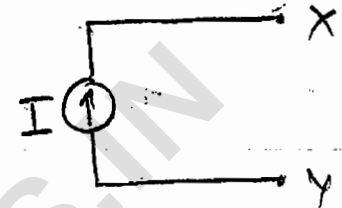
$$R_{Th} = R_N = \infty$$

$V_{Th} \rightarrow$ cannot be determined

Norton theo.

$$I_N = I$$

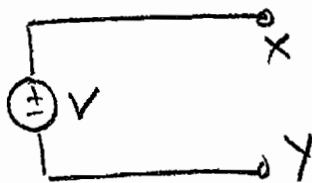
$$R_N = \infty$$



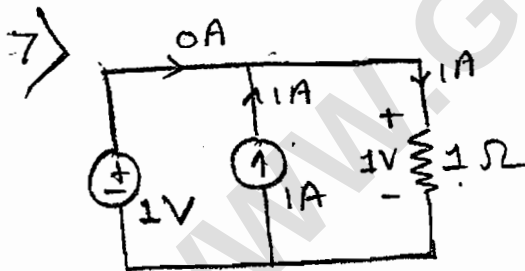
$$R_{Th} = R_N = 0 \Omega$$

$$V_{Th} = V$$

$I_N \rightarrow$ cannot be determined



Here Thevenin theo. $\Rightarrow$ equiv. is possible.

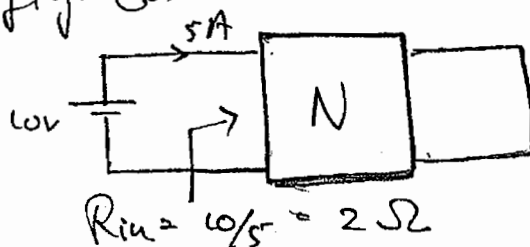


1 V delivers 0 W

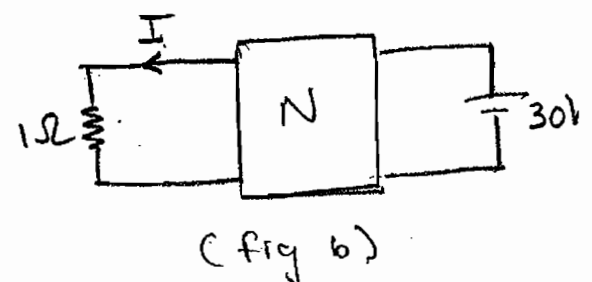
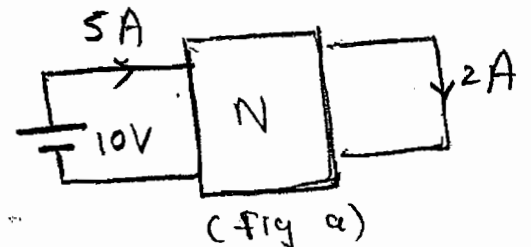
1 A delivers 1 W

1 Ω absorbs 1 W

8) The n/w N consists of only the resistor. Use the data given in fig (a) to find current I in fig-(b)



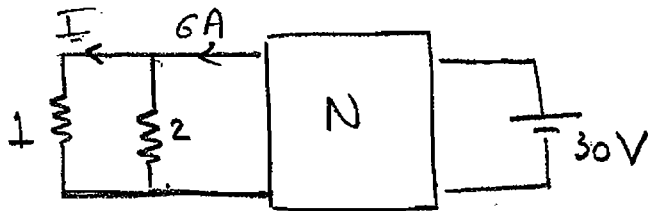
$$R_{in} = 10/5 = 2 \Omega$$



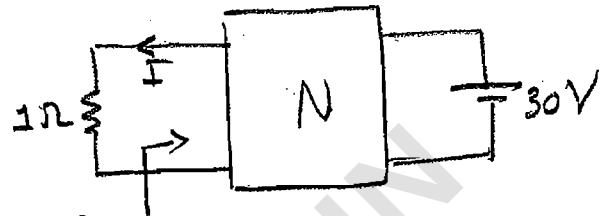
Using reciprocity :-



When no resis. is present then ~~we~~ we get 6A current (using homogeneity) where R_{in} does not appear



$$I = 6 \left(\frac{2}{3} \right) = 4A$$



R_{in} appears.
(with addition of R)

Addition of 1 Ω resistance will allow to appear the input / port resistance.

PROPERTIES OF INDUCTORS

Since $V = L \frac{di}{dt}$

① For dc excitation, $\frac{di}{dt} = 0$,

$V_L = 0 \rightarrow$ inductor is S.C for ideal D.C

② An inductor never allows sudden change in current through it.

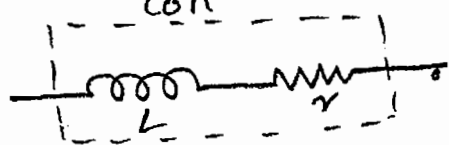
Eg:-

This is the principle of operation of choke coil in fluorescent lamp.

If inductor really allows sudden change in current through it, we get huge impulse voltages across it.

③ An ideal inductor is a coiled wire with zero internal resis. so power dissipated is zero. $L (r=0)$

④ Practical inductors will have small internal resistance & they are represented as coil shown below which allow some power losses.



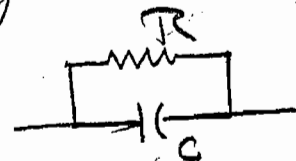
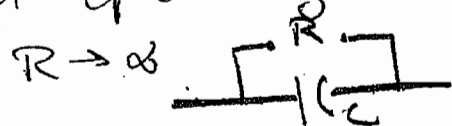
⑤ Inductors are available in diff. shapes & sizes & they are classified on the basis of the type of core material on which winding is done.

- ⑥ Inductors are used as filters, current limiting reactors, ~~var~~ ^{var} compensators, etc in communication & power sys.

PROPERTIES OF CAPACITORS

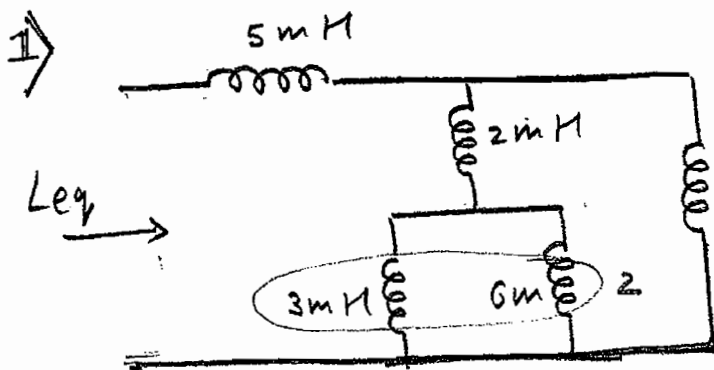
Since $i = C \frac{dV}{dt}$

- ① For DC excitation, $\frac{dV}{dt} = 0$, $i_c = 0$
→ Capacitor acts as O.C for ideal D.C (in steady state)
- ② Capacitor never allows sudden change in vltg across it.
- ③ Ideal capacitors are considered to have infinite dielectric ^{resis} capacitance b/w electrodes so dielectric losses are zero but conduction is through polarization.
- ④ Practical capacitors are considered to have very large dielectric resis. b/w electrodes so they undergo losses.



- ⑤ Capacitors are available in diff shapes & sizes & they are classified on the basis of dielectric material b/w the electrodes.
- ⑥ They are used as filters, compensators, power factor correcting equipments,

signal conditioning & wave shaping, etc in communication & power sys.

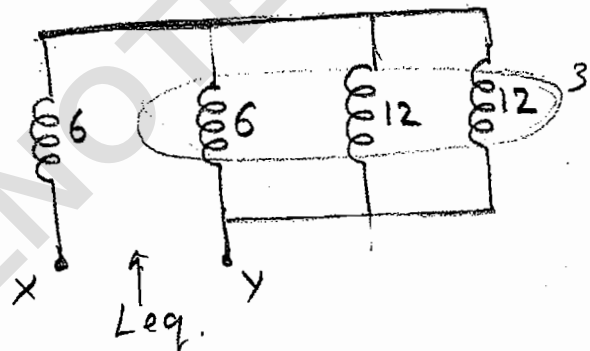
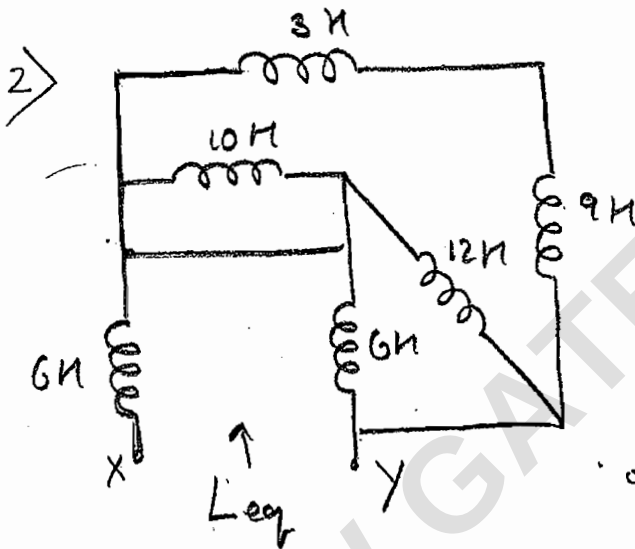


$$L_{eq} = 5 + \left[\frac{(3 \parallel 6) + 2}{4} \right]$$

$$\therefore L_{eq} = 5 + \left[\frac{(2+2) \parallel 4}{1} \right]$$

$$= 5 + 2$$

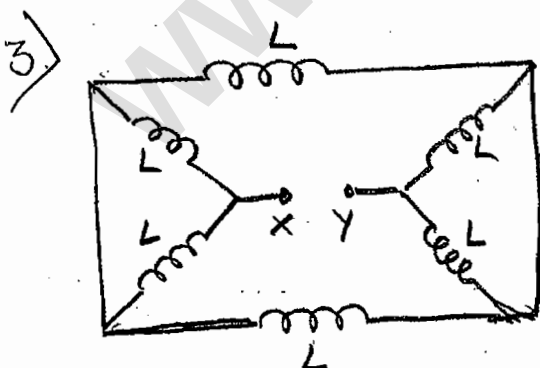
$$= 7 \text{ mH.}$$



$$\therefore L_{eq} = 6 + [6 \parallel 12 \parallel 12]$$

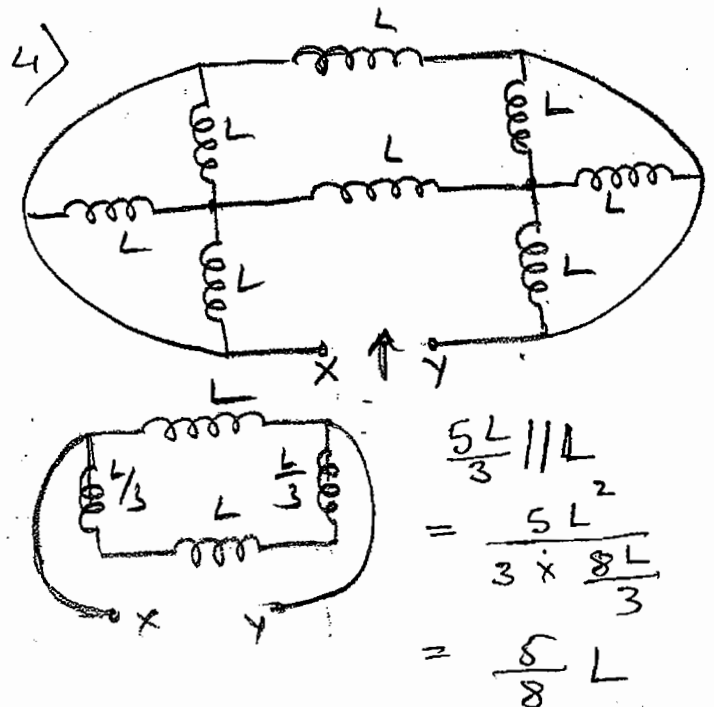
$$= 6 + 3$$

$$= 9 \text{ H}$$



$$L_{eq} = \frac{L}{2} + \frac{L}{2} + \frac{L}{2}$$

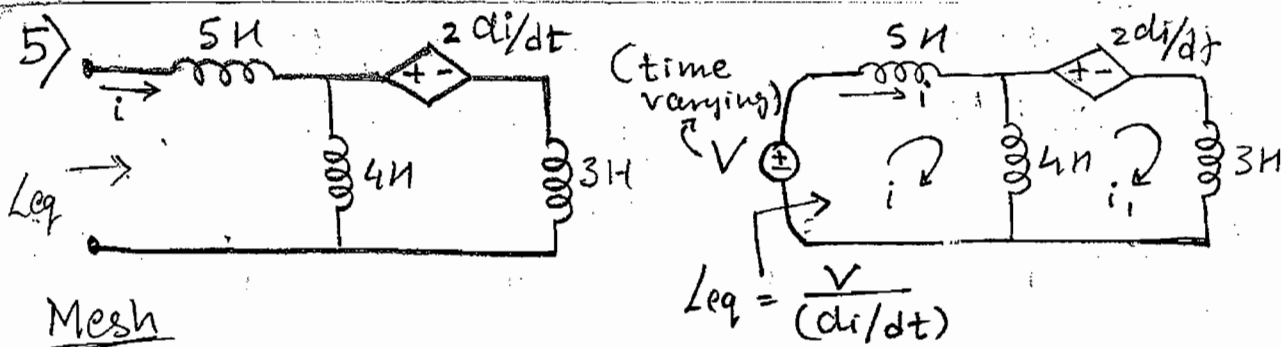
$$L_{eq} = \frac{3L}{2}$$



$$\frac{5L}{3} \parallel L$$

$$= \frac{5L}{3 \times \frac{8L}{3}}$$

$$= \frac{5}{8} L$$



Mesh

$$-V + 5 \frac{di}{dt} + 2 \left[\frac{di}{dt} - \frac{di_1}{dt} \right] = 0$$

$$9X - 4Y = V \quad \text{--- (1)}$$

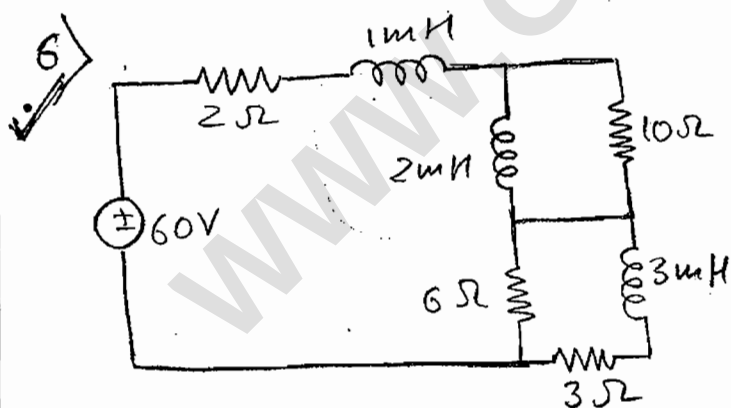
$$4 \left[\frac{di_1}{dt} - \frac{di}{dt} \right] + 2 \frac{di}{dt} + 3 \frac{di_1}{dt} = 0$$

$$-2X + 7Y = 0 \quad \text{--- (2)}$$

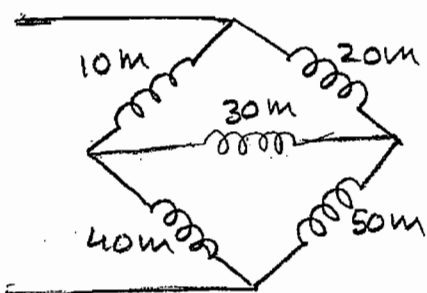
$$X \left[9 - 4 \times \frac{2}{7} \right] = V$$

$$\therefore V = \left[\frac{63-8}{7} \right] X = \frac{55}{7} \frac{di}{dt}$$

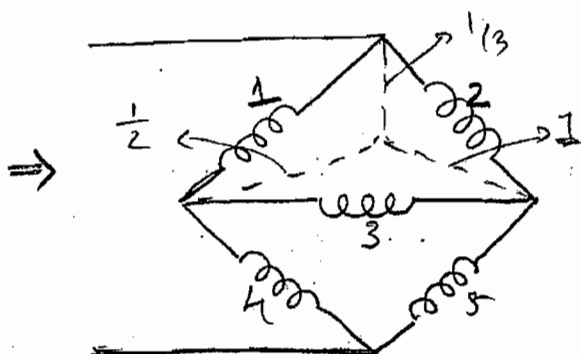
$$\therefore L_{eq} = \frac{55}{7} \text{ H}$$

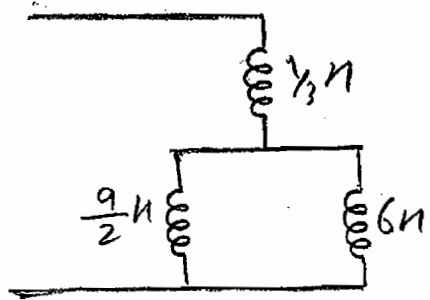


Find the steady state current through the inductors & also energy stored in it.



Scaling 'L' by 10mH

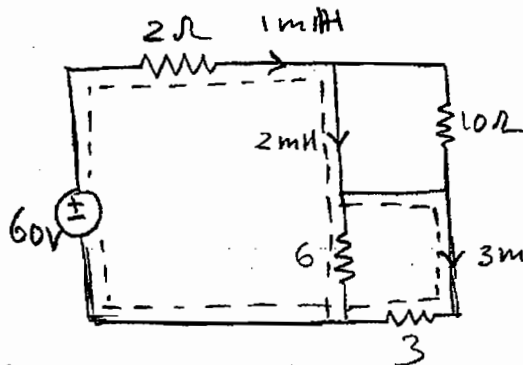




$$L_{eq} = \left[\frac{1}{3} + \left(\frac{9}{2} \parallel 6 \right) \right] \times \frac{10 \text{ mA}}{\downarrow \text{scaled product}}$$

$$= 2.9 \times 10 \text{ mH}$$

$$= 29 \text{ mH}$$



$$\rightarrow i_{1m} = 15 \text{ A}$$

$$E_L = \frac{1}{2} (1 \text{ m}) (15)^2 = 112.5 \text{ mJ}$$

$$\rightarrow i_{2m} = 15 \text{ A}$$

$$E_L = \frac{1}{2} (3 \text{ m}) (15)^2 = 225 \text{ mJ}$$

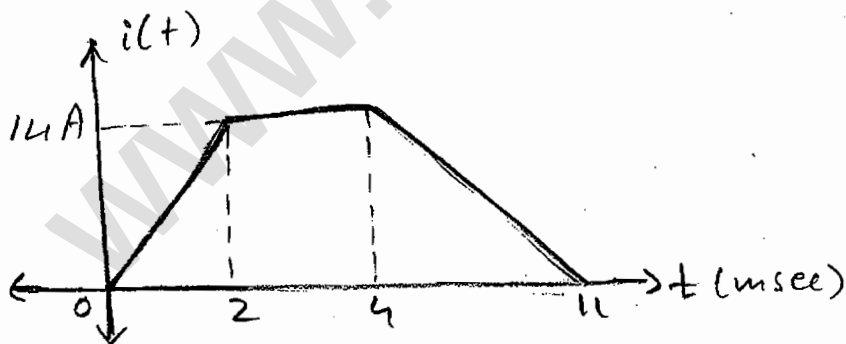
$$i_{3m} = \frac{6}{9} (15) = 10 \text{ A}$$

$$\rightarrow i_{3m} = 10 \text{ A}$$

$$E_L = \frac{1}{2} (3 \text{ m}) (10)^2 = 150 \text{ mJ}$$

These energies are in electromagnetic form (DC flux)

7) If the current flowing through 2 H inductor is as shown, plot the vltg across it.



$$0 < t < 2 \text{ m}$$

$$i(t) = 7t \Rightarrow v = 2 \frac{d}{dt} (7t) = +14 \text{ V pulse}$$

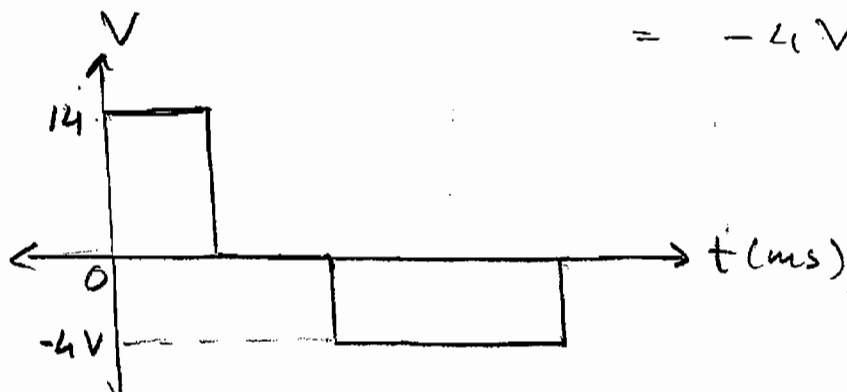
$$2 \text{ m} < t < 4 \text{ m}$$

$$i(t) = 14 \Rightarrow v = 2 \frac{d}{dt} (14) = 0 \text{ V}$$

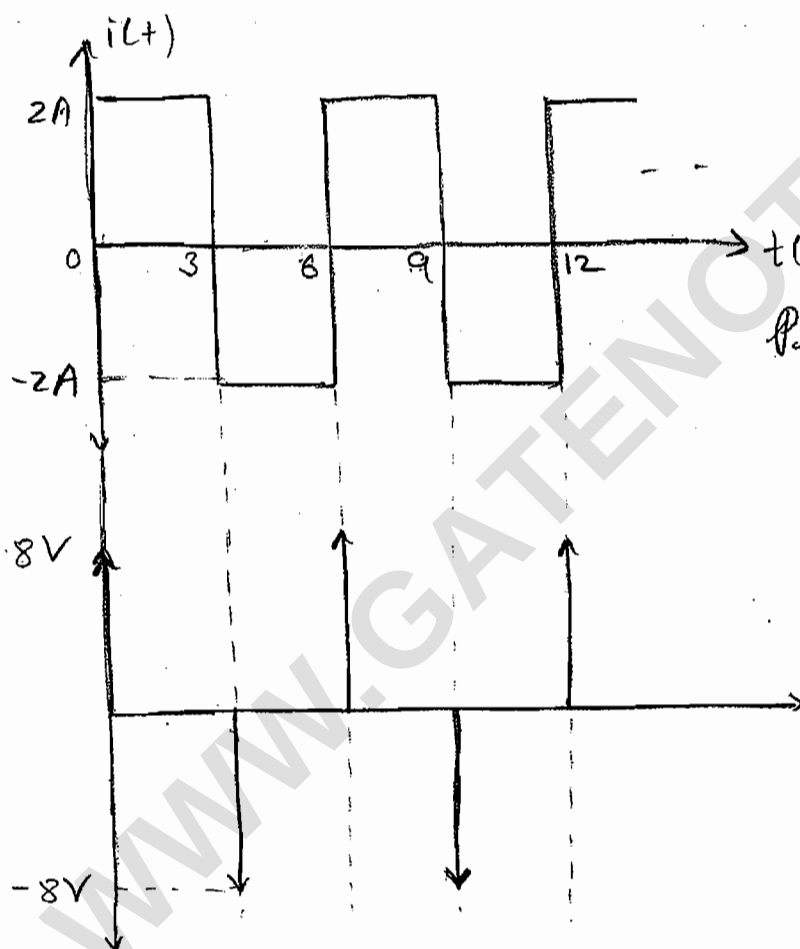
(For DC L is shorted)

$$4\text{m} < t < 11\text{m}$$

$$i(t) = -2t + c \Rightarrow v = 2 \frac{d}{dt} (-2t + c) = -4\text{V}$$



8)



Plot of current flowing through 2H inductor is shown. Plot vltg across it.

$$V = L \frac{di}{dt}$$

impulse response $\rightarrow dt \rightarrow 0$

Numerator restricts magnitude

$$V = L \frac{[\Delta I]}{\Delta t} = L \left[\frac{i_{\text{final}} - i_{\text{initial}}}{\Delta t} \right]$$

impulse $\rightarrow \Delta t \rightarrow 0$

At $t = 0$ msec

$$\underset{\substack{\downarrow \\ \text{impulse}}}{V} = 2 \frac{\underset{\substack{\downarrow \\ 0}}{[2-0]}}{\Delta t} = 4V \text{ impulse}$$

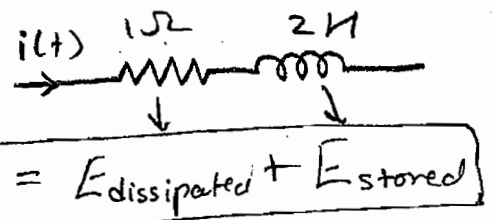
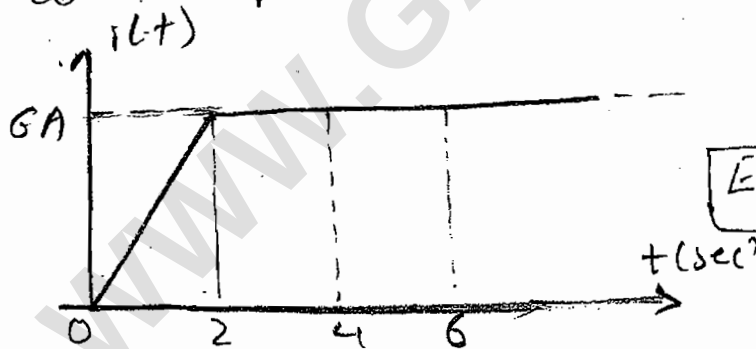
At $t = 3$ msec

$$\underset{\substack{\downarrow \\ \text{impulse}}}{V} = 2 \left[\frac{-2-2}{\underset{\substack{\downarrow \\ 0}}{\Delta t}} \right] = -8V$$

At $t = 6$ msec

$$V = 2 \left[\frac{2-(-2)}{\Delta t} \right] = +8V$$

9) A practical coil has an inductance of $2H$ & resis. of 1Ω . If this coil is excited with the current as shown below find the total energy absorbed by the coil upto 1^{st} 4 seconds.



[R]

$$\begin{aligned} E_{\text{diss}} &= \int T_R dt = \int_0^4 [i(t)]^2 R dt \\ &= \int_0^2 (3t)^2 \times 1 \cdot dt + \int_2^4 6^2 dt \\ &= 3 [t^3]_0^2 + 36 [4-2] \\ &= 24 + 72 \\ &= 96 \text{ J} \end{aligned}$$

Q2

$$E_{\text{stored}} = \int P_L dt = \int_0^4 L i \frac{di(t)}{dt} dt$$

$$= \int_0^2 2(3t) \frac{d}{dt}(3t) dt + \int_2^4 2 \cdot 6 \frac{d}{dt}(6) dt$$

$$= 9 [t^2]_0^2 = \boxed{36 \text{ J}}$$

$$E_{\text{absorbed}} = 96 + 36 = 132 \text{ J}$$

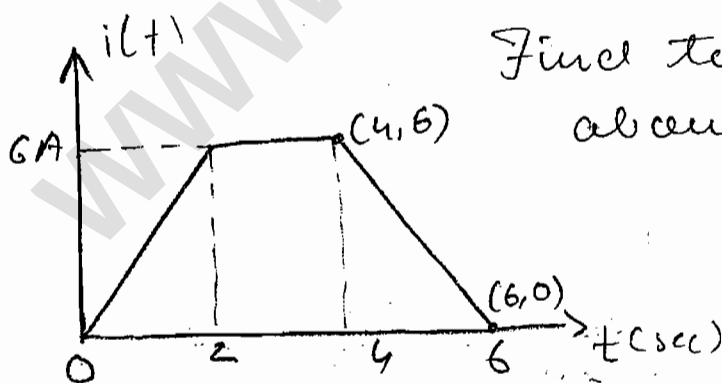
NOTE

An inductor stores energy for some time variance occurring at any instance in the ckt. & retains this energy as long as excitation is given.

So energy stored here upto first 4 sec is the energy stored at the 4th sec

$$E_L = \frac{1}{2} L i^2 = \frac{1}{2} (2) (6)^2 = \boxed{36 \text{ J}}$$

10)



Find total energy about 6th second.

$$\underline{0 < t < 2}$$

$$y = mx = i(t) = 3t$$

$$\underline{2 < t < 4}$$

$$i(t) = 6$$

$$\underline{4 < t < 6}$$

$$(i(t) - 0) = \frac{(6-0)}{(4-6)} (t-6) \Rightarrow i(t) = -3t + 18$$

In the above problem adding the limits from 4 to 6, we get

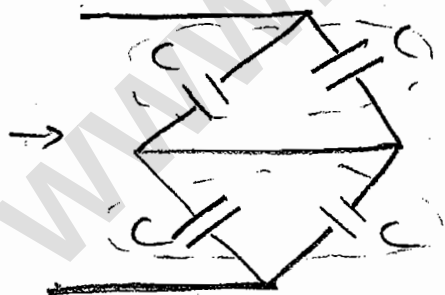
$$\begin{aligned}
 E_{\text{dissi.}} &= 96 + \int_4^6 (-3t + 18)^2 dt \\
 &= 96 + \int_4^6 9(t^2 - 12t + 36) dt \\
 &= 96 + 24 \\
 &= 120 \text{ J}
 \end{aligned}$$

~~$E_{\text{absor.}}$~~

$$\begin{aligned}
 E_{\text{stored}} &= 36 + \int_4^6 \frac{1}{2} (-3t + 18) \frac{d}{dt} (-3t + 18) dt \\
 &= 36 + 18 \int_4^6 (-t + 6) \left(-\frac{t}{2} + 18t \right) dt \\
 &= 36 - 36 = 0 \text{ J}
 \end{aligned}$$

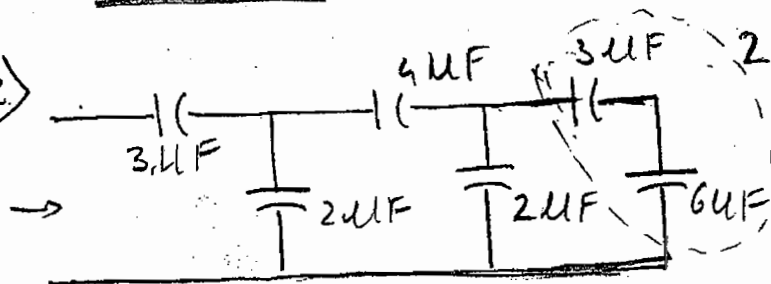
∴ $E_{\text{absorbed}} = \underline{\underline{120 \text{ J}}}$

11)

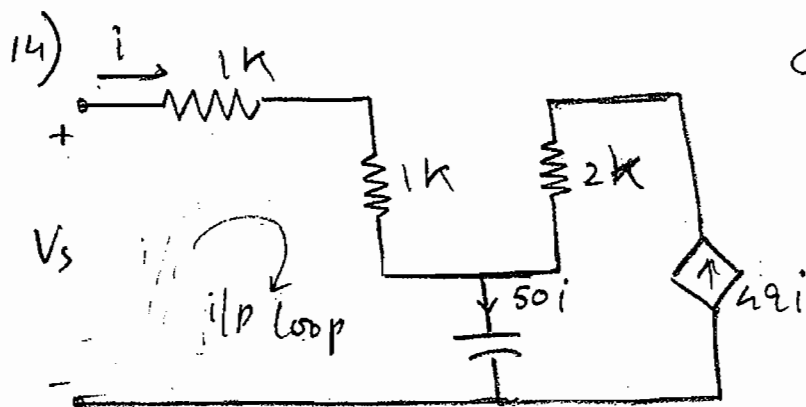
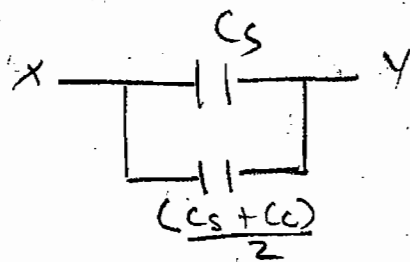
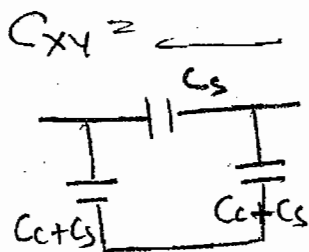
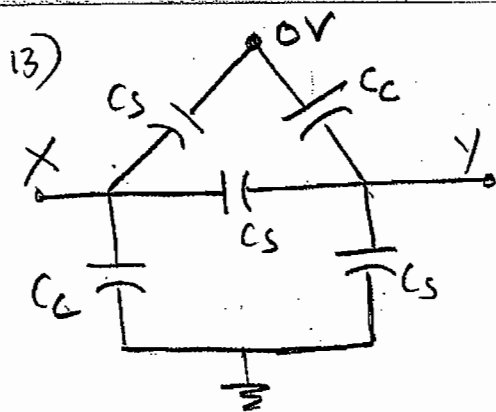


$$\begin{aligned}
 C_{\text{eq}} &= \frac{2C \times 2C}{4C} \\
 &= \frac{4C^2}{4C} = C
 \end{aligned}$$

12)



$$\begin{aligned}
 C_{\text{eq}} &= \left[\frac{(2+2) \times 4}{8} + 2 \right] \text{ series } (3\mu\text{F}) \\
 &= \frac{4 \times 3}{7} = \frac{12}{7} \mu\text{F}
 \end{aligned}$$



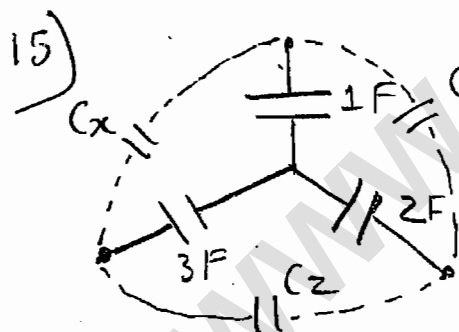
Find the i/p loop capacitance.

KVL

$$V_s = (2000) i + \frac{1}{100\mu} \int 50i dt$$

$$\therefore V_s = \frac{2000}{R} i + \frac{1}{e^{2\mu}} \int i dt$$

$$\therefore C_{loop} = 2\mu F \quad \& \text{ also } R_{loop} = 2000 \Omega$$



$$\Rightarrow \frac{1}{C_x} = \frac{1}{1} + \frac{1}{3} + \frac{\frac{1}{1} \times \frac{1}{3}}{\frac{1}{2}} = 1 + \frac{1}{3} + \frac{2}{3} = 2$$

$$\therefore C_x = \frac{1}{2} F$$

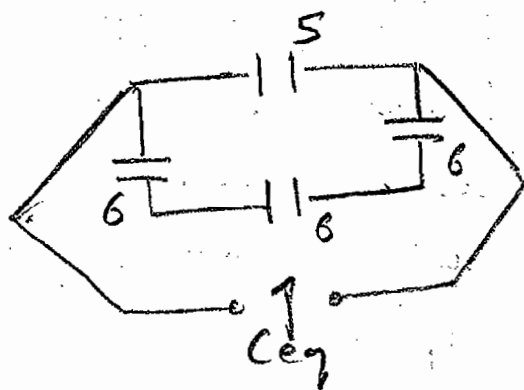
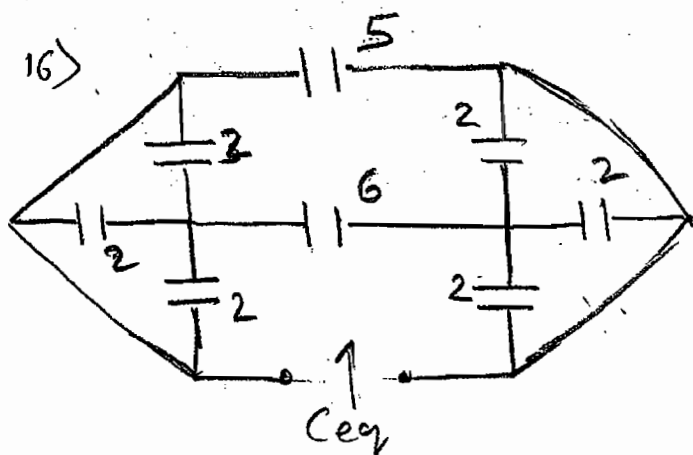
Convert star to delta

$$\Rightarrow \frac{1}{C_y} = \frac{1}{1} + \frac{1}{2} + \frac{\frac{1}{1} \times \frac{1}{2}}{\frac{1}{3}} = 1 + \frac{1}{2} + \frac{3}{2}$$

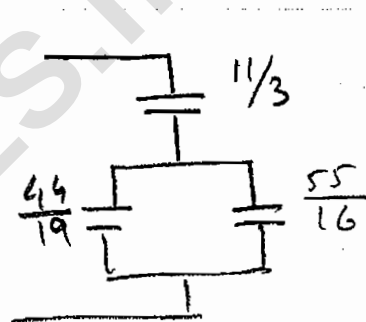
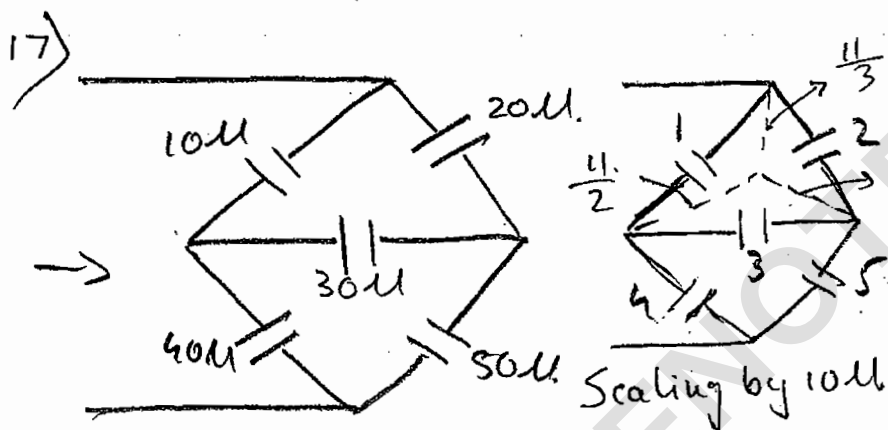
$$\frac{1}{C_y} = \frac{6}{2} = 3 \Rightarrow C_y = \frac{1}{3} F$$

$$\Rightarrow \frac{1}{C_z} = \frac{1}{3} + \frac{1}{2} + \frac{\frac{1}{3} \times \frac{1}{2}}{\frac{1}{1}} = \frac{1}{3} + \frac{1}{2} + \frac{1}{6}$$

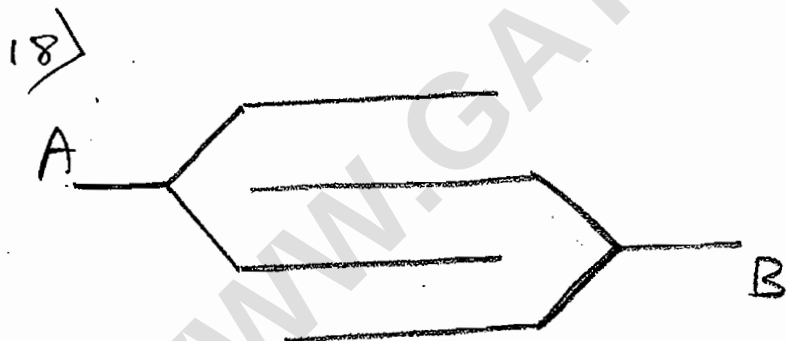
$$\frac{1}{C_z} = 1 \Rightarrow C_z = 1 F$$



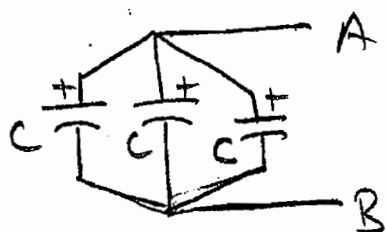
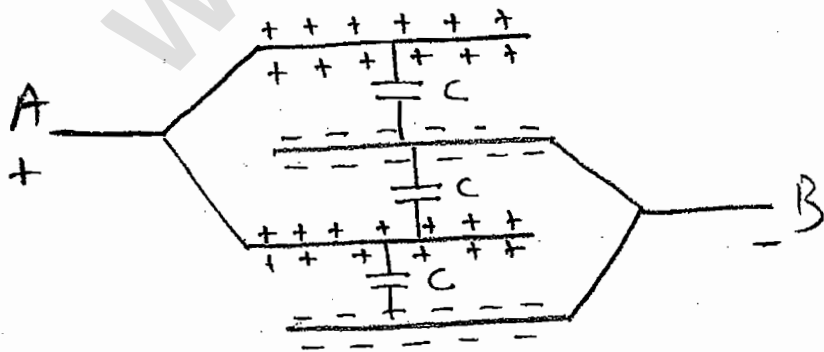
$$\therefore C_{eq} = \frac{5 \times 2}{7} = \frac{10}{7} F$$



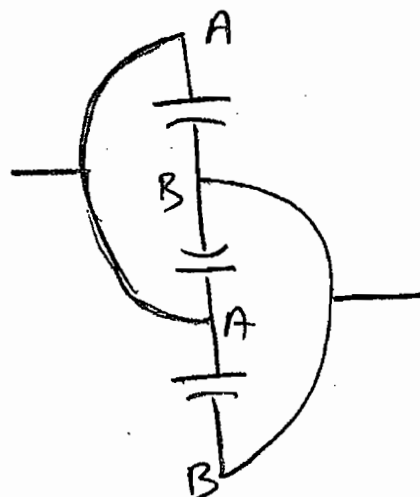
$$C_{eq} = 2.23 \times 10 = 22.3 \mu F$$



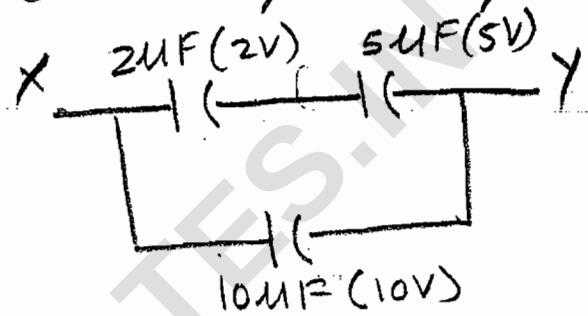
If equivalent capacitance b/w the plates is 'C' then $C_{AB} =$ _____.



$$\Rightarrow C_{AB} = 3C$$



1) Three capacitors are arranged in fig. as shown below. The value of vltgs in bracket indicates their breakdown vltg limit. What the max vltg upto which entire n/w can work w/o breakdown of any capacitor. & hence determine max. storage charge in the n/w.



For $10\mu F$
 $V = 10V$

For $5\mu F$
 $V \left[\frac{2}{7} \right] = 5 \Rightarrow V = 17.5 V$

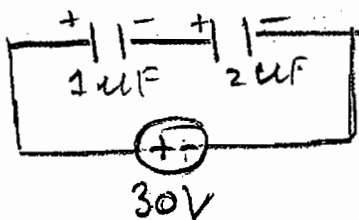
For $2\mu F$
 $V \left[\frac{5}{7} \right] = 2 \Rightarrow V = 2.8 V$

$\therefore V_{max} = 2.8 V$

Hence $C_{eq} = \frac{10}{7} + 10 = \frac{80}{7} \mu F$

$q_{max} = C_{eq} V_{max} = \frac{80}{7} \times \frac{2.8}{10} = 32 \mu C$

2) Two capacitors of $1 \mu F$ & $2 \mu F$ are connected in series across a $30V$ DC source. Find their steady vltgs & charge on each. Now if these 2 capacitors are disconnected from supply & connected with like polarities together, now determine steady state vltg & charge on it each.



$$V_{1\mu} = \frac{30(2)}{3} = 20 \text{ V}$$

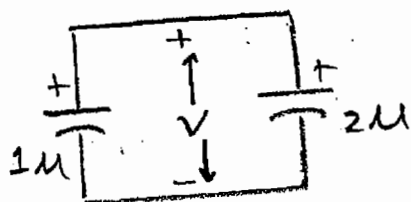
$$V_{2\mu} = \frac{30(1)}{3} = 10 \text{ V}$$

$$q_{1\mu} = 1 \times 20 = 20 \mu\text{C}$$

$$q_{2\mu} = 2 \times 10 = 20 \mu\text{C}$$

In current electricity if current is equal in series connected elements then in static electricity the charges will be equal/same in series connected capacitors.

$$\therefore q_1 = q_2 = 20 \mu\text{C}$$



$$\text{Here, } V_1 = V_2 = V$$

$$\therefore \frac{q_1}{C_1} = \frac{q_2}{C_2} \Rightarrow \frac{q_1}{1} = \frac{q_2}{2}$$

$$\therefore q_2 = 2q_1$$

From law of conservation of charge

$$q_1 + q_2 = 40$$

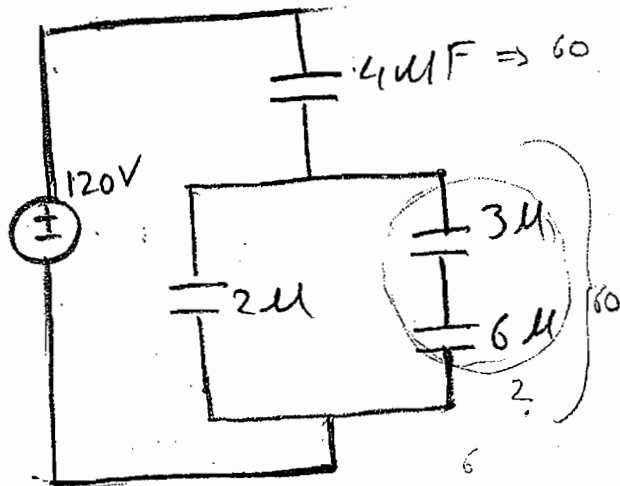
$$\therefore q_1 + 2q_1 = 40 \Rightarrow q_1 = \frac{40}{3} \mu\text{C}$$

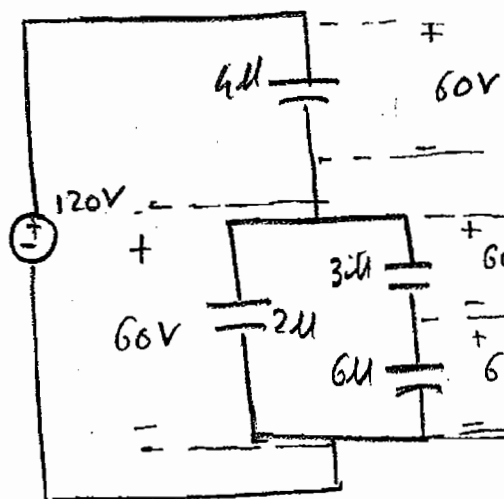
$$\Rightarrow q_2 = \frac{80}{3} \mu\text{C}$$

$$\therefore V_1 = V_2 = \frac{q_1}{C_1} = \frac{q_2}{C_2}$$

$$= \frac{40}{3} \text{ volts}$$

3) Determine steady state vltgs across each capacitor & energy stored in it each





$$q_4 = CV = 4 \times 60 = 240 \mu C$$

$$E_4 = \frac{1}{2} (4^2) (60)^2 = 7200 \mu J$$

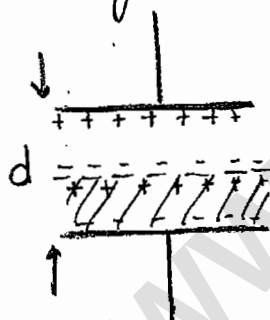
$$q_2 = 2 \times 60 = 120 \mu C$$

$$E_2 = \frac{1}{2} (2^2) (60)^2 = 3600 \mu J$$

$$q_3 = q_5 = 3 \times 40 = 120 \mu C$$

$$E_3 = E_5 = \frac{1}{2} (3^2) (40)^2 = 2400 \mu J$$

4) A capacitor having a common cross section area b/w the plates of $A \text{ m}^2$ & dist. b/w the plates as 'd' metres is now dipped in ethyl alcohol upto $d/2 \text{ m}$. What is the ratio of capacitance before & after immersing it into the ethyl alcohol. [Consider $\epsilon_r = 25$ for ethyl alcohol]



Before: $C = \frac{A \epsilon_0 \epsilon_r}{d}$

$$C = \frac{A \epsilon_0}{d} \quad (\because \epsilon_r = 1 \text{ (air)})$$

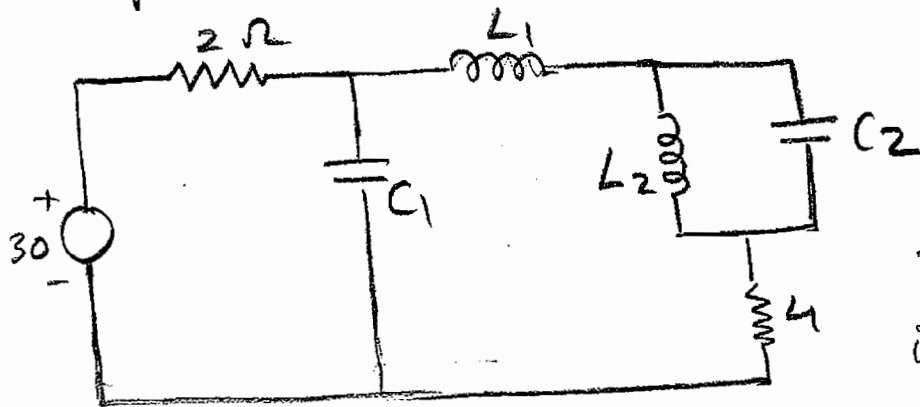
After:

$$C = \frac{\frac{A \epsilon_0 \epsilon_r}{d/2} \times \frac{A \epsilon_0 \epsilon_r}{d/2}}{2 \times \frac{A \epsilon_0 \epsilon_r}{d/2}} = \frac{A \epsilon_0 (25)}{d}$$

$$C' = \frac{\frac{A \epsilon_0}{d/2} \times \frac{A \epsilon_0 \epsilon_r}{d/2}}{\frac{A \epsilon_0}{d/2} [1 + \epsilon_r]} = \frac{\left(\frac{A \epsilon_0}{d} \right) \times \frac{25 \times 2}{2 \times 26}}{C}$$

$$\therefore \frac{C}{C'} = \frac{26}{50}$$

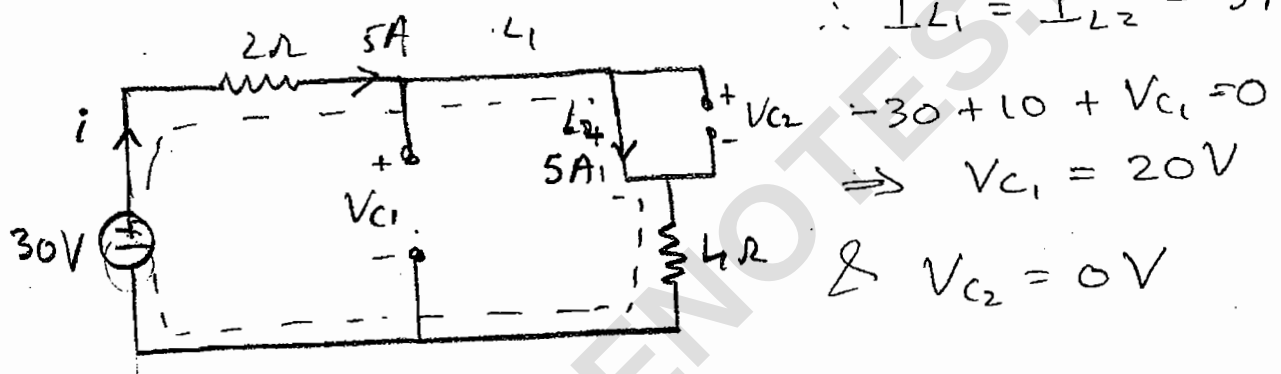
- 1) Determine steady state vltgs across Capacitors & current through inductors



$$R_T = 2 + 4 = 6$$

$$i = \frac{30}{6} = 5A$$

$$\therefore I_{L1} = I_{L2} = 5A$$

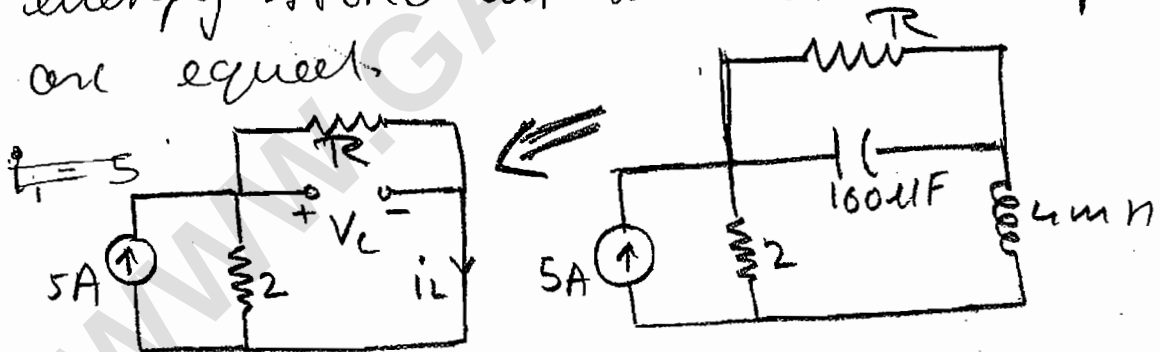


$$-30 + 10 + V_{C1} = 0$$

$$\Rightarrow V_{C1} = 20V$$

$$\& V_{C2} = 0V$$

- 2) What is value of R for which the energy stored in inductor & capacitor are equal.



$$i_L = 5 \times \frac{2}{R+2} = \frac{10}{2+R}$$

$$V_C = i_L \times R = \frac{10R}{2+R}$$

Now

$$E_L = E_C$$

$$\frac{1}{2} L i_L^2 = \frac{1}{2} C V_C^2$$

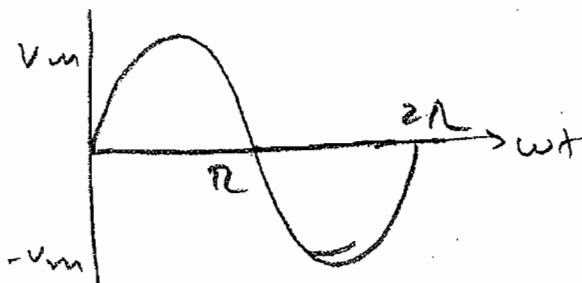
$$\therefore \frac{1}{2} \times (4m) \left(\frac{100}{(2+R)^2} \right) = \frac{1}{2} \frac{(160\mu)(100)^2 R^2}{(2+R)^2}$$

$$R^2 = \frac{1 \text{ m}}{40 \text{ m}} = 25 \Rightarrow R = 5 \Omega$$

STEADY STATE AC circuit Analysis : —

Radian

$$v = V_m \sin \omega t$$

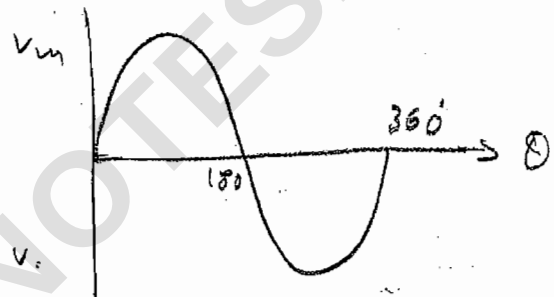


$V_m \rightarrow$ amplitude

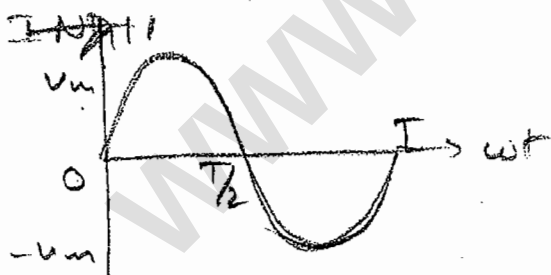
$\omega \rightarrow$ angular freq. (rad/sec)

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Degree



$$v = V_m \sin \theta$$



Time

$$v = V_m \sin \left(\frac{2\pi}{T} \cdot t \right)$$

Example

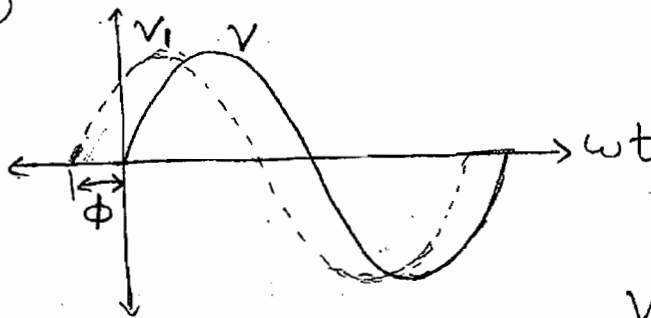
Power freq = 50 Hz $\Rightarrow f = \frac{1}{50} = 20 \text{ ms}$

$$1T = 2\pi = 360^\circ = 20 \text{ ms}$$

Standard sine wave : —

$$V_x = V_m \sin(\omega t + \phi) \quad ; \quad \phi = \text{phase shift (deg)} \\ \rightarrow \text{time shift}$$

①



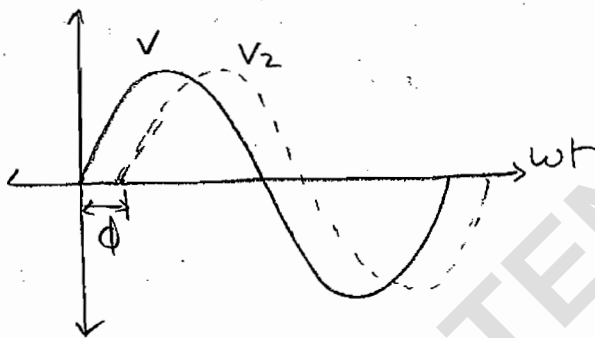
$$V_1 = V_m \sin(\omega t + \phi)$$

V_1 comes early than V by ϕ°

V_1 leads V by ϕ°

(+) $\rightarrow$ leading

②



$$V_2 = V_m \sin(\omega t - \phi)$$

V_2 lags V by ϕ°

(-) $\rightarrow$ lagging

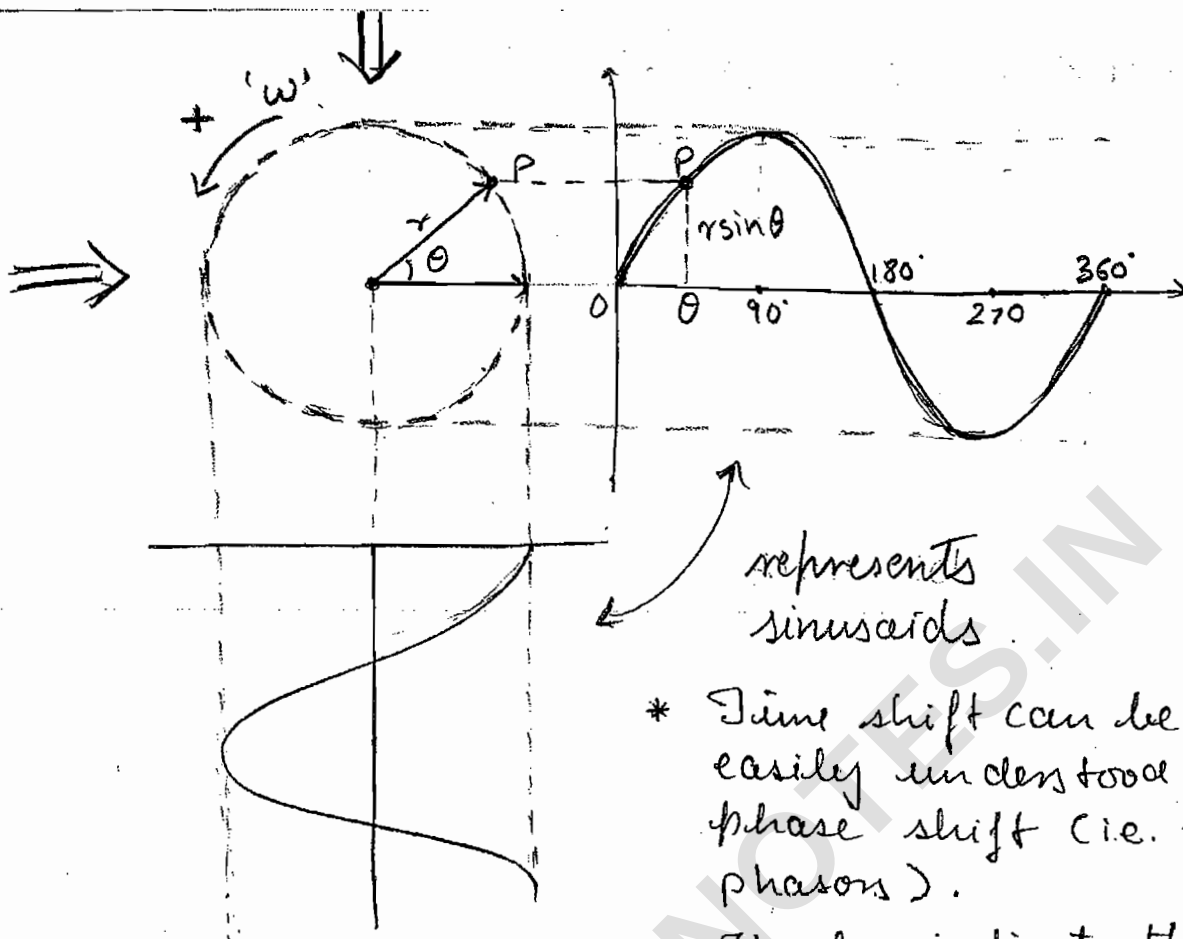
INDIA , $\phi = 60^\circ$

$360^\circ \rightarrow 20 \text{ msec}$

$60^\circ \rightarrow t_{\text{shift}}$

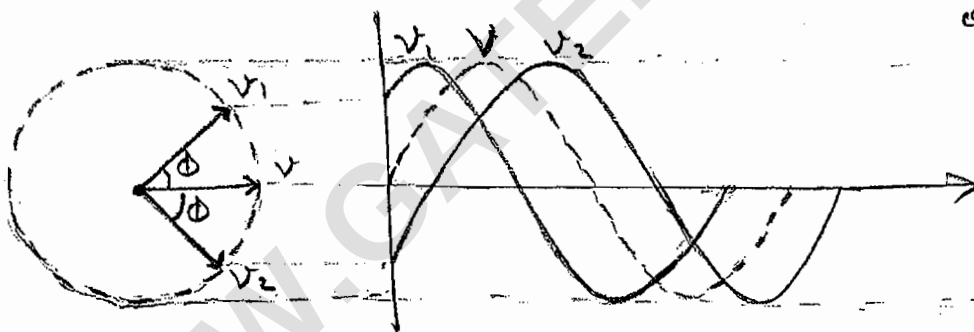
$$t_{\text{shift}} = \frac{60}{360} \times 20 \text{ msec} = 3.33 \text{ msec}$$

$$\omega = 2\pi f = 2\pi(50) = 100\pi \text{ rad/sec}$$



* Time shift can be easily understood from phase shift (i.e. using phasors).

It also indicates the amt of shift.

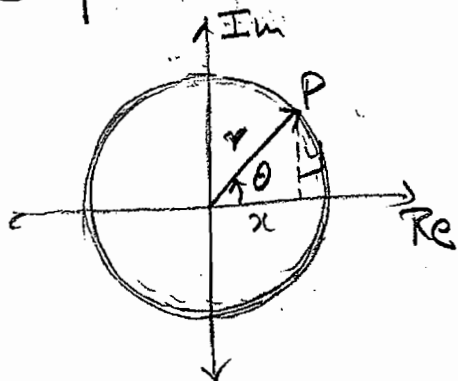


$$V = |V_m| \angle 0^\circ$$

$$V_1 = |V_m| \angle \phi$$

$$V_2 = |V_m| \angle -\phi$$

S-plane



$$[P] = x + jy \rightarrow \text{Rectangular form}$$

$$[P] = r \angle \theta \rightarrow \text{Polar form}$$

$$[P] = r e^{j\theta} \rightarrow \text{Euler's form}$$

$$\boxed{R} \longrightarrow \boxed{P}$$

$$0 + j0 \longrightarrow 0 \angle 0^\circ$$

$$1 + j0 \longrightarrow 1 \angle 0^\circ$$

$$\boxed{0 + j1} \longrightarrow \boxed{1 \angle 90^\circ}$$

$$1 + j1 \longrightarrow \sqrt{2} \angle 45^\circ$$

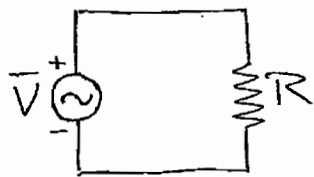
$$0 - j1 \longrightarrow 1 \angle -90^\circ$$

$j \rightarrow$ powerful operator

$+j \rightarrow$ leads

$-j \rightarrow$ lags

Phasor relationship b/w voltages & Current in passive elements :—

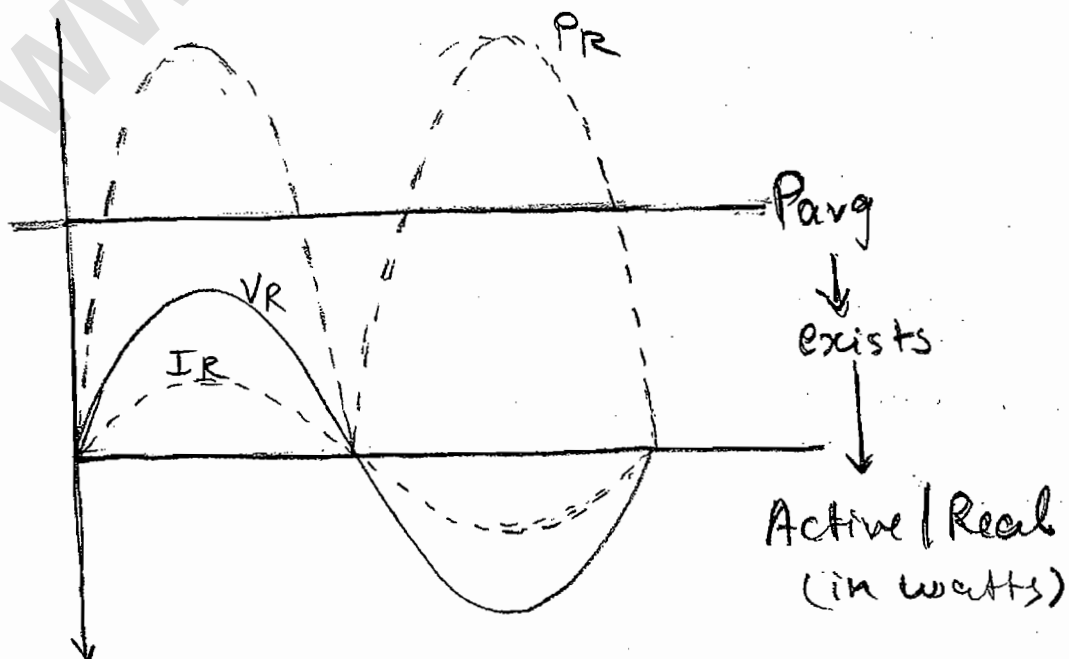


$$\text{Let } \bar{V} = V_m \sin \omega t \quad \text{--- (1)}$$

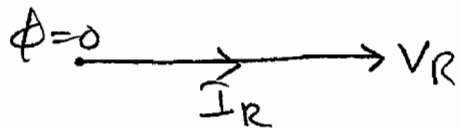
$$\boxed{V = IR}$$

$$\therefore \bar{I} = \frac{\bar{V}}{R} = \frac{V_m \sin \omega t}{R} = \frac{V_m}{R} \sin \omega t$$

$$\therefore \bar{I} = I_m \sin \omega t \quad \text{--- (2)}$$



Phasor diagram



Power factor

$$\cos \phi = \cos 0^\circ = 1 \text{ (UPF)}$$

$$P_{avg} = \frac{1}{T} \int_0^T v(t) i(t) dt$$

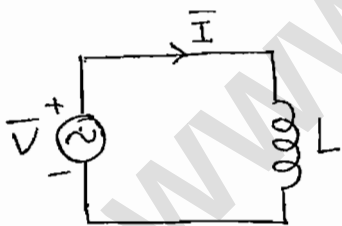
$$\begin{aligned} P_{avg} &= V_m \sin \omega t * I_m \sin \omega t \\ &= V_m I_m \left(\frac{1 - \cos 2\omega t}{2} \right) \\ &= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\omega t}{2} \end{aligned}$$

$\xrightarrow{\quad} 0$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}}$$

$$P_{avg} = V_{RMS} \cdot I_{RMS} \text{ watts.}$$

Inductor:—



$$\text{Let } \bar{I} = I_m \sin \omega t \quad \text{--- (1)}$$

$$V = L \frac{dI}{dt}$$

$$V = L \frac{d(I_m \sin \omega t)}{dt} = \omega L I_m \cos \omega t$$

$$= \omega L I_m \sin(\omega t + 90^\circ) \quad \text{--- (2)}$$

$$= \omega L I_m \sin(\omega t) [j]$$

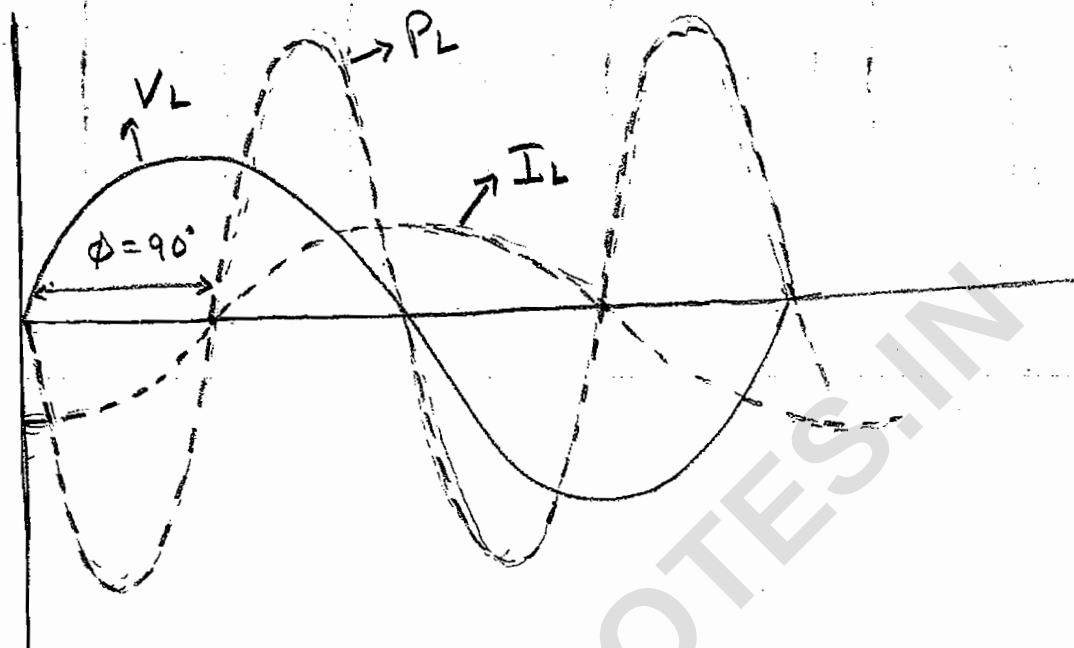
$$\bar{V} = j\omega L \bar{I}$$

Voltage is "j operator" times the current.

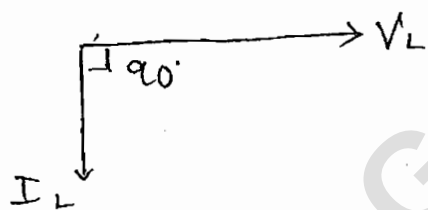
$$\bar{V} = +j X_L \bar{I}$$

i.e. V leads I by 90°

Where $X_L = \omega L = 2\pi fL$
 $\downarrow$
 inductive reactance.



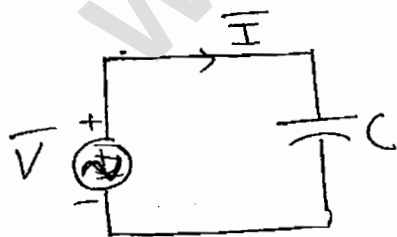
Phasor diag



Power factor

$$PF = \cos \phi = \cos 90^\circ = 0$$

Capacitor



Let $\bar{V} = V_m \sin \omega t$ — (1)

$$\boxed{I = C \frac{dV}{dt}}$$

$$I = C \frac{d}{dt} (V_m \sin \omega t) = C \cdot \omega V_m \cos \omega t$$

$$= \omega C V_m \sin (\omega t + 90^\circ) \text{ — (2)}$$

$$= \omega C V_m \sin \omega t [j]$$

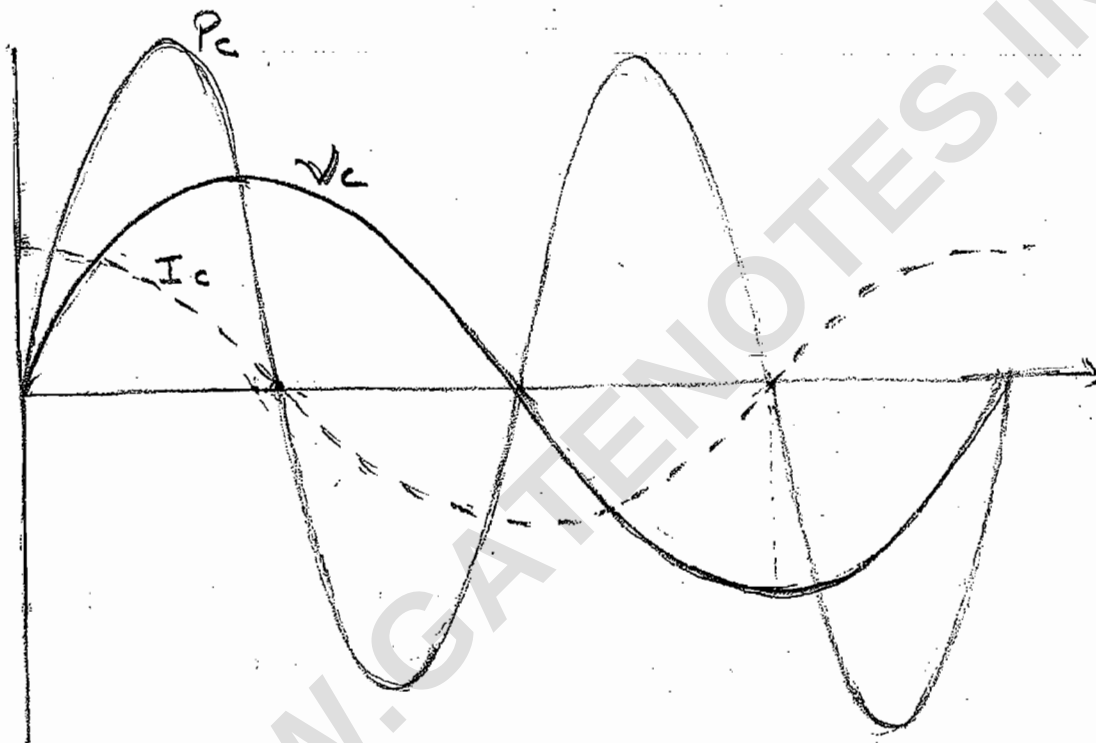
$$I = j \omega C V$$

$$\bar{V} = \frac{\bar{I}}{j\omega C} = \frac{-j}{\omega C} \bar{I}$$

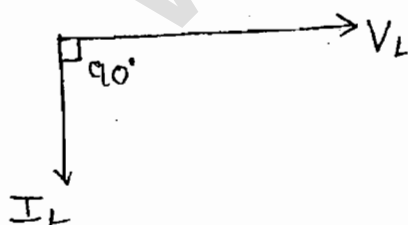
$$\bar{V} = -j X_C \bar{I}$$

where, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

↓
capacitive reactance



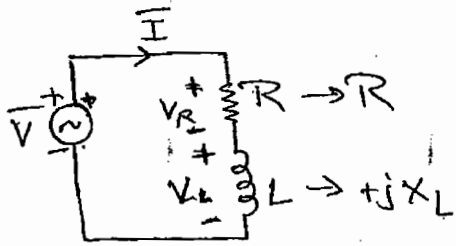
Phasor diag



Power factor

$$P.F. = \cos \phi = \cos 90^\circ = 0$$

Series R-L



KVL

$$-\bar{V} + \bar{V}_R + \bar{V}_L = 0$$

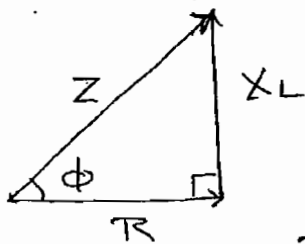
$$\bar{V} = \bar{I}R + jX_L \bar{I}$$

$$\bar{V} = \bar{I}[R + jX_L]$$

$$\boxed{\bar{V} = \bar{I} Z}$$

; $Z = R + jX_L$
 ↓
 impedance (Ω)

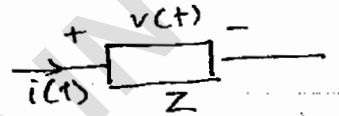
Impedance Δle



$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1} \left[\frac{X_L}{R} \right] = \tan^{-1} \left[\frac{\omega L}{R} \right]$$

↓
Impedance of ckt



Power factor

$$\cos \phi = \frac{R}{|Z|}$$

$$\sin \phi = \frac{X_L}{|Z|}$$

$$\bar{I}^2 Z = \bar{I}^2 R + j X_L \bar{I}^2$$

$$V \bar{I} = \bar{I}^2 R + j \bar{I}^2 X_L$$

Generate Do business Control

$$\boxed{S = P + j Q_L}$$

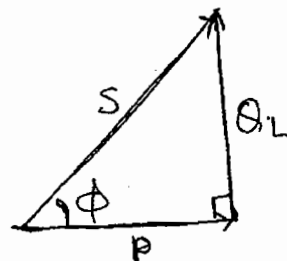
In AC ckt, all the convertible part of energy is represented by Resistance
 eg: electricity → light
 → heat
 → sound

→ Reactive power (VAR's)
 (flux, charge)
 → Active/Real/True Power (watts)
 (consumable part of energy)
 → Total/Apparent Power (VA's)
 (associated with source)

Power Δle

$$|S| = \sqrt{P^2 + Q_L^2}$$

$$\phi = \tan^{-1} \left[\frac{Q_L}{P} \right]$$



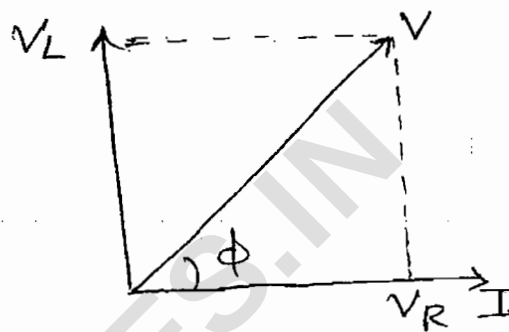
$$\cos \phi = \frac{P}{S} \Rightarrow P = \cos \phi \times S = VI \cos \phi \text{ watts}$$

$$\sin \phi = \frac{Q_L}{S} \Rightarrow Q_L = S \sin \phi = VI \sin \phi \text{ VAR's}$$

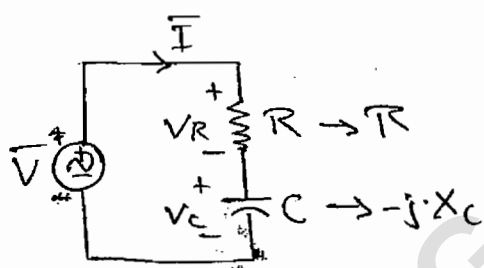
Here $V, I \rightarrow$ rms value

Phasor diagram :

I lags V by $\phi < 90^\circ$



Series RC



KVL $-\bar{V} + \bar{V}_R + \bar{V}_C = 0$

$$\bar{V} = \bar{I}R - jX_C \bar{I}$$

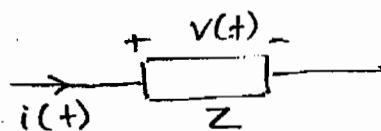
$$\bar{V} = \bar{I} [R - jX_C]$$

$$\boxed{\bar{V} = \bar{I} Z}$$

Here,

$$Z = R - jX_C$$

↓
impedance (Ω)

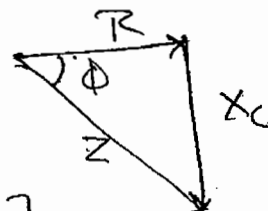


Impedance Δ le :-

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = \tan^{-1} \left[\frac{X_C}{R} \right] = \tan^{-1} \left[\frac{1}{\omega RC} \right]$$

↓
impedance of ckt



Power factor :-

$$\cos \phi = \frac{R}{|Z|}$$

$$\sin \phi = \frac{X_C}{|Z|}$$

$$IZ^2 = I^2 R - j X_c I^2$$

$$VI = I^2 R - j I^2 X_c$$

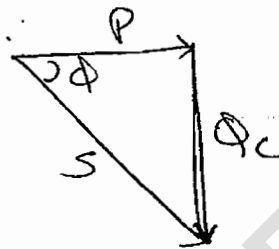
$$\boxed{S = P - j Q_c}$$

→ Reactive power (VAR's)

Power Ale

$$|S| = \sqrt{P^2 + Q_c^2}$$

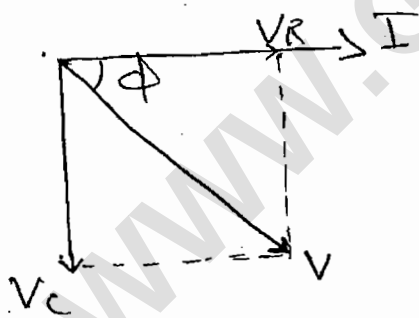
$$\phi = \tan^{-1} \left[\frac{Q_c}{P} \right]$$



$$\cos \phi = \frac{P}{S} \Rightarrow P = S \cos \phi = VI \cos \phi \text{ (watts)}$$

$$\sin \phi = \frac{Q_c}{S} \Rightarrow Q_c = S \sin \phi = VI \sin \phi \text{ (VAR's)}$$

$V, I \rightarrow$ rms value



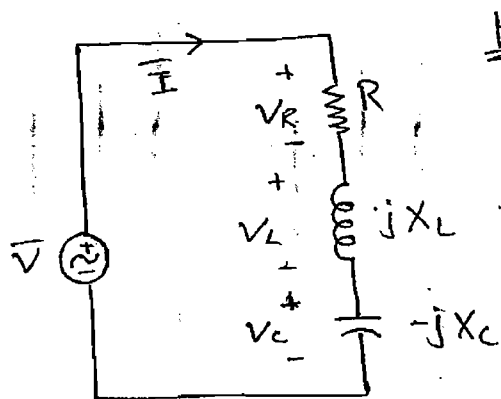
Phasor diag.

I leads V by $\phi < 90^\circ$

$\oplus Q_L \rightarrow$ absorbing VAR's
(lagging VAR's)

$\ominus Q_c \rightarrow$ generating VAR's
(leading VAR's)

R-L-C circuit



KVL $-\bar{V} + \bar{V}_R + \bar{V}_L + \bar{V}_C = 0$

$$\bar{V} = \bar{I}R + jX_L\bar{I} - jX_C\bar{I}$$

$$\bar{V} = \bar{I} [R + j(X_L - X_C)]$$

$$V = IZ$$

Here $Z = R + j[X_L - X_C]$

$X_{\text{net}} \rightarrow$ net reactances

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \left[\frac{X_L - X_C}{R} \right]$$

$\downarrow$
net impedance angle

Power factor

$$\cos \phi = \frac{R}{|Z|}$$

$$\sin \phi = \frac{|X_L - X_C|}{|Z|}$$

Case 1 If $X_L > X_C \Rightarrow$ (General nature of electrical sys.)

$$Z = R + jX_{\text{net}}$$

$\hookrightarrow$ series RL ckt

I lags V by $\phi < 90^\circ$ (lagging PF)

Case 2 If $X_L < X_C$

$$Z = R - jX_{\text{net}}$$

$\hookrightarrow$ series RC ckt

I leads V by $\phi < 90^\circ$ (leading PF)

Case 3 If $X_L = X_C$

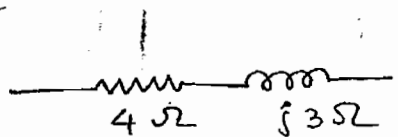
$$Z = R \Rightarrow \text{purely resistive}$$

I in phase with V $\Rightarrow \phi = 0^\circ$, P.F. = 1 (V.P.F.)

$$Z = R \pm jX$$

$$\begin{aligned} \rightarrow Z &= R + jX_L \\ \rightarrow Z &= R - jX_C \end{aligned}$$

Ex:



$$Z = 4 + j3$$

$$Y = \frac{1}{Z} \rightarrow \text{admittance (S) or } S$$

$$Y = \frac{1}{4 + j3} \times \frac{4 - j3}{4 - j3} = \frac{4 - j3}{25}$$

$$\therefore Y = (0.16 - j0.12) \text{ S}$$

$$Y = G \pm jB$$

$$\rightarrow Y = G - jB_L$$

$$\text{where } B_L = \frac{1}{X_L} = \frac{1}{\omega L} = \frac{1}{2\pi f L}$$

$\rightarrow$ inductive susceptance (S)

$$\rightarrow Y = G + jB_C$$

$$\text{where } B_C = \frac{1}{X_C} = \frac{1}{1/\omega C} = \omega C = 2\pi f C$$

$\rightarrow$ capacitive susceptance (S)

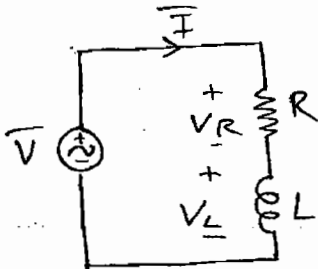
impedance $\rightarrow$ admittance

reactance $\rightarrow$ susceptance

Phasor diagrams : —

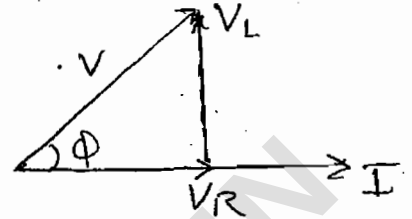
(1) Series circuits $\Rightarrow I \rightarrow \text{'ref'}$

(a) Series R-L ckt :



$$V_R = IR \angle 0^\circ$$

$$V_L = +j X_L I \\ = I \cdot X_L \angle +90^\circ$$

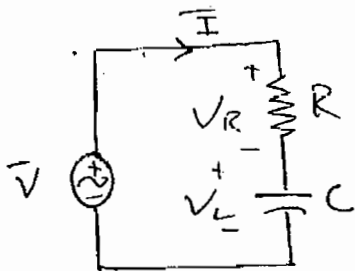


$$|V| = \sqrt{V_R^2 + V_L^2}$$

$$\phi = \tan^{-1} \left(\frac{V_L}{V_R} \right)$$

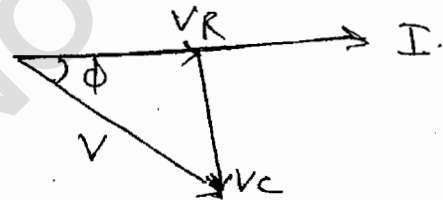
$$\cos \phi = \frac{V_R}{V} \quad (\text{lagging})$$

(b) Series R-C ckt



$$V_R = IR \angle 0^\circ$$

$$V_C = -j X_C I \\ = X_C I \angle -90^\circ$$

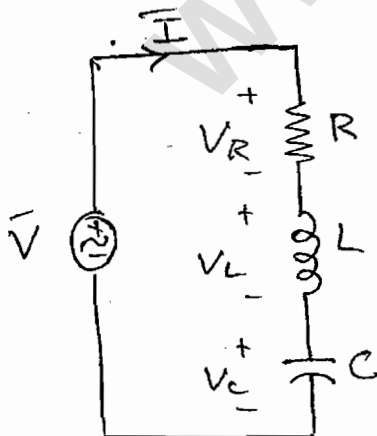


$$|V| = \sqrt{V_R^2 + V_C^2}$$

$$\phi = \tan^{-1} \left(\frac{V_C}{V_R} \right)$$

$$\cos \phi = \frac{V_R}{V} \quad (\text{leading})$$

(c) Series R-L-C



$$V_R = IR \angle 0^\circ$$

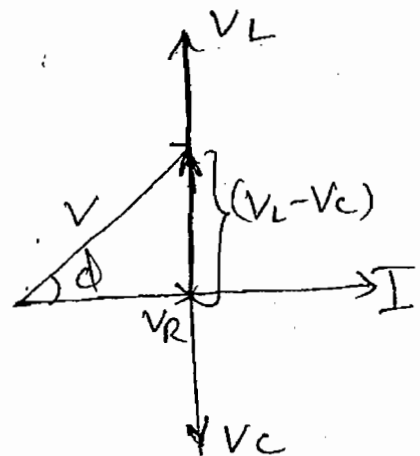
$$V_L = I X_L \angle 90^\circ$$

$$V_C = I X_C \angle -90^\circ$$

$$|V| = \sqrt{V_R^2 + (V_L - V_C)^2}$$

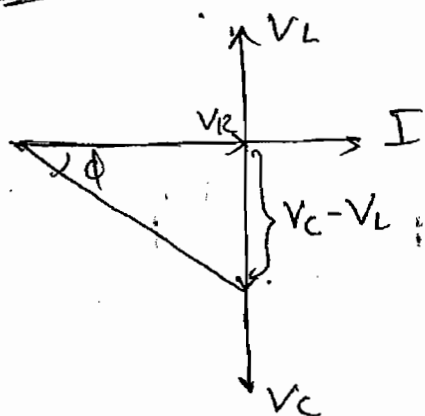
$$\phi = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

$$\cos \phi = \frac{V_R}{V} \quad (\text{lagging})$$



(Case 1)

Case 2: $X_L < X_C \rightarrow V_L < V_C$

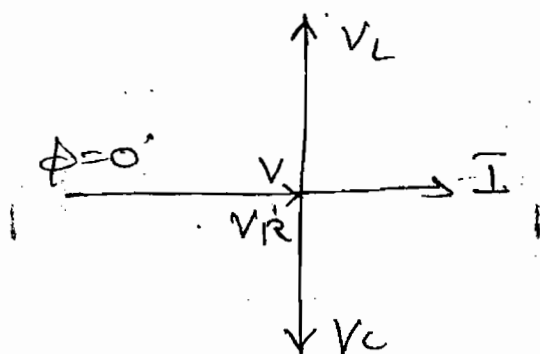


$$|V| = \sqrt{V_R^2 + (V_C - V_L)^2}$$

$$\phi = \tan^{-1} \left(\frac{V_C - V_L}{V_R} \right)$$

$$\cos \phi = \frac{V_R}{V} \text{ (leading)}$$

Case 3: $X_L = X_C \rightarrow V_L = V_C$



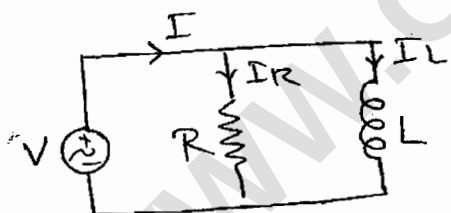
$$|V| = V_R$$

$$\phi = 0^\circ$$

$$\text{P.F.} = \cos \phi = \cos 0^\circ = 1 \text{ (UPF)}$$

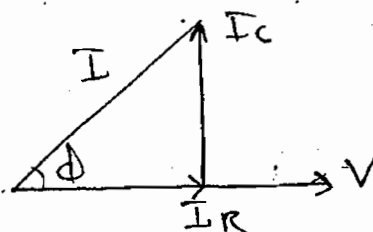
(2) Parallel ckts $\Rightarrow V \rightarrow \text{'ref'}$

(a) Parallel R-L



$$I_R = \frac{V}{R} \angle 0^\circ$$

$$I_L = \frac{V}{jX_L} = \frac{V}{X_L} \angle -90^\circ$$

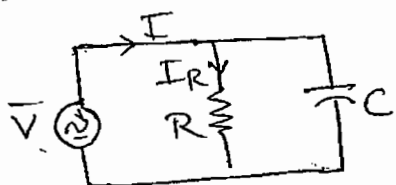


$$|I| = \sqrt{I_R^2 + I_L^2}$$

$$\phi = \tan^{-1} \left(\frac{I_L}{I_R} \right)$$

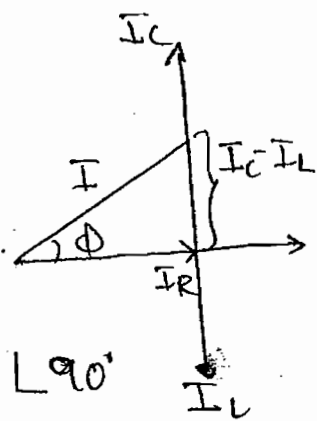
$$\cos \phi = I_R / I \text{ (lagging)}$$

(b) Parallel R-C



$$I_R = \frac{V}{R} \angle 0^\circ$$

$$I_C = \frac{V}{-jX_C} = \frac{V}{X_C} \angle 90^\circ$$

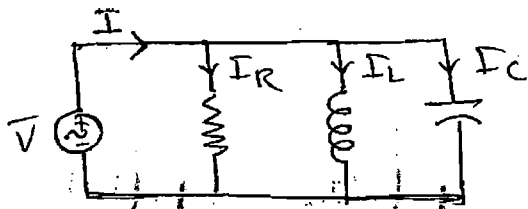


$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1} \left(\frac{I_C - I_L}{I_R} \right)$$

$$\cos \phi = \frac{I_R}{I} \text{ (leading)}$$

(c) Parallel R-L-C

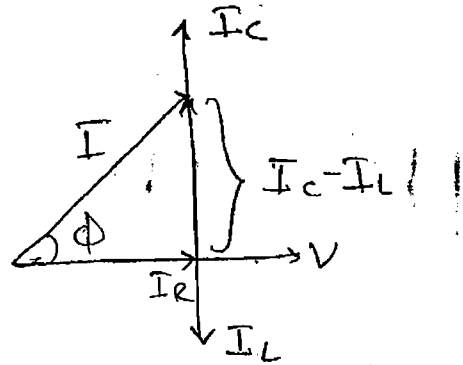


$$I_R = \frac{V}{R} \angle 0^\circ$$

$$I_L = \frac{V}{X_L} \angle -90^\circ$$

$$I_C = \frac{V}{X_C} \angle 90^\circ$$

Case (1) : $X_L > X_C \Rightarrow I_L < I_C$

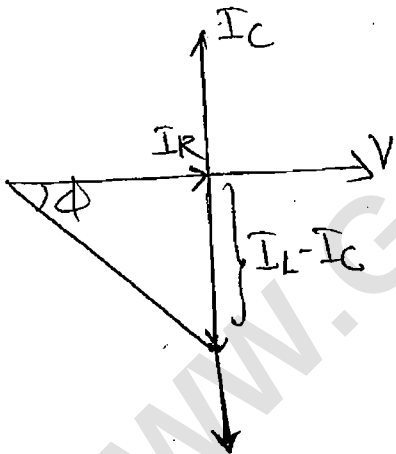


$$|I| = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$\phi = \tan^{-1} \left[\frac{I_C - I_L}{I_R} \right]$$

$$\cos \phi = \frac{I_R}{I} \quad (\text{leading})$$

Case (2) : $X_L < X_C \Rightarrow I_L > I_C$



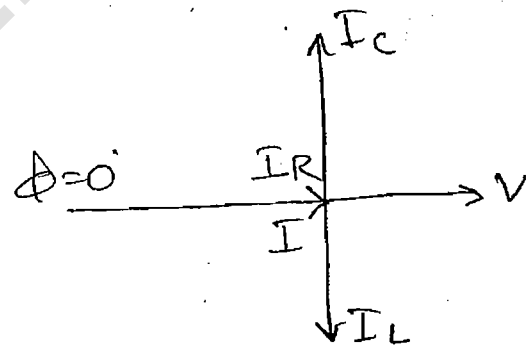
$$|I| = \sqrt{I_R^2 + (I_L - I_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{I_L - I_C}{I_R} \right)$$

$$\cos \phi = \frac{I_R}{I}$$

(lagging)

Case (3) : $X_L = X_C \Rightarrow I_L = I_C$



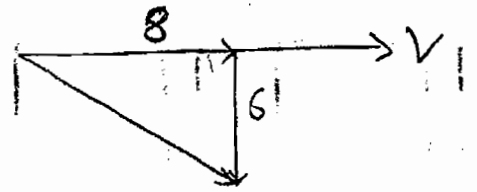
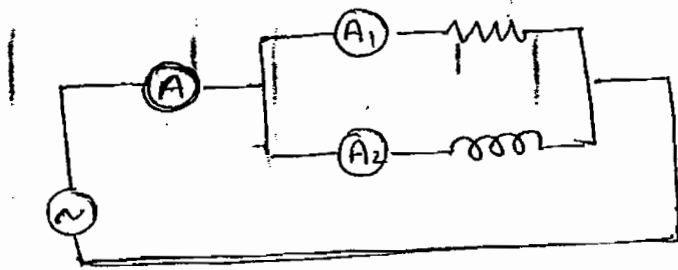
$$|I| = I_R$$

$$\phi = 0^\circ$$

$$\cos \phi = \cos 0^\circ = 1 \quad (\text{UPF})$$

1) If $A_1 \rightarrow \text{reads} \rightarrow 8A$
 $A_2 \rightarrow \text{reads} \rightarrow 6A$
 $A \rightarrow \text{reads} = \underline{\hspace{2cm}}$

ckt P.F. = $\underline{\hspace{2cm}}$

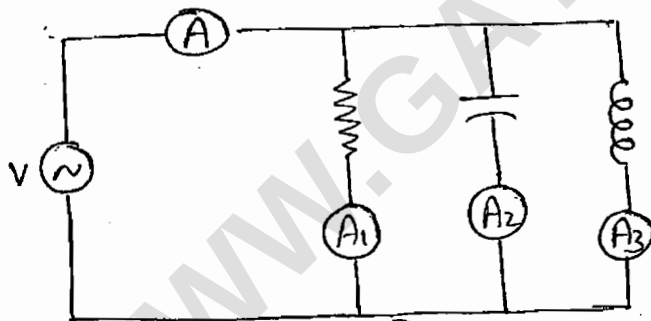


$$I = \sqrt{I_R^2 + I_L^2} = \sqrt{64 + 36} = \sqrt{100} = 10$$

$$\text{P.F.} = \frac{I_R}{I} = \frac{8}{10} = 0.8 \text{ (lagging)}$$

2) If $A_1 \rightarrow \text{reads} \rightarrow 6A$
 $A_2 \rightarrow \text{reads} \rightarrow 18A$
 $A_3 \rightarrow \text{reads} \rightarrow 10A$
 $A \rightarrow \text{reads} = \underline{\hspace{2cm}}$

ckt P.F. = $\underline{\hspace{2cm}}$



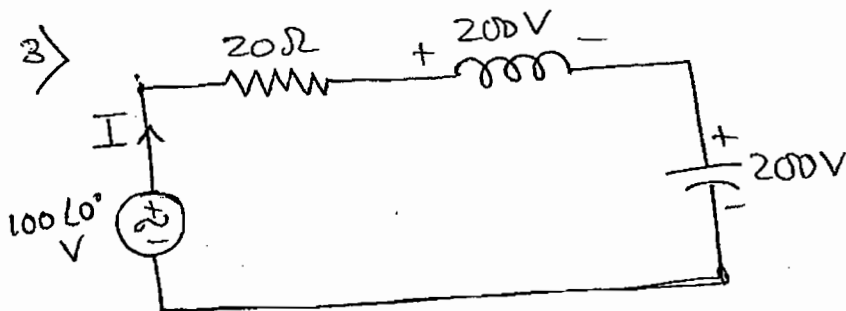
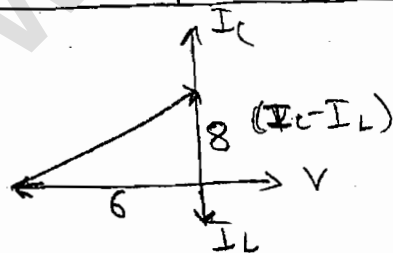
$$I = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$= \sqrt{36 + 64}$$

$$= \sqrt{100} = 10$$

$$\text{P.F.} = \frac{I_R}{I} = \frac{6}{10}$$

$$= 0.6 \text{ (leading)}$$



Find $I = \underline{\hspace{2cm}}$

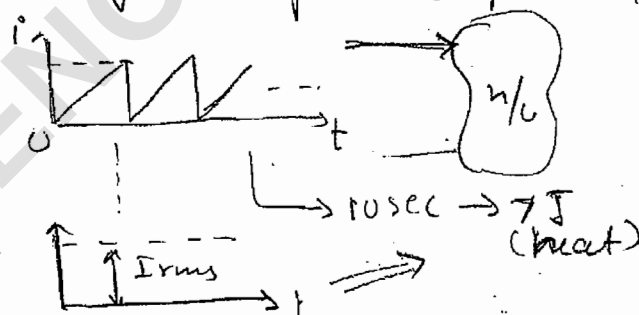
ckt PF = $\underline{\hspace{2cm}}$

$\phi = 0$
 $I_R = \frac{V_R}{R} = \frac{100 \angle 0^\circ}{20} = 5 \text{ A} = I$
 $\cos \phi = \cos 0 = 1 \text{ (UPF)}$

RMS value / True / Effective value : —

It is that steady value of a time varying vltg or current waveform which could produce the same ^{value} of heat as given by the original waveform for definite period of time.

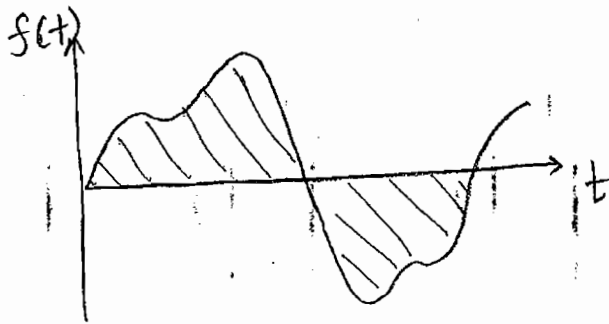
$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T [v(t)]^2 dt}$$



Average value / Mean value

It is that steady value of equivalent value of time varying vltg or current waveform which could develop same amount of charge as given by the original waveform for a definite period of time in a ckt.

Concept of Symmetry :-

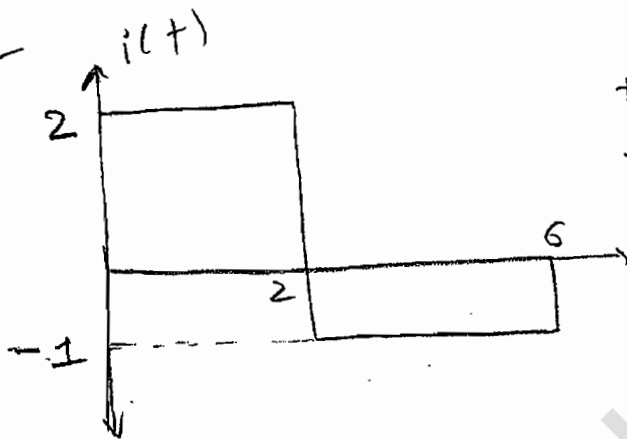


$$|A_1| = |A_2| \Rightarrow \text{Symmetrical}$$

$$|A_1| \neq |A_2|$$

$\Rightarrow$ Asymmetrical

Eg:-



$$\text{+ve area} = |2 \times 2| = 4$$

$$\text{-ve area} = |1 \times (-4)| = 4$$

$\Rightarrow$ Symmetrical

The average value of any symmetrical waveform for one full cycle is always zero.

(a) For symmetrical waveform :-

$$V_{\text{avg}} = \begin{cases} 0 & \rightarrow \text{full cycle} \\ \frac{1}{(T/2)} \int_0^{T/2} v(t) dt & \rightarrow \text{half cycle} \end{cases}$$

(b) For asymmetrical waveform :-

$$V_{\text{avg}} = \frac{1}{T} \int_0^T v(t) dt \rightarrow \text{Full cycle}$$

Peak factor / Crest Factor :—

$$= \frac{V_{\max}}{V_{\text{RMS}}}$$

Form Factor / Shape Factor :—

$$= \frac{V_{\text{RMS}}}{V_{\text{avg}}}$$

Peak to Peak value :—

$$V_{\text{p-p}} = |V_{\max} - V_{\min}|$$

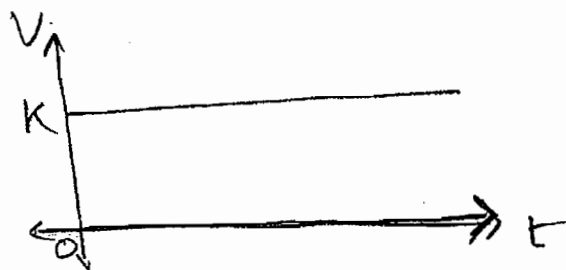
Note:-

Most of our electrical utilities application involves heat generation so we talk RMS values in general.

eg:- 1ϕ , Domestic supply in India
= 230 V $\rightarrow$ RMS value.

However applications like battery charging, electroplating, electrorefining process, etc involves charge, so we calculate "avg" values.

Standard waveforms :—



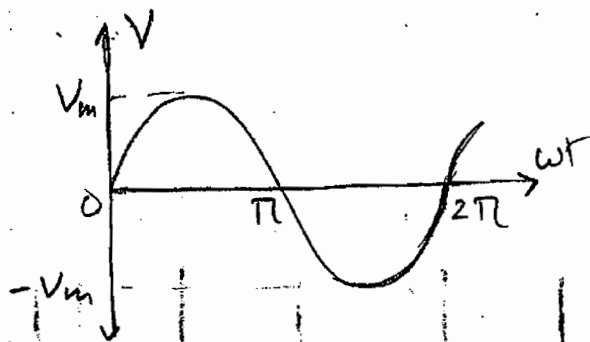
$$V_{\text{RMS}} = K$$

$$V_{\text{avg}} = K$$

$$\text{P.F.} = 1$$

$$\text{F.F.} = 1$$

$$V_{\text{pp}} = K$$



$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

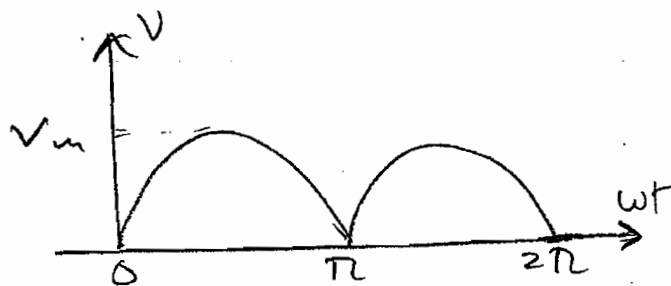
$$V_{avg} = 0 \rightarrow \text{Full cycle}$$

$$= \frac{2V_m}{\pi} \rightarrow \text{Half cycle}$$

$$P.F = \sqrt{2}$$

$$F.F = \frac{\pi}{2\sqrt{2}} = 1.11$$

$$V_{pp} = 2V_m$$



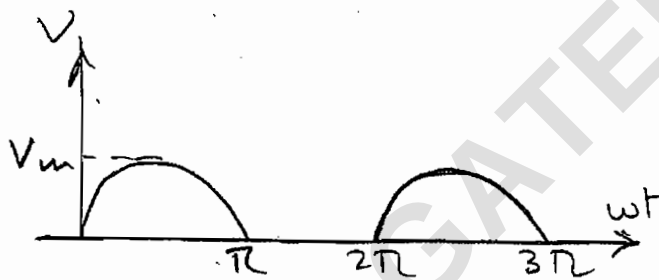
$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_{avg} = \frac{2V_m}{\pi}$$

$$P.F = \sqrt{2}$$

$$F.F = 1.11$$

$$V_{pp} = V_m$$



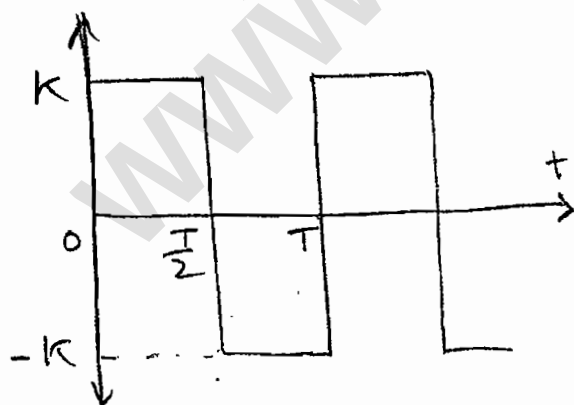
$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$V_{avg} = \frac{V_m}{\pi}$$

$$P.F = 2$$

$$F.F = 1.57$$

$$V_{pp} = V_m$$



$$V_{RMS} = K$$

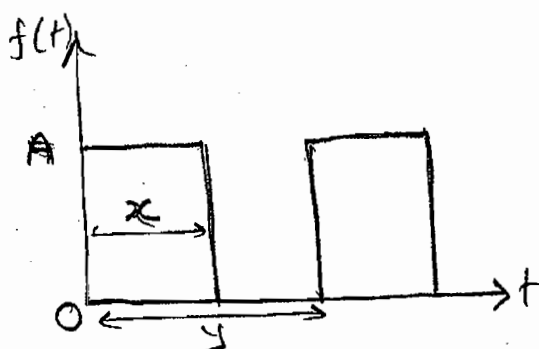
$$V_{avg} = 0 \rightarrow \text{Full cycle}$$

$$= K \rightarrow \text{Half cycle}$$

$$P.F = 1$$

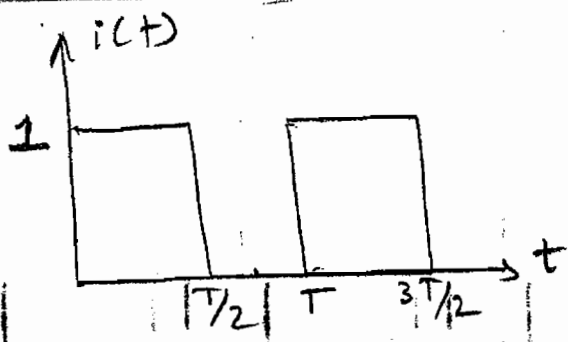
$$F.F = 1.57$$

$$V_{pp} = V_m$$



$$f_{RMS} = A \sqrt{\frac{x}{y}}$$

$$f_{avg} = A \left[\frac{x}{y} \right]$$



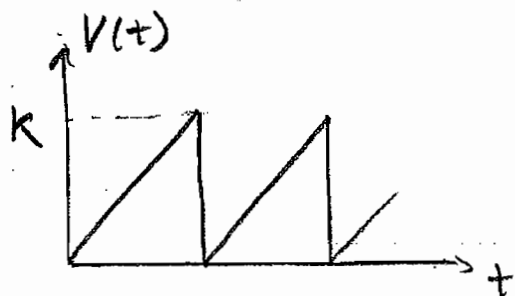
$$I_{RMS} = \frac{I_0}{\sqrt{2}}$$

$$I_{avg} = \frac{I_0}{2}$$

$$P.F. = \sqrt{2}$$

$$F.F. = \sqrt{2}$$

$$I_{pp} = I_0$$



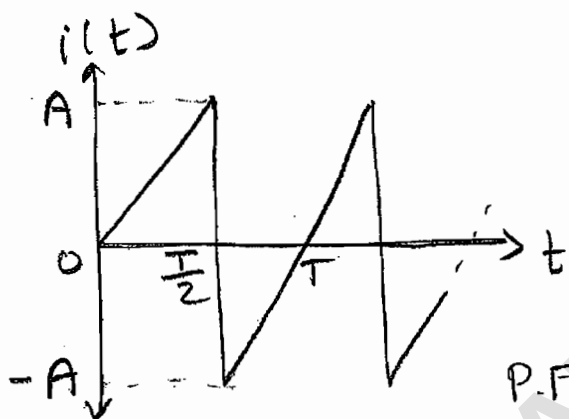
$$V_{RMS} = \frac{k}{\sqrt{2}}$$

$$V_{avg} = \frac{k}{2}$$

$$P.F. = \sqrt{3}$$

$$F.F. =$$

$$V_{pp} = k$$



$$I_{rms} = \frac{A}{\sqrt{3}}$$

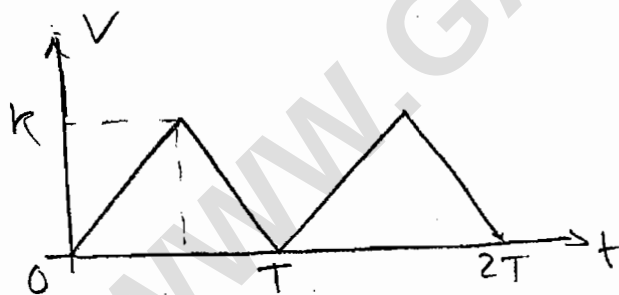
$$I_{avg} = 0 \rightarrow \text{full cycle}$$

$$= \frac{A}{2} \rightarrow \text{half cycle}$$

$$P.F. = \sqrt{3}$$

$$F.F. = \frac{2}{\sqrt{3}}$$

$$I_{pp} = 2A$$



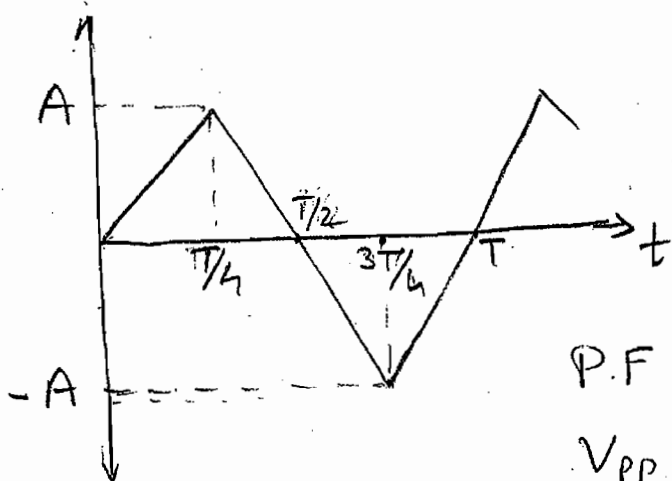
$$V_{RMS} = \frac{k}{\sqrt{3}}$$

$$V_{avg} = \frac{k}{2}$$

$$P.F. = \sqrt{3}$$

$$FF = \frac{2}{\sqrt{3}}$$

$$V_{pp} = k$$



$$V_{RMS} = \frac{A}{\sqrt{3}}$$

$$V_{avg} = 0 \rightarrow \text{Full cycle}$$

$$= \frac{A}{2} \rightarrow \text{Half cycle}$$

$$P.F. = \sqrt{3}$$

$$FF = \frac{2}{\sqrt{3}}$$

$$V_{pp} = 2A$$

NOTE:-

AC Analog Meters

↓
Moving Iron type

↓
RMS values

DC Analog Meters

↓
PMMC Type

↓
Average value

Rectifier Type

$$\left[\begin{array}{c} \text{final} \\ \text{value} \end{array} \right] = \left[\begin{array}{c} \text{Avg.} \\ \text{value} \end{array} \right] \times \underset{\substack{\downarrow \\ \text{1.11}}}{\text{F.F.}} \quad \text{(of sinusoidal waveform)}$$

→ Practical waveforms are not std.

So, we use Fourier series expansion to express these practical vltg & current waveforms in terms of sine (or) cosine.

eg: $v(t) = V_0 + V_1 \sin \omega t + V_2 \sin 2\omega t + \dots$

$$V_{\text{avg}} = V_0$$

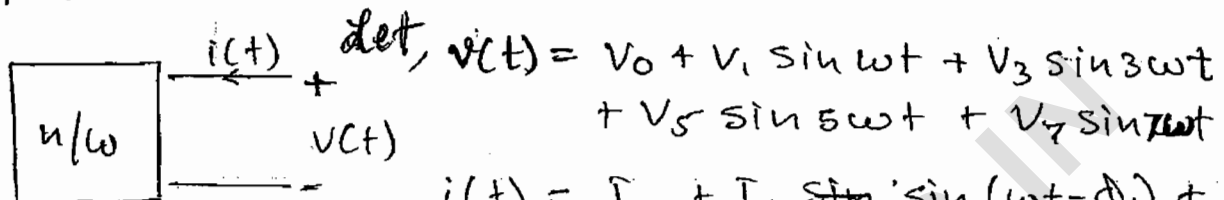
$$V_{\text{rms}} = \sqrt{V_0^2 + \left(\frac{V_1}{\sqrt{2}}\right)^2 + \left(\frac{V_2}{\sqrt{2}}\right)^2 + \dots}$$

$$i(t) = I_0 + I_1 \cos(\omega t - \phi_1) + I_3 \cos(3\omega t + \phi_3) + \dots$$

$$\therefore I_{avg} = I_0$$

$$I_{RMS} = \sqrt{I_0^2 + \left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_3}{\sqrt{2}}\right)^2 + \dots}$$

> Non sinusoidal excitation :-



$$i(t) = I_0 + I_1 \sin(\omega t - \phi_1) + I_3 \sin(3\omega t + \phi_3) + I_7 \cos(7\omega t - \phi_7) + I_7 \sin(90^\circ + 7\omega t - \phi_7)$$

What is avg. power in n/w ?

NOTE:-

If freq. are same, don't use fourier series concept.

$$v(t) = V_1 \sin(\omega t + \phi) + V_2 \sin(\omega t - \phi)$$

$$= \underbrace{V_1 \angle \phi + V_2 \angle -\phi}_{\text{phasor addition}}$$

$$\Rightarrow V_{RMS} = \frac{V_{max}}{\sqrt{2}}$$

Solⁿ

$$P_{avg} = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt$$

$$P_{avg} = V_0 I_0 + \frac{V_1}{\sqrt{2}} \cdot \frac{I_1}{\sqrt{2}} \cos \phi_1 + \frac{V_3}{\sqrt{2}} \cdot \frac{I_3}{\sqrt{2}} \cos \phi_3 + 0 + \frac{V_7}{\sqrt{2}} \cdot \frac{I_7}{\sqrt{2}} \cos(90 - \phi_7)$$

Avg. power is avg value of
as power waveform by itself the product
of vltg & current waveform.

This avg power exist in resistive part of
the n/w which is the consumable/
convertible part.

So, simply avg power means
active power in watts.

→ From power the avg power/active
power for any combination of
load can be expressed as.

$$P = V_{rms} \cdot I_{rms} \cos \phi$$

$$\begin{aligned} 2) \quad v(t) &= 10 + 5\sqrt{2} \cos \omega t + 3\sqrt{2} \sin 3\omega t \\ v(t) &= 10 + 5\sqrt{2} \sin(90^\circ + \omega t) + 3\sqrt{2} \sin 3\omega t \end{aligned}$$

$$\begin{aligned} V_0 &= 10 \\ V_{RMS} &= \sqrt{100 + (5)^2 + (3)^2} \end{aligned}$$

$$= \sqrt{134} = 11.57$$

$$V_{avg} = 10$$

$$3) i(t) = 10 + 7 \cos(\omega t - 10^\circ) + 5 \cos[3\omega t + 30^\circ]$$

$$I_{avg} = 10$$

$$I_{rms} = \sqrt{(10)^2 + \left(\frac{7}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2}$$

$$= \sqrt{137} = \underline{11.7}$$

$$4) i(t) = 20 + 10 \sin(\omega t - 30^\circ) + 7 \cos(\omega t + 40^\circ)$$

$$= 20 + 10 \sin(\omega t - 30^\circ) + 7 \sin(\omega t + 130^\circ)$$

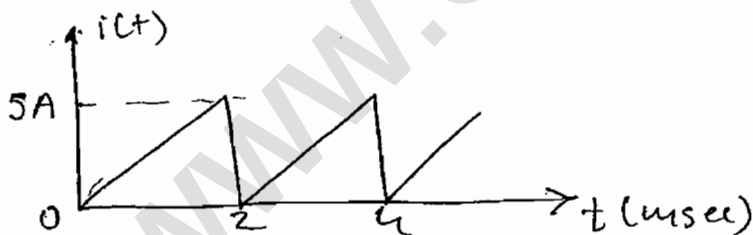
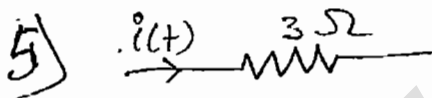
$$= 20 + [10 \angle -30^\circ] + [7 \angle +130^\circ]$$

$$\checkmark = 20 + 8.66 - 5j + (-4.5 + 5.36j)$$

$$= 24.16 + 0.36j$$

$$= 20 + [4.17 \angle 4.94^\circ]$$

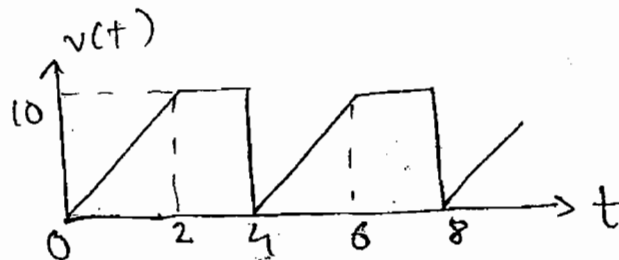
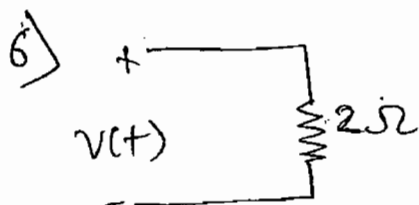
$$I_{rms} = \sqrt{(20)^2 + \left(\frac{4.17}{\sqrt{2}}\right)^2} = \underline{20.21} \quad I_{avg} = 20$$



$$P_{lost} = I_{rms}^2 \times R$$

$$= \left(\frac{5}{\sqrt{3}}\right)^2 \times 3$$

$$= 25W$$



$$0 < t < 2 \quad v(t) = 5t$$

$$2 < t < 4 \quad v(t) = 10$$

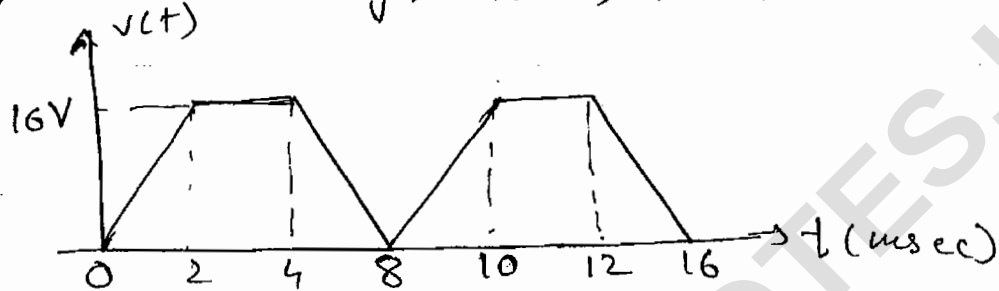
$$P_{lost} = \left[\frac{V_{rms}^2}{R} \right]$$

$$V_{RMS} = \sqrt{\frac{1}{4} \left[\int_0^2 (5t)^2 dt + \int_2^4 (10)^2 dt \right]}$$

$$= \sqrt{\frac{1}{4} \left[25 \left(\frac{8}{3} \right) + 100 \times 2 \right]} = \sqrt{66.66}$$

$$P_{lost} = \frac{(V_{RMS})^2}{2} = \frac{66.66}{2} = 33.33 \text{ W}$$

7) Find V_{avg} , V_{RMS} , P.F., F.F., V_{PP}



→ asymmetrical

→ calculate for full cycle

$$0 < t < 2$$

$$v(t) = 8t$$

$$4 < t < 8$$

$$(y-0) = \frac{(16-0)}{(4-8)} (x-8)$$

$$2 < t < 4$$

$$v(t) = 16$$

$$v(t) = -4t + 32$$

$$V_{avg} = \frac{1}{8} \left[\int_0^2 8t dt + \int_2^4 16 dt + \int_4^8 (-4t + 32) dt \right]$$

$$= \frac{1}{8} [4(2) + 16(2) + 32(4) - 2(48)] = 10 \text{ V}$$

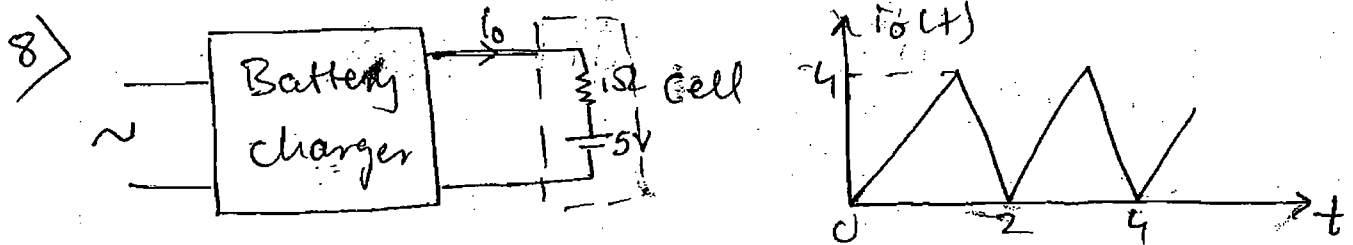
$$V_{RMS} = \sqrt{\frac{1}{8} \left[\int_0^2 (8t)^2 dt + \int_2^4 (16)^2 dt + \int_4^8 (-4t + 32)^2 dt \right]}$$

$$= \sqrt{\frac{1}{8} \left[\frac{64}{3} (8) + (16)^2 (2) + (32)^2 (4) + \frac{16}{2} (48) - 128(48) \right]}$$

$$= 8\sqrt{2}$$

$$PF = \frac{16}{8\sqrt{2}} = \sqrt{2}$$

$$F.F. = \frac{8\sqrt{2}}{10} = 1.13$$



Total power absorbed by cell = ?

$$P_{\text{absorbed}} = (i_{\text{RMS}})^2 R + (i_{\text{avg}}) E$$

$$i_{\text{RMS}} = \frac{4}{\sqrt{3}} \quad i_{\text{avg}} = \frac{4}{2} = 2$$

$$\begin{aligned} \therefore P_{\text{abs}} &= \left(\frac{4}{\sqrt{3}}\right)^2 \times 1 + (2) \times 5 \\ &= \frac{16}{3} + 10 = \frac{46}{3} \\ &= 15.33 \text{ W} \end{aligned}$$

9) If a current of $i(t) = 5 \cos(1000t + 100^\circ)$ A is flowing through an impedance of $(4 + j3)\Omega$ the avg. power is — ?

$$i(t) = 5 \cos(1000t + 100^\circ)$$

$$Z = 4 + j3 \Rightarrow R = 4$$

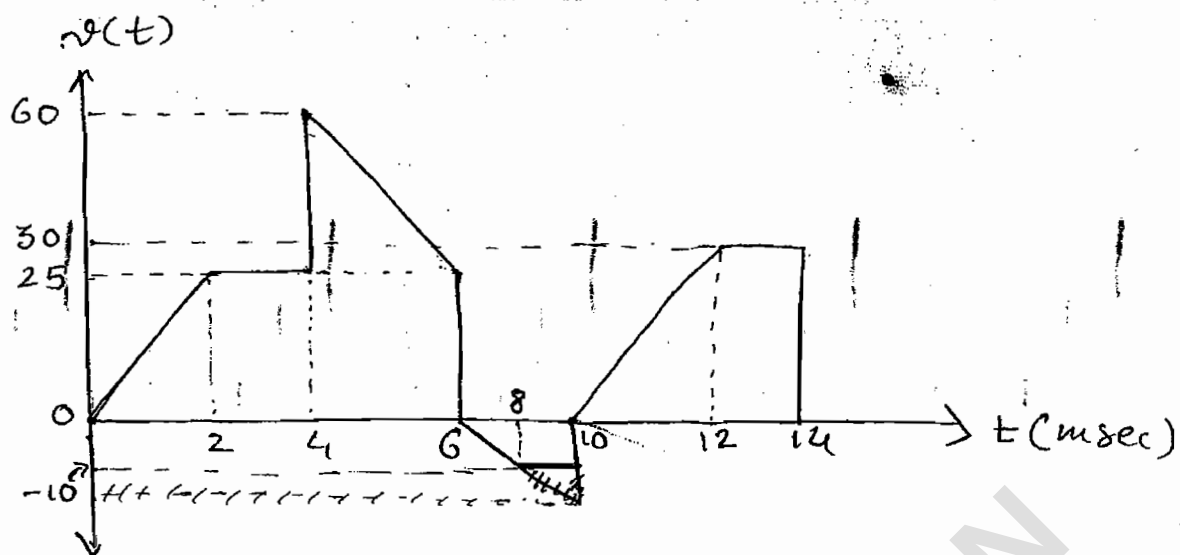
$$X_L = 3$$

$$P_{\text{avg}} = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt \rightarrow \text{exist only for resistive element}$$

active power.

$$\begin{aligned} \therefore P_{\text{avg}} &= (i_{\text{rms}})^2 \times R \\ &= \left(\frac{5}{\sqrt{2}}\right)^2 \times 4 \\ &= \frac{25}{2} \times 4 \\ &= \underline{\underline{50 \text{ W}}} \end{aligned}$$

10)



If the periodic vltg waveform shown is given to a ~~PMMC~~ ^{PMMC} type voltmeter then what is its readings.

PMMC $\rightarrow$ Avg value
provides

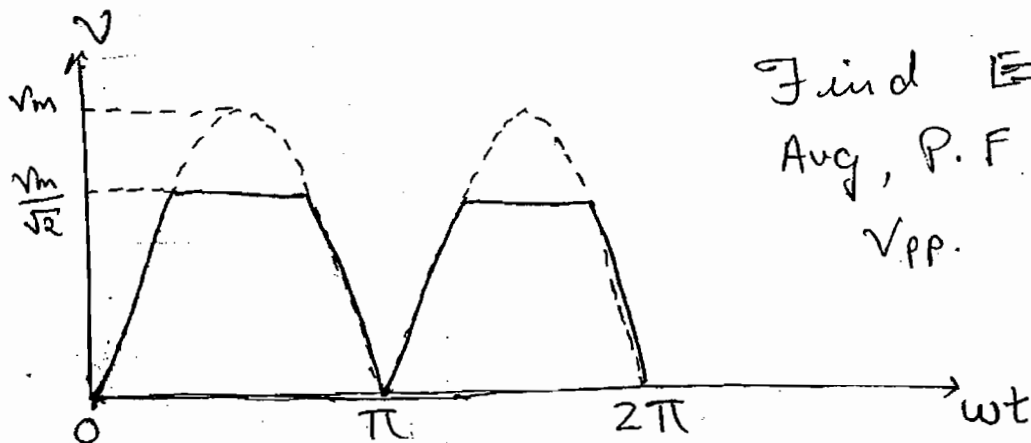
$$\therefore V_{avg} = \frac{1}{T} \int_0^T v(t) dt \quad \text{which is area under the curve } v(t)$$

$$\therefore V_{avg} = \left\{ \frac{1}{2} \times 25 \times 2 + 2 \times 25 + \frac{1}{2} \times 2 \times 35 + 2 \times 25 + \frac{1}{2} \times 2 \times 25 \right\} - \left\{ \frac{1}{2} \times 2 \times 10 + 2 \times 10 \right\}$$

$$\therefore V_{avg} = \text{Net area} = 250 - 30 = 220$$

$$\therefore V_{avg} = \frac{\text{Area}}{T} = \frac{220}{14} = 15.71 \text{ V}$$

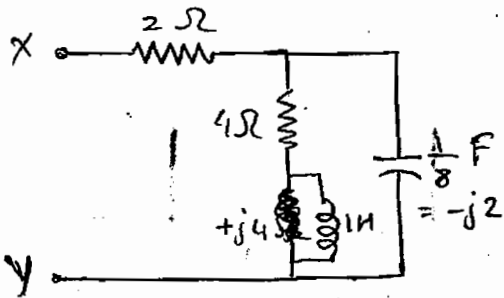
11)



Find ~~EFF~~, RMS, Avg, P.F., FF, V_{pp} .

WWW.GATENOTES.IN

1) If $\omega = 4 \text{ rad/sec}$ find n/w P.F.



$$Z_{eq} = 2 + (4 + j4) \parallel (-j2)$$

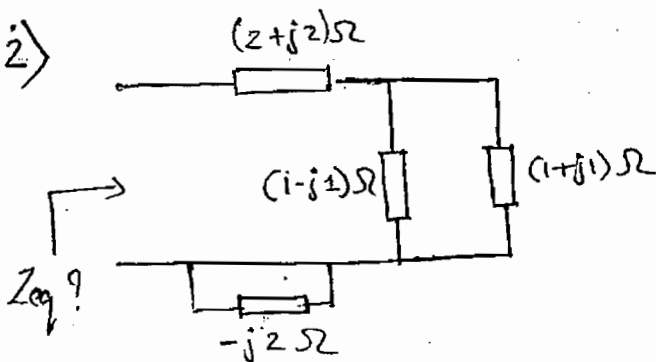
$$= (2.8 - j2.4) \Omega$$

Overall R & n/w

$$\cos \phi = \frac{R}{|Z|} = \frac{2.8}{\sqrt{(2.8)^2 + (2.4)^2}}$$

$$\therefore \text{P.F.} = 0.76 \text{ (leading)}$$

2)

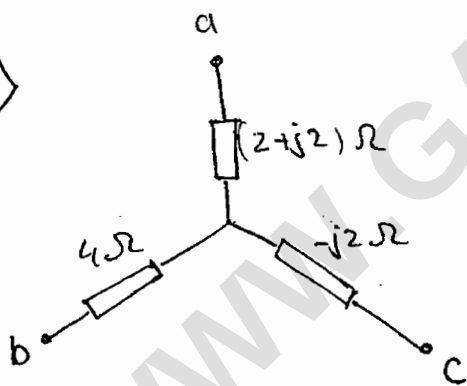


$$Z_{eq} = (2 + j2) + [(1 - j1) \parallel (1 + j1)]$$

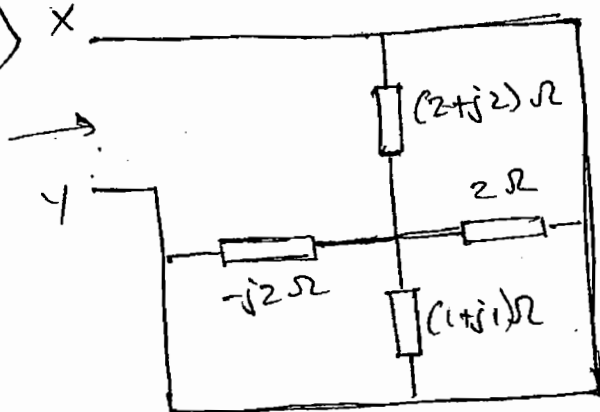
$$= 2 + j2 + 1$$

$$= 3 + j2$$

3)

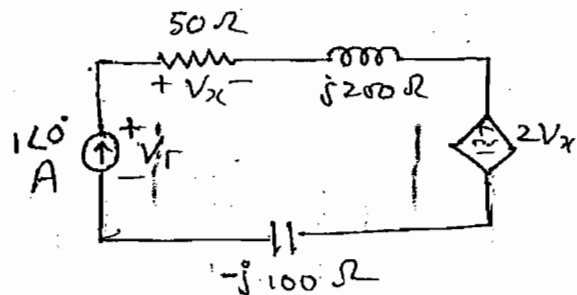
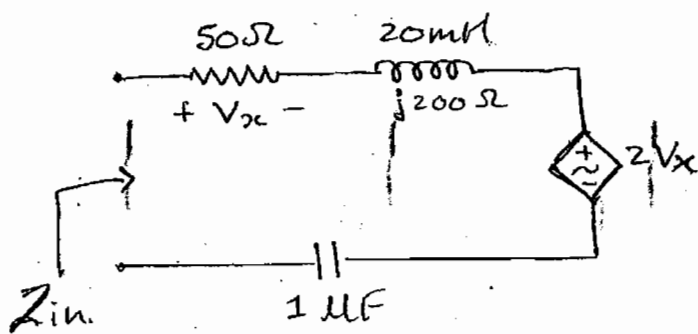


4)



$$Z_{eq} = 0$$

5) $\omega = 10 \text{ k rad/sec}$, $Z_{in} =$ _____

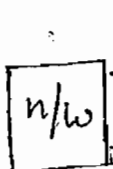


$$\therefore V_{xL} = 50(1) = 50 \text{ V}$$

$$\text{KVL} \quad -V_T + [50 + j200 + -j100](1 \angle 0^\circ) + 100 \angle 0^\circ = 0$$

$$\therefore V_T = 150 + j100$$

$$Z_{in} = \frac{V_T}{I} = (150 + j100) \Omega$$

6)  $v(t) = 20 \cos(100t - 40^\circ)$
 $i(t) = 8 \sin(100t + 10^\circ)$
 Determine n/w elements.

Here,

$$v(t) = 20 \cos(100t - 40^\circ)$$

$$= 20 \sin(100t + 50^\circ) \Rightarrow 20 \angle 50^\circ$$

$$i(t) = 8 \sin(100t + 10^\circ) \Rightarrow 8 \angle 10^\circ$$

$$Z = \frac{V}{I} = \frac{20 \angle 50^\circ}{8 \angle 10^\circ} = 2.5 \angle 40^\circ$$

$$= 1.91 + j1.6 \Omega$$

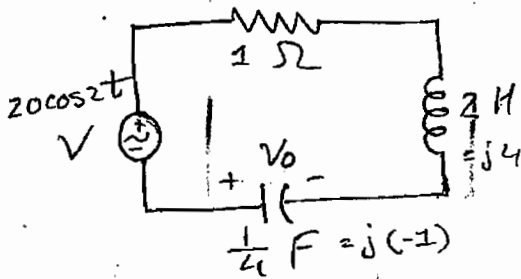
Simple form of n/w could be R-L series ckt

$$\therefore R = 1.91 \Omega$$

$$X_L = 1.6 \Rightarrow \omega L = 1.6 \Rightarrow L = 16 \text{ mH}$$

It can also be a R-L-C n/w when $R \cdot X_L > X_C$

7) Find $V_o =$ _____



Here $V = 20 \cos 2t$

$\therefore \omega = 2, \text{ rad/sec}$

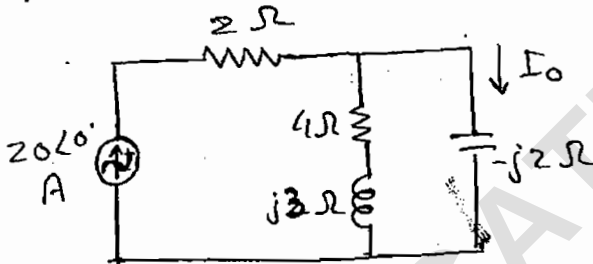
$V_m = 20 \angle 0^\circ$

$V_o = - \left[\frac{20 \angle 0^\circ}{\sqrt{2}} \right] \left[\frac{-j2}{1+j4-j2} \right]$
 $\downarrow$ RMS value V_{rms}

$\therefore V_o = \frac{(10\sqrt{2} \angle 0^\circ)(-j2)}{(1+j2)}$

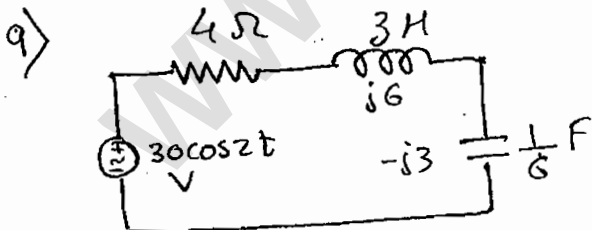
$\therefore V_o = (11.3 + 5.6j) \text{ V}$

8) Find $I_o =$ _____



$I_o = \frac{(20 \angle 0^\circ)(4+j3)}{4+j1} =$

In phasor representation of polar quantity ; $r \angle \theta$; 'r' is clearly representing the max. value.



Find: ① Z ② Y ③ I

④ V_R, V_L, V_C

⑤ verify KVL

⑥ P, Q, S.

⑧ P.F

⑦ Phasor diag.

① $Z = 4 + j8 - j3 = 4 + j5$

$|Z| = 5 \Omega$

② $Y = \frac{1}{4+j5} = \frac{4-j5}{25} = (0.16 - j0.2) \text{ S}$

$\therefore |Y| = \frac{1}{|Z|} = 0.2 \text{ S}$

$$\textcircled{c} \quad \underline{I} = \frac{\frac{30}{\sqrt{2}} \angle 0^\circ}{4 + j3} = \frac{V}{Z} = \frac{30 \angle 0^\circ}{5\sqrt{2} \angle 36.86^\circ}$$

$$\begin{array}{l} \downarrow \\ \text{RMS} \\ \downarrow \\ = I_R = I_L = I_C \end{array} = 4.24 \angle -36.86^\circ \text{ A}$$

→ impedance angle

$$\textcircled{d} \quad V_R = I_R R = (4.24 \angle -36.86^\circ) \times 4$$

$$= 16.96 \angle -36.86^\circ \text{ V}$$

$$V_L = I_L [jX_L] = (4.24 \angle -36.86^\circ) (j6)$$

$$= 25.44 \angle 53.14^\circ \text{ V}$$

$$V_C = I_C (-jX_C) = (4.24 \angle -36.86^\circ) (-j3)$$

$$= \cancel{4.24 \angle -36.86^\circ} 12.72 \angle -126.86^\circ \text{ V}$$

$$\textcircled{e} \quad \bar{V}_S = \bar{V}_R + \bar{V}_L + \bar{V}_C \quad [\text{KVL} \rightarrow \text{RMS value}]$$

$$\underline{\text{RHS}} \quad |\bar{V}_R| + |\bar{V}_L| + |\bar{V}_C|$$

$$= \cancel{16.96} + \cancel{25.44} + \cancel{12.72} = 21.16$$

To be written in rect. form

$$\underline{\text{LHS}} \quad V_S = \frac{30 \angle 0^\circ}{\sqrt{2}} = 15\sqrt{2} = 21.2$$

$$\therefore \text{LHS} = \text{RHS}$$

$\therefore$ KVL verified.

$\textcircled{f}$ For calculating power consider RMS mag. only

$$P = I_R^2 R = \frac{V_R^2}{R} = V_R \cdot I_R$$

$$= \cancel{(16.96)^2} (4.42)^2 = \frac{(16.96)^2}{4} = (16.96)(4.42)$$

$$= 71.91 \text{ watt}$$

$$Q_{\text{net}} = +Q_L - Q_C$$

$$Q_L = I_L^2 \cdot X_L = \frac{V_L^2}{X_L} = V_L \cdot I_L$$

$$= (4.42)^2 (6) = \frac{(25.44)^2}{6} = (25.44)(4.42)$$

$$= 107.86 \text{ VAR (absorbing)}$$

$$Q_C = I_C^2 \cdot X_C = \frac{V_C^2}{X_C} = V_C \cdot I_C$$

$$= (4.42)^2 (3) = \frac{(12.72)^2}{3} = (12.72)(4.42)$$

$$= 53.93 \text{ VAR's (generating)}$$

$$\therefore Q_{\text{net}} = 107.86 - 53.93$$

$$= +53.93 \text{ VAR's (absorbing / lagging)}$$

⑧ Total Power

$$S = V_S \cdot I_S = |I_S|^2 Z = \frac{|V_S|^2}{Z}$$

$$= (21.21)(4.24) = (4.04)^2 (5) = \frac{(21.21)^2}{5}$$

$$= 89.93 \text{ VA's}$$

$$\textcircled{9} \text{ Power factor } = \cos \phi = \frac{R}{Z} = \frac{P}{S}$$

$$= \cos(36.36) = \frac{4}{5} = \frac{71.91}{89.93}$$

$$= 0.8 \text{ (lagging)}$$

Also check;

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

$$|S| = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\phi = \tan^{-1} \left(\frac{Q_L - Q_C}{P} \right)$$

$$P = S \cos \phi = V_S I_S \cos \phi$$

$$Q_{\text{net}} = S \sin \phi = V_S I_S \sin \phi$$

Phasor diagram

$$V_s = 21.21 \angle 0^\circ \leftarrow \text{Ref}$$

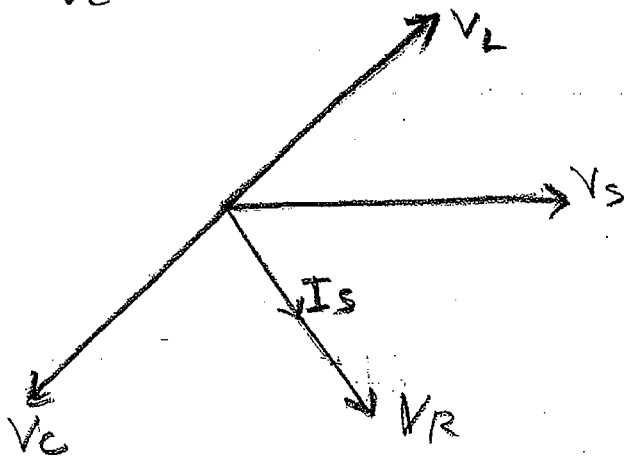
$$I_s = I_R = I = I_C = 4.42 \angle -36.86^\circ \text{ A}$$

$$V_R = 16.96 \angle -36.86^\circ \text{ V}$$

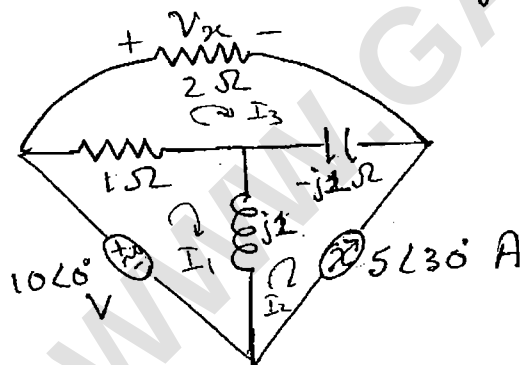
$$V_L = 25.44 \angle 53.14^\circ \text{ V}$$

$$V_C = 12.72 \angle -126.86^\circ \text{ V}$$

Normally we take I as ref. but here we take V_s as ref, becoz all calculations are done w.r.t. V_s .



10) Find V_{oc} using mesh & nodal analysis.



Mesh

$$-[10 \angle 0^\circ] + 1[I_1 - I_3] + j1[I_1 - I_2] = 0$$

$$(1+j)I_1 - jI_2 - I_3 = 10$$

$$I_2 = -[5 \angle 30^\circ] \quad \text{--- (2)}$$

$$2I_3 - j1[I_3 - I_2] + 1[I_3 - I_1] = 0$$

$$\therefore -I_1 + jI_2 + (3-j)I_3 = 0 \quad \text{--- (3)}$$

$$(1+j)I_1 - I_3 = \underbrace{10 - 5 \angle 120^\circ}_{12.5 - j4.33} \quad \text{--- (A)}$$

$$-I_1 + (3-j)I_3 = \underbrace{5 \angle 120^\circ}_{-2.5 + j4.33} \quad \text{--- (B)}$$

$$(A) \Rightarrow (1+j1) I_1 - I_3 = 12.5 - j4.33$$

$$(B) (1+j) \Rightarrow -(1+j1) I_1 + (4+2j) I_3 = -6.83 + j1.83$$

$$(3+j2) I_3 = 5.67 - j2.5$$

$$\therefore I_3 = 1.71 \angle -57.48^\circ \text{ A}$$

$$\text{then, } V_{oc} = 2 I_3 = 3.42 \angle -57.48^\circ \text{ V}$$

Nodal

$$V_1 = 10 \quad \text{--- (1)}$$

$$\frac{V_2 - 10}{1} + \frac{V_2}{j2} + \frac{V_2 - V_3}{-j1} = 0$$

$$V_2 - 10 - jV_2 + j(V_2 - V_3) = 0$$

$$\therefore V_2 - jV_3 = 10 \quad \text{--- (2)}$$

$$- [5 \angle 30^\circ] + \frac{V_3 - V_2}{-j1} + \frac{V_3 - 10}{2} = 0$$

$$j \frac{(V_3 - V_2)}{1} + \frac{(V_3 - 10)}{2} = 5 \angle 30^\circ$$

$$-j2 V_2 + (1+j2) V_3 = 10 + 10 \angle 30^\circ$$

$$= 18.66 + j5 \quad \text{--- (3)}$$

$$(2) \times j2 \Rightarrow j2 V_2 + 2 V_3 = j20$$

$$(3) \Rightarrow -j2 V_2 + (1+j2) V_3 = 18.66 + j5$$

$$V_3 = \frac{18.66 + j5}{3 + j2}$$

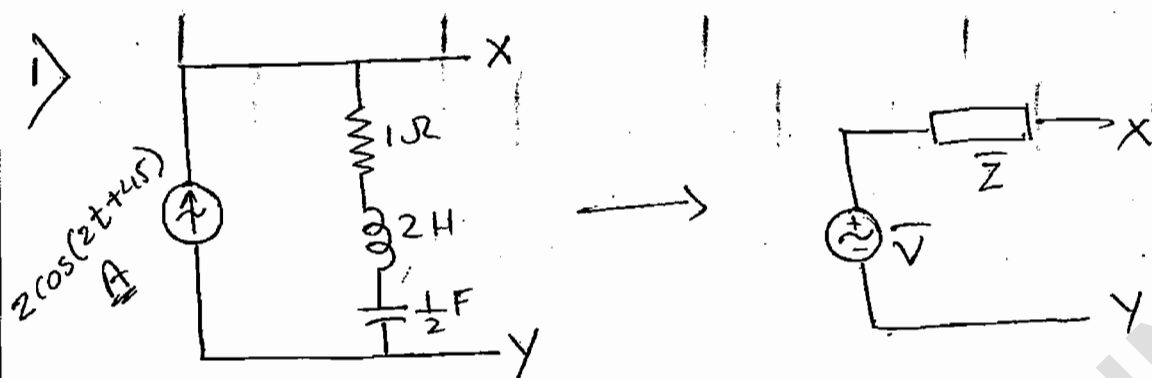
$$\therefore V_3 = 8.65 \angle 19.57^\circ$$

$$V_{oc} = \bar{V}_1 - \bar{V}_3 = 10 - [8.66 \angle 19.57^\circ]$$

$$= 3.43 \angle -57.44^\circ \text{ V}$$

Theorem 1

Source Transformation Technique :-



Here $I = 2 \angle 45^\circ$; $\omega = 2$

$$\bar{Z} = (1 + j4) - j1 = (1 + j3) \Omega$$

$$\bar{V} = \bar{I} \cdot \bar{Z} = 2 \angle 45^\circ \cdot (1 + j3)$$

$$= 2 \angle 45^\circ \times 3.16 \angle 71.56^\circ$$

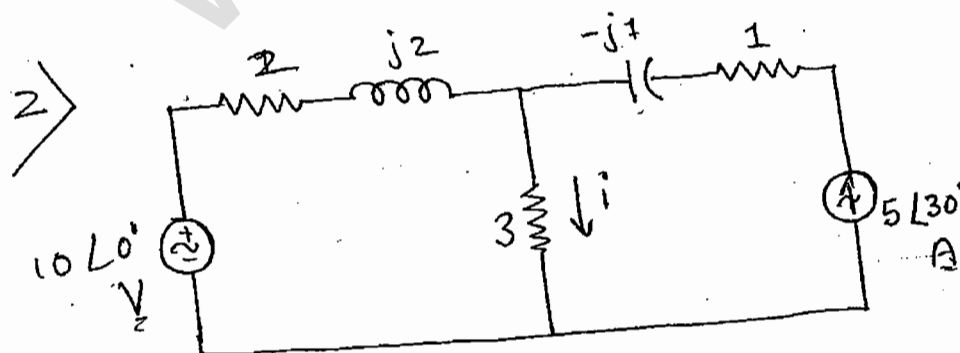
$$= 6.32 \angle 116.56^\circ \text{ V}$$

$$\therefore \bar{V} = 6.32 \cos [2t + 116.56] \text{ V}$$

Theorem 2: Superposition theo.

Theorem 3: Thevenin's theo.

Theorem 4: Norton's theo.



Find i using
superposition,
Thevenin
Norton.

(1) Superposition :-

① 10 V only : $i' = \frac{10 \angle 0^\circ}{5 + j2} = \frac{10 \angle 0^\circ}{5.38 \angle 21.80^\circ}$

$$\therefore i' = 1.857 \angle -21.8^\circ$$

② 5A only;

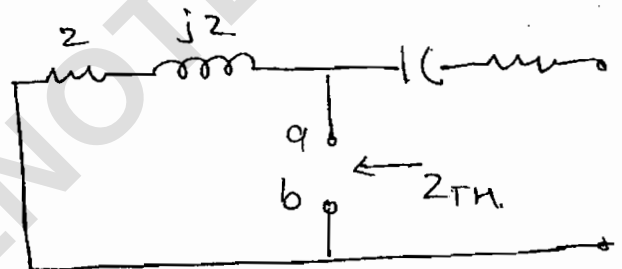
$$i'' = 5 \angle 30^\circ \left[\frac{2+j2}{5+j2} \right] = 5 \angle 30^\circ \times (0.482 + j0.2j) \\ = 2.61 \angle 53.19^\circ$$

$$\therefore i = i' + i'' \\ = 3.58 \angle 23.2^\circ \text{ A}$$

<2> Thevenin

① ~~by only~~

$$Z_{TH} = (2+j2) \Omega$$

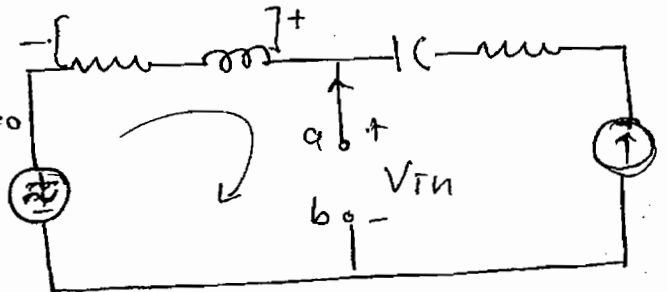


②

KVL

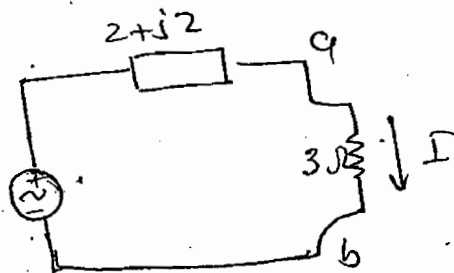
$$-[10 \angle 0^\circ] + V_{TH} - (2+j2)(5 \angle 30^\circ) = 0$$

$$\therefore V_{TH} = 19.32 \angle 45^\circ$$



$$I = \frac{19.32 \angle 45^\circ}{5+j2}$$

$$= 3.58 \angle 23.2^\circ \text{ A}$$

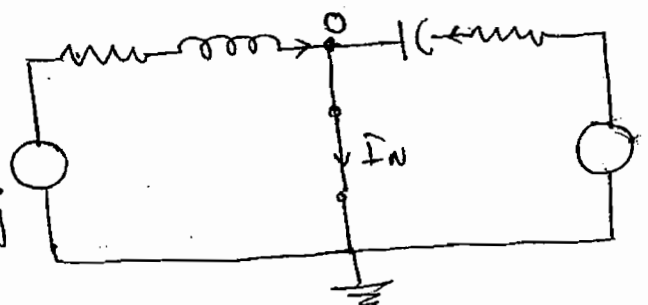


<3> Norton

$$I_N = \frac{10 \angle 0^\circ}{(2+j2)} + 5 \angle 30^\circ$$

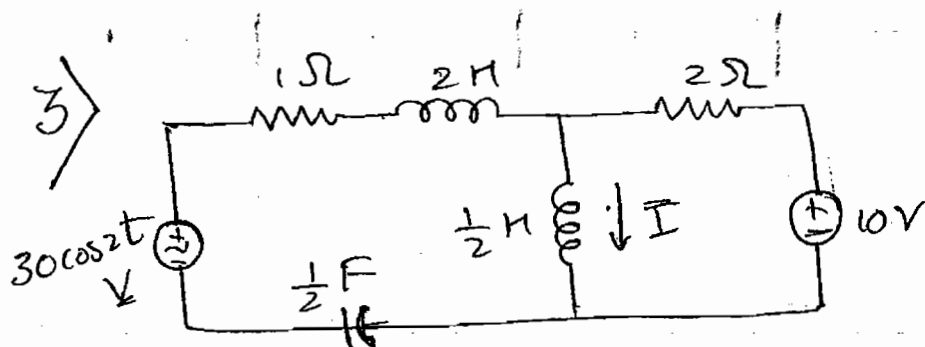
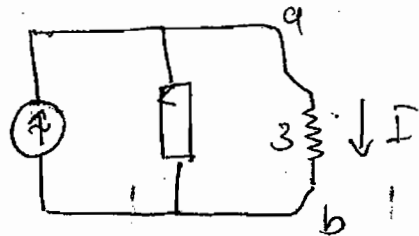
$$= \frac{10 \angle 0^\circ}{2.82 \angle 45^\circ} + 4.33 + 2.5j$$

$$= 6.83 \angle 23.2^\circ \text{ A}$$



$$I = 6.83 \angle 0 \left[\frac{2+j2}{5+j2} \right]$$

$$= 3.58 \angle 23.2^\circ \text{ A}$$



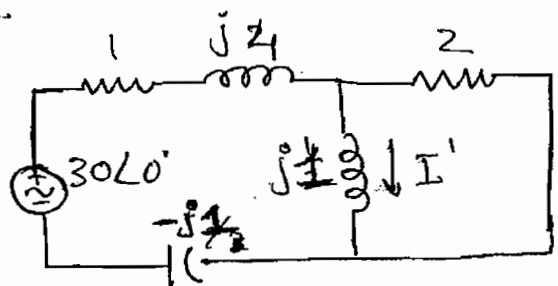
Find I

This ckt is multi frequency excited.
So in time domain only superposition theorem can provide the solution.
But we can also do this problem in freq. domain by applying Laplace Transforms.

① 30 V AC source only

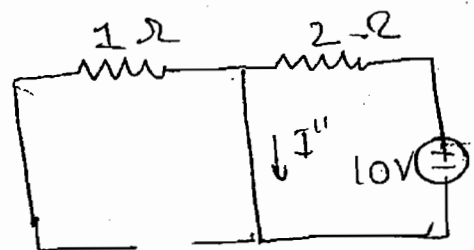
$$I' = \frac{30 \angle 0^\circ}{(1+j3) + [2 \parallel j4]} \cdot \frac{2}{2+j1}$$

$$= 6.63 \angle -96^\circ \text{ A}$$



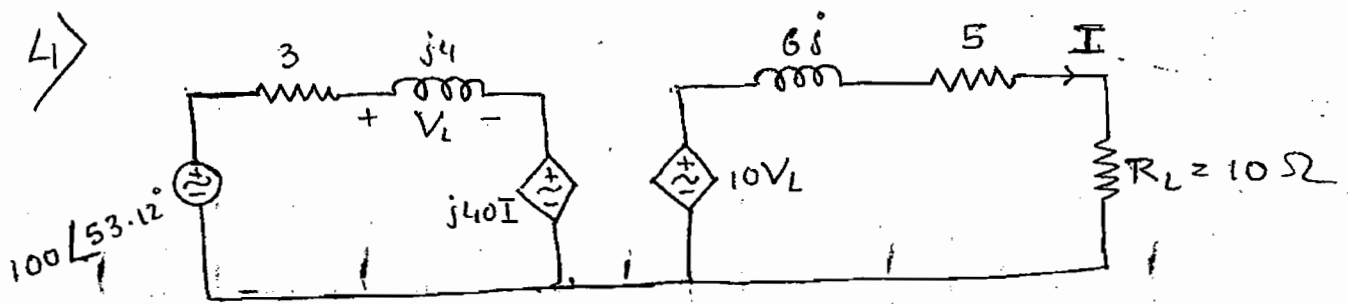
② 10V DC source only

$$I'' = \frac{10}{2} = 5 \text{ A}$$

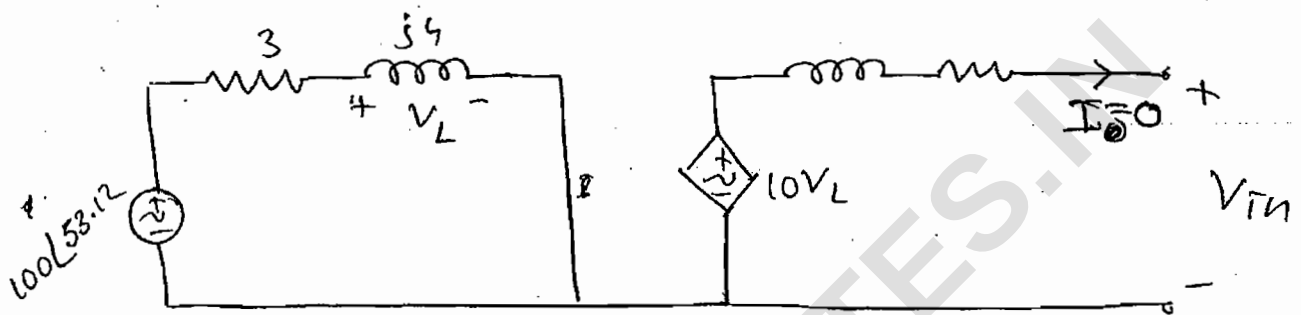


By SPT

$$I = 5 + 6.63 \cos(2t - 96^\circ) \text{ A}$$

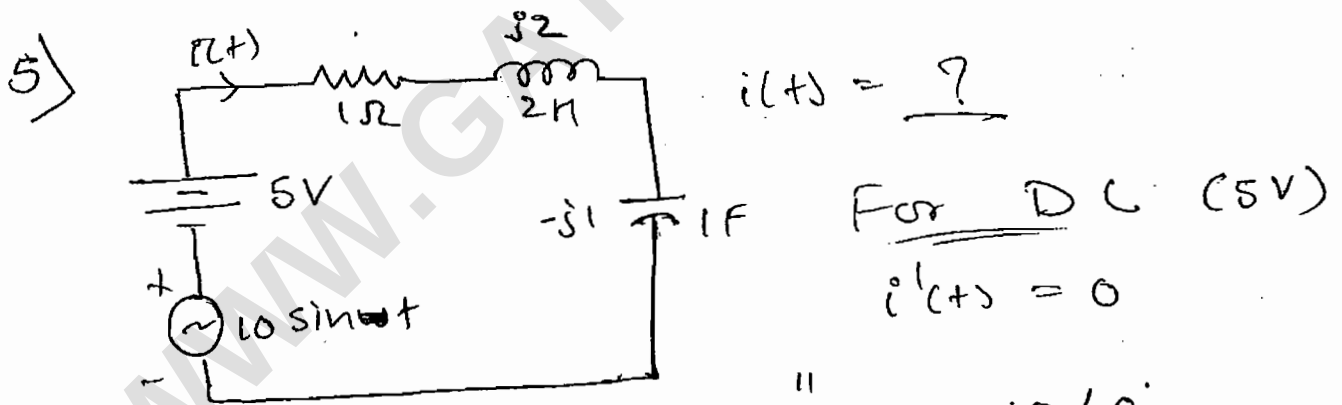


Therinin equivalent r(ty across the load.



$$V_L = 100 \angle 53.12^\circ \left[\frac{j4}{3 + j4} \right] = 80 \angle 90^\circ$$

$$\therefore V_{Th} = 10 V_L = 800 \angle 90^\circ$$



For 10V ac source :

$$i''(t) = \frac{10 \angle 0^\circ}{1 + j1} = 7.07 \angle -45^\circ$$

By SPT $i(t) = 7.07 \sin(t - 45^\circ)$

$$\begin{aligned}
 6) \sqrt{\frac{(10 \angle 20^\circ)(30 \angle 40^\circ)}{50 \angle 70^\circ}} &= \sqrt{6 \angle -10^\circ} = \sqrt{6} e^{-j10^\circ} \\
 &= \sqrt{6} e^{-j5^\circ} = \sqrt{6} \angle -5^\circ
 \end{aligned}$$

Theorem 5

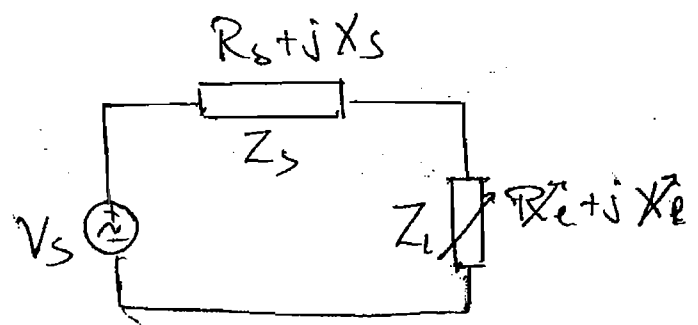
Maximum Power Transfer theo.

Though there are 3 types of physical power existing in AC steady state n/ws, the power that is consumable / utilisable / convertible in any other form is Active power or Real power in watts.

So max. power transfer theo. is confined to active power only.

General case :

In this case both the resistive part & reactive part are controllable.



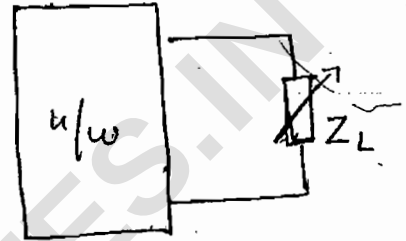
Since our target is to maximise active power in the load. ($\vec{I} \cdot \vec{R}_L$), here if we can compensate the net reactance so Total power = Active power
 so P_{max} occurs in the load if load

impedance is complex conjugate of equivalent source impedance seen by it. i.e. $Z_L = Z_S^* = R_S - jX_S$ &

$$P_{\max} = \frac{|V_S|^2}{4R_S} \quad \xrightarrow{\text{rms value}} \quad \text{(heat concept for power)}$$

In General :—

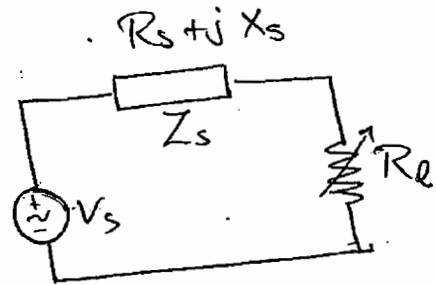
$P_{\max}$ occurs in the load when $Z_L = Z_{TH}^*$



and, $P_{\max} = \frac{|V_{TH}|^2}{4R_{TH}}$; $R_{TH} = \text{Real part } [Z_{TH}^*]$

Special case :— (a)

Since load is purely resistive but source has some reactance;



which here we cannot avoid reactive power in the u/w. Since phase balancing of impedances is not possible so to extract max. power atleast balance the magnitude so $P_{\max}$ occurs in the load. i.e.

$$R_L = |Z_L| = \sqrt{R_S^2 + X_S^2}$$

Now to calculate $P_{\max}$, we substitute the calculated value of R_L to find rms current in it. Then calculate

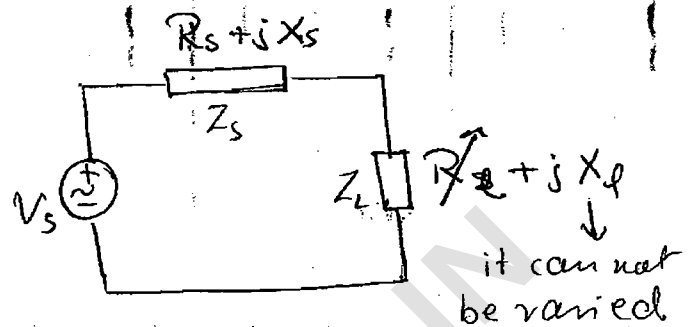
$$P_{\max} = I_{\text{rms}}^2 \times R_L$$

Use Thevenin's concept to all other general circuit.

Special Case :- (b)

$$R_e = |R_s + jX_s + jX_e|$$

$$= \sqrt{R_s^2 + (X_e + X_s)^2}$$

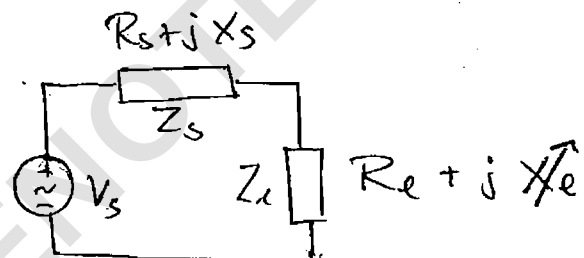


Special Case :- (c)

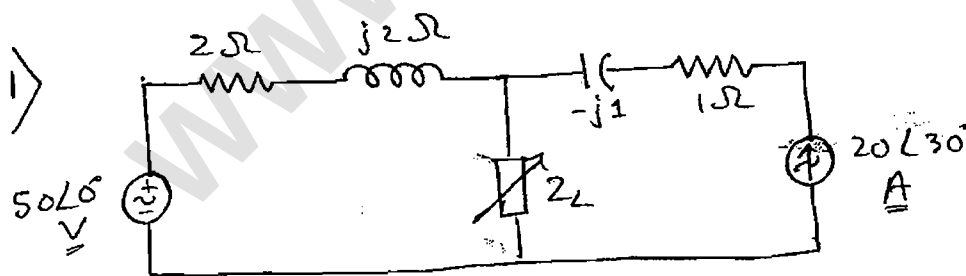
For P_{max} ;

$$X_e = -X_s$$

$$X_e + X_s = 0$$



Here, we can nullify the net reactance & reactive power but still resistance can not be controlled so P_{max} occurs in the load if $X_e = -X_s$.



Value of Z_L for which P_{max} occurs in it & also find P_{max} .

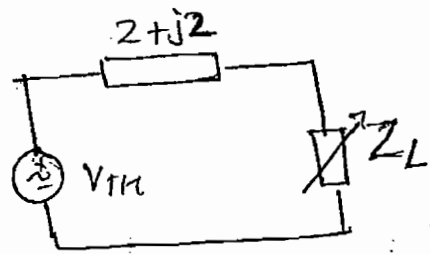
→ Determine Thevenin equi. across Z_L .

① $Z_{TH} = (2 + j2) \Omega$

② KVL $-[50\angle 0^\circ] + -(2 + j2)(20\angle 30^\circ) + V_{TH} = 0$

$\therefore V_{TH} = 84.64 \angle 40.2^\circ$

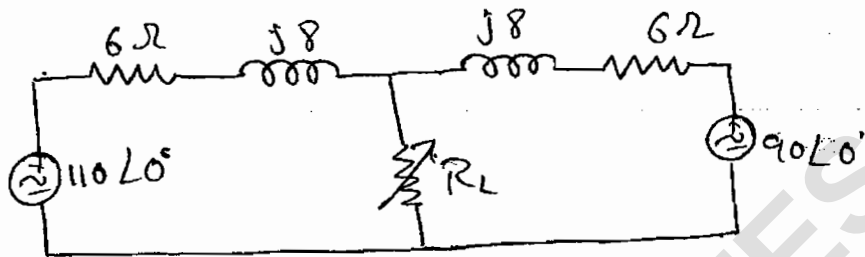
$$Z_L = Z_{TH}^* = (2 - j2) \Omega$$



$$P_{max} = \frac{|V_{TH}|^2}{4 R_{TH}} = \frac{|186.64|^2}{4 \times 2}$$

$$= \underline{895 \text{ W}}$$

2)



What is P_{max} transferred to load.

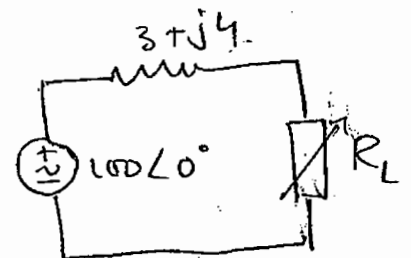
$$① Z_{TH} = (6 + j8) // (6 + j8) = (3 + j4) \Omega$$

$$② i \neq 100 \angle 0^\circ \dots \text{Nodal Analysis :-}$$

$$\frac{(V_{TH} - 110 \angle 0^\circ)}{6 + j8} + \frac{V_{TH} - 90 \angle 0^\circ}{6 + j8} = 0$$

$$\therefore 2V_{TH} = 200 \angle 0^\circ$$

$$V_{TH} = 100 \angle 0^\circ$$



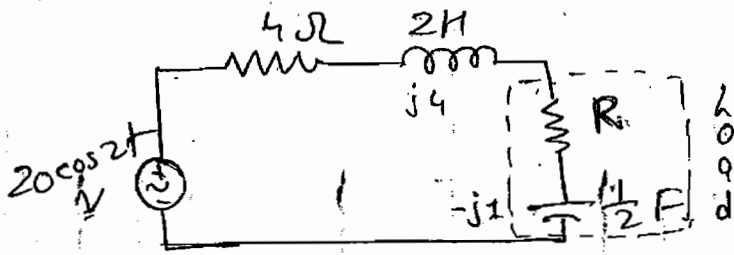
$$R_L = |Z_{TH}| = \sqrt{3^2 + 4^2} = 5 \Omega$$

$$I_{5\Omega} = \frac{100 \angle 0^\circ}{(3 + j4 + 5)} = 11.18 \angle -26.56^\circ$$

$$P_{max} = 11.18^2 \times 5$$

$$= \underline{625 \text{ W}}$$

3) What is P_{max} transferred to load

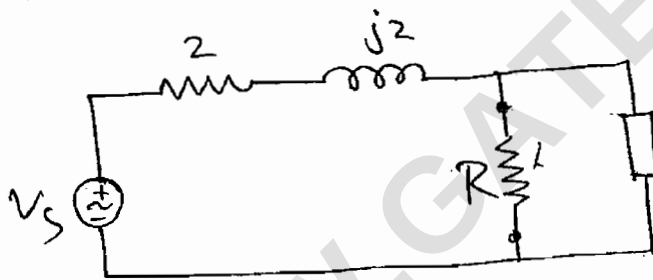


$$R = |4 + j4 - j1| = \sqrt{16 + 9} = 5 \Omega$$

$$I_{Rms} = \frac{\frac{20}{\sqrt{2}} \angle 0^\circ}{(4 + j4 + 5 - j1)} = \frac{14.142 \angle 0^\circ}{9.486 \angle 18.43^\circ} = 1.49 \angle -18.43^\circ$$

$$P_{max} = 1.49^2 \times 5 = \underline{11.10 \text{ W}}$$

4)

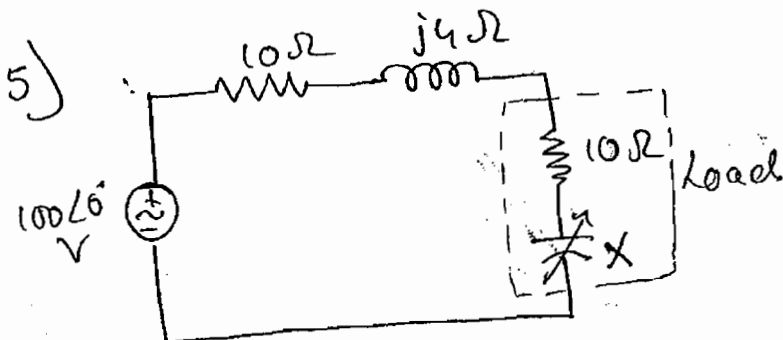


What is value of R for which P_{max} occurs in it.

$$R = |Z_{TH}| =$$

$$Z_{TH} = (2 + j2) \parallel (2 - j2) = \frac{8}{4} = 2$$

$$\therefore R = |2| = 2 \Omega$$



P_{max} transferred to load?

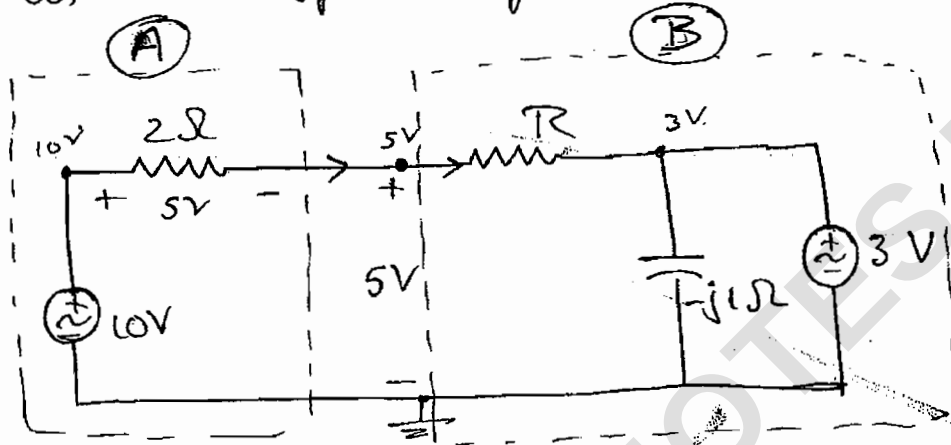
$$Z_{TH} = 10 + j4$$

$$V_{TH} = 100 \angle 0^\circ \text{ then } X = -j4$$

$$P_{\max} = I^2 R$$

$$= \left[\frac{\cos 40^\circ}{20} \right]^2 \times 10 = \underline{\underline{250 \text{ W}}}$$

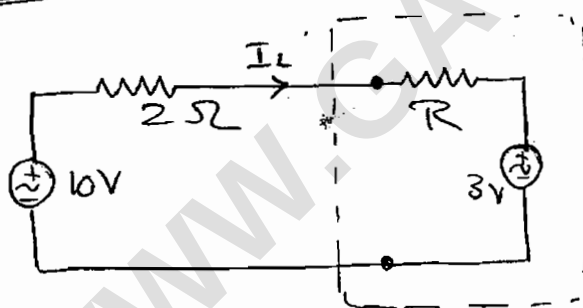
c) For what value of R max. power is transferred from ckt A to ckt B.



Short cut

$$\frac{10-5}{2} = \frac{5-3}{R} \Rightarrow R = \frac{2 \times 2}{5} = 0.8 \Omega$$

Exact proof



→ T.E. of ckt (B) as seen by ckt (A)

$$I_L = \frac{(10-3)}{2+R} = \frac{7}{2+R}$$

$$P_L = I_L^2 \cdot R = \frac{49}{(2+R)^2} \times R + 3 \left(\frac{7}{2+R} \right)$$

$$\therefore P_L = \frac{7}{2+R} \left[\frac{7R}{2+R} + 3 \right] = \frac{70R + 42}{(2+R)^2}$$

For $P_{\max}$; $\frac{dP_L}{dR} = 0$

$$\left[\frac{(2+R)^2(70) - (70R+42) \cdot (2)(2+R)}{(2+R)^4} \right] = 0$$

$$\therefore (2+R)^2(70) = 2(2+R)(70R+42)$$

$$\therefore 140 + 70R = 140R + 84$$

$$\therefore 70R = 56$$

$$\therefore \boxed{R = 0.8 \Omega}$$

Concept of Complex Power :- (S^*)

$$S^* = VI^*$$

It is a mathematical concept which removes the confusion in calculating the 3 physical powers: S , P , Q in any n/w or element.

It will be very essential to calculate instantaneous power in AC ckt to estimate, design, control & analyse stability, fault calculations, etc in electrical power system since load is very dynamic in nature.

~~I~~ To verify Tellegen's theorem, to calculate power at any instant, we need complex power.

Here, $I^* \rightarrow$ complex conjugate I

$$\text{eg: } I = 10 \angle 20^\circ \text{ A} \Rightarrow I^* = 10 \angle -20^\circ \text{ A}$$

Units of S^* : Volt-Ampere (VA's)

We can also write.

$S^* = V^* I$ but generally we write

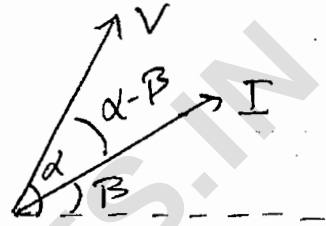
$S^* = V I^*$ as $V \rightarrow$ is the cause &
 $I \rightarrow$ is the effect.

Ex:1

In one element

$$V = |V| \angle \alpha$$

$$I = |I| \angle \beta$$



$\rightarrow$ If we want to calculate inst. power

$$V \cdot I = |V| |I| \angle \alpha + \beta \quad \times$$

$\rightarrow$ Correct way of calculating inst. power

$$V \cdot I^* = |V| |I| \angle \alpha - \beta \quad \checkmark$$

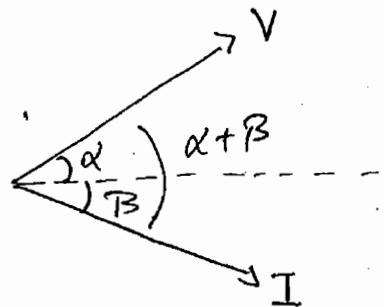
Ex:2

$$V = |V| \angle \alpha$$

$$I = |I| \angle -\beta$$

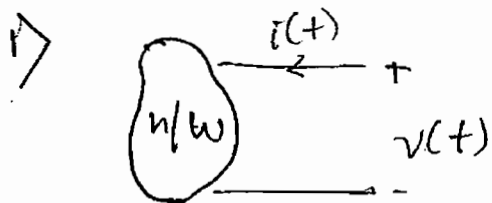
$\rightarrow$ just $V I$ product

$$|V| |I| \angle \alpha - \beta \quad \times$$



$\rightarrow$ Consider $S^* = V I^*$

$$= |V| |I| \angle \alpha + \beta \quad \checkmark$$



Here, $v(t) = 20 + j12$

$i(t) = 5 + j4$

Calculate: P, Q in the n/w

$V(t) \rightarrow 20 + j12 \rightarrow 23.32 \angle 30.96^\circ \text{ V}$

$i(t) \rightarrow 5 + j4 \rightarrow 6.4 \angle 38.65^\circ \text{ A}$

Complex Power $S^* = V I^*$

$= 149.248 \angle -7.69^\circ$

$= 147.9 - j19.97 \text{ VA}$

$\downarrow \quad \downarrow$
 $P \quad Q_c$

For our n/w:

$P = 147.9 \text{ watts.}$

$Q_c = 19.97 \text{ VAR's (generating.)}$

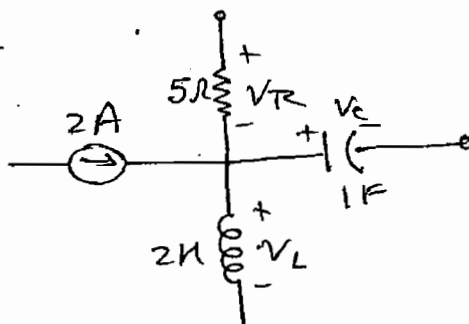
2) If $V_R = 5 \text{ V}$, $V_C = 4 \sin 2t \text{ V}$, $V_L = ?$

$2 + \frac{V_R}{5} = \frac{dV_C}{dt} + \frac{1}{2} \int V_L dt$

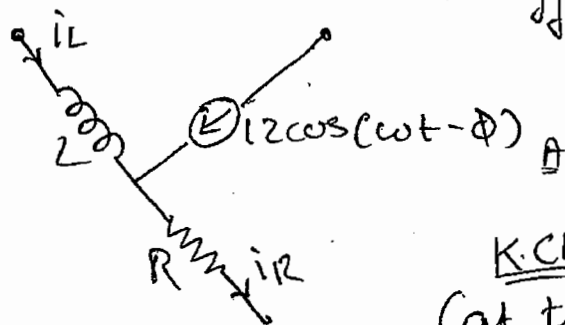
$\frac{1}{2} \int V_L dt = 2 + 1 - 8 \cos 2t$

$V_L = \frac{d}{dt} (6 - 16 \cos 2t)$

$V_L = 32 \sin 2t \text{ V}$



3)



If $i_R = (4e^{-3t} + 3e^{-4t}) \text{ A}$

$i_L(0) = 1 \text{ A.}$

$\phi = ?$

KCL: $i_L + 12 \cos(\omega t - \phi) = i_R$

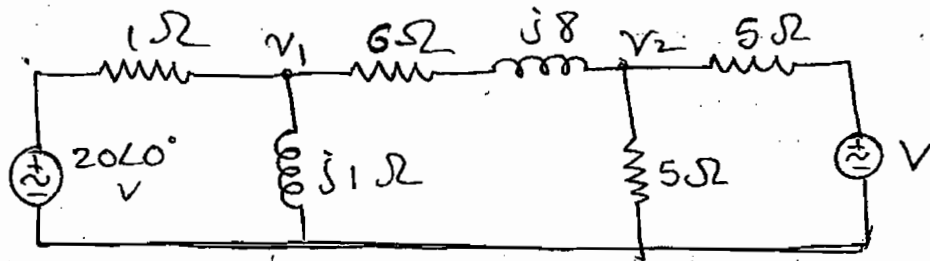
(at $t=0$)

$i_L(0) + 12 \cos(-\phi) = 4 + 3$

$1 + 12 \cos \phi = 7$

$\therefore \cos \phi = \frac{6}{12} = \frac{1}{2} \Rightarrow \boxed{\phi = 60^\circ}$

4)



power dissipated in 6Ω resis. is $0W$ then what is 'V'.

Here, $P_{6\Omega} = 0W \Rightarrow I$ across 6Ω resis. is zero.

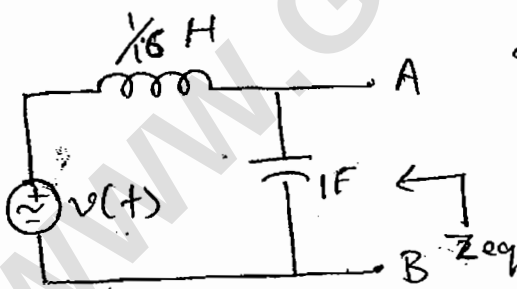
$\therefore V_1 = V_2$ (same in mag. & phase).

$$V_1 = 20\angle 0^\circ \left[\frac{j1}{1+j1} \right] = 10\sqrt{2} \angle 45^\circ$$

$$V_2 = 10\sqrt{2} \angle 45^\circ = V \left[\frac{5}{5+2} \right]$$

$$\therefore V = 2V_2 \Rightarrow V = 20\sqrt{2} \angle 45^\circ$$

5)

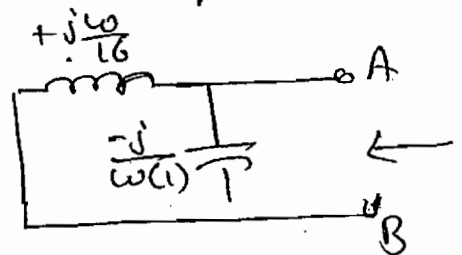


For what ckt frequency the n/w acts as ideal current source b/w A & B.

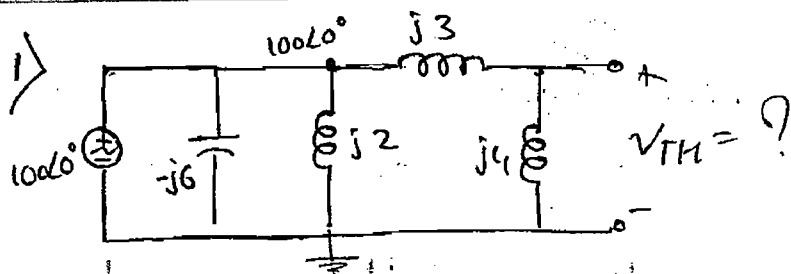
Ideal current source $\Rightarrow Z_{eq} = \infty$

$$Z_{eq} = \left(\frac{j\omega}{16} \right) \parallel \left(-\frac{j}{\omega} \right)$$

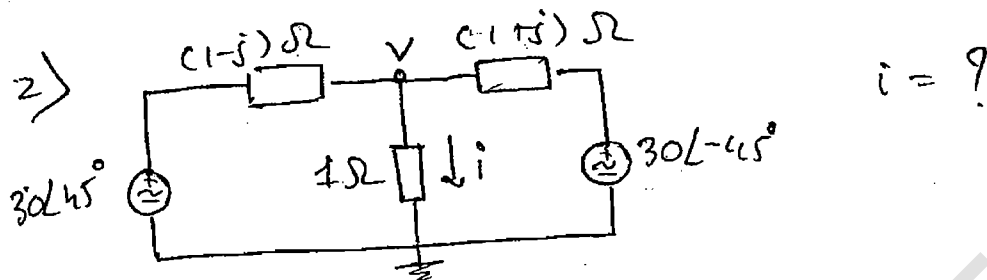
$$= \frac{1/16}{j \left[\frac{\omega}{16} - \frac{1}{\omega} \right]} \rightarrow \infty$$



$$\therefore \frac{j\omega}{16} = \frac{1}{\omega} \Rightarrow \omega^2 = 16 \Rightarrow \boxed{\omega = 4}$$



$$V_{TH} = (100 \angle 0^\circ) \cdot \frac{j4}{j5 + j4} = \frac{400}{9} \angle 0^\circ \text{ V}$$



$$i = ?$$

Nodal

$$\frac{V - [30 \angle 45^\circ]}{1-j} + \frac{V - [30 \angle -45^\circ]}{1+j} + \frac{V}{1} = 0$$

$$\therefore \frac{[V - (30 \angle 45^\circ)](1+j) + [V - (30 \angle -45^\circ)](1-j) + V}{2} = 0$$

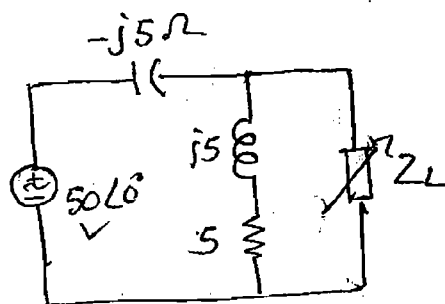
$$\therefore 4V = (30 \angle 45^\circ)(1+j) + (30 \angle -45^\circ)(1-j)$$

$$= (30\sqrt{2} \angle 90^\circ) + (30\sqrt{2} \angle -90^\circ)$$

$$4V = 0 \Rightarrow V = 0 \Rightarrow \boxed{i = 0 \text{ A}}$$

3) What is the max. power transferred to the load impedance.

It is a general case prob.



$$\therefore Z_{TH} = (5 + j5) \parallel (-j5)$$

$$= \underline{5 - j5 \Omega}$$

For P_{max} : $Z_L = Z_{TH}^* = \underline{5 + j5 \Omega}$ Δ

$$P_{max} = \frac{|V_{TH}|^2}{4 R_{TH}}$$

$$V_{in} = (50 \angle 0^\circ) \cdot \left[\frac{5 + j5}{4 + j5} \right] = 50\sqrt{2} \angle 45^\circ \text{ V}$$

$$\therefore P_{max} = \frac{(50\sqrt{2})^2}{4 \times 5} = \underline{\underline{250 \text{ W}}}$$

21

WWW.GATENOTES.IN

7) Find I .

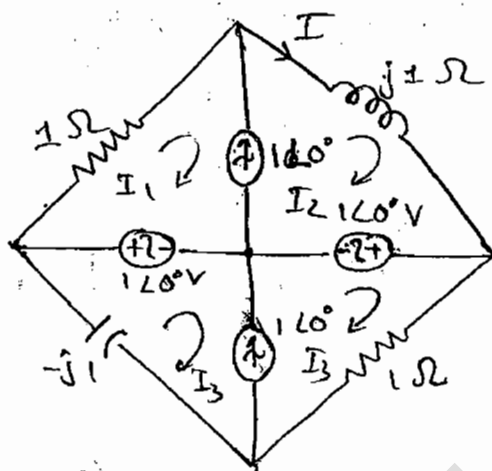
KVL

$$I_1 + j I_2 = 0$$

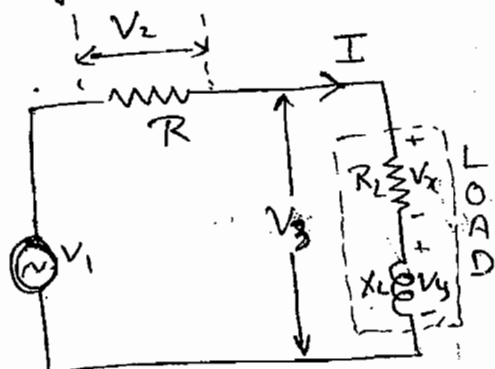
$$-I_1 + I_2 = 1$$

$$(1 + j) I_2 = 1$$

$$\therefore I_2 = \left(\frac{1}{1+j} \right) A$$



8) If $V_1 = 220V$, $V_2 = 122V$, $V_3 = 136V$



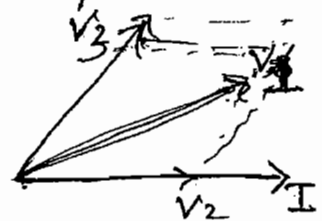
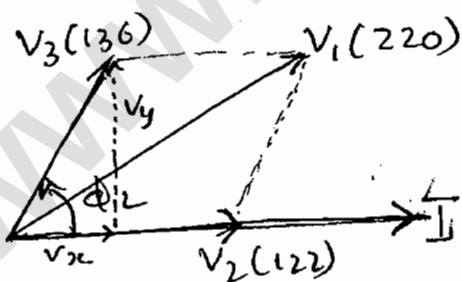
(a) Find Load Power factor

(b) If $R_L = 5\Omega$, what is avg power in the load. (i.e. active power in watts)

Load Power factor: $\cos \phi_L = \cos (V_3, I)$

N/w Power factor: $\cos \phi = \cos (V_1, I)$

→ This is based on phasor diagram:



Formula: $R_T = \sqrt{R_1^2 + R_2^2 + 2R_1R_2 \cos(\phi_1, \phi_2)}$

$$\therefore 220 = \sqrt{(122)^2 + (136)^2 + 2(122)(136) \cos \phi_L}$$

$$\therefore \cos \phi_L = 0.45 \text{ (lagg)}$$

Now, $R_L = 5 \Omega$

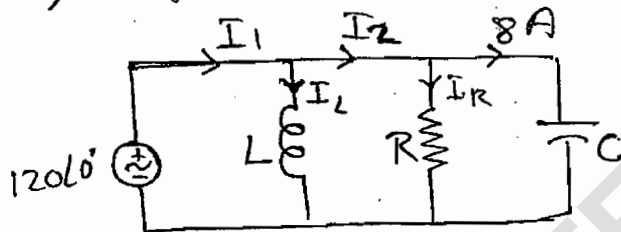
$$P = \frac{|V_{xL}|^2}{R} = |I|^2 R = |I| |V|$$

also, $\cos \phi_L = \frac{V_{xL}}{V_y}$

$$\therefore V_x = V_3 \cos \phi_L = 136 \times 0.45 = 61.2$$

$$P = \frac{(61.2)^2}{5} = 750 \text{ W}$$

9) 9. $|I_1| = |I_2| = 10 \text{ A}$ determine I_R &

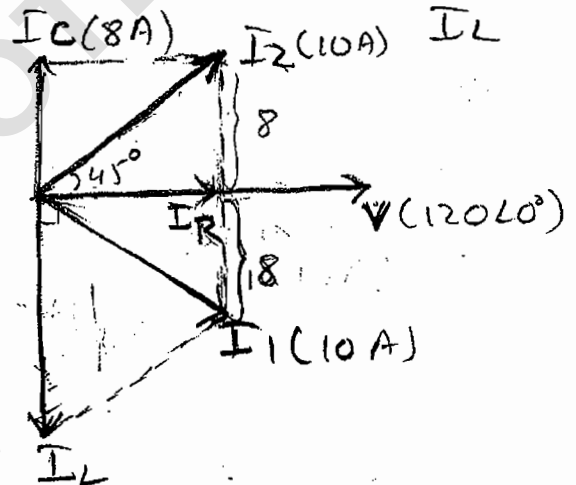


$$I_R = \sqrt{100 - 64} = \sqrt{36}$$

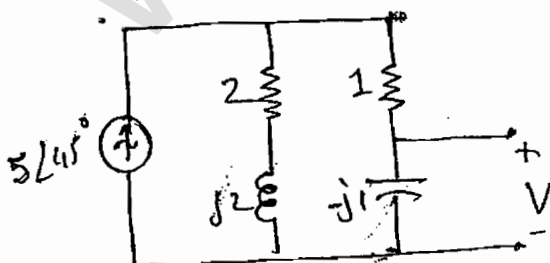
$$I_R = 6 \text{ A}$$

From phasor diag.

$$I_2 = 8 + 8 = 16 \text{ A}$$



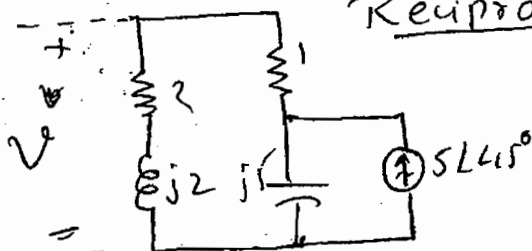
10) Verify Reciprocity theorem in find vltg V



Step 1

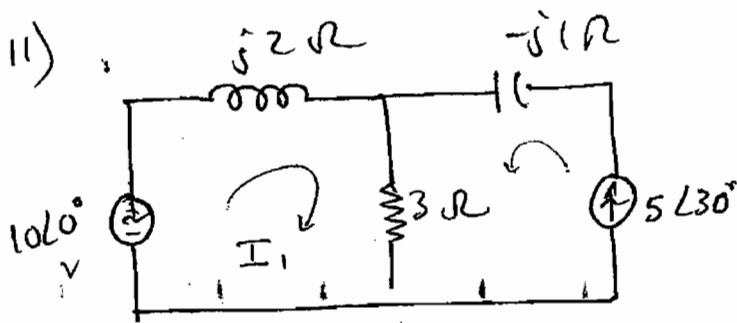
$$V = \left[5 \angle 45^\circ \left(\frac{2 + j2}{3 + j1} \right) \right] (-j1)$$

Step 2



Reciprocal n/w

$$V = (2 + j2) \left[\frac{5 \angle 45^\circ (-j1)}{(3 + j1)} \right]$$



Verify Tellegen's thm.

Here we verify instantaneous power theory.

So we find complex power S^* in each element.

Element	V	I	I^*	VI^*	Remarks
V_s	$10\angle 0^\circ$	$2.23\angle -145.4^\circ$		$22.3\angle 145.4^\circ$	Source $\rightarrow$ Apparent power
I_s	$10\angle -3.63^\circ$	$5\angle 30^\circ$	$5\angle -30^\circ$	$50\angle -33.63^\circ$	Source $\rightarrow$ Apparent power
R	$8.34\angle 26.31^\circ$	$2.78\angle 26.31^\circ$		$23.18\angle 0^\circ$	Sink $\rightarrow$ Active power
L	$4.46\angle -55.4^\circ$	$2.23\angle -145.4^\circ$		$9.94\angle 90^\circ$	Sink $\rightarrow$ Reactive power (+Q _L)
C	$5\angle -60^\circ$	$5\angle 30^\circ$	$5\angle -30^\circ$	$25\angle -120^\circ$	Sink $\rightarrow$ Reactive power (-Q _C)

$$\text{KVL} \quad -[10\angle 0^\circ] + j2I_1 + 3I_1 + 3(5\angle 30^\circ) = 0$$

$$\therefore (3 + j2)I_1 + 15\angle 30^\circ = 0$$

$$I_1 = \frac{-10\angle 0^\circ - 15\angle 30^\circ}{3 + j2} = \frac{2.99 + 7.5j}{3 + j2} = 2.23\angle -145.4^\circ$$

$$I_R = I_1 + 5\angle 30^\circ = 2.78\angle 26.31^\circ$$

$$V_R = 3(I_R) = 8.34\angle 26.31^\circ$$

$$I_L = 2.23\angle -145.4^\circ$$

$$V_L = (j2)I_L = (2\angle 90^\circ)(2.23\angle -145.4^\circ) = 4.46\angle -55.4^\circ$$

$$V_C = (-j)I_C = (1\angle -90^\circ)(5\angle 30^\circ) = 5\angle -60^\circ$$

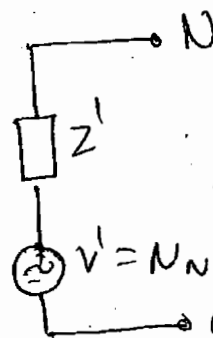
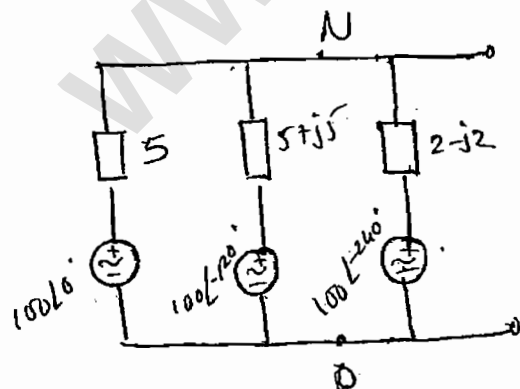
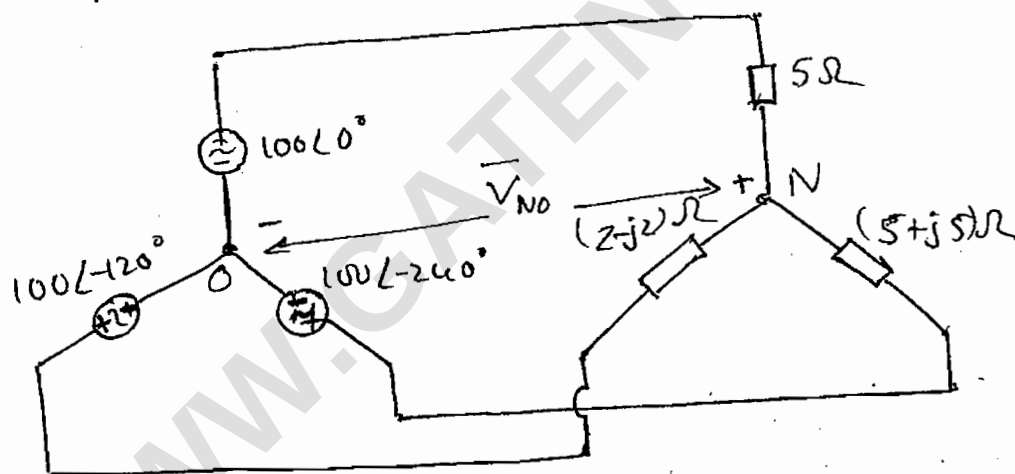
To verify Tellegen's Theorem;

$$\sum VI^* |_{\text{source}} = \sum VI^* |_{\text{sink}}$$

$$\begin{aligned} LHS &= [22.3 \angle 145.4^\circ] + [50 \angle -33.60^\circ] \\ &= 27.7 \angle -32.8^\circ \text{ VA} \end{aligned}$$

$$\begin{aligned} RHS &= [23.18 \angle 0^\circ] + [9.94 \angle 90^\circ] + [25 \angle -90^\circ] \\ &= 27.7 \angle -32.8^\circ \text{ VA} \end{aligned}$$

12) Find the shift in the neutral voltage using Millman's theo. for the unbalanced 3-phase ckt shown.



$V' = N_N0 \rightarrow$ shift in neutral voltage.

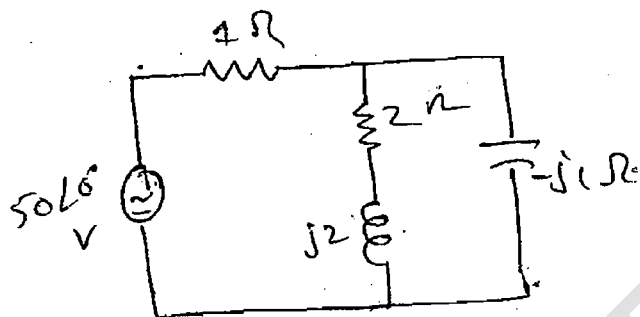
$$V' = \frac{\left(\frac{100 \angle 0^\circ}{5}\right) \left(\frac{100 \angle -120^\circ}{5+j5}\right) \left(\frac{100 \angle -240^\circ}{2-j2}\right)}{\frac{1}{5} + \frac{1}{5+j5} + \frac{1}{2-j2}}$$

$$V = \frac{20 \angle 0^\circ + 14.14 \angle -165^\circ + 35.36 \angle -195^\circ}{0.2 + 0.1 - 0.1i + 0.25 + 0.25i}$$

$$= \frac{-27.813 \angle 168.82^\circ}{}$$

$$= 49.72 \angle 153.7^\circ \text{ V}$$

12) Find change in current in capacitor when 1Ω resistor is increased to 3Ω compensation theo.

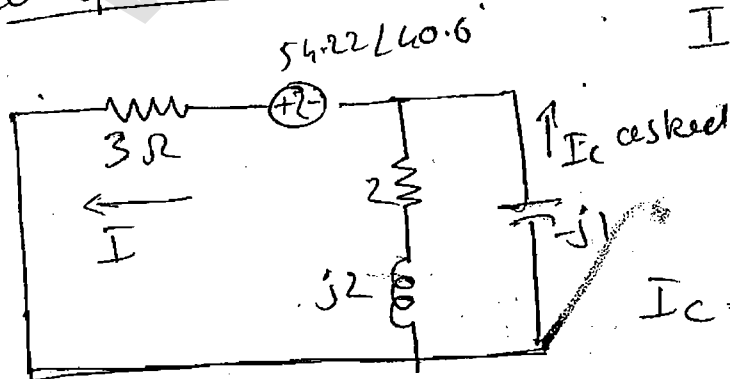


$$\begin{aligned} I_{1\Omega} &= \frac{50 \angle 0^\circ}{1 + [(2+j2) \parallel (-j1)]} \\ &= \frac{50 \angle 0^\circ}{1 + 0.4 - 1.2j} \\ &= 27.11 \angle 40.6^\circ \end{aligned}$$

Now Find compensated vltg

$$\begin{aligned} \bar{V}_c &= \bar{I} [\Delta Z] \\ &= [27.11 \angle 40.6^\circ] [3 - 1] \\ &= 54.22 \angle 40.6^\circ \end{aligned}$$

Compensated vltg



$$\begin{aligned} \bar{I} &= \frac{54.22 \angle 40.6^\circ}{3 + [(2+j2) \parallel (-j1)]} \\ &= 15.0 \angle -60^\circ \end{aligned}$$

$$\bar{I}_c = \bar{I} \left[\frac{2 + j2}{2 + j2 - j1} \right]$$

$$= 18.96 \angle 78^\circ \text{ A}$$

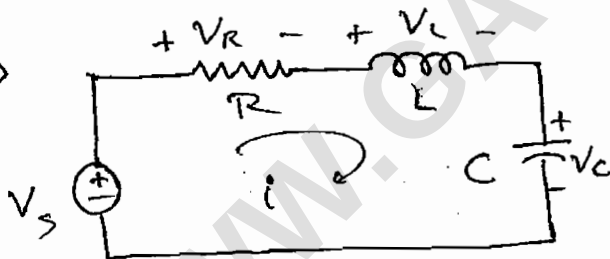
DUALS & DUALITY

Two ckt are duals of each other if the mesh eqⁿ that characterize one of them has the same mathematical form as the nodal eqⁿ that characterize the other.

Principle of Duality :—

Identical behaviour & pattern observed bet 2 vltgs & currents of 2 independent n/ws illustrate the principle of Duality.

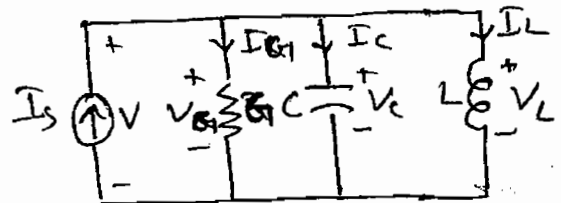
Eg)



Mesh-KVL

$$-V_s + V_R + V_L + V_C = 0$$

$$V_s = iR + L \frac{di}{dt} + \frac{1}{C} \int i dt$$



Nodal-KCL

$$-I_s + I_R + I_C + I_L = 0$$

$$\therefore I_s = V \cdot G + C \frac{dV}{dt} + \frac{1}{L} \int V dt$$

Some Dual Elements

$$V \longleftrightarrow I$$

$$v(t) \longleftrightarrow i(t)$$

$$V_m \sin \omega t \longleftrightarrow I_m \sin \omega t$$

$$R \longleftrightarrow G$$

$$L \longleftrightarrow C$$

$$KVL \longleftrightarrow KCL$$

$$\text{Series} \longleftrightarrow \text{Parallel}$$

$$\text{Mesh} \longleftrightarrow \text{Node}$$

Φ

$$T_L \longleftrightarrow T$$

$$\text{Tree} \longleftrightarrow \text{Co-Tree}$$

$$* \text{Twig} \longleftrightarrow \text{Link / chord}$$

$$\frac{di}{dt} \longleftrightarrow \frac{dv}{dt}$$

$$S_{vd} \longleftrightarrow S_{id}$$

$$O.C \longleftrightarrow S.C$$

$$\text{Thevenin} \longleftrightarrow \text{Norton}$$

$$* \Phi \longleftrightarrow q$$

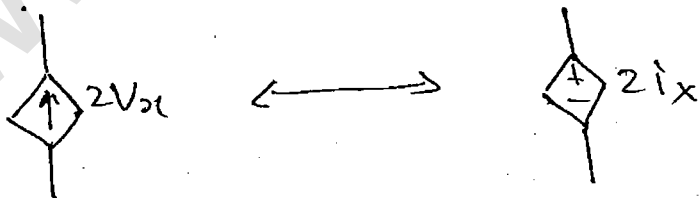
$$Y \longleftrightarrow \Delta$$

$$\text{cut set} \longleftrightarrow \text{Tie-set}$$

$$Z \longleftrightarrow Y$$

$$X \longleftrightarrow B$$

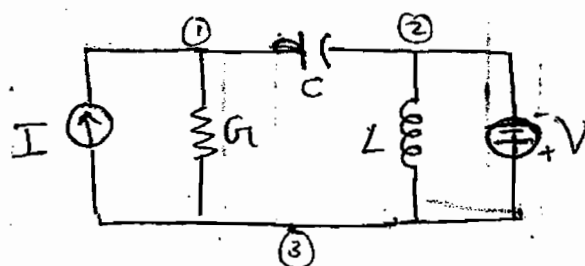
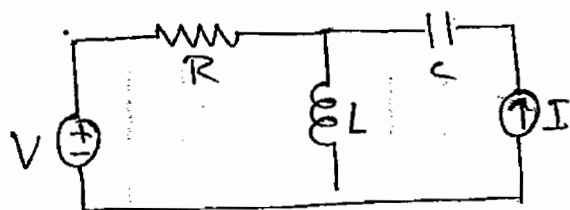
s/w in series (getting closed) $\longleftrightarrow$ s/w in parallel (getting opened)



polarity (voltage) $\longleftrightarrow$ direction (current)

Eg Dual of 2Ω resistance $\Rightarrow$ 2 Siemens conductance

1) Construct the dual of ckt shown below & verify by writing n/w equations.



Mesh

$$-V + i_1 R + L \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] = 0$$

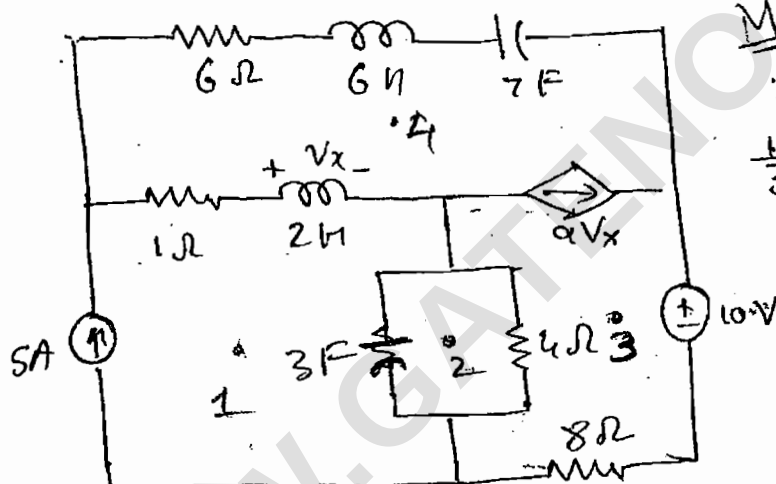
$$i_2 = -I$$

Nodal

$$-I + G V_1 + C \left[\frac{dV_1}{dt} - \frac{dV_2}{dt} \right] = 0$$

$$V_2 = -V$$

2)



Mesh

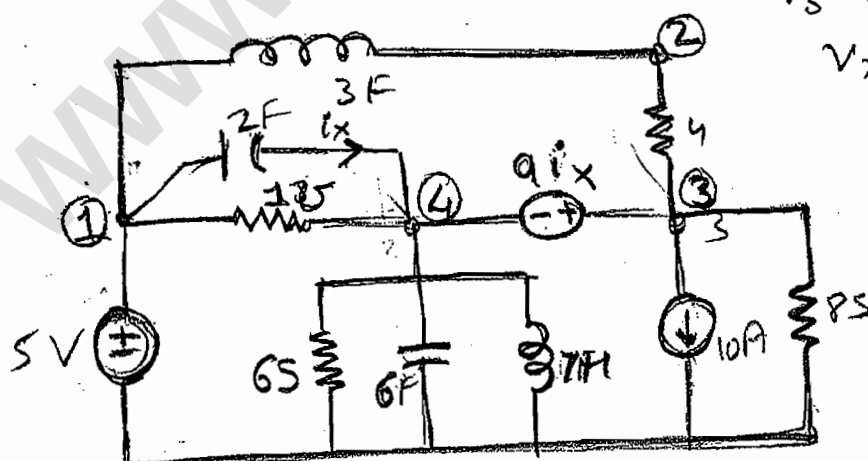
$$i_1 = 5 \quad \text{--- (1)}$$

$$\frac{1}{3} \int (i_2 - i_1) dt + 4 [i_2 - i_3] = 0 \quad \text{--- (2)}$$

$$4 [i_3 - i_2] + 2 \left[\frac{di_4}{dt} - \frac{di_1}{dt} \right] + 1 [i_4 - i_1] + 5 i_4 + 6 \frac{di_4}{dt} + \frac{1}{7} \int i_4 dt + 10 + 8 i_3 = 0 \quad \text{--- (3)}$$

$$i_3 - i_4 = 4V_x \quad \text{--- (4)}$$

$$V_x = 2 \left[\frac{di_1}{dt} - \frac{di_4}{dt} \right] \quad \text{--- (5)}$$



Nodal

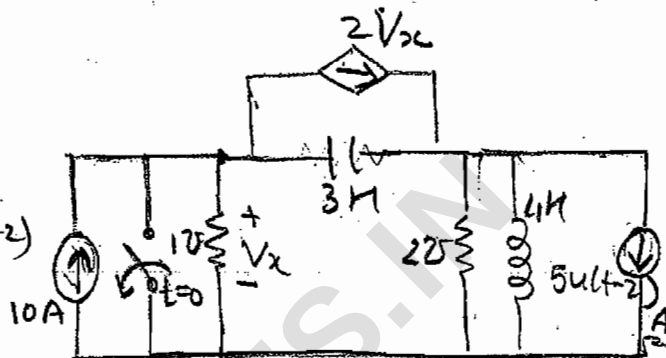
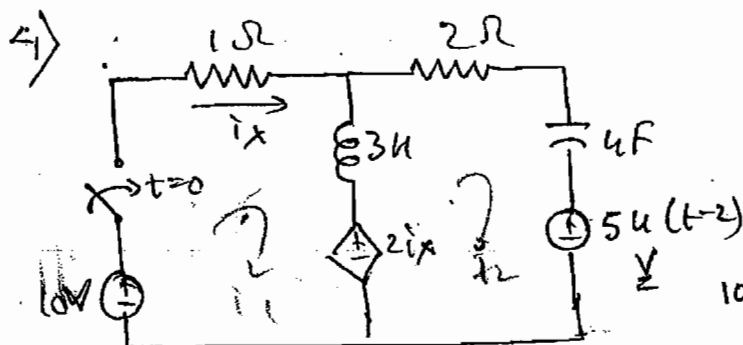
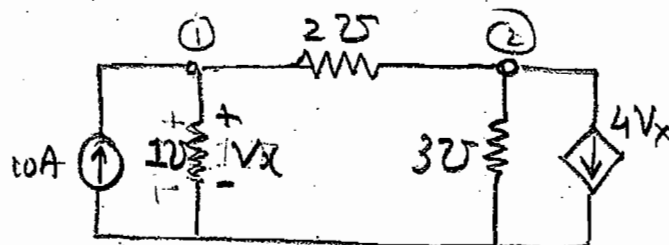
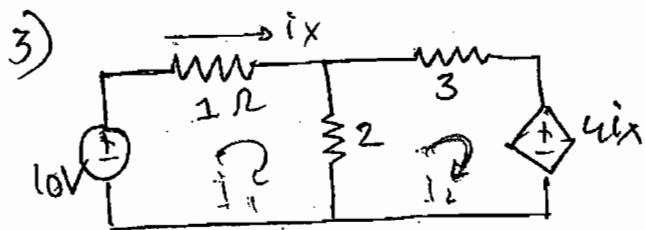
$$V_1 = 5 \quad \text{--- (1)}$$

$$\frac{1}{3} \int (V_2 - V_1) dt + 4 [V_2 - V_3] = 0 \quad \text{--- (2)}$$

$$4 [V_3 - V_2] + 2 \left[\frac{dV_4}{dt} - \frac{dV_1}{dt} \right] + 1 [V_4 - V_1] + 5 V_4 + 6 \frac{dV_4}{dt} + \frac{1}{7} \int V_4 dt + 10 + 8 V_3 = 0 \quad \text{--- (3)}$$

$$V_3 - V_4 = 4 i_x \quad \text{--- (4)}$$

$$i_x = 2 \left[\frac{dV_1}{dt} - \frac{dV_4}{dt} \right] \quad \text{--- (5)}$$

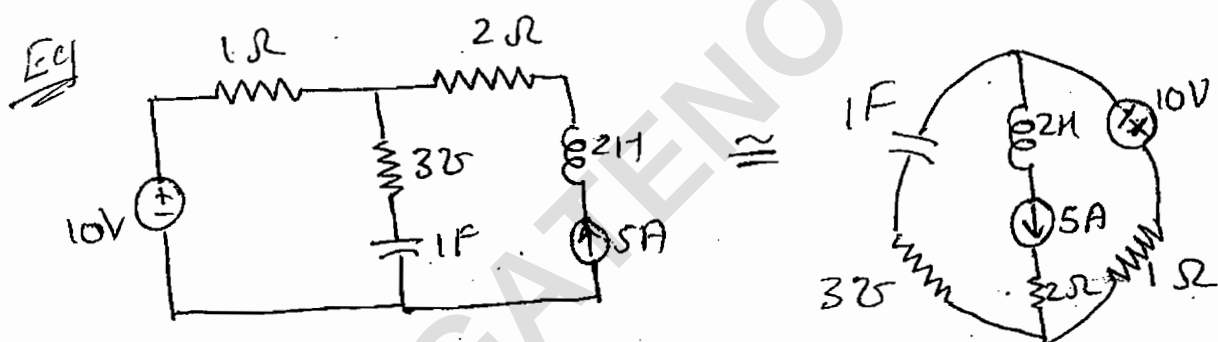


NETWORK TOPOLOGY or

GRAPH THEORY

Topology

Topology is a branch of geometry applicable to electrical ckt where even by bending, stretching, swapping, thrashing, tying in knots, making ckt upside down, etc will not disturb the ckt property.



⇒ Shape of a n/w ideally will not affect circuit analysis.

Graph

A graph is a skeleton representation of a n/w where every element is suppressed by its nature & represented as a simple line-segment.

NOTE

Ideal vltg source $\rightarrow$ S.C

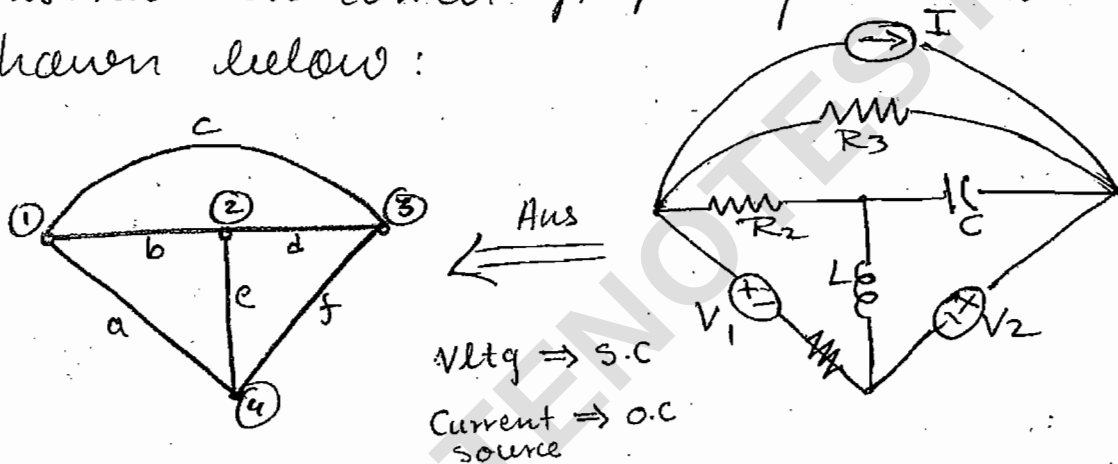
" current " $\rightarrow$ O.C

→ To simplify the order of matrix we generally consider principle node only.

→ Nodes → vertices
Branches → edges.

→ Nodes are numbers. → ①, ②, ③, ...
Branches are named → a, b, c, ...

Ex Construct the correct graph of the n/w shown below:



Node (n)

Branch (b)

Mesh (m) $\rightarrow$ independent loops
 $[l_i = b - (n - 1)]$

Loop (l)

$$n = 4$$

$$b = 6$$

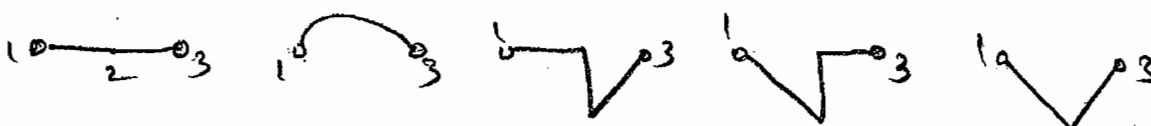
$$m = 3 \rightarrow [6 - (4 - 1)]$$

$$l = 3 + 4 = 7$$

Path

A path is a traversal from one node to another node without crossing the same node twice.

Q No. of possible path for the above graph from node ① to node ③ is 5

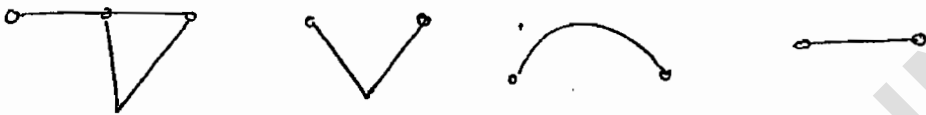


Sub graph

A sub-graph consists of some ^{the} nodes & branches of main graph.

~~Ex~~ Even a single edge can be subgraph of the main graph.

Ex



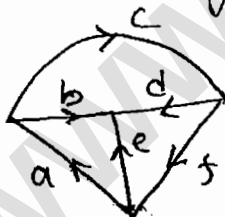
Directed Graph

A graph is said to be directed if every edge is given a reference dirⁿ which is indicated by placing an arrow on every branch.

Note:-

This reference orientation need not necessarily indicate current dirⁿ.

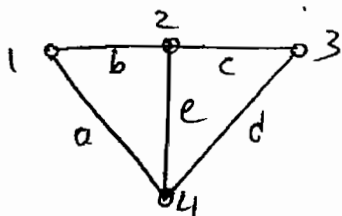
Ex



Connected Graph.

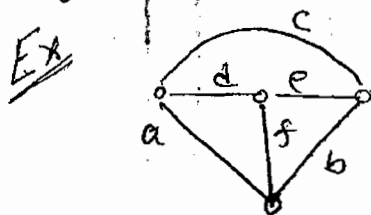
A graph is said to be connected if there exist atleast one path from every node to every other node.

Ex



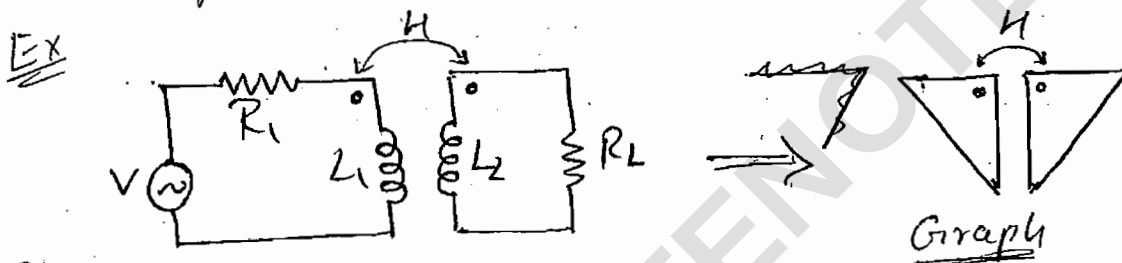
Complete Graph / Completely Connected Graph

A graph is said to be complete graph if a direct path from every node to every other node.



Unconnected Graph :-

- 1) Wireless Communications networks
- 2) Magnetic circuits



1) The min. no. of edges to make a graph complete with 'n' nodes is $\frac{nC_2}{2}$

<u>Ex</u>	<u>nodes</u>	<u>Graph</u>	<u>Edges</u>
	2		$1 = \frac{2(2-1)}{2}$
	3		$3 = \frac{3(3-1)}{2}$
	4		$6 = \frac{4(4-1)}{2}$
	5		$10 = \frac{5(5-1)}{2}$

Tree

A Tree is a sub graph of main graph which connects all the nodes without forming closed loops.

The rank of a tree with 'n' nodes is $(n-1)$.

Any corresponding tree of a given graph with 'n' nodes will have $(n-1)$ edges.

$$\text{No. of trees} = \begin{cases} n^{(n-2)} & ; \text{ for } n \geq 2 \rightarrow \text{complete graph only} \\ \det[A_r][A_r]^T & ; \text{ for any graph} \end{cases}$$

where, $[A_r]$ = Reduced Incidence Matrix

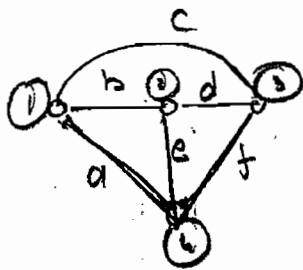
The branch of a tree is specifically called as a Twig indicated by thick line segment. Any tree with 'n' nodes has $(n-1)$ twigs.

Co-Tree (Compliment of Tree)

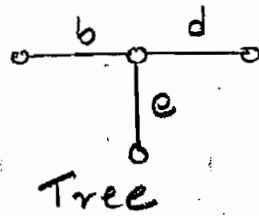
This set of branches other than Tree branches in a graph collectively form a co-tree.

Link / Chord :

The branch of Co-Tree is specifically called as a link which is indicated by dotted line. For any corresponding Co-Tree we have $b - (n-1)$ links.



=



Tree

b, d, e → Twigs

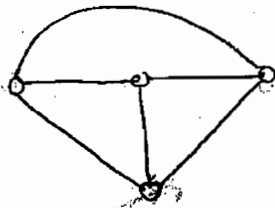


Co-Tree

a, c, f → links

$$\begin{array}{ccccc}
 \text{GRAPH} & = & \text{TREE} & + & \text{CO-TREE} \\
 \downarrow & & \downarrow & & \downarrow \\
 \text{edges} & & \text{twigs} & & \text{links} \\
 \downarrow & & \downarrow & & \\
 b & = & (n-1) & + & b-(n-1)
 \end{array}$$

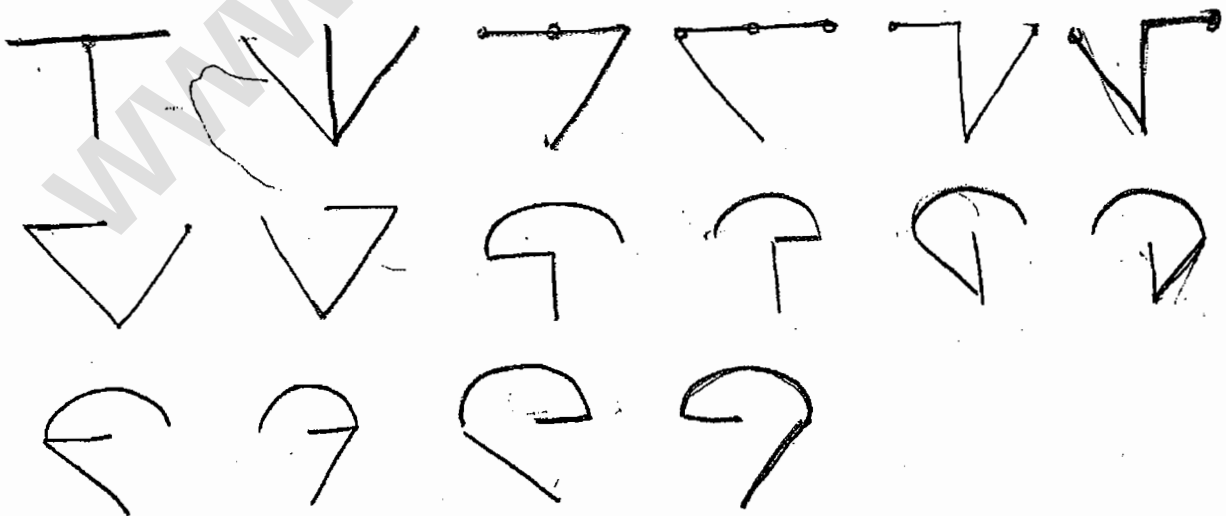
1) The no. of possible f-trees for the n/w graph shown below is _____ & draw separately.



→ This is a complete graph

$$\begin{aligned}
 \therefore \text{No. of Trees} &= n^{(n-2)} \\
 &= 4^2 = 16
 \end{aligned}$$

These are



Incidence Matrix [A]

It is the matrix that gives relation b/w no. of nodes & no. of branches & the orientation of a particular branch w.r.t a node.

The order of this matrix is $(n \times b)$ or $(\overset{\text{vertices}}{\uparrow} \times \overset{\text{edges}}{\uparrow})$

The ~~rank~~^{rank} of Incidence Matrix with 'n' nodes is $(n-1)$

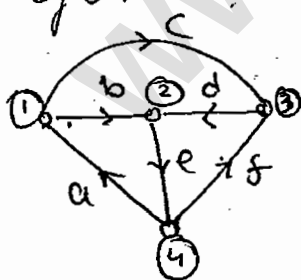
The elements of this matrix $[A] = [a_{ij}]_{n \times b}$ where $a_{ij} = +1$

if j^{th} branch is incident with i^{th} node & oriented away from it.

$a_{ij} = -1$; if incidented towards

$a_{ij} = 0$; if not incident

⇒ Construct the complete Incidence Matrix for the oriented graph shown below:

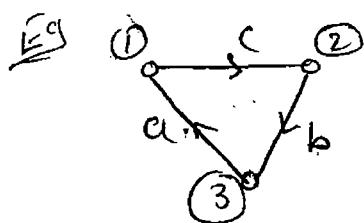


$$[A] = \begin{matrix} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix} & \begin{bmatrix} -1 & +1 & +1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & +1 & 0 & -1 \\ +1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{matrix}_{4 \times 6}$$

The algebraic sum of the elements of every column vertically is zero

1) The determinant of Incidence Matrix of a closed loop graph is 0

Incidence matrix of a closed loop graph is of order $n \times n$



$$[A] = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\det |A| = -1(-(-1)) + 1(1) \\ = -1 + 1 = \underline{0}$$

⇒ In the Incidence Matrices of 2 independent n/w graphs are identical then they are said to obey the principle of Isomorphism.

Reduced Incidence Matrix $[A_r]$

If one of the node in a given graph is considered as reference & that particular row is neglected while writing the incidence matrix, then it is a reduced incidence matrix, its order is ~~$(n-1) \times b$~~
 $(n-1) \times b$

In computer methods of electrical ckt analysis by considering A_r the memory space requirement & iteration time for solutions will be decreased.

Eg From above graph if node ③ is considered as ref. & that particular row ③ is neglected, then the reduced incidence

matrix is :

$$[A_r] = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & -1 \end{bmatrix}$$

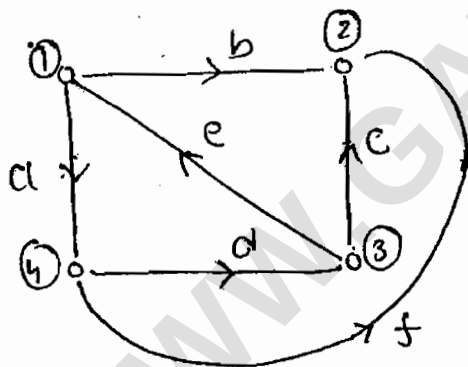
- 2) Construct the oriented graph of a n/w whose Incidence Matrix is given as: $\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \end{bmatrix}$

→ The given is an $[A_r]$

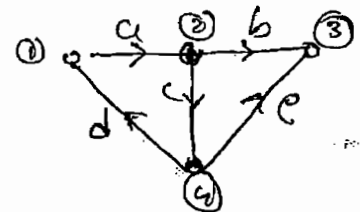
So make it complete $[A]$

$$\Rightarrow \begin{matrix} & \begin{matrix} a & b & c & d & e & f \end{matrix} \\ \begin{matrix} ① \\ ② \\ ③ \\ ④ \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

∴ The n/w graph has nodes = 4, edges = 6



- 3) The no. of possible trees from n/w graph shown below is _____ & draw them separately.



→ Just connected graph.

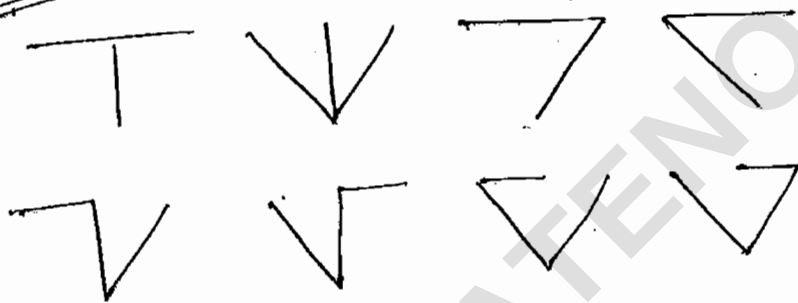
$$\text{No. of trees} = \det |[A_r][A_r]^T|$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

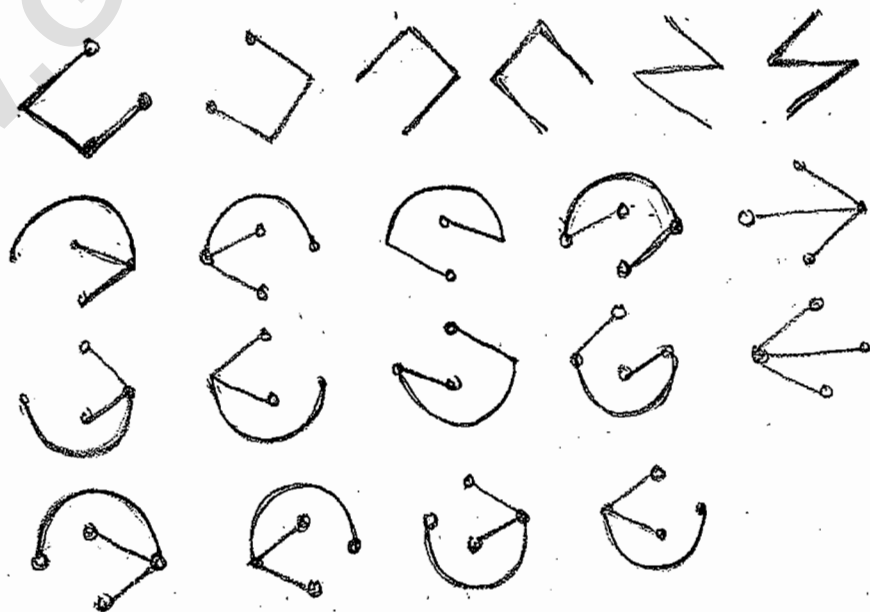
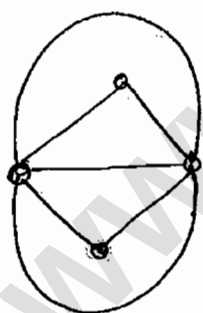
$$[A_r][A_r]^T = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{bmatrix}$$

$$\det [A_r][A_r]^T = 2(6-1) - 1(2) = 10 - 2 = 8 \text{ possible trees.}$$

These Trees are



HW
41



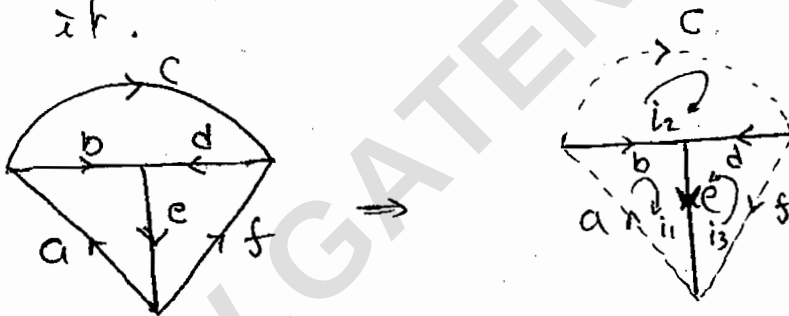
Concept of Fundamental Loops & Tie Set Currents. :-

Fundamental loops are closed path of the graph which are formed by only one link & rest of them as twigs.

The no. of F-loops for any given graph = no. of links i.e. $b - (n - 1)$

These fundamental loops currents are called Tie-set current & their orientation is governed by the link in it.

Eg:-

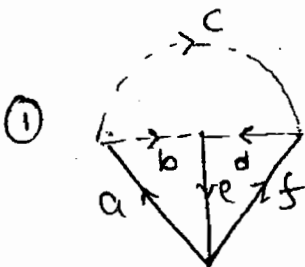


$$\text{No. of f-loops} = 6 - (4 - 1) = 3$$

$$fl_1 = a, b, e \rightarrow i_1 \downarrow$$

$$fl_2 = b, c, d \rightarrow i_2 \downarrow$$

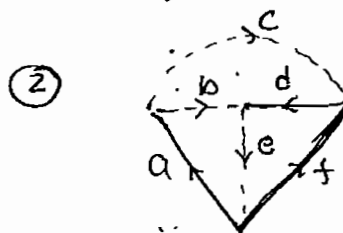
$$fl_3 = d, e, f \rightarrow i_3 \downarrow$$



$$fl_1 = a, b, e \rightarrow i_1 \downarrow$$

$$fl_3 = d, e, f \rightarrow i_2 \uparrow$$

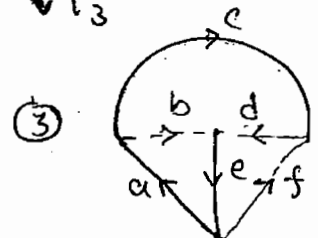
$$fl_2 = a, c, f \rightarrow i_3 \downarrow$$



$$fl_1 = a, b, d, f \rightarrow i_1 \downarrow$$

$$fl_2 = a, c, f \rightarrow i_2 \downarrow$$

$$fl_3 = d, e, f \rightarrow i_3 \uparrow$$



$$fl_1 = a, b, c, e \rightarrow i_1 \downarrow$$

$$fl_2 = a, c, d, e \rightarrow i_2 \downarrow$$

$$fl_3 = a, c, f \rightarrow i_3 \uparrow$$

Tie-Set Matrix $[M]$:-

It is a matrix that gives the relation b/w the branch currents & Tie-set currents where every branch current can be expressed in terms of Tie-set currents.

The order of this matrix is (links) $\times$ (branches)
i.e. $[b-(n-1)] \times b$

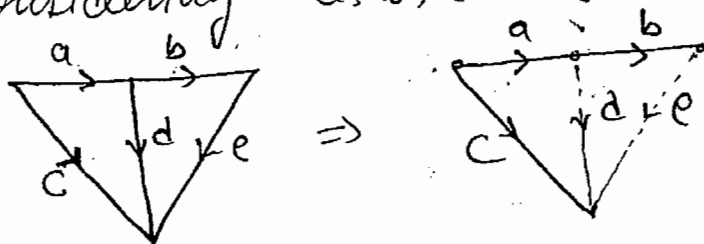
The no. of Tie-set Matrices possible for any graph = no. of Trees.

The elements of this matrix $[M] = [a_{ij}]$ links $\times$ branches
where $a_{ij} = +1$; if j^{th} branch current is incident with i^{th} Tie set current & oriented in same dirⁿ

$a_{ij} = -1$; If incident & opp.

$a_{ij} = 0$; if not incident.

1) Construct the Tie-set Matrix & the write the equilibrium eqⁿ in std. KVL form for n/w graph shown below by considering a, b, c as tree branches.



$$\begin{aligned}\text{No. of f. loops} &= 5 - (4 - 1) \\ &= 2\end{aligned}$$

$$fl_1 \rightarrow a, c, d \rightarrow i_1$$

$$fl_2 \rightarrow a, b, e \rightarrow i_2$$

$$[M] = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} i_1 \\ i_2 \end{matrix} & \begin{bmatrix} +1 & 0 & -1 & +1 & 0 \\ +1 & +1 & -1 & 0 & +1 \end{bmatrix} \end{matrix}_{2 \times 5}$$

Equilibrium Equations

Let $i_a, i_b, i_c, i_d, i_e \rightarrow$ Branch Currents

$V_a, V_b, V_c, V_d, V_e \rightarrow$ Branch voltages

Set I KVL $[\rightarrow]$

$$[M][V_b] = [0]$$

$$\begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix}_{(2 \times 5)} \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \end{bmatrix}_{5 \times 1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2 \times 1}$$

Set II Relation b/w 'j' & 'i' $[\downarrow]$

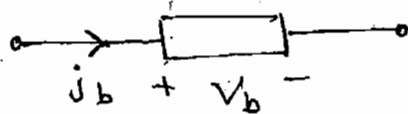
$$[M]^T [I_e] = [J_b]$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{5 \times 2} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}_{2 \times 1} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5 \times 1}$$

$$\begin{bmatrix} i_1 + i_2 \\ i_2 \\ -i_1 - i_2 \\ i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}$$

NOTE :-

In graph theory if graph is given, we consider every edge as a local set of reference (by default)

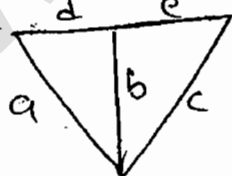


Concept of cut-set :-

A cut set represents set of branches which when removed in a graph can be divided into 2 parts.

Note The no. of cut-sets simply represent the no. of possible ways that a graph can be divided in 2 parts.

1) Which of foll. set of branches represent a proper cut set for the graph shown below.

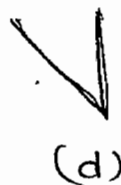
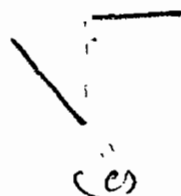
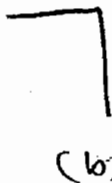


(1) a c d

(2) a c e

☒ (3) b d c

(4) c d e



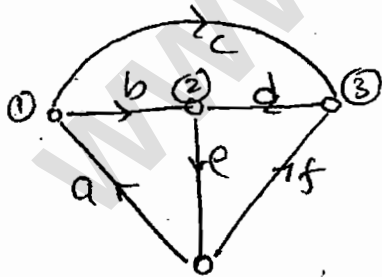
Concept of ~~fundamental loops~~ ^{cut-set} & Cut-set voltages : —

Fundamental cut set are cut through of a graph which can divide in 2 parts in any dir but in path of cutting it should cut only one twig & rest of them as links.

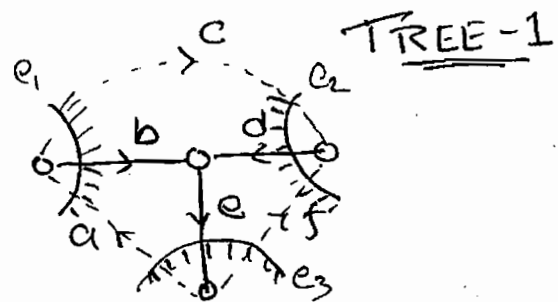
The no. of f-cut-set = no. of twigs
(ie $n-1$)

These f-cut sets form isopotential lines & their voltages are termed as cut set vltges.

The orientation of f-cut sets are governed by the twig in it.



GRAPH



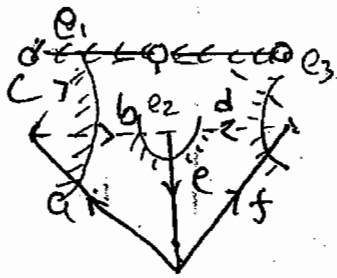
No. of ^{f-cutsets} ~~loops~~ = $4-1=3$

$fC_1 \rightarrow a, b, c \rightarrow e_1$

$fC_2 \rightarrow c, d, f \rightarrow e_2$

$fC_3 \rightarrow a, e, f \rightarrow e_3$

TREE 2

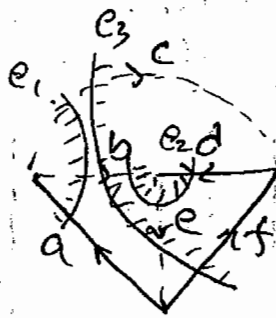


$$fC_1 \rightarrow a, b, e$$

$$fC_2 \rightarrow b, d, e$$

$$fC_3 \rightarrow c, d, f$$

TREE -3

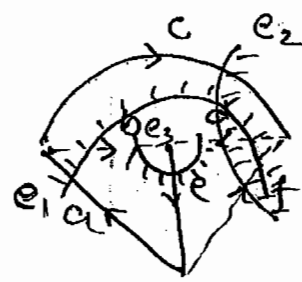


$$fC_1 \rightarrow a, b, c$$

$$fC_2 \rightarrow b, d, e$$

$$fC_3 \rightarrow b, c, e, f$$

TREE -4



$$fC_1 \rightarrow a, b, d, f$$

$$fC_2 \rightarrow c, d, f$$

$$fC_3 \rightarrow b, d, e$$

Cut-set Matrix $[C]$:-

It is a matrix that gives relation b/w branch vltgs & cut-set vltgs where every branch vltg can be expressed in terms of cut-set vltgs.

The order of this matrix = (twigs) $\times$ (branches)
i.e. $(n-1) \times b$

The no. of possible cut set-matrix for any given graph = no. of trees.

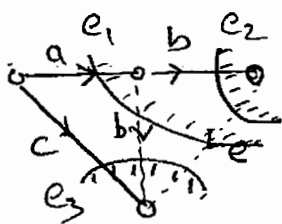
The elements of this matrix $[C] = [a_{ij}]_{(n-1) \times b}$

where $a_{ij} = 1$; if j^{th} branch vltg is incident to i^{th} cut set vltg & oriented in same dirⁿ.

$a_{ij} = -1$; if incident & opp.

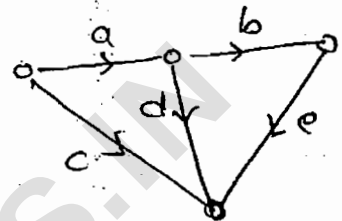
$a_{ij} = 0$; if not incident.

i) Construct the cut-set matrix & write the equilibrium eqⁿ in std. KCL form for the oriented graph shown below by considering branches a, b, c as tree branches.



No. of ~~cut sets~~ ^{f-cut sets}
 $= 4 - 1 = 3$

$$\begin{aligned} fC_1 &= a, d, e \rightarrow \vec{e}_1 \\ fC_2 &= b, e \rightarrow \vec{e}_2 \\ fC_3 &= c, d, e \rightarrow \vec{e}_3 \end{aligned}$$



$$[C] = \begin{matrix} & a & b & c & d & e \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & +1 \end{bmatrix} \end{matrix}_{3 \times 5}$$

Let, $j_a, j_b, j_c, j_d, j_e \rightarrow$ Branch currents
 $V_a, V_b, V_c, V_d, V_e \rightarrow$ Branch voltages.

5-1 KCL $[\rightarrow]$

$$[C][T_b] = [0]$$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}_{3 \times 5} \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \end{bmatrix}_{5 \times 1} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1}$$

$$\begin{bmatrix} j_a - j_d - j_e \\ j_b - j_e \\ j_c + j_d + j_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

S-2 Relation b/w 'v' & 'e' [↓]

$$[c]^T [e_t] = [v_b]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 1 \end{bmatrix}_{5 \times 3} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} = \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \end{bmatrix}_{5 \times 1}$$

$$\begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ -e_1 + e_3 \\ -e_1 - e_2 + e_3 \end{bmatrix} = \begin{bmatrix} v_a \\ v_b \\ v_c \\ v_d \\ v_e \end{bmatrix}$$

2) Mention the relation b/w Tie-set & cut-set matrices.

⇒ For the same given m/w ; for that particular tree, for that particular orientation only we can compare Tie-set & Cut-set matrices.

Tie set Matrix $\rightarrow$ Links $\rightarrow$ KVL $\rightarrow$ Mesh
(f-loops)

$$[M] = \begin{matrix} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} i_1 \\ i_2 \end{matrix} & \begin{bmatrix} 1 & 0 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$\downarrow$ M_{twigs}
 $\downarrow$ U_{twigs}

Cut-set Matrix $\rightarrow$ Twigs $\rightarrow$ KCL $\rightarrow$ Nodal

$$[C] = \begin{matrix} & \begin{matrix} a & b & c \end{matrix} & \begin{matrix} d & e \end{matrix} \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \end{matrix}$$

$\downarrow$ U_{twigs}
 $\downarrow$ C_{links}

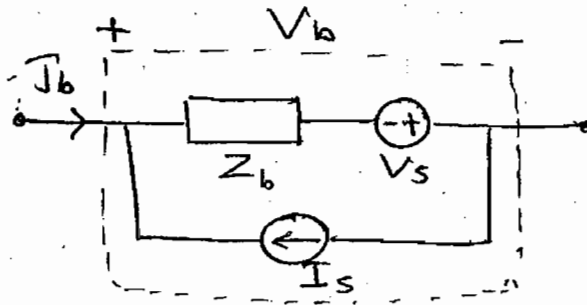
also,

$$[M_{\text{twigs}}] = -[C_{\text{links}}]^T$$

$$\text{or } [C_{\text{links}}] = -[M_{\text{twigs}}]^T$$

Application of Nlw Topology in Electrical ckt Analysis :-

Concept of stel. branch :



[I] Solution by KVL Eqⁿ [TIE-SET]

$$V_b = \underbrace{Z_b [I_b + I_s]}_{\text{Impedance form}} - V_s$$

But,

$$[M][V_b] = 0$$

$$\therefore M Z_b I_b + M Z_b I_s - M V_s = 0$$

But,

$$[M]^T [I_e] = [I_b]$$

$$[M][Z_b][M]^T [I_e] = [M][V_s] - [M][Z_b][I_s]$$

↑
Solve this

Then branch currents can be calculated

$$[M]^T [I_e] = [I_b]$$

↑
substitute this

→ final ans.

[II] Solution by KCL Eqⁿ [Cut-set]

$$I_b = Y_b [V_b + V_s] - I_s$$

Admittance form.

But,

$$[C][I_b] = 0$$

$$C Y_b V_b + C Y_b V_s - C I_s = 0$$

But,

$$[C]^T [e_+] = [V_b]$$

$$[C]^T [Y_b] [C]^T [e_+] = [C][I_s] - [C][Y_b][V_s]$$

↑
Solve this

Then branch voltages can be determined by:

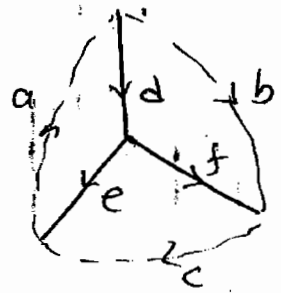
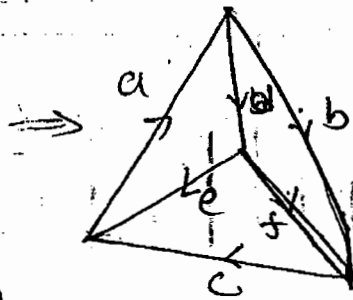
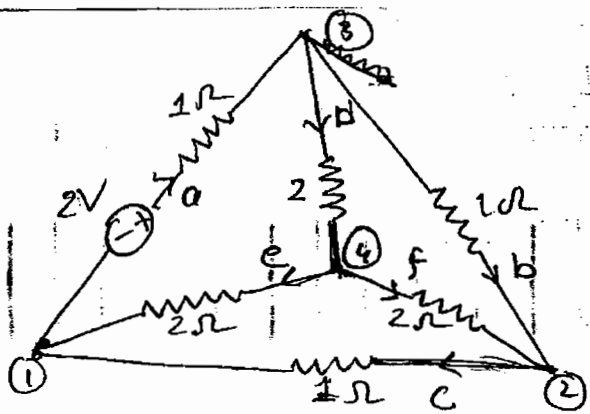
$$[C]^T [e_+] = [V_b]$$

↑
Substitute this

↳ Final answer

Q

- 1) Solve the n/w to find branch currents by writing the eqⁿ in std KVL form
- 2) Solve the n/w to find all the branch vltges by writing eqⁿ in std KCL form



1) TIE SET

$$[M][Z_b][M]^T[I_t] = [M][V_s] + [M][Z_b][I_s]$$

$$[M] = \begin{matrix} i_1 \\ i_2 \\ i_3 \end{matrix} \begin{bmatrix} a & b & c & d & e & f \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}_{3 \times 6}$$

$$[Z_b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6 \times 6}$$

$$[I_t] = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}_{3 \times 1} \quad [V_s] = \begin{bmatrix} +2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1} \quad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1}$$

Ans

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 2 & 0 \\ 0 & 1 & 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 0 & -2 & 2 \end{bmatrix}_{3 \times 6} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}_{6 \times 3} = \begin{bmatrix} 5 & -2 & -2 \\ -2 & 5 & -2 \\ -2 & -2 & 5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

$$\underline{\text{RHS}} \quad [M][V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}_{3 \times 1}$$

Final LHS

$$\begin{bmatrix} 5i_1 - 2i_2 - 2i_3 \\ -2i_1 + 5i_2 - 2i_3 \\ -2i_1 - 2i_2 + 5i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \quad \text{Final RHS}$$

Solving

$$i_1 = \frac{6}{7} \text{ A}, \quad i_2 = \frac{4}{7} \text{ A}, \quad i_3 = \frac{4}{7} \text{ A}$$

Then, Branch currents can be calculated by

$$[M]^T [I_c] = [J_b]$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 6/7 \\ 4/7 \\ 4/7 \end{bmatrix} = \begin{bmatrix} j_a \\ j_b \\ j_c \\ j_d \\ j_e \\ j_f \end{bmatrix}$$

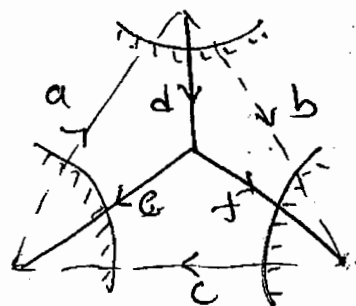
Final Ans

$$j_a = \frac{6}{7}, \quad j_b = \frac{4}{7}, \quad j_c = \frac{4}{7}, \quad j_d = \frac{2}{7}$$

$$j_e = \frac{2}{7}, \quad j_f = 0$$

2) CUT-SET

$$[C][Y_b][C]^T [e_s] = [C][I_s] - [C][Y_b][V_s]$$



$$[C] = \begin{matrix} & a & b & c & d & e & f \\ \begin{matrix} e_1 \\ e_2 \\ e_3 \end{matrix} & \begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix} \end{matrix}_{3 \times 6}$$

$$[Y_b] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 \end{bmatrix}_{3 \times 6}$$

$$[e_t] = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}_{3 \times 1} \quad [V_s] = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1} \quad [I_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6 \times 1}$$

LHS

$$\begin{bmatrix} -1 & 1 & 0 & 1/2 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1/2 \end{bmatrix}_{3 \times 6} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{6 \times 3} = \begin{bmatrix} 5/2 & 1 & 1 \\ 1 & 5/2 & 1 \\ 1 & -1 & 5/2 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}$$

RHS

$$-[C][Y_b][V_s]$$

$$-\begin{bmatrix} -2 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} +2 \\ +2 \\ 0 \end{bmatrix}$$

Final LHS

$$\begin{bmatrix} 2.5e_1 + e_2 + e_3 \\ e_1 + 2.5e_2 + e_3 \\ e_1 - e_2 + 2.5e_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

Final RHS

Solving

$$\begin{aligned} e_1 &= 4/7 \\ e_2 &= 4/7 \\ e_3 &= 0 \end{aligned}$$

Then, Branch voltages can be calculated by

$$[C]^T [e_s] = [V_b]$$

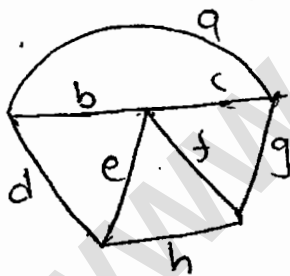
$$\begin{bmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4/7 \\ 4/7 \\ 0 \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \end{bmatrix}$$

Final Ans → this indicates presence of source in branch

$$V_a = -\frac{8}{7} V, \quad V_b = \frac{4}{7} V, \quad V_c = \frac{4}{7} V, \quad V_d = \frac{4}{7} V$$

$$V_e = \frac{4}{7} V, \quad V_f = 0 V$$

1) Which of the foll. set of branches is not a tree for the graph shown below.



(a) a g h e

(b) b c g h

(c) d e f g

✓ (d) a b f g

2) No. of trees = 5



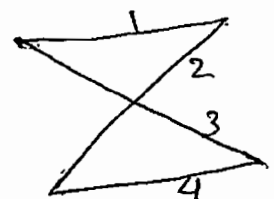
3) For the ^{n/w} ~~oriented~~ graph shown below if no of trees = p & no. of cut sets = q. then

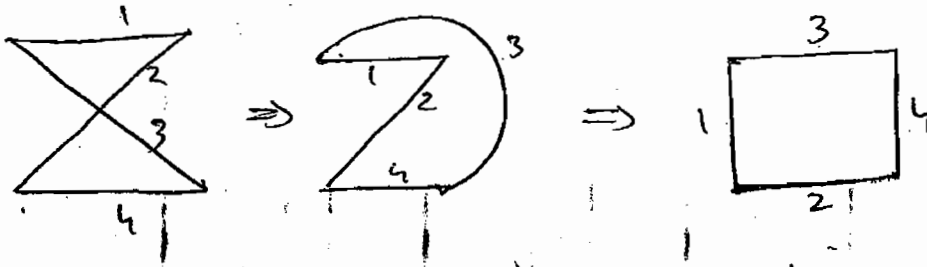
(a) p = 2, q = 2

(c) p = 4, q = 6

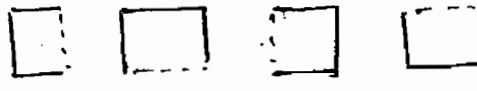
(b) p = 4, q = 4

(d) p = 4, q = 16





Here numbering indicates that there are 4 nodes only & the center one is not a node.

No. of trees = 4 

No of cut sets = 6

i.e. no. of ways that the graph can be cut into 2 parts.



Proper Tree :-

↳ Same defⁿ of Tree + $\begin{bmatrix} C, V \rightarrow \text{Twigs} \\ L, I \rightarrow \text{Links} \end{bmatrix}$

C & V to be linked with twigs.

L & I to be linked with links.

Spanning Graph :-

It is a sub-graph which connects all the nodes.

All trees are spanning graphs., but all spanning graphs are not trees.

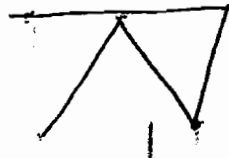
Ex



Graph

→ Spanning graph

→ but not Tree



→ Spanning graph

→ Also Tree

✓ 1) A n/w with 'n' nodes & 'b' branches.
has _____ no. of node-pair voltages.

Aus $nC_2 = \frac{n(n-1)}{2}$

ELECTRICAL

RESONANCE

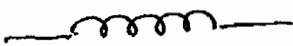
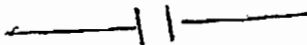
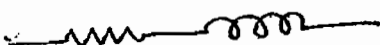
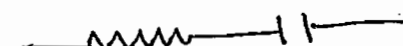
- Resonance is the freq. response of a ckt / n/w when the ckt operates at its natural freq called resonance freq.
- Under resonance total supply vltg & supply current are in phase. So, $\phi = 0^\circ$
& $PF = \cos \phi = 1$ (UPF)
- Under resonance the net impedance of the ckt becomes purely resistive & max power will be transferred to the ckt from source

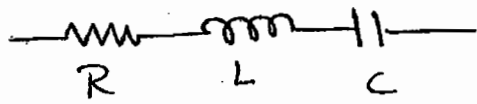
- Resonance can occur in any electrical n/w provided we have 2 similar but opposite natured energy storage components i.e. L & C .
- To undergo a good observable resonance for practical applications we need a good quality in these energy storage components which is measured as Quality factor or figure of merit given by:

$$Q\text{-factor} = 2\pi * \left[\frac{\text{Max. energy stored per cycle of supply}}{\text{Energy dissipated per cycle of supply}} \right]$$

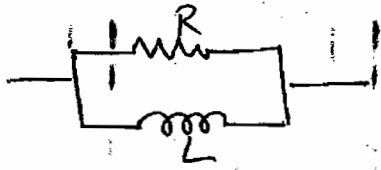
In practical applications $Q \geq 10$

- Resonance phenomenon is useful in designing of passive filters, & antennas, receivers, SONARS, etc.

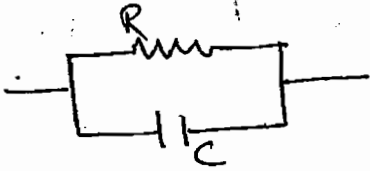
Element	Q-Factor
	0
	∞
	∞
	$\frac{\omega L}{R}$
	$\frac{1}{\omega RC}$



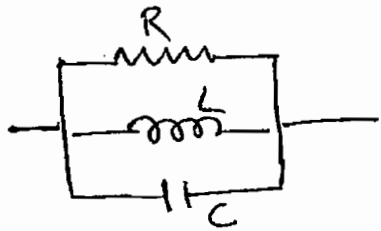
$$Q_0 = \frac{1}{R} \sqrt{\frac{L}{C}}$$



$$\frac{R}{\omega L}$$

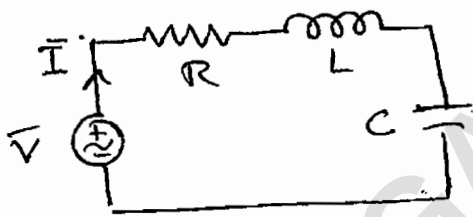


$$\omega RC$$



$$Q_0 = R \sqrt{\frac{C}{L}}$$

<1> Series Resonance :-



At resonance ; $\omega = \omega_0$

V & I in phase

$$\phi = 0^\circ ; Z = R$$

But, $Z = R + j[X_L - X_C]$

but at resonance, net reactance = 0

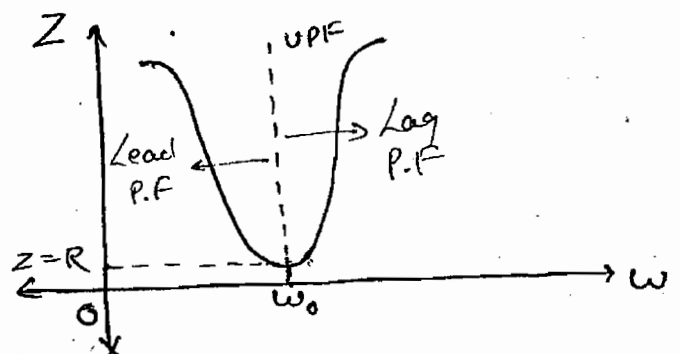
$$\therefore X_L - X_C = 0 \Rightarrow X_L = X_C$$

$$\therefore \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} ; f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

Graph ①

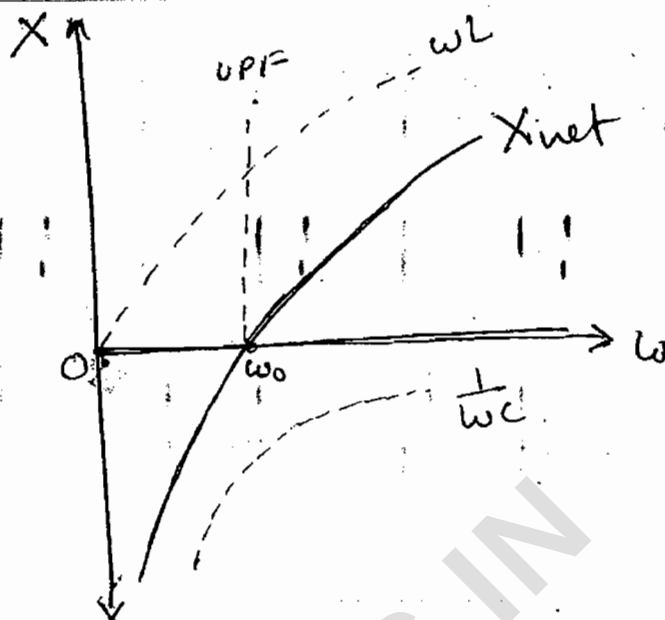
Z vs ω



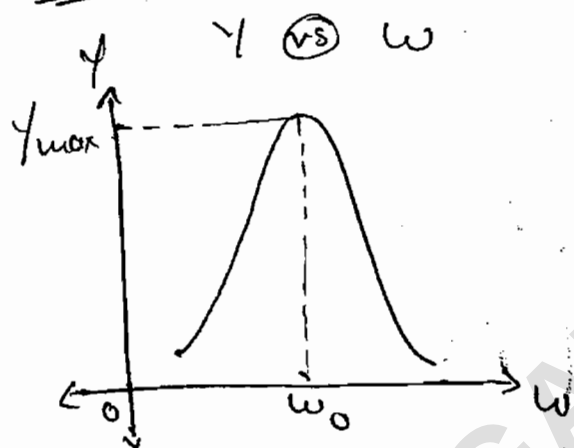
Graph ② $X \text{ vs } \omega$

$$Z = R + j \left[\omega L - \frac{1}{\omega C} \right]$$

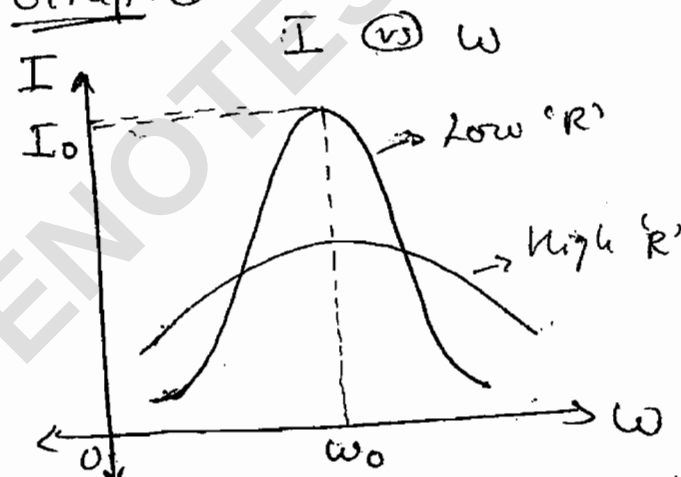
$\downarrow$
 X_{net}



Graph ③



Graph ④



Phasor diagram :-

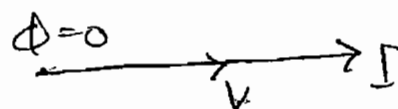
(a) $\omega = \omega_0$

$$Z = R$$

$\hookrightarrow$ purely resist.

'I' in phase with 'V'

$$\phi = 0^\circ \text{ [UPF]}$$

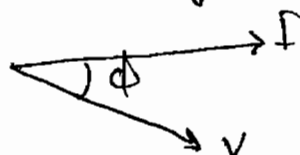


(b) $\omega < \omega_0$

$$Z = R - jX_{net}$$

$\hookrightarrow$ R-C ckt

'I' leads 'V'
by $\phi < 90^\circ$
(Leading PF)



ϕ is -ve with
respect to I (neg)

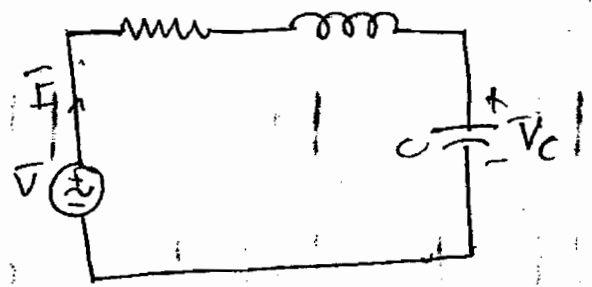
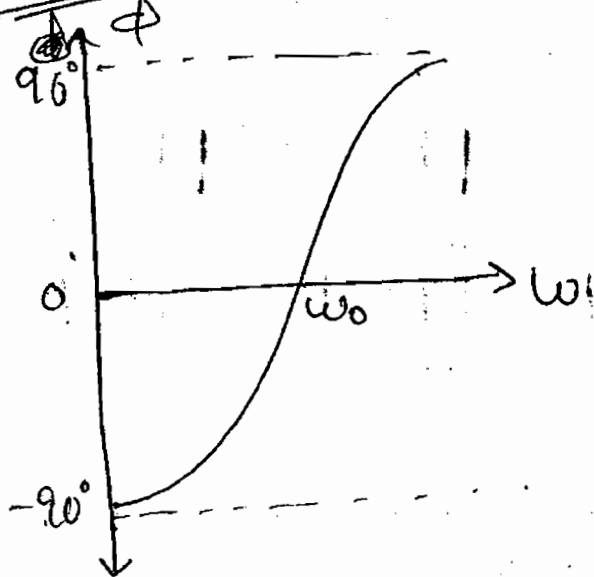
(c) $\omega > \omega_0$

$$Z = R + jX_{net}$$

$\hookrightarrow$ R-L ckt

'I' lags 'V'
by $\phi < 90^\circ$
(Lagging PF)

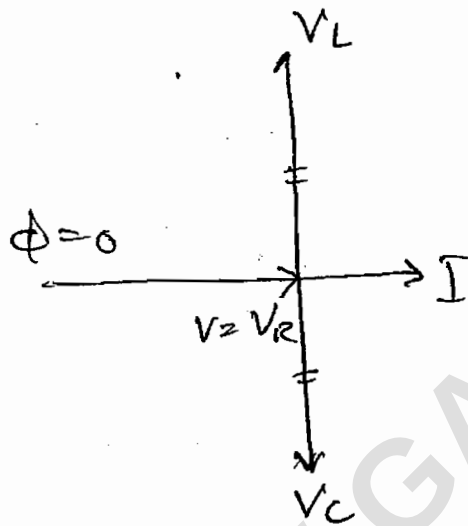
GRAPH - 5



at Resonance $\omega = \omega_0$

$$|X_L| = |X_C|$$

$$|V_L| = |V_C|$$



Q - Factor at Resonance.

$$Q_0 = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R}$$

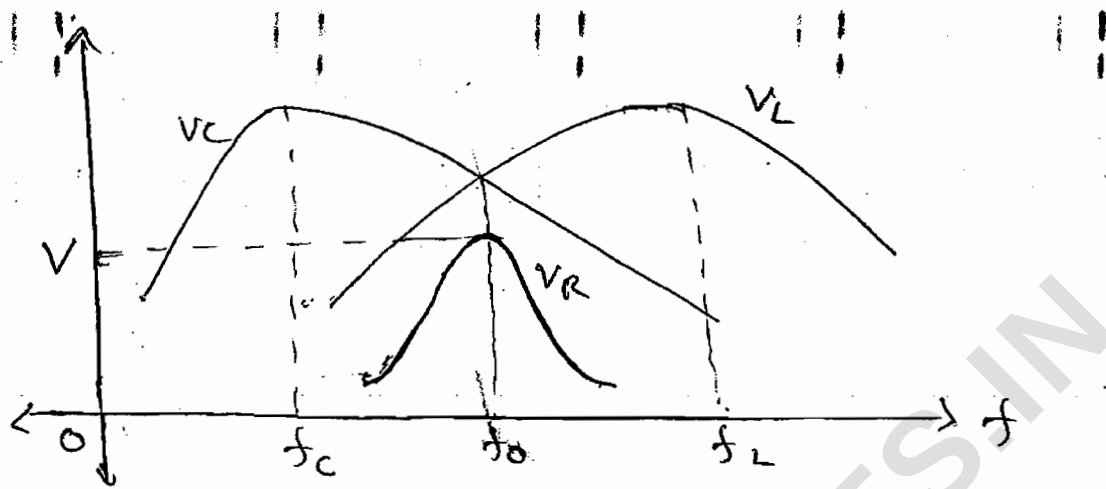
But $\omega_0 = \frac{1}{\sqrt{LC}}$

→ Under series resonance net impedance is min, so current is max.

Hence it is called as acceptor ckt.

→ At series resonance freq. it is as if the total supply vltg appears across resistor. Hence series resonance is called vltg amplification ckt.

Variation of voltages across passive elements with change in freq.



The freq. at which max. vltg appears across capacitor

$$f_c = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}} = \frac{1}{2\pi\sqrt{LC}} \sqrt{1 - \frac{R^2C}{2L}}$$

$$f_c = f_0 \sqrt{1 - \frac{R^2C}{2L}} \text{ Hz}$$

The freq. at which max. vltg appears across inductor:

$$f_L = \frac{1}{2\pi \sqrt{LC - \frac{R^2C^2}{2}}} = \frac{1}{2\pi\sqrt{LC} \left[1 - \frac{R^2C}{2L}\right]}$$

$$f_L = \frac{f_0}{\sqrt{1 - \frac{R^2C}{2L}}} \text{ Hz}$$

From circuit

$$\bar{V} = \bar{I} \bar{Z} \Rightarrow \bar{I} = \frac{|\bar{V}|}{|\bar{Z}|}$$

But

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$|I| = \frac{|\bar{V}|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

But at $\omega = \omega_0$

$$|I| = \frac{|\bar{V}|}{\sqrt{R^2 + 0^2}} \Rightarrow \text{maximum}$$

So,

$$|I_0| = \frac{|\bar{V}|}{R}$$

So, Power transfer also 'maximum'

$$P_0 = I_0^2 R = \frac{|\bar{V}|^2}{R} \quad \text{W}$$

at $\omega = \omega_0$

$$\boxed{R} \quad V_R = I_R \cdot R = I_0 R = \frac{|\bar{V}|}{R} \cdot R = \boxed{|\bar{V}| = V_R}$$

$$\begin{aligned} \boxed{L} \quad V_L &= +j X_L I = j \omega_0 L I_0 = +j \omega_0 L \times \frac{|\bar{V}|}{R} \\ &= +j \left[\frac{\omega_0 L}{R} \right] |\bar{V}| \Rightarrow \boxed{V_L = +j Q_0 |\bar{V}|} \end{aligned}$$

$$\begin{aligned} \boxed{C} \quad V_C &= -j X_C I = \frac{-j}{\omega_0 C} I_0 = \frac{-j}{\omega_0 C} \frac{|\bar{V}|}{R} = -j \left[\frac{1}{\omega_0 R C} \right] |\bar{V}| \\ &\Rightarrow \boxed{V_C = -j Q_0 |\bar{V}|} \quad Q_0 \rightarrow \text{voltage magnification factor.} \end{aligned}$$

Bandwidth:—

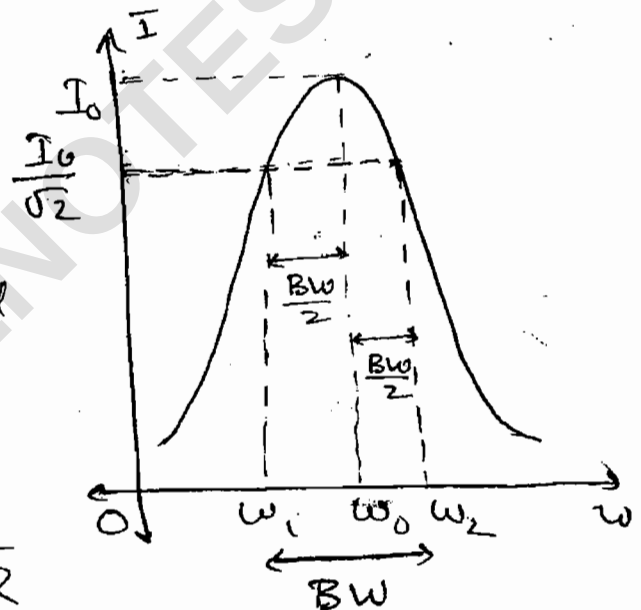
Bandwidth represents the range of frequencies for which the power level in the signal is at half of the signal max. power.

→ Half of max. power frequencies:

$$\frac{P_0}{2} = \frac{I_0^2 R}{2} = \left[\frac{I_0}{\sqrt{2}} \right]^2 R = (0.707 I_0)^2 R$$

$\omega_1 \rightarrow$ lower cut-off /
roll off ~~freq.~~ ^{corner} freq.

$\omega_2 \rightarrow$ upper cut-off / roll
off corner freq.



$$\frac{|V|}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{I_0}{\sqrt{2}} = \frac{V}{\sqrt{2} R}$$

$$\therefore R^2 + (\omega L - \frac{1}{\omega C})^2 = 2R^2$$

$$(\omega L - \frac{1}{\omega C})^2 = R^2 \Rightarrow \omega L - \frac{1}{\omega C} = \pm R$$

At $\omega = \omega_1$

$$\omega_1 L - \frac{1}{\omega_1 C} = -R \quad \text{--- (1)}$$

At $\omega = \omega_2$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \quad \text{--- (2)}$$

Bandwidth :

$$\left. \begin{aligned} \omega_2 - \omega_1 &= \frac{R}{L} \quad \text{rad/sec} \\ f_2 - f_1 &= \frac{R}{2\pi L} \quad \text{Hz} \end{aligned} \right\} \begin{aligned} \text{B.W} &\propto R \\ \text{BW} &\text{ is independent to 'f}_0\text{'} \end{aligned}$$

→ Resonance freq. is geometric mean of Bandwidth freq.

$$\omega_0 = \sqrt{\omega_1 \omega_2}$$

$$f_0 = \sqrt{f_1 f_2}$$

also,

$$\omega_0 - \omega_1 = \frac{\text{BW}}{2} \Rightarrow \omega_1 = \omega_0 - \frac{R}{2L} \quad \text{rad/sec}$$

$$f_1 = f_0 - \frac{R}{4\pi L} \quad \text{Hz}$$

$$\omega_2 - \omega_0 = \frac{\text{BW}}{2} \Rightarrow \omega_2 = \omega_0 + \frac{R}{2L} \quad \text{rad/sec}$$

$$f_2 = f_0 + \frac{R}{4\pi L} \quad \text{Hz}$$

⇒ Resonance freq. is independent of resistor (R).

Selectivity :- (S)

→ Selectivity is the ability of a ckt/ulw to distinguish or discriminate desired & undesired freq.

$$\rightarrow S \propto \frac{1}{\text{BW}}$$

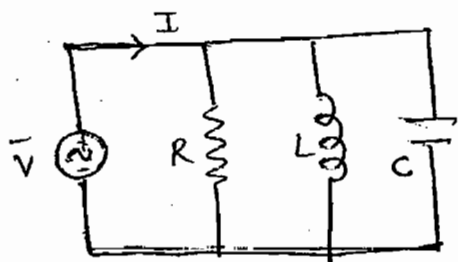
$$S = \frac{f_0}{|f_2 - f_1|} = \frac{\omega_0}{|\omega_2 - \omega_1|}$$

$$S = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi}} = \frac{1}{R} \sqrt{\frac{L}{C}} = Q!$$

→ The value of selectivity is Q-factor under Resonance.

<2> Parallel Resonance : —

① General ckt



At resonance

$V \propto I$ (in phase)

$\phi = 0^\circ$, $PF = 1$ (UPF)

$$Y_T = Y_R + Y_L + Y_C$$

$$Y_T = \frac{1}{R} + \frac{1}{jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} + \frac{j}{X_C} - \frac{j}{X_L} = \frac{1}{R} + j \left[\frac{1}{X_C} - \frac{1}{X_L} \right]$$

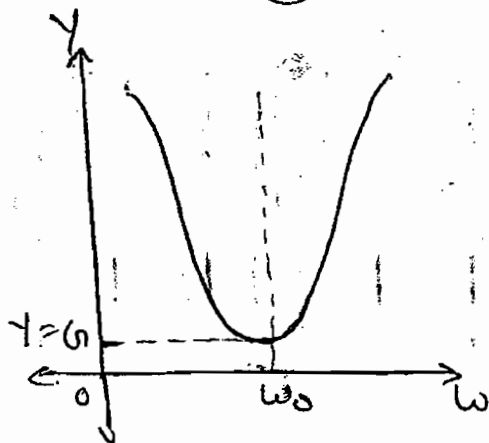
At resonance ($\omega = \omega_0$)

Net susceptance = 0

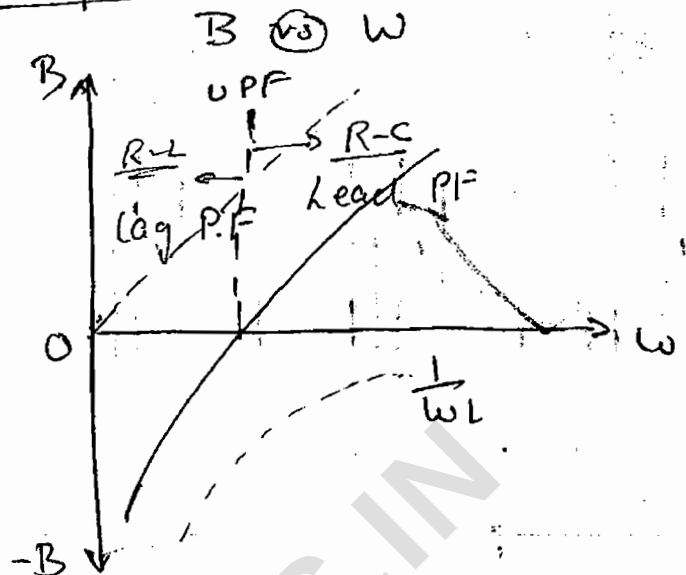
$$\frac{1}{X_C} - \frac{1}{X_L} = 0 \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

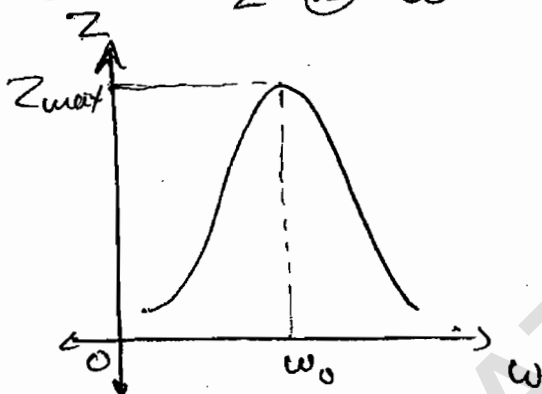
Graph ① $Y \text{ vs } \omega$



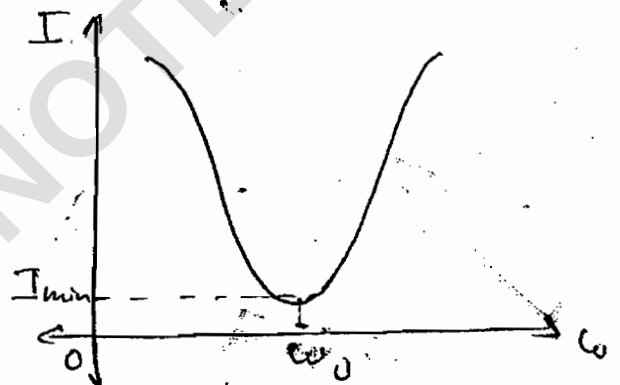
Graph ② $B \text{ vs } \omega$



Graph ③ $Z \text{ vs } \omega$



Graph ④ $I \text{ vs } \omega$



Phasor diagrams:

(a) $\omega = \omega_0$

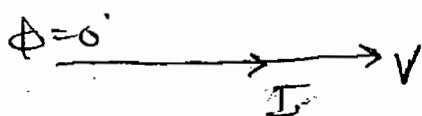
$$Y = G$$

$\hookrightarrow$ pure resis.

'I' in phase with 'V'.

$$\phi = 0^\circ$$

$$PF = 1 \text{ (UPF)}$$



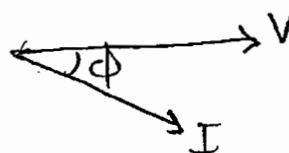
(b) $\omega < \omega_0$

$$Y = G - jB_{net}$$

$\hookrightarrow$ R-L parallel ckt

'I' lags 'V' by $\phi < 90^\circ$

(lagging PF)



ϕ is -ve w.r.t V (ref.)

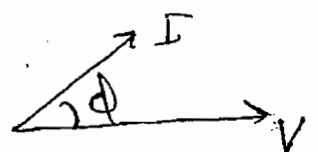
(c) $\omega > \omega_0$

$$Y = G + jB_{net}$$

$\hookrightarrow$ R-C parallel ckt

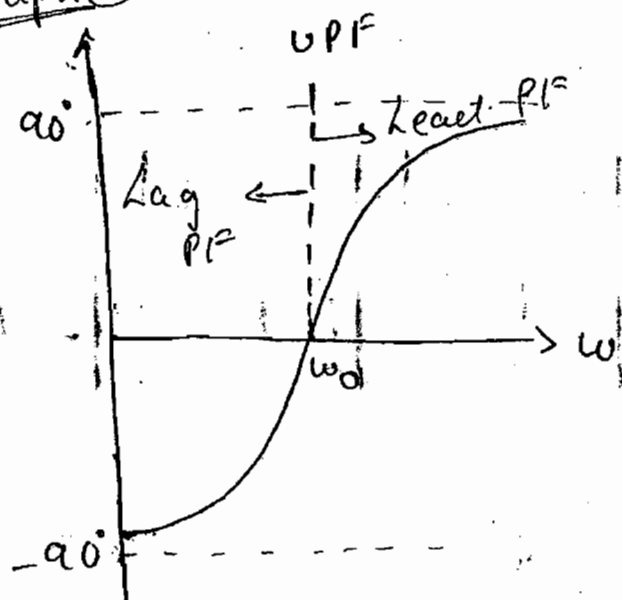
'I' leads 'V' by $\phi < 90^\circ$

(leading PF)

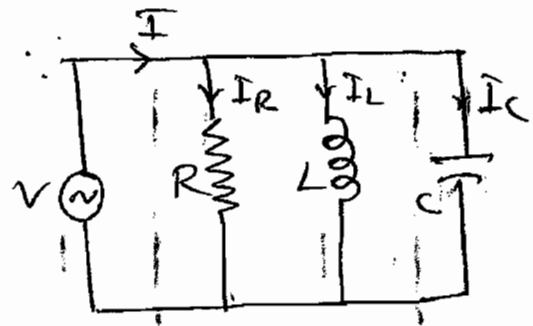


ϕ is +ve w.r.t V (ref.)

Graph ⑤



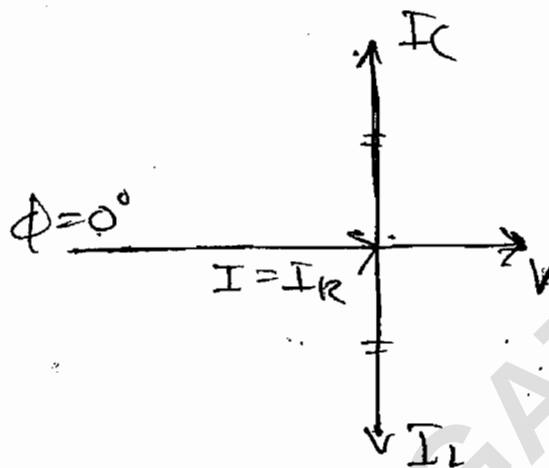
Complete phasor diagram



At resonance

$$|B_L| = |B_C|$$

$$|I_L| = |I_C|$$



Q-factor at Resonance

$$Q_0 = \frac{R}{\omega_0 L} = \omega_0 RC$$

But $\omega_0 = \frac{1}{\sqrt{LC}}$

$$Q_0 = \frac{R}{\frac{1}{\sqrt{LC}} \times L} = \frac{1}{\sqrt{LC}} \times RC$$

$$I = I_R$$

$$\phi = 0^\circ$$

$$PF = 1 \text{ (UPF)}$$

$$Q_0 = R \sqrt{\frac{C}{L}}$$

At parallel resonance condition, net impedance is max. so current is minimum. Hence it is called rejector ckt.

At 11th resonance freq. it is as if the total current flows only through resis. Hence it is called as current amplification circuit.

So, from circuit

$$|V| = |I| |Z| \Rightarrow |V| = \frac{|I|}{|Y|}$$

But

$$|Y| = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

$$|V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}}$$

At $\omega = \omega_0$

$$|V| = \frac{|I|}{\sqrt{\frac{1}{R^2} + 0^2}}$$

So,

$$|V| = |I| R$$

At $\omega = \omega_0$

[R] $I_R = \frac{V_R}{R} = \frac{V}{R} = \frac{|I| R}{R} \Rightarrow I_R = |I|$

[L] $I_L = \frac{V_L}{+jX_L} = \frac{V}{+j\omega_0 L} = -j \left[\frac{R}{\omega_0 L} \right] \cdot I$

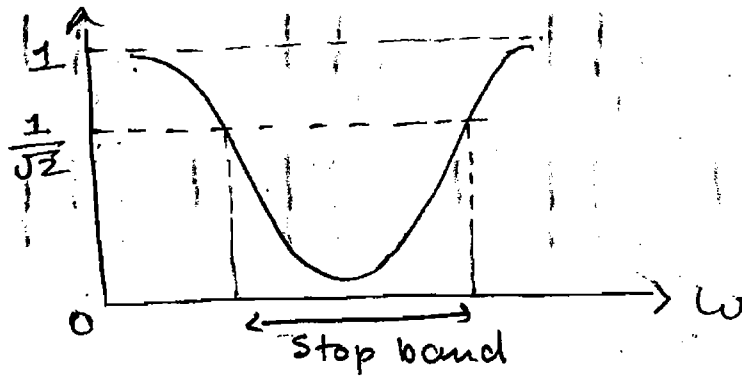
$$\Rightarrow I_L = -j Q_0 |I|$$

[C] $I_C = \frac{V_C}{-jX_C} = \frac{V}{-j/\omega_0 C} = +j [\omega_0 R C] \cdot I$

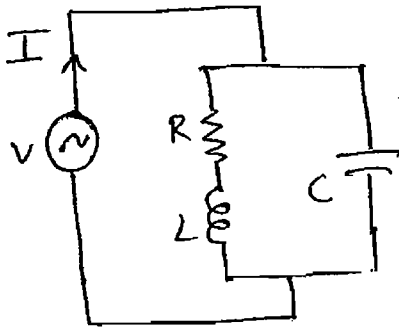
$$\Rightarrow I_C = +j Q_0 |I|$$

$Q_0 \rightarrow$ current amplification factor.

Parallel resonance phenomenon is used in design of Band Stop Filter.



Practical Parallel Resonance :-
(Tank Circuit)



$$Y_T = Y_1 + Y_2$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R + jX_L} = \frac{R - jX_L}{R^2 + X_L^2}$$

$$Y_2 = \frac{1}{Z_2} = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y_T = \frac{R}{(R^2 + X_L^2)} + j \left[\frac{1}{X_C} - \frac{X_L}{R^2 + X_L^2} \right]$$

At Resonance $\omega = \omega_0$

⇒ Net susceptance = 0

$$\frac{1}{X_C} = \frac{X_L}{R^2 + X_L^2} \Rightarrow R^2 + X_L^2 = \omega_0 L \times \frac{1}{\omega_0 C}$$

$$\therefore R^2 + X_L^2 = \frac{L}{C}$$

$$\text{So, } R^2 + \omega_0^2 L^2 = \frac{L}{C}$$

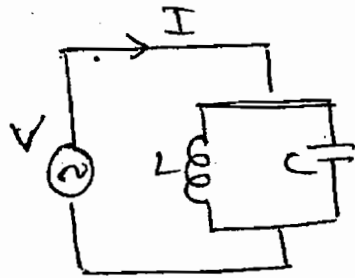
$$\omega_0^2 L^2 = \frac{L}{C} - R^2 \Rightarrow \omega_0^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \omega_0 = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{Hz}$$

Ideal Tank circuit: —

($R=0$)



$$Y_T = Y_L + Y_C$$

$$Y_L = \frac{1}{jX_L} = \frac{-j}{X_L}$$

$$Y_C = \frac{1}{-jX_C} = \frac{j}{X_C}$$

$$Y_T = +j \left[\frac{1}{X_C} - \frac{1}{X_L} \right]$$

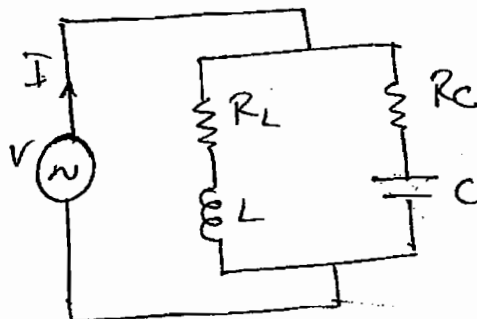
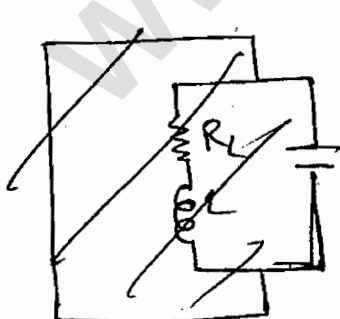
At resonance $\omega = \omega_0$

$\Rightarrow$ Net susceptance = 0

$$\frac{1}{X_C} = \frac{1}{X_L} \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz}$$

1) Determine $f_0 =$ —



$$f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_L^2 - (L/C)}{R_C^2 - (L/C)}}$$

Concept of Dynamic Impedance (Dynamic Resistance)

→ It is the resis. offered by the ckt to the i/p at resonant freq.

① Series R-L-C ckt

$$Z_{dyn} = R$$

② Generally parallel R-L-C ckt.

$$Z_{dyn} = R$$

③ Tank ckt : $Z_{dyn} = \frac{L}{RC}$

$$\text{Practically } Z_{dyn} \gg R$$

④ Ideal tank ckt :

$$Z_{dyn} = \infty \rightarrow 0^\circ C$$

1) Two practical coils with internal resistance R_1, R_2 have Q-factor Q_1, Q_2 resp. If these coils are connected in series then total Q-factor is —

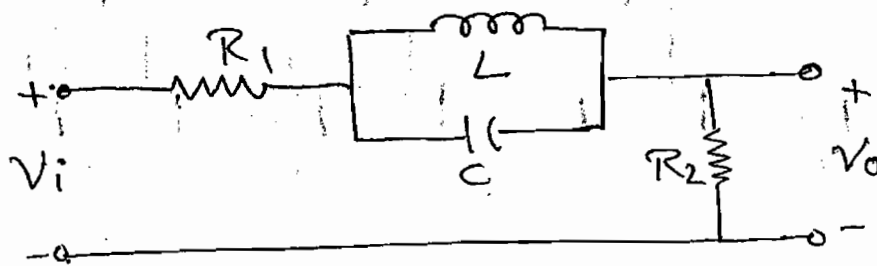
$$Q_1 = \frac{\omega L_1}{R_1}$$

$$Q_2 = \frac{\omega L_2}{R_2}$$

$$Q_T = \frac{\omega (L_1 + L_2)}{R_1 + R_2} = \frac{\frac{\omega L_1}{R_1} \times R_1 + \frac{\omega L_2}{R_2} \times R_2}{R_1 + R_2}$$

$$Q_T = \frac{Q_1 R_1 + Q_2 R_2}{R_1 + R_2}$$

2) The n/w below acts as _____



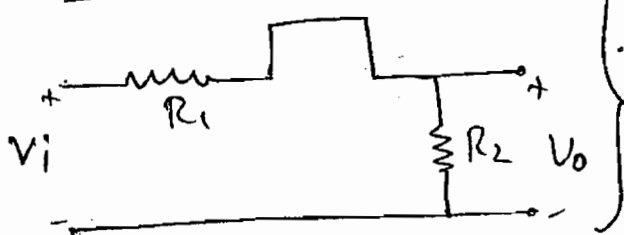
(a) Low pass filter

(b) High pass "

(c) Band pass "

✓ (d) Band stop "

At $\omega = 0$



$$V_o = V_i \left[\frac{R_2}{R_1 + R_2} \right]$$

$$H(\omega) = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

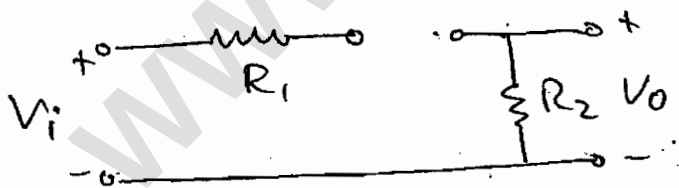
At $\omega = \infty$



$$V_o = V_i \left[\frac{R_2}{R_1 + R_2} \right]$$

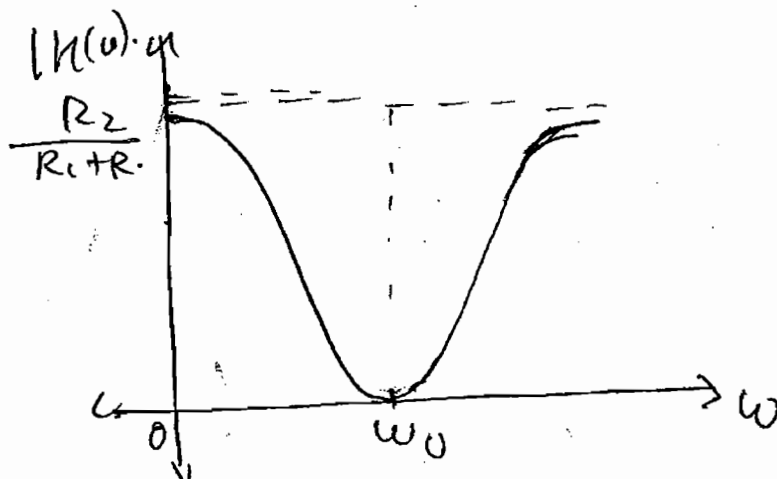
$$H(\omega) = \frac{V_o}{V_i} = \frac{R_2}{R_1 + R_2}$$

At $\omega = \omega_0$



$$V_o = 0$$

$$H(\omega) = 0$$

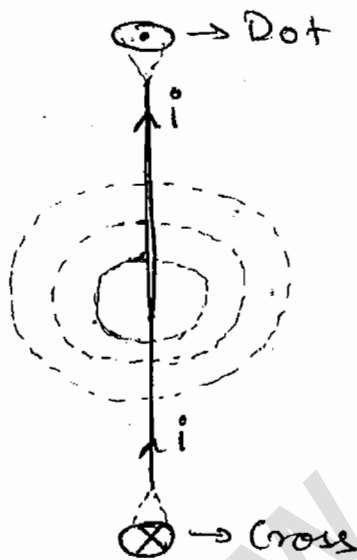


MAGNETIC

CIRCUITS

Charges at rest produce only Electro-static field but charges in motion produce both Electric & Magnetic field.

Ampere's Right Hand Thumb Rule :-



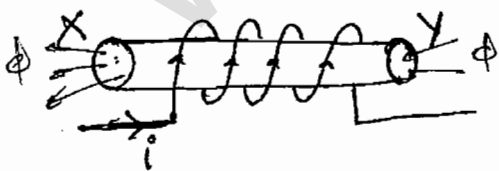
Permanent Magnet

→ Rare Earth material

→ Alnicos

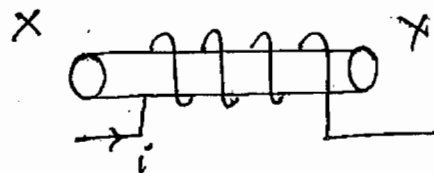
→ High Retentivity

Electro-magnet



X = N pole

Y = S pole



X = S pole

Y = N pole



Φ = clock-wise oriented



Φ = anti clock wise oriented.



X = S pole



Y = N pole

Analogy b/w Electric & Magnetic ccts

Electric Circuit

Magnetic Circuit -

- ① Voltage / EMF is the 'cause'
units: volts

- ① MMF → Magneto motive force → cause

$$\boxed{\text{MMF} = N \cdot i}$$

units: Ampere-turns (AT)

- ② 'Current' is the 'effect' I
units: Ampere (A)

- ② 'Flux' is the 'effect' Φ , units: webers (wb)

- ③ Ohms Law :

$$R = \frac{V}{I}$$

↳ Resistance ; unit: Ω

- ③ Ohms Law :

$$\mathcal{S} = \frac{\text{MMF}}{\Phi} = \frac{N \cdot i}{\Phi}$$

↳ Reluctance ; units: $\frac{\text{AT}}{\text{wb}}$

- ④ $\boxed{R = \frac{\rho l}{a}}$ → Electrical Material

- ④ $\boxed{\mathcal{S} = \frac{l}{\mu a}}$ → Magnetic material

- ⑤ Electric Field Intensity :

$$\boxed{E = \frac{V}{d}}$$

unit: volt/m

- ⑤ Magnetic Field intensity

$$\boxed{H = \frac{\text{MMF}}{l} = \frac{N \cdot i}{l}}$$

units: AT/m

⑥ Electric Current Density

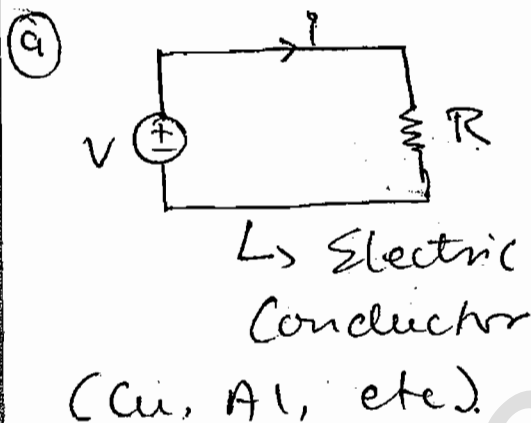
$$\boxed{J = \frac{I}{a}} \quad \text{unit: } \frac{A}{m^2}$$

⑦ $\boxed{D = \epsilon E}$
 ↳ Electric flux density

$$\boxed{\epsilon = \epsilon_0 \epsilon_r}$$

⑧ $R_s = R_1 + R_2$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$



- ⑩
- KVL
 - KCL
 - VDR (series)
 - CDR (H^{el})

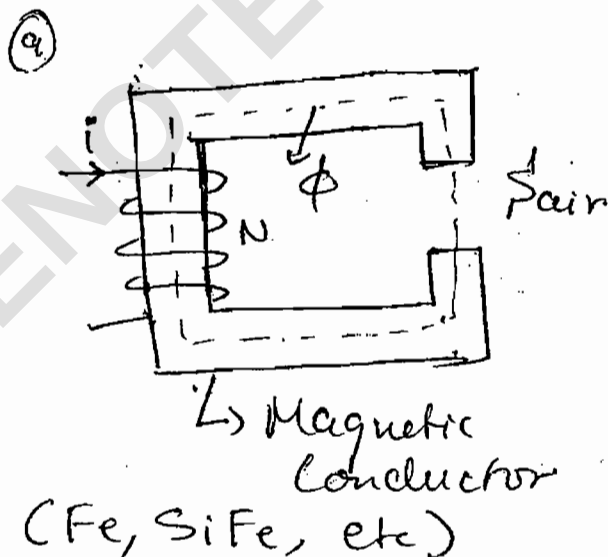
⑥ Magnetic Flux Density

$$\boxed{B = \frac{\phi}{a}} \quad \text{units: } \frac{wb}{m^2} \text{ (or) Tesla (T)}$$

⑦ $\boxed{B = \mu H}$

$$\boxed{\mu = \mu_0 \mu_r}$$

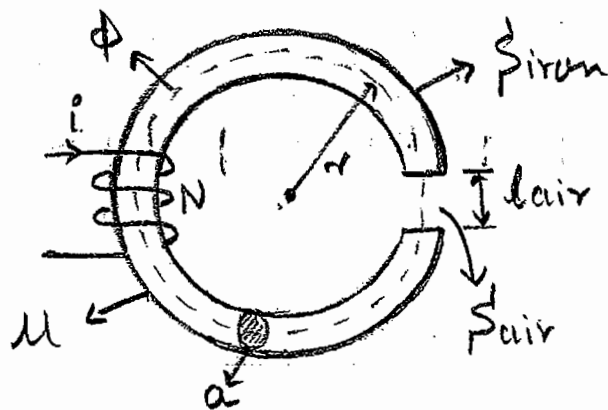
⑧ $S_s = S_1 + S_2$
 $\frac{1}{S_p} = \frac{1}{S_1} + \frac{1}{S_2}$



- ⑩
- K MMF L
 - K Flux L
 - MMF · D · R (Series)
 - Flux · D · R (H^{el})

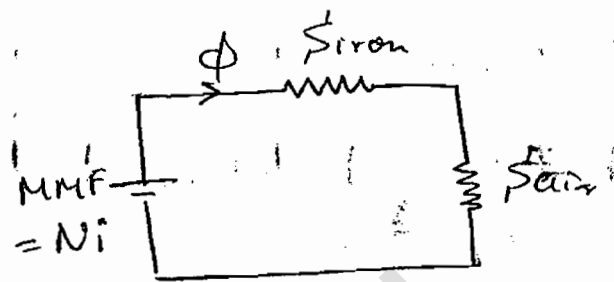
Composite Magnetic Circuits :—

Series Magnetic Circuit



$$S_{\text{iron}} = \frac{l_{\text{iron}}}{\mu_0 \mu_r a}$$

$$S_{\text{air}} = \frac{l_{\text{air}}}{\mu_0 a}$$

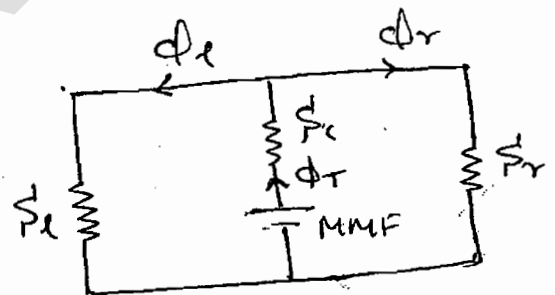
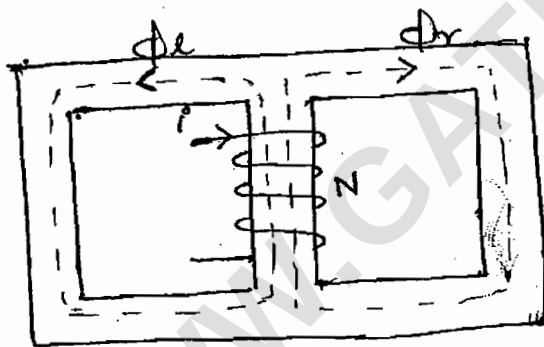


$$MMF = Ni = \Phi [S_T]$$

$$Ni = \Phi [S_{\text{iron}} + S_{\text{air}}]$$

$$l = 2\pi r$$

$$l_{\text{iron}} + l_{\text{air}} = 2\pi r$$



$$[MMF - \Phi_r S_c] = \Phi_e S_e = \Phi_r S_r$$

Faraday's Law of Electromagnetic Induction :—

- ① "There should be change in the flux linkage with the coil in order to induce emf in it."
- ② "The magnitude of this induced emf is proportional to rate of change of flux."

Mathematically Faradays law is given by:

$$e = - \frac{d\psi}{dt}$$

But, $\psi = N\phi \Rightarrow$ Flux linkages (wb-Turns)

$$e = - N \frac{d\phi}{dt}$$

$\rightarrow$ Rate of change of flux (wb/sec)
 $\rightarrow$ No. of turns.
 $\rightarrow$ -ve sign due to Lenz's law
 $\rightarrow$ induced emf in coil (volts)

$$e = - N \left[\frac{\phi_{\text{final}} - \phi_{\text{initial}}}{\Delta t} \right]$$

Dynamically Induced emf :-

Ex Motors, generator.

Statically Induced emf :-

Ex Transformers.

$\Rightarrow$ Flux is a function of current

$$\text{MMF} = Ni = \phi \oint$$

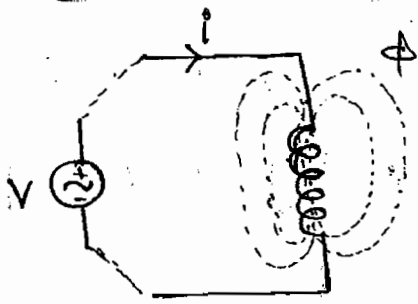
$$\therefore \frac{1}{\oint} \approx \frac{\phi}{i} = \frac{N \rightarrow \text{const}}{\oint \rightarrow \text{const.}}$$

$$\therefore \boxed{\frac{\phi}{i} = k} \Rightarrow \phi = ki$$

$$\phi \propto i$$

$\rightarrow$ Flux, a func of current

Concept of self induced emf & Self inductance :—



$$\Phi = \frac{\Phi}{i} \times i$$

$$\frac{d\Phi}{dt} = \frac{\Phi}{i} \frac{di}{dt}$$

$$e = -N \frac{d\Phi}{dt}$$

$$\therefore e = -\frac{N\Phi}{i} \cdot \frac{di}{dt}$$

$$e = -L \frac{di}{dt} \text{ volts} \quad (v = -e = +L \frac{di}{dt})$$

$$L = \frac{N\Phi}{i} = \frac{\Psi}{i} \quad (\text{self inductance H})$$

$$e = -L \left[\frac{i_{\text{final}} - i_{\text{initial}}}{\Delta t} \right] \text{ volts.}$$

$$\boxed{\Psi = Li} \longrightarrow \text{Ohm's Law}$$

also

$$\text{MMF} = \Phi S$$

$$Ni = \Phi S$$

$$\therefore N \left[\frac{N\Phi}{L} \right] = \Phi S$$

$$L = \frac{N^2}{S}$$

$$L \propto N^2$$

also

$$S = \frac{l}{\mu a}$$

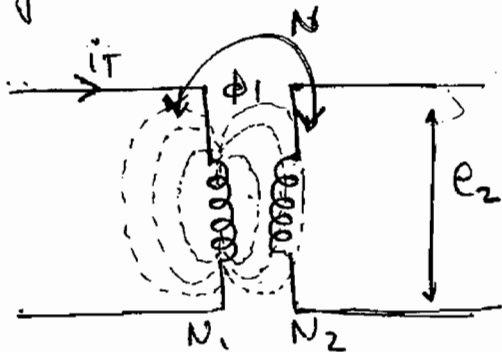
$$L = \frac{N^2}{l/\mu a}$$

$$L = \frac{\mu N^2 a}{l} \text{ H}$$

Concept of Mutual Inductance & Mutually Induced Emf :—

Self inductance is w.r.t the same coil & its own turns, current & flux.

However mutual inductance is b/w sep. of coils (min. two).



$$\Phi_{12} = k \Phi_1$$

$$0 \leq k \leq 1$$

$$\Phi_{12} = \Phi_{12} \times \frac{i_1}{i_1}$$

$$\Phi_{12} = \frac{\Phi_{12}}{i_1} \cdot i_1$$

$$\frac{d\Phi_{12}}{dt} = \left[\frac{\Phi_{12}}{i_1} \right] \frac{di_1}{dt}$$

$$\Phi_1 = \Phi_{l1} + \Phi_{12}$$

Total flux
Leakage flux
Common flux (Mutual flux)

$$e_2 = -N \frac{d\Phi_{12}}{dt} \quad \text{--- (2)}$$

$$e_2 = - \left[\frac{N\Phi_{12}}{i_1} \right] \frac{di_1}{dt} \Rightarrow \boxed{e_2 = -[M_{12}] \frac{di_1}{dt}}$$

↳ Mutually induced emf (volts)

$$M_{12} = \frac{N_2 \Phi_{12}}{i_1} = \frac{k \Phi_1 N_2}{i_1}$$

Mutual inductance in H.

Also
vice-versa

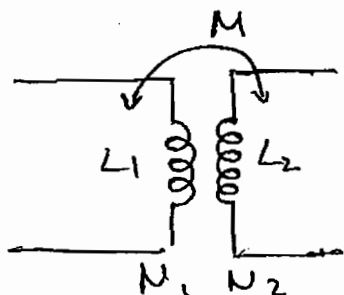
$$\Phi_{21} = k \Phi_2$$

$$e_1 = - \left[\frac{N_1 \Phi_{21}}{i_2} \right] \frac{di_2}{dt}$$

$$e = -[M_{21}] \frac{di_2}{dt}$$

$$M_{21} = \frac{N_1 \Phi_{21}}{i_2} = \frac{k \Phi_2 N_1}{i_2}$$

* If dist. b/w coils & permeability of medium b/w coils is const. then: $M_{12} = M_{21} = M$



$$L_1 = \frac{N_1 \Phi_1}{i_1}$$

$$L_2 = \frac{N_2 \Phi_2}{i_2}$$

$$M = \frac{k \Phi_1 N_2}{i_1} = \frac{k \Phi_2 N_1}{i_2} \quad \left. \vphantom{\frac{k \Phi_1 N_2}{i_1}} \right\} \text{Mutual inductance b/w coils.}$$

Relation b/w self & Mutual inductances :-

$$M * M = \left[\frac{k \Phi_1 N_2}{i_1} \right] * \left[\frac{k \Phi_2 N_1}{i_2} \right]$$

$$M^2 = k^2 \left[\frac{N_1 \Phi_1}{i_1} \right] \left[\frac{N_2 \Phi_2}{i_2} \right]$$

$$M^2 = k^2 [L_1][L_2]$$

$$\boxed{M = k \sqrt{L_1 L_2}}$$

But, $0 \leq k \leq 1$

$$\Rightarrow \boxed{M \leq \sqrt{L_1 L_2}}$$

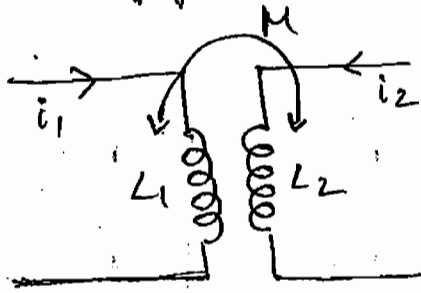
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$\rightarrow$ co. eff. of coupling

For ideal transf.

$$k = 1$$

Energy stored in system of 2-coils :-



$$E_T = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 \pm M i_1 i_2$$

+ $\rightarrow$ Mutually adding flux
- $\rightarrow$ Mutually opposing flux

Mutual inductance is always a +ve quantity.

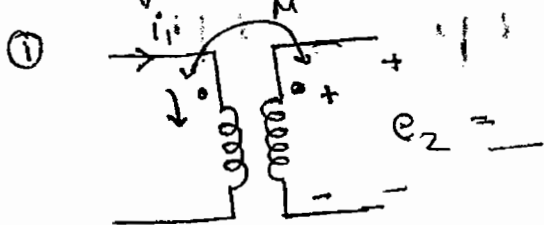
But, mutually induced emf may be +ve & -ve

Determining the correct polarity of mutual induced emf is not possible directly & hence we use Dot convention.

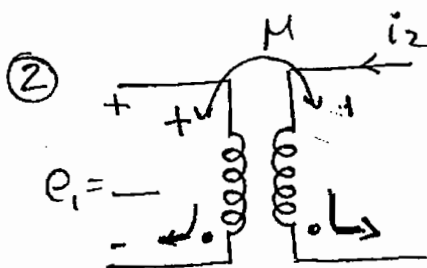
- ① If current enters the dotted terminal of the 1st coil, then the polarity of mutual v_{xy} is +ve at the dotted terminal of 2nd coil.
- ② If current leaves dotted terminal of 1st coil, then the polarity of mutual v_{xy} is -ve at the dotted terminal of the 2nd coil.

Ex
1)

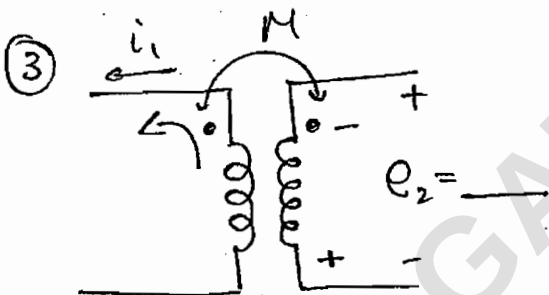
Determine the correct magnitude & polarity of the mutual voltage w.r.t the given reference voltage for the sys. of coils shown below.



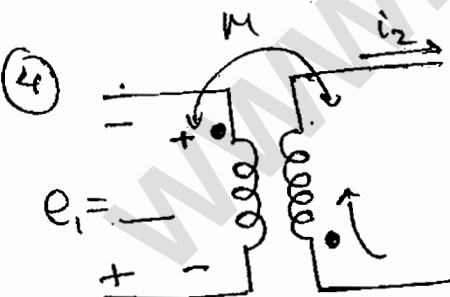
$$e_2 = + M \frac{di_1}{dt}$$



$$e_1 = + M \frac{di_2}{dt}$$



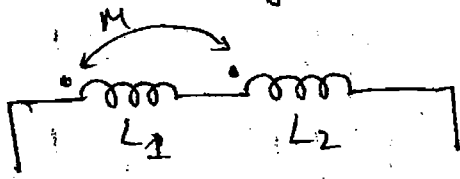
$$e_2 = - M \frac{di_1}{dt}$$



$$e_1 = - M \frac{di_2}{dt}$$

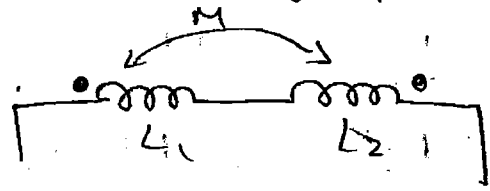
Coils in Series

Mutually adding



$$L_{eq} = L_1 + L_2 + 2M$$

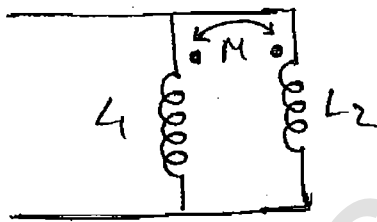
Mutually opposing



$$L_{eq} = L_1 + L_2 - 2M$$

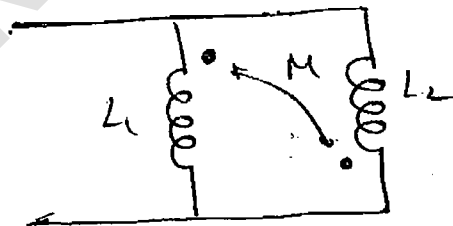
Coils in Parallel

Mutually adding

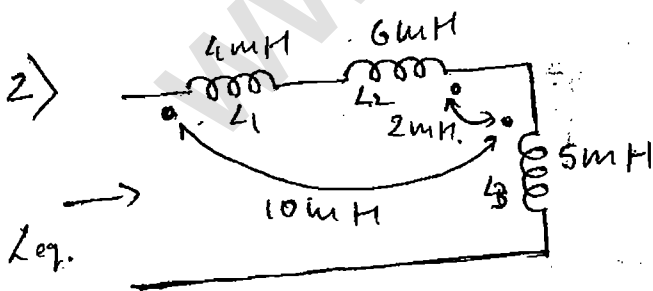


$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

Mutually opposing



$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$



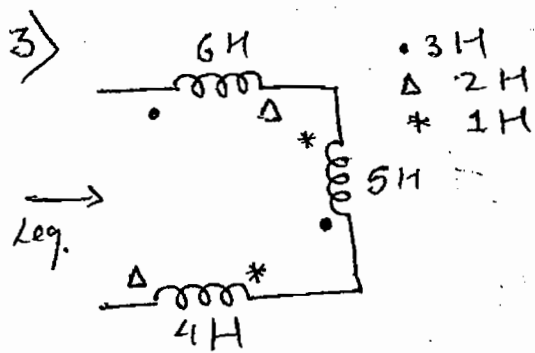
$$L_1 + M_{13} - M_{12} + L_2 - M_{21} - M_{23} + L_3 + M_{31} - M_{32} \quad (\text{or})$$

$$= (4 + 6 + 5) + 2(10) - 2(2)$$

$$= 15 + 20 - 4$$

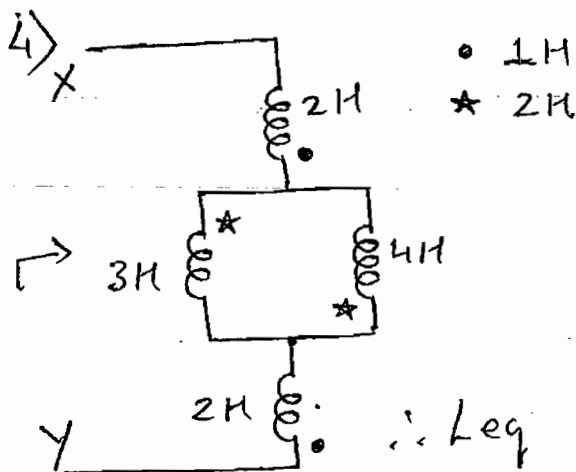
$$= 31 \text{ mH}$$

$$\left. \begin{aligned} V_1 &= 4 + 10 = 14 \\ V_2 &= 6 - 2 = 4 \\ V_3 &= 5 + 10 - 2 = 13 \end{aligned} \right\} 31 \text{ mH}$$



$$\begin{aligned}
 Leq &= (6 - 3 + 2) + \\
 &\quad (5 - 3 + 1) + \\
 &\quad (4 + 1 + 2) \\
 &= 5 + 3 + 7
 \end{aligned}$$

$$Leq = 15 H$$



S.A

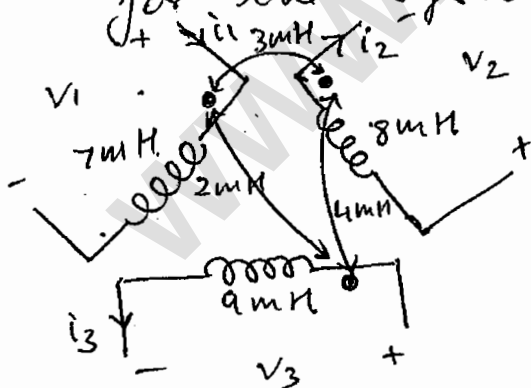
$$2 + 2 + 2 = 6H$$

P.O

$$\frac{12 - 4}{3 + 4 + 4} = \frac{8}{11}$$

$$\therefore Leq = 6 + \frac{8}{11} = \frac{74}{11} H$$

5) Write the complete inductance Matrix for the systems of coils below

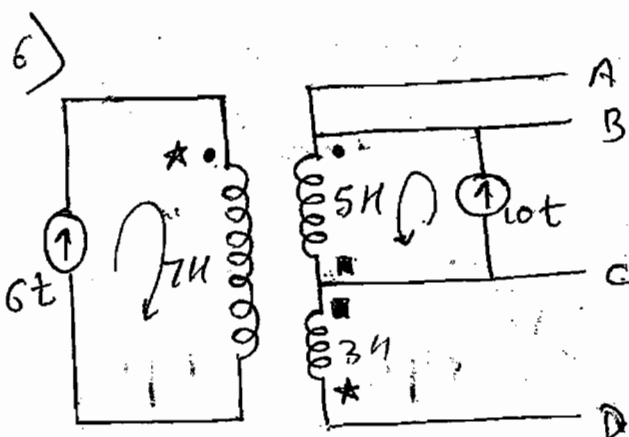


$$V_1 = 7 \frac{di_1}{dt} - 3 \frac{di_2}{dt} + 2 \frac{di_3}{dt}$$

$$V_2 = -3 \frac{di_1}{dt} + 8 \frac{di_2}{dt} - 4 \frac{di_3}{dt}$$

$$V_3 = 2 \frac{di_1}{dt} - 4 \frac{di_2}{dt} + 9 \frac{di_3}{dt}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 2 \\ -3 & 8 & -4 \\ 2 & -4 & 9 \end{bmatrix} \begin{bmatrix} di_1/dt \\ di_2/dt \\ di_3/dt \end{bmatrix}$$



- 4H
- ★ 2H
- 1H

$$V_{BD} = ?$$

$$V_{BD} = V_{BC} + V_{CD}$$

$$V_{BC} = V_S + V_M(\cdot)$$

$$= 5 \frac{d}{dt}(10t) + 4 \frac{d}{dt}(6t)$$

$$= 50 + 24 = 74 \text{ volts}$$

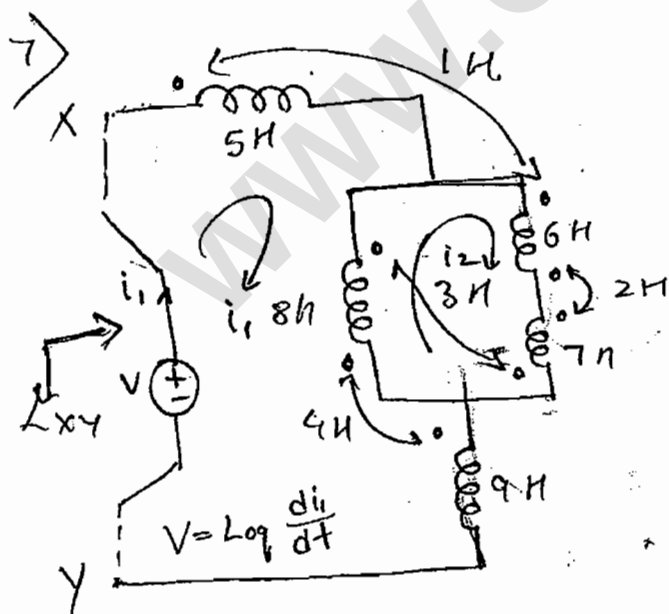
$$V_{CD} = V_M(\cdot) + V_M(\cdot)$$

$$= -2 \frac{d}{dt}(6t) - 1 \frac{d}{dt}(10t)$$

$$= -12 - 10 = -22 \text{ volts}$$

self inductance
(No individual
as current does not
flow)

$$\therefore V_{BD} = 74 - 22 = \underline{52 \text{ volts}}$$



Rules [Mutual Vltg]

- ① Which Mesh?
- ② Which two dots?
- ③ $\oplus$ or $\ominus$
- ④ Resultant current

M-I

$$-V + 5 \frac{di_1}{dt} + 8 \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] + 9 \frac{di_1}{dt} + \left(\frac{di_2}{dt} - 3 \frac{di_2}{dt} \right)$$

$$- 4 \frac{di_1}{dt} - 4 \left[\frac{di_1}{dt} - \frac{di_2}{dt} \right] = 0$$

$$\therefore 14X - 6Y = V \quad \text{--- (1)}$$

M-2

$$8 \left[\frac{di_2}{dt} - \frac{di_1}{dt} \right] + 6 \frac{di_2}{dt} + 7 \frac{di_2}{dt} + 4 \frac{di_1}{dt} + 3 \frac{di_2}{dt} + \frac{di_1}{dt} - 2 \frac{di_2}{dt} + 3 \left[\frac{di_2}{dt} - \frac{di_1}{dt} \right] - 2 \frac{di_2}{dt} = 0$$

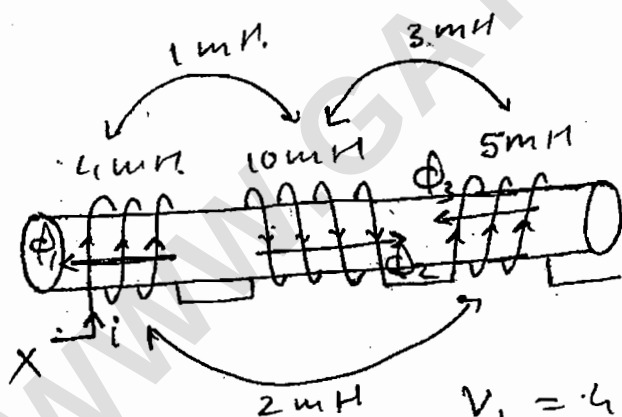
$$\therefore -6X + 23Y = 0 \quad \text{--- (2)}$$

From (1) & (2)

$$X \left[14 - \frac{6 \times 6}{23} \right] = V$$

$$\therefore V = \left[\frac{286}{23} \right] \frac{di_1}{dt} \Rightarrow L_{eq} = \frac{286}{23} \text{ H.}$$

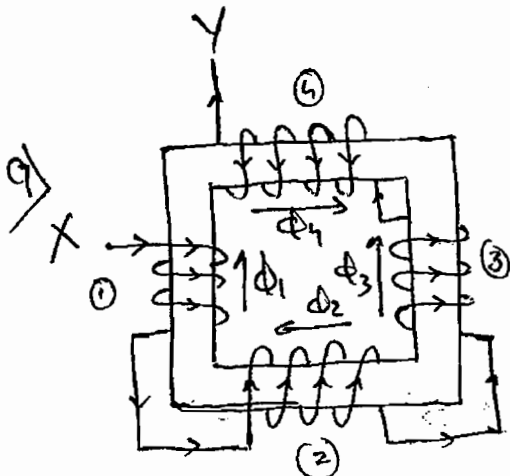
8)



$L_{xy} = ?$

Using Right Hand Thumb rule we decide the dirⁿ of flux

$$\left. \begin{aligned} V_1 &= 4 - 1 + 2 = 5 \\ V_2 &= 10 - 1 - 3 = 6 \\ V_3 &= 5 + 2 - 3 = 4 \end{aligned} \right\} 15 \text{ mH}$$



Find L_{xy} i.e.:

$$L_{11} = L_{22} = L_{33} = L_{44} = 5 \text{ mH}$$

$$M_{12} = M_{23} = M_{34} = 2 \text{ mH}$$

$$M_{13} = M_{14} = M_{24} = 1 \text{ mH}$$

$$V_1 = 5 + 2 - 1 + 1 = 7$$

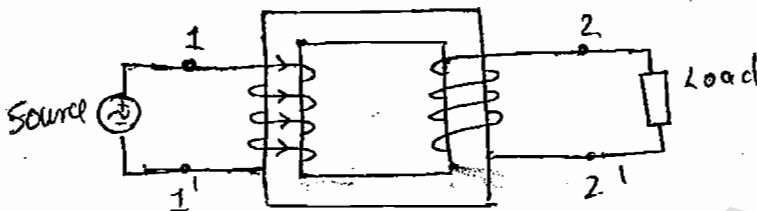
$$V_2 = 5 + 2 - 2 + 1 = 6$$

$$V_3 = 5 - 1 - 2 - 2 = 0$$

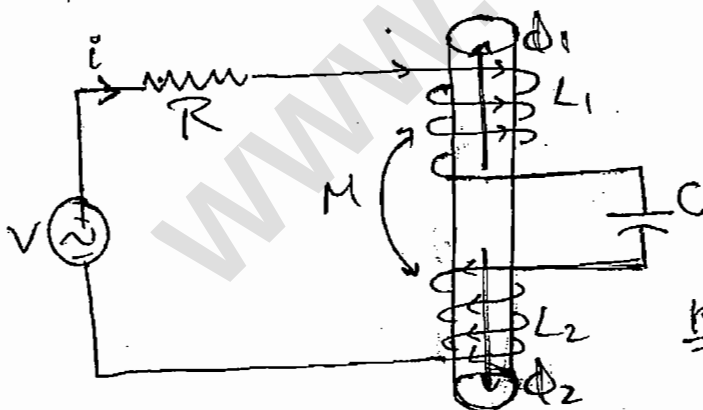
$$V_4 = 5 + 1 + 1 - 2 = 5$$

$$\therefore Z_{xy} = 7 + 6 + 0 + 5 = 18 \text{ mH}$$

10) Place correct dot convention b/w 2 coils



ii) Write KVL governing the circuit below & find resonance frequency ' f_0 ' if $R = 10 \Omega$, $L_1 = L_2 = 10 \text{ mH}$, $M = 2 \text{ mH}$, $C = 0.1 \mu\text{F}$



KVL (Exact form)

$$-V + iR + (L_1 + L_2 - 2M) \frac{di}{dt} + \frac{1}{C} \int i dt = 0$$

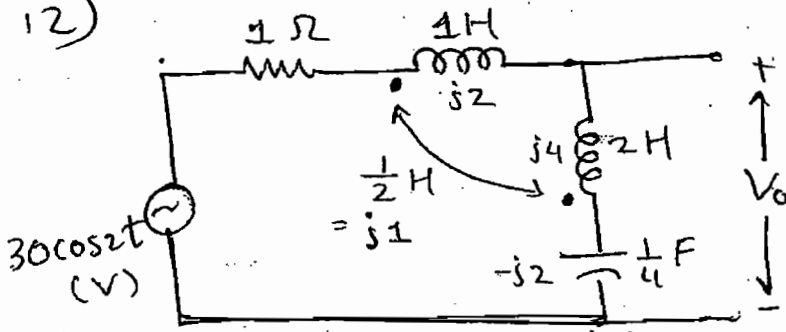
KVL (S.S A.C form)

$$-V + IR + j\omega(L_1 + L_2 - 2M)I - \frac{j}{\omega C} I = 0$$

$$f_0 = \frac{1}{2\pi \sqrt{(L_1 + L_2 - 2M)C}}$$

=

12)



KVL $-[30\angle 0^\circ] + (1 + j2 + j4 - j2) I - (j1 \times 2) I = 0$

$$(1 + j2) I = 30\angle 0^\circ$$

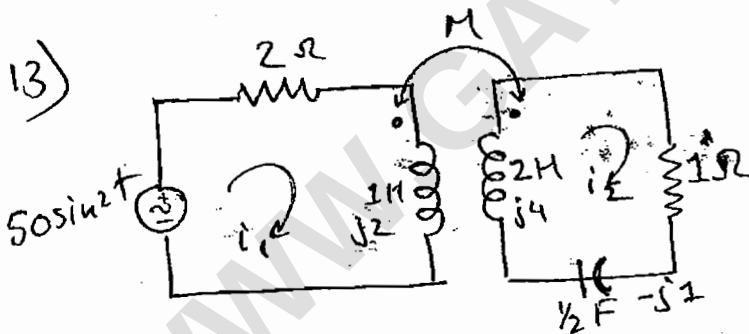
$$\therefore I = \frac{30\angle 0^\circ}{1 + j2}$$

$$V_0 = I [+j4 - j2] - j1 [I] = j I$$

$$= \frac{30\angle 90^\circ}{1 + j2} = 13.41\angle 26.56^\circ$$

$$\therefore V_0 = 13.41 \cos(2t + 26.56)$$

13)



Find power lost in 1Ω resis.

$$-[50\angle 0^\circ] + (2 + j2) I_1 - j1 [I_2] = 0 \quad \text{--- (1)}$$

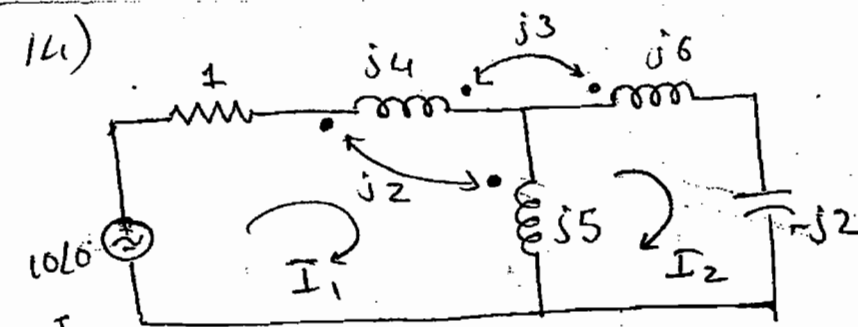
$$(1 + j4 - j1) I_2 - j1 [I_1] = 0 \quad \text{--- (2)}$$

Solving for $I_2 =$ _____

$$I_2 (\text{rms}) = \frac{\quad}{\sqrt{2}}$$

$$P_{1\Omega} = |I_2|_{\text{rms}}^2 \times R$$

14)



Write Mesh eqⁿ governing the circuit below.

M-I

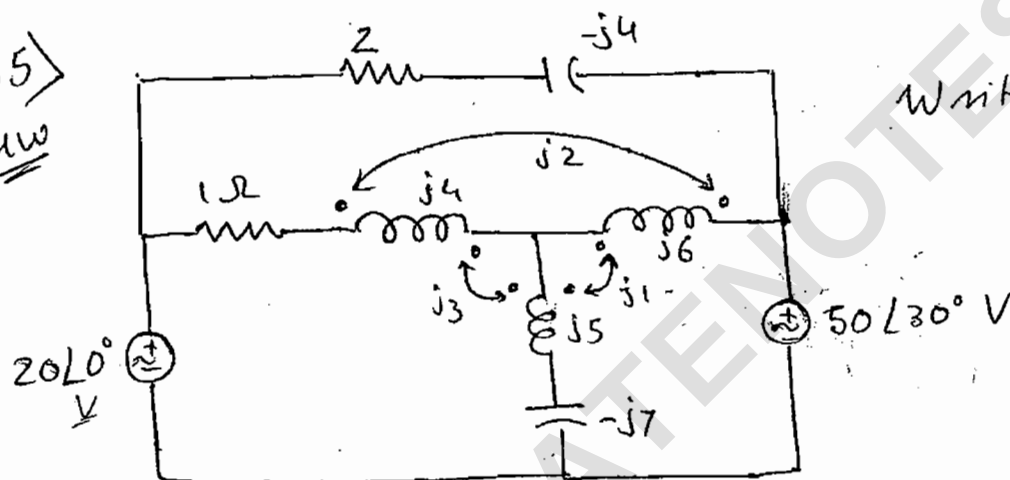
$$-10\angle 0^\circ + (4 + j9)I_1 - j5I_2 + j2[I_1 - I_2] - j3[I_2] + j2[I_1] = 0$$

M-II

$$j5[I_2 - I_1] + j6I_2 - j2I_2 - j2[I_1] - j3[I_2] = 0$$

15)

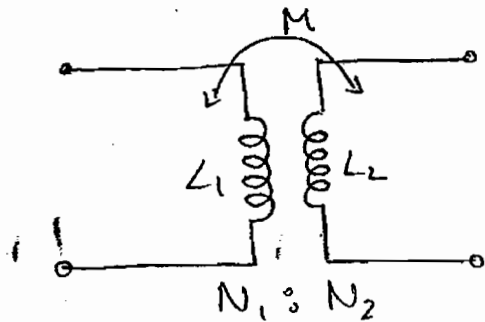
HW



Write mesh eq^s.

WWW.GATENOTES.N

IDEAL TRANSFORMER (Circuit Concept)



it can store / transfer ideally electrical energy

$$L_1 \rightarrow \infty, \quad L_2 \rightarrow \infty, \quad M \rightarrow \infty$$

$k = 1 \rightarrow$ co. eff. of coupling.

$\rightarrow$ No losses

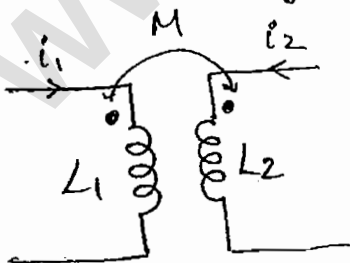
$\rightarrow$ 100% η

$$L_1 : L_2 : M = N_1^2 : N_2^2 : N_1 N_2$$

Turns Ratio: $\frac{N_1}{N_2} = \sqrt{\frac{L_1}{L_2}} = \frac{L_1}{M} = \frac{M}{L_2}$

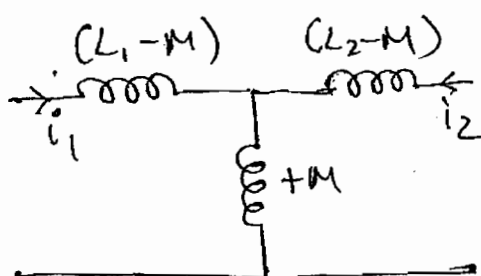
T-equivalent representation of Ideal transformer (Circuit concept)

(A) Mutually Adding



KVL : $V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$

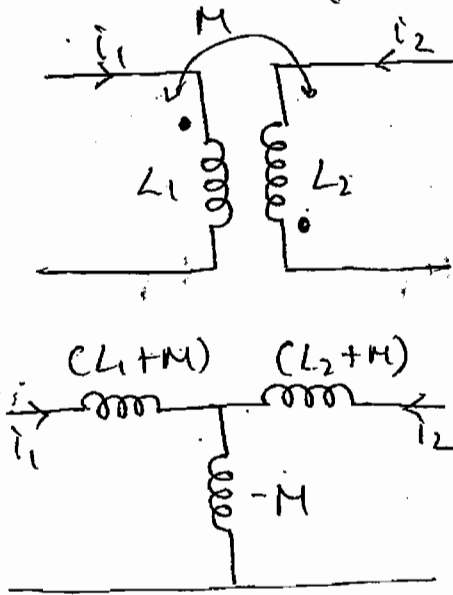
$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$



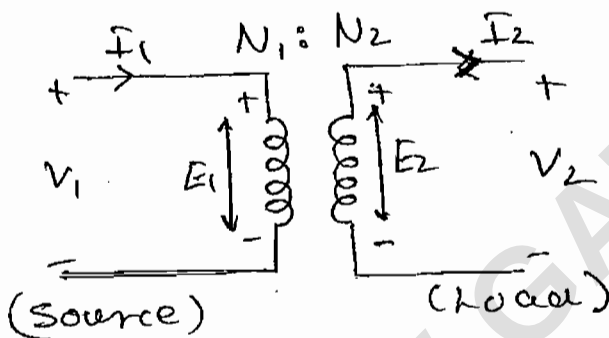
KVL
 $V_1 = (L_1 - M) \frac{di_1}{dt} + M \left[\frac{di_1}{dt} + \frac{di_2}{dt} \right]$

$$V_2 =$$

(b) Mutually opposing.



IDEAL TRANSFORMER Machine Concept :-



But,

$$a = \frac{1}{K} = \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1}$$

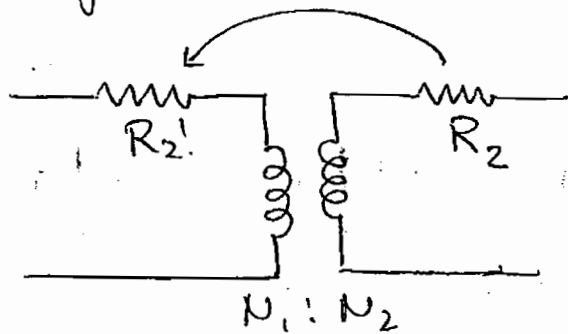
↳ Turns ratio

Here,

$$K = \frac{N_2}{N_1} = \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{I_1}{I_2}$$

↳ Voltage transformation ratio

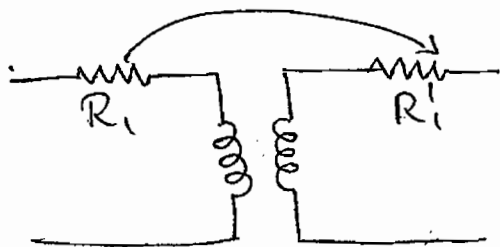
Concept of Reflecting Primary (or) Secondary circuit parameters to simplify the T/f as a single ckt:



$$R_2' = \frac{R_2}{k^2} \quad \left(k = \frac{N_2}{N_1} \right)$$

↳ secondary res.
reflected to primary

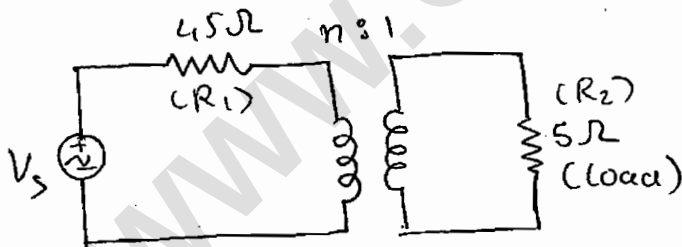
Also; $X_2' = \frac{X_2}{k^2}$, $Z_2' = \frac{Z_2}{k^2}$



$$R_1' = k^2 R_1$$

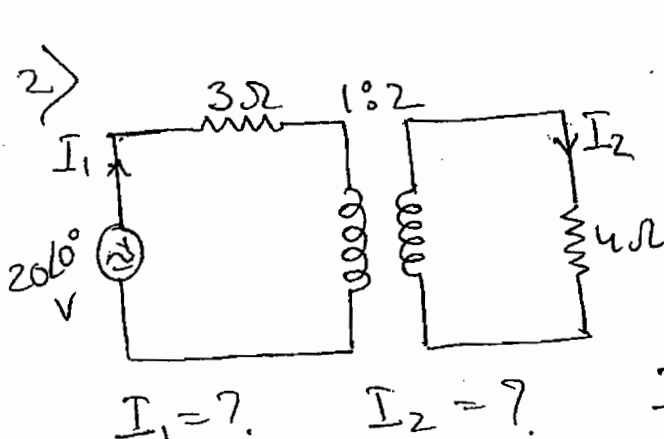
↳ primary res.
reflected to secondary

1) What is the value of 'n' for which max. power is transferred to the load.



$$R_2' = \frac{R_2}{k^2} = \frac{R_2}{(1/n)^2} = 5n^2$$

For P_{max}
 $R_L = R_s \Rightarrow 5n^2 = 45 \Rightarrow n^2 = 9 \Rightarrow n = 3$

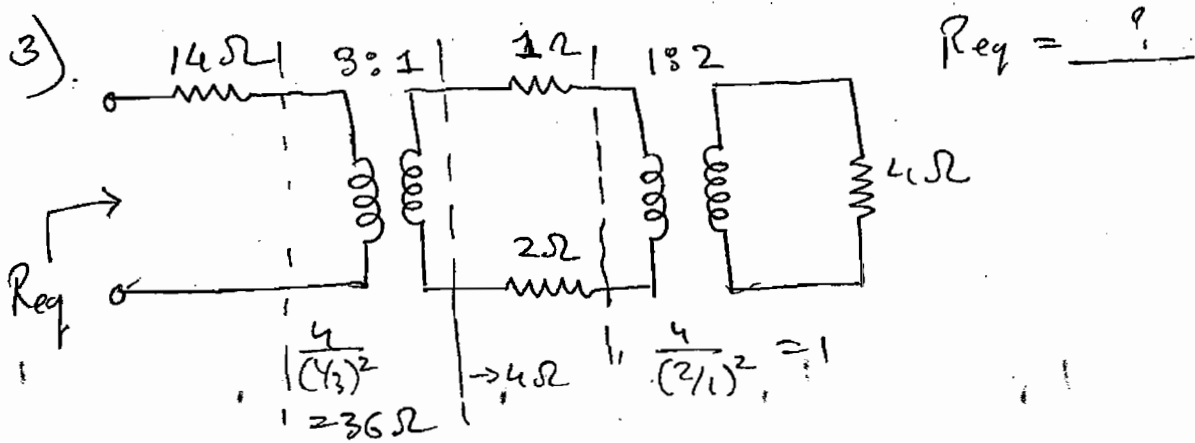


$$R_1 + R_2' = 3 + \frac{4}{(2/1)^2} = 4 \Omega$$

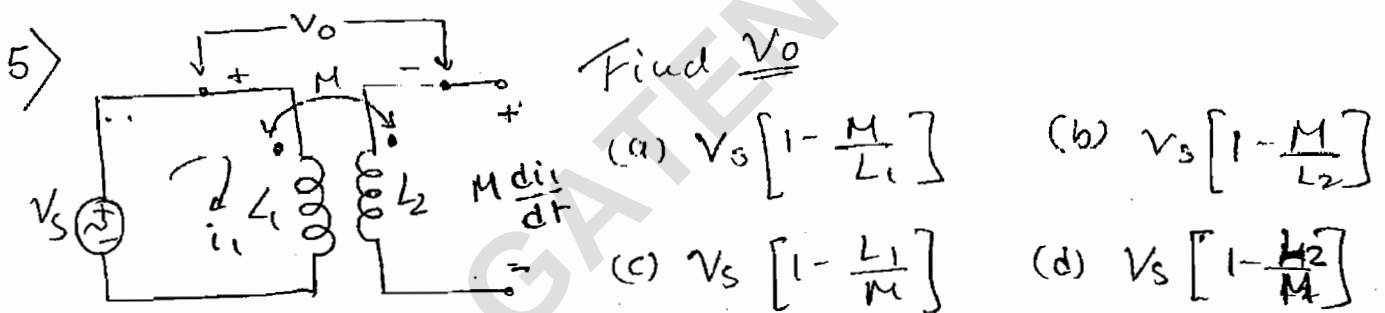
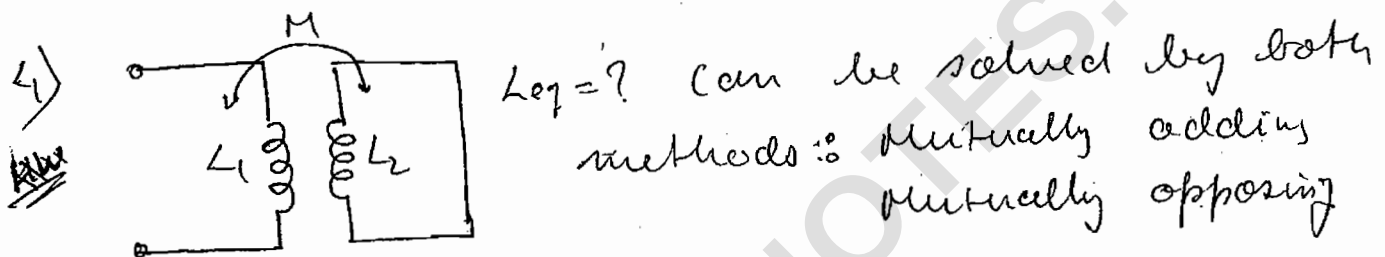
$$\frac{V}{I_1} = \frac{V_s}{R_T} = \frac{20 \angle 0^\circ}{4} = 5 \angle 0^\circ A$$

$$k = \frac{I_1}{I_2} = \frac{2}{1}$$

$$I_2 = \frac{I_1}{2} = \frac{5 \angle 0^\circ}{2} = 2.5 \angle 0^\circ A$$



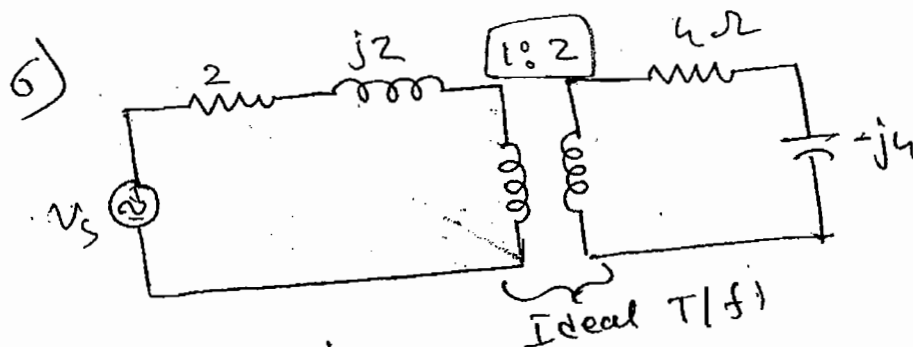
$$R_{eq} = 14 + 36 = 50\Omega$$



$$V_s = L_1 \frac{di_1}{dt}$$

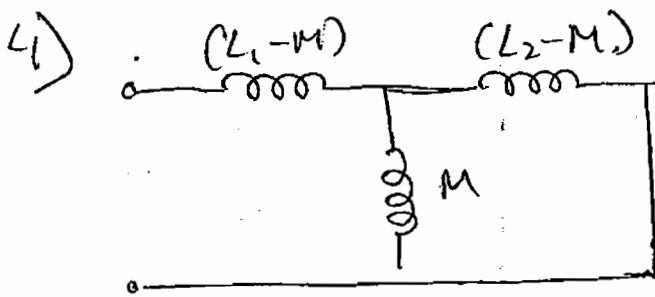
$$V_0 = V_s - M \frac{di_1}{dt} = L_1 \frac{di_1}{dt} - M \left(\frac{V_s}{L_1} \right)$$

$$V_0 = V_s \left[1 - \frac{M}{L_1} \right]$$



$$Z_T = Z_1 + Z_2'$$

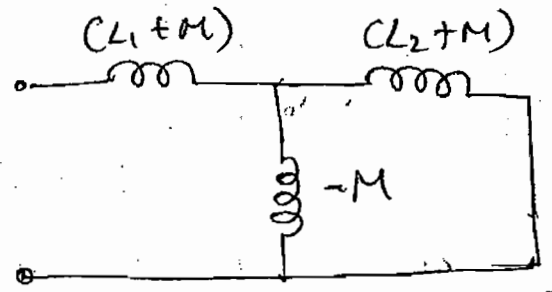
$$= (2 + j2) + \frac{(4 - j4)}{(2/1)^2} = (3 + j)\Omega$$



$$Z_{eq} = L_1 - M + \left[\frac{M(L_2 - M)}{M + L_2 - M} \right]$$

$$= L_1 - M + \frac{ML_2 - M^2}{L_2}$$

$$= L_1 - \frac{M^2}{L_2}$$



$$Z_{eq} = L_1 + M + \left[\frac{-M(L_2 + M)}{-M + L_2 + M} \right]$$

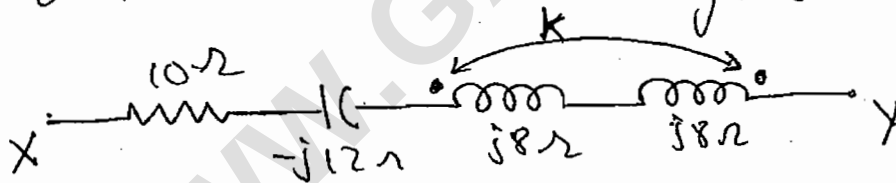
$$= L_1 + M - \frac{ML_2}{L_2} - \frac{M^2}{L_2}$$

$$= L_1 - \frac{M^2}{L_2}$$

⇒ Dot convention will not affect circuit performance.

Its just used for analysis purpose.

5) What is the value of k for which the branch undergoes resonance.



For resonance: $X_L = X_C$

$$\therefore -j12 = (j8 + j8 - 2M) \omega$$

$$X_{Leq} = \omega L_{eq}$$

$$= \omega [L_1 + L_2 - 2M]$$

$$= \omega L_1 + \omega L_2 - 2\omega k \sqrt{L_1 L_2}$$

$$= \omega L_1 + \omega L_2 - 2k \sqrt{(\omega L_1)(\omega L_2)}$$

$$= 8 + 8 - 2k \sqrt{8 \times 8}$$

$$= 16 - 16k$$

$$\text{So, } X_L = X_C$$

$$\therefore 16 - 16k = 12 \Rightarrow 16k = 4$$

$$\boxed{k = 0.25}$$

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ELECTRICAL

TRANSIENTS

- Transients are considered as sudden changes in the state of a ckt or n/w which are indicated by the switch operation.
- Transients occur in any electrical n/w or sys. as none of the system's are adaptable for quick sudden changes in time.
- Transients are also considered as an argument b/w the i/p command to o/p response as a n/w changes from previous steady state to next steady state w.r.t the state variable.
(I, L or V, C)
- Though transients occur for very short duration in time, their impact is huge in determining the entire steady state resp.
- Though customers look into steady state performance of an electrical device or n/w, the designers are more interested in transient state performance as they give the critical design specification values.

State Variables :-

These are the critical parameters that must be observed to determine the transient state solution in any n/w.

(a) In a capacitor :-

$$i = C \frac{dv}{dt}$$

(KCL $\rightarrow$ Nodal) $\rightarrow$ Voltage across capacitor is correct S.V.

(b) In an inductor :-

$$V = L \frac{di}{dt}$$

(KVL $\rightarrow$ Mesh) $\rightarrow$ Current through inductor is correct S.V.

$\rightarrow$ The response of any ckt/n/w when a source is present is called as forced response.

This response is independent to the nature of passive elements & it can be different for diff. types of i/p's.

Eg:- DC ckt Analysis, AC ckt analysis, etc.

The response of any ckt or n/w without any source is called as Natural resp. This response is independent to i/p but purely depends upon the nature of passive elements & it is always unique determined by the characteristic eqⁿ.

governing the n/w.

This source free response is possible, provided the n/w has initial stored energy (inductor stores energy in electromagnetic form : $\psi = Li \rightarrow I_0$).

capacitor stores energy in electrostatic form : $q = CV \rightarrow V_0$)

So,

$$\begin{array}{c} \left[\begin{array}{c} \text{Complete resp.} \\ \text{of a sys.} \end{array} \right] = \left[\begin{array}{c} \text{Forced} \\ \text{resp.} \end{array} \right] + \left[\begin{array}{c} \text{Natural} \\ \text{resp.} \end{array} \right] \\ \downarrow \qquad \qquad \qquad \downarrow \\ \text{Zero state} \qquad \qquad \text{Zero i/p} \\ \text{response} \qquad \qquad \text{response} \end{array}$$

Any n/w or sys. reaches steady state after overcoming transient state. So in this chapter, as we are determining state response, all the solutions of voltage, current, power, energy, etc are w.r.t time.

Initial Conditions :—

These are critical values of vltg across capacitor & current through inductor from the previous steady state of a n/w which are specifically indicated as :

$t = 0^- \rightarrow$ instant just before s/w operation
(steady state before s/w operation)
 $t = 0^+ \rightarrow$ instant just after s/w operation,
(TRANSIENT state operation)

$t \rightarrow \infty \Rightarrow$ Steady state after slow operation

Order of ckt or n/w :—

The no. of energy storage components available in distributed form in any ckt represents its order.

Eg: $R-L, R-C \rightarrow 1^{st}$ order n/w

$R-L-C, L-L-R, C-C-R, L-C \rightarrow 2^{nd}$ order n/w

NOTE:—

① A capacitor will never allow sudden change in vltg across it.

$$V_C(0^-) = V_C(0) = V_C(0^+)$$

② A current with inductor will never allow sudden change in current through it.

$$i_L(0^-) = i_L(0) = i_L(0^+)$$

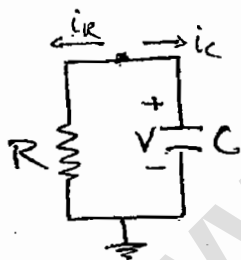
Behaviour of passive elements in Transient state in comparison to Steady state :—

If analyzing of n/w as $t \rightarrow 0^+$, is considered as transient solution then, analysing the same n/w as $s \rightarrow \infty$ (Steady state ^{soln} freq. resp.) is also considered as Transient solution. Hence L.T. are powerfull tools to analyze the n/w during TRANSIENT STATE.

Element	D.C SS ($s=0$)	A.C SS ($s=j\omega$)	Transient State ($t \rightarrow 0^+$) ($s \rightarrow \infty$)	$s = j\omega$ $\rightarrow$ complex freq.
R	R	R	R	$Z_R = R$
L	S.C	'i' lags 'v' 'v' lags 'i' $\phi = 90^\circ$	O.C	$Z_L = +j\omega L$ $= sL$
C	O.C	'i' leads 'v' 'v' lags 'i' $\phi = 90^\circ$	S.C	$Z_C = \frac{1}{j\omega C}$ $= \frac{1}{sC}$

[I] Source Free 1st Order :—

(a) R-C circuit :—



Let $V(0) = V_0$

KCL $i_L + i_C = 0$

$\therefore C \frac{dV}{dt} + \frac{V}{R} = 0$

$\therefore C \frac{dV}{dt} = -\frac{V}{R} \Rightarrow \int \frac{dV}{V} = \int -\frac{dt}{RC}$

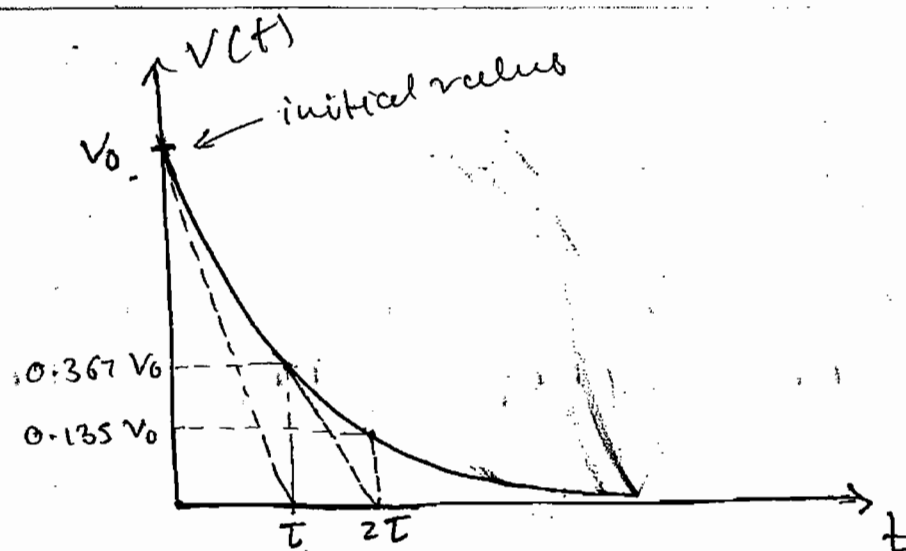
$\therefore \ln[V] = -\frac{t}{RC} + \ln[A]$

$\therefore \ln\left[\frac{V}{A}\right] = -\frac{t}{RC} \Rightarrow V = A e^{-\frac{t}{RC}}$

But at $t=0$, $V=V_0$ then $A=V_0$

$\therefore V(t) = V_0 e^{-\frac{t}{RC}}$ $V(t) = V_0 e^{-\frac{t}{\tau}}$

$\tau = RC \rightarrow$ Time const. of R-C n/w



$$V(t=0) \rightarrow V_0$$

$$V(t=\tau) = e^{-1} V_0 = 0.367 V_0$$

$$V(t=2\tau) = e^{-2} V_0 = 0.135 V_0$$

$$V(t=3\tau) = e^{-3} V_0 = 0.049 V_0$$

$$V(t=4\tau) = e^{-4} V_0 = 0.018 V_0$$

$$V(t=5\tau) = e^{-5} V_0 = 0.006 V_0 \rightarrow t \geq 5\tau$$

$0 < t \leq 4\tau$
Transient state

(steady state)

Expression of current through capacitor,

$$i_c = C \frac{dV}{dt} = C \frac{d}{dt} [V_0 e^{-t/\tau}] = C V_0 e^{-t/\tau} \left(-\frac{1}{\tau} \right)$$

$$i_c = -\frac{V_0}{R} e^{-t/\tau}$$

Expression of power dissipated through R

$$P_R(t) = \frac{[V(t)]^2}{R} = \frac{[V_0 e^{-t/\tau}]^2}{R}$$

$$= \frac{V_0^2}{R} e^{-t/(\tau/2)}$$

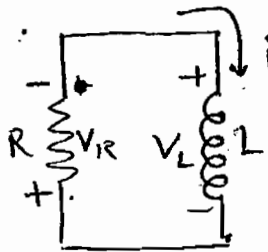
Expression for energy decay in the capacitor:

$$E_c(t) = \frac{1}{2} C [V(t)]^2$$
$$= \frac{1}{2} C V_0^2 e^{-t/(\tau/2)}$$

Expression for charge

$$q(t) = C V(t)$$
$$q = C V_0 e^{-t/\tau}$$

(b) R-L circuit:—



Let $i(0) = I_0$

KVL

$$V_R + V_L = 0$$

$$iR + L \frac{di}{dt} = 0$$

$$\therefore L \frac{di}{dt} = -iR$$

$$\int \frac{di}{i} = \int -\frac{R}{L} dt$$

$$\ln[i] = -\frac{R}{L} t + \ln[A]$$

$$\therefore \ln\left[\frac{i}{A}\right] = -\frac{R}{L} t$$

$$\therefore i = A e^{-R/L t}$$

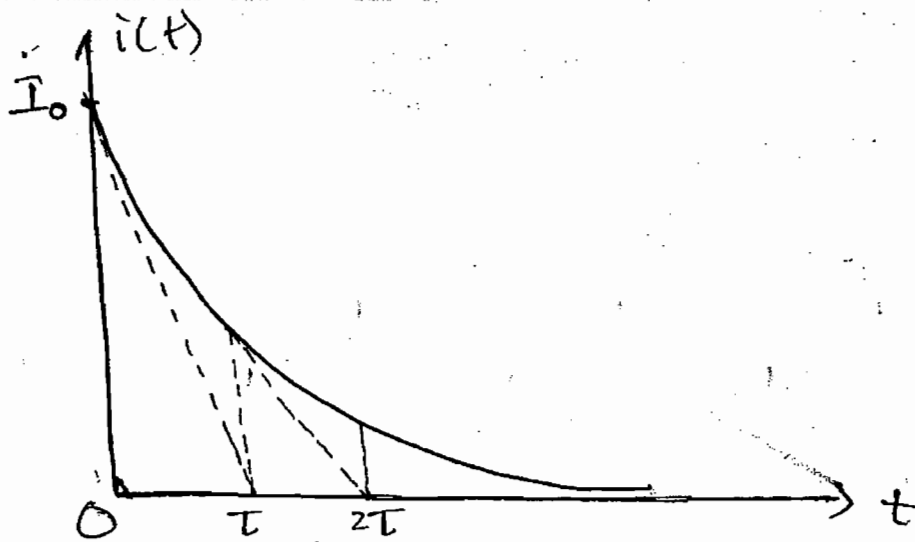
At $t=0$, $i = I_0$ then $A = I_0$

$$i(t) = I_0 e^{-R/L t} = I_0 e^{-t/(\tau/2)}$$

$$i(t) = I_0 e^{-t/\tau}$$

$$\tau = \frac{L}{R}$$

time const. of R-L u/w



Expression of vltg across inductor.

$$V_L = L \frac{di}{dt} = L \frac{d(I_0 e^{-t/\tau})}{dt}$$

$$= -I_0 R e^{-t/\tau}$$

$$V_R + V_L = 0$$

$$V_R(t) = -V_L(t) = I_0 R e^{-t/\tau}$$

Expression for power dissipated in the resistor.

$$P_R = [i(t)]^2 R = (I_0 e^{-t/\tau})^2 R$$

$$P_R(t) = I_0^2 R e^{-t/(\tau/2)} \quad \underline{\underline{W}}$$

Expression for energy decay in inductor

$$E_L(t) = \frac{1}{2} L [i(t)]^2 = \frac{1}{2} L (I_0 e^{-t/\tau})^2$$

$$= \frac{1}{2} L I_0^2 e^{-t/(\tau/2)} \quad \underline{\underline{J}}$$

Expression for Flux linkages

$$\Psi(t) = L i(t)$$

$$= L I_0 e^{-t/\tau} \quad \text{wb-T}$$

Here the power dissipated by the resis. & energy decay in inductor or capacitor is 2 times faster than current or vltg respectively.

Time Constant :- It is the time taken by the resp to reach 36.7% of its initial value or It is also defined as time taken by the resp. to reach 63.4% to its final value.

The unit of Time const. is : seconds

Then, w.r.t to units only :-

$$\text{sec} = \text{sec}$$

$$L/R = RC$$

$$\boxed{\frac{L}{R} = RC}$$

$$\rightarrow \text{Unit of } \frac{L}{RC} = \Omega$$

$$\text{Unit of } R^2 C = H$$

$$\text{Units of } \frac{L}{R^2} = F$$

$$\text{Units of } \frac{RC}{L} = V$$

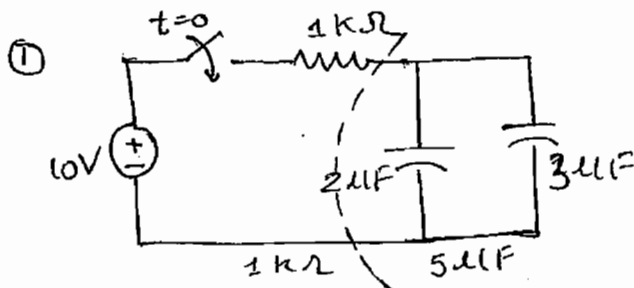
units of $\frac{L}{C} = (\Omega)^2$

units of $\sqrt{\frac{L}{C}} = \Omega$

units of $\frac{R^2 C}{L} = 1$

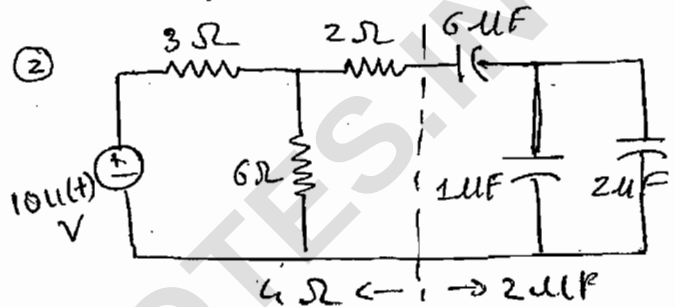
Time const. determines the time with which the state variables responds.

1) Determine the time const. of ckt shown below



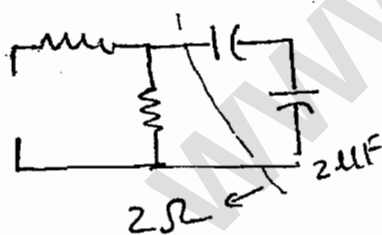
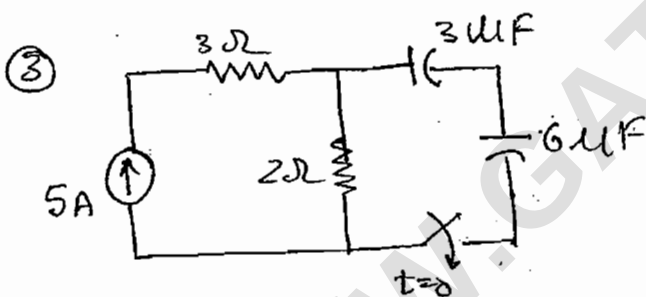
$$\tau = R_{eq} C_{eq}$$

$$= 1k \times 5\mu = 5 \text{ msec}$$

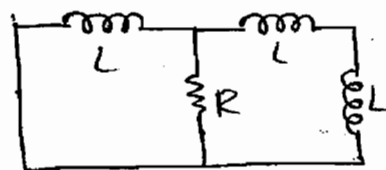
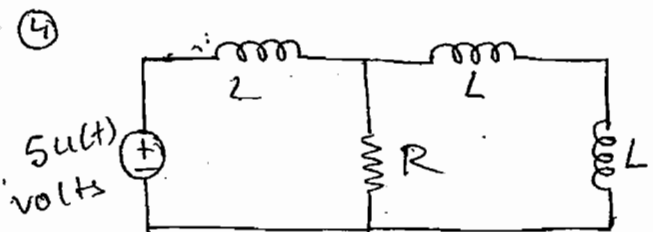


$$\tau = 4 \times 2$$

$$= 8 \text{ msec}$$

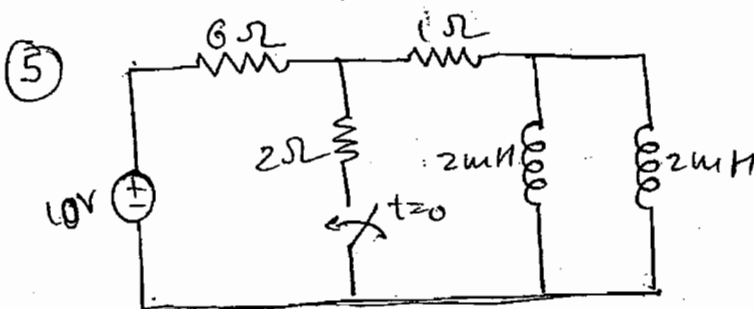


$$\tau = 2 \times 2 = 4 \text{ msec}$$

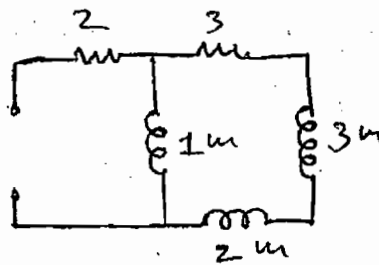
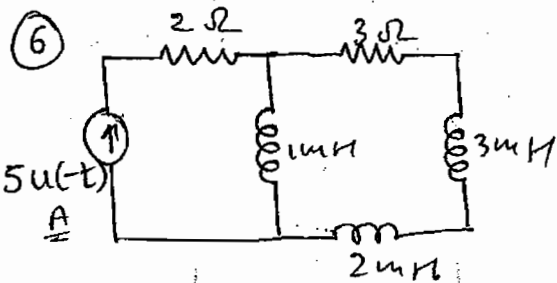


$$L_{eq} = L || 2L = \frac{2}{3} L$$

$$\tau = \frac{L}{R} = \frac{2}{3} \frac{L}{R}$$



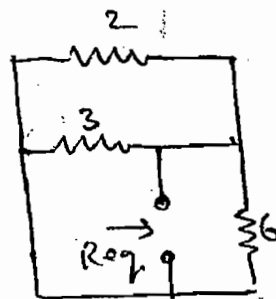
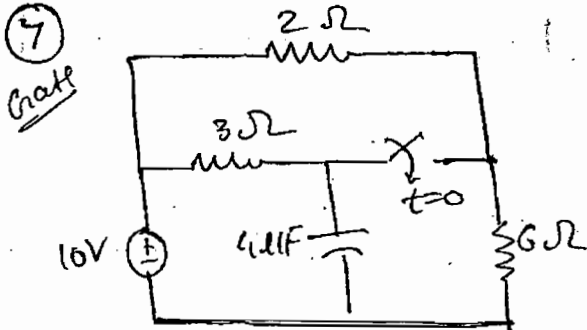
$$\tau = \frac{1m}{7/2} = \frac{2}{7} \text{ msec}$$



$$L_{eq} = 6 \text{ mH}$$

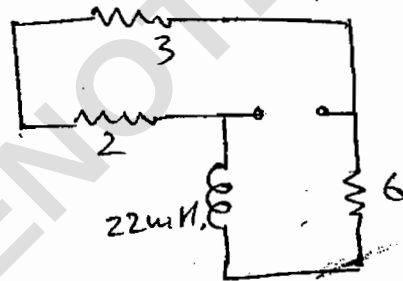
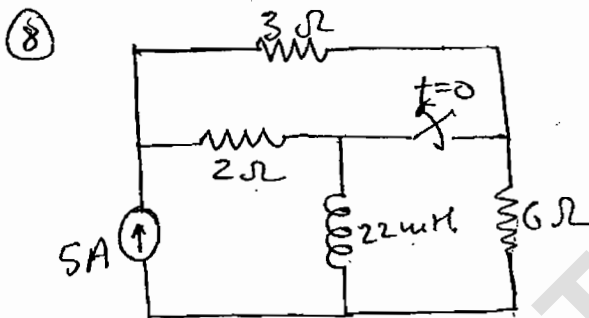
$$R = 3 \Omega$$

$$\tau = 2 \mu\text{sec}$$

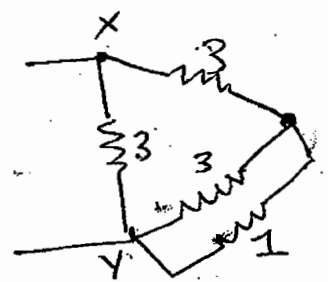
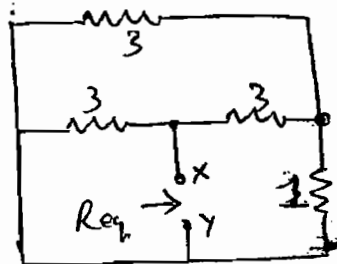
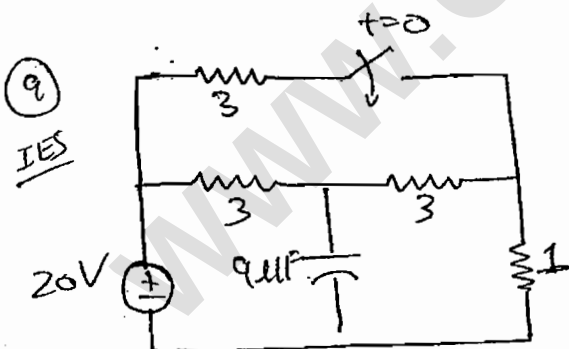


$$R_{eq} = 6 || 3 || 2 = 1 \Omega$$

$$\tau = R_{eq} \cdot C = 1 \cdot 4 = 4 \mu\text{sec}$$

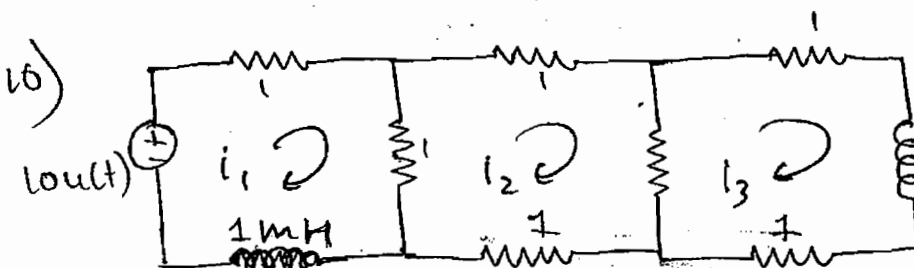


$$\tau = \frac{22 \text{ mH}}{11} = 2 \text{ msec}$$



$$R_{eq} = 3 || [3 + \frac{3 \cdot 1}{3+1}] = 3 || \frac{15}{4} = \frac{45}{4} \times \frac{4}{27} = \frac{5}{3} \Omega$$

$$\tau = \frac{45}{3} \times 9 = 15 \mu\text{sec}$$

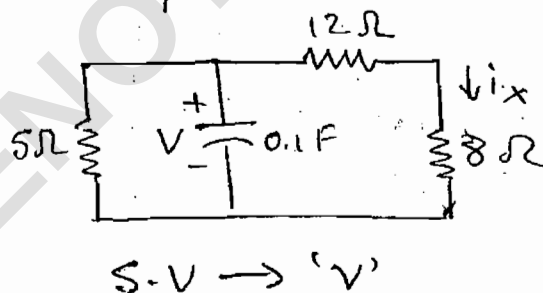


Here the inductors & capacitors cannot be lumped together. This is a second order L-L-R circuit.

This ckt has multiple time const. in multiple segments & the solution to these state variables can be determined by solving simultaneous differential eqⁿ or in a simple way by using Laplace Transforms (L.T.)

- ⑪ If $V(0) = 15\text{ V}$ find complete expression for ' i_x '.

This is a source-free 1st order R-C n/w



$$V(t) = V_0 e^{-t/\tau}$$

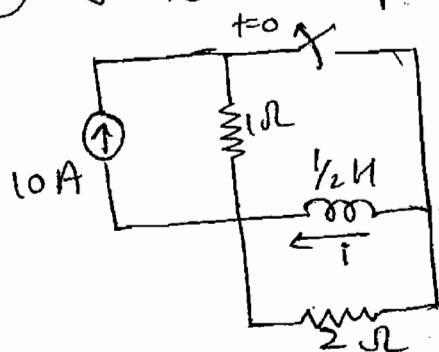
$$V_0 = 15\text{ V (given)}$$

$$\tau = R_{eq} \cdot C = (5 \parallel 12) \cdot 0.1 = 0.4\text{ sec}$$

$$\therefore V(t) = 15 \cdot e^{-t/0.4} = 15 e^{-2.5t} \text{ V}$$

$$\text{But } i_x(t) = \frac{V(t)}{12+8} = 0.75 e^{-2.5t} \text{ A}$$

- ⑫ Find complete expression for I .



This is a source free, 1st order R-L ckt.

$$\text{S.V} \rightarrow 'i'$$

$$i(t) = I_0 e^{-t/\tau}$$

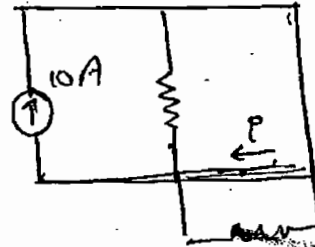
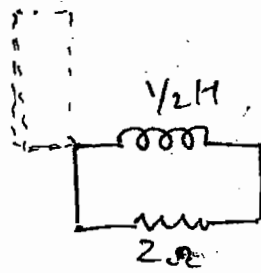
$$\tau = \frac{L}{R_{eq}} \quad R_{eq} = 2\Omega$$

$$\tau = \frac{L}{R} = \frac{1/2}{2}$$

$$\tau = \frac{1}{4} \text{ sec}$$

$$i(t) = 10 e^{-t/4}$$

$$= 10 e^{-4t} \text{ A}$$

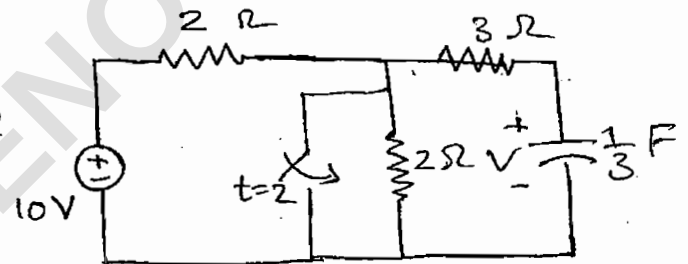


$$i(0) = 10 \text{ A} = I_0$$

($\because$ inductor does not allow sudden change in current)

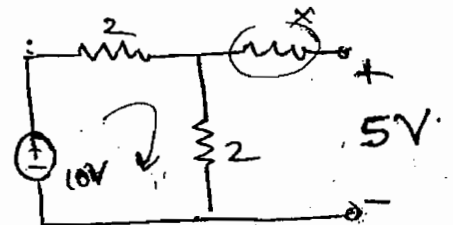
- 1) Determine the complete expression for V &
2) determine the energy stored in the capacitor upto 3rd second.

This is a source free
1st order RC n/w
(delayed)



State variable $\rightarrow V$

$0 < t \leq 2 \rightarrow$ Steady state only.



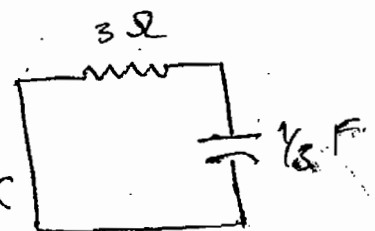
$t > 2 \rightarrow$ Transient solution.

$$V = V_0 e^{-t/\tau}$$

Here $V_0 \rightarrow$ means $\rightarrow V(2^-)$ (from previous state)

$$\therefore V_0 = 5 \text{ V}$$

τ ; $t > 2$: $\tau = R \cdot C = \frac{1}{3} \times 3 = 1 \text{ sec}$



$$V(t) = 5 e^{-\frac{(t-2)}{1}}$$

The complete expression.

$$V(t) = \begin{cases} 5V & ; 0 \leq t \leq 2 \\ 5e^{-(t-2)} & ; t > 2 \end{cases}$$

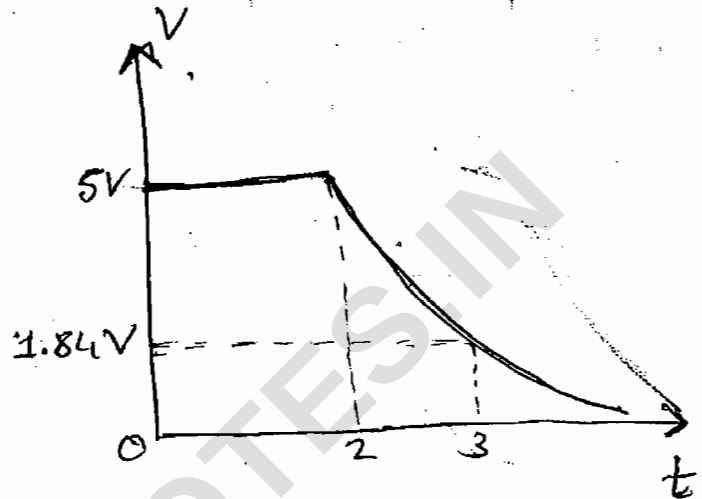
$$E_C(t=3) = \frac{1}{2} C(V)^2 \quad \xrightarrow{t=3}$$

Now, $V(t=3) = 5e^{-(3-2)}$

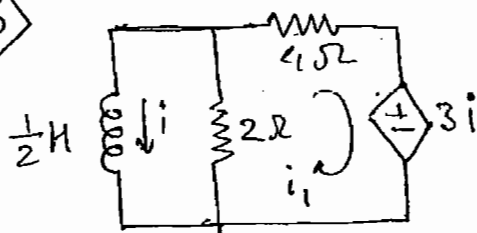
$$\therefore = 1.84 V$$

$$E_C = \frac{1}{2} \times \frac{1}{3} \times (1.84)^2$$

$$= 0.56 J$$



3)



Given $i(0) = 10 A$, find its complete expression.

This is a 1st order, source free R-L ckt. $S.V \rightarrow i$

Classical Method

$$\frac{1}{L} \frac{di}{dt} + 2[i + i_1] = 0 \quad \text{--- (1)}$$

$$\therefore 4i_1 + 3i + 2[i + i_1] = 0 \quad \text{--- (2)}$$

$$\therefore 5i = -6i_1 \quad \text{--- (2)}$$

$$\frac{1}{L} \frac{di}{dt} + 2i + 2\left(-\frac{5}{6}\right)i = 0$$

$$\int \frac{1}{L} \frac{di}{dt} = \int -\frac{i}{3} \Rightarrow \ln[i] = -\frac{2}{3}t + \ln[A]$$

$$\therefore i = Ae^{-\frac{2}{3}t}$$

At $t=0$, $i=10$

So $A=10$

$$\therefore i(t) = 10e^{-\frac{2}{3}t} A$$

Shortcut

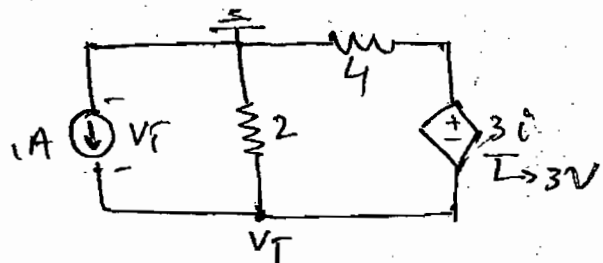
$$i(t) = I_0 e^{-t/\tau}$$

$$I_0 = 10 \text{ A (given)}$$

$$\tau = \frac{L}{R_{eq}}$$

$$= \frac{1/2}{1/3} = \frac{3}{2} \text{ sec}$$

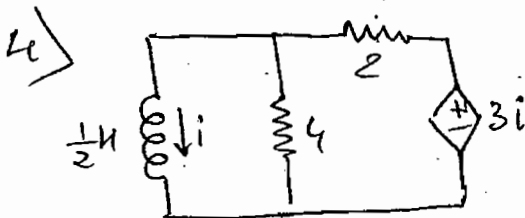
$$\therefore i(t) = 10 e^{-\frac{2}{3}t} \text{ A}$$



$$-1 + \frac{V_T}{2} + \frac{V_T+3}{4} = 0$$

$$\therefore 3V_T = 1 \Rightarrow V_T = \frac{1}{3}$$

$$\Rightarrow R = \frac{1}{3} \Omega$$



$$i(t) = I_0 e^{-t/\tau}$$

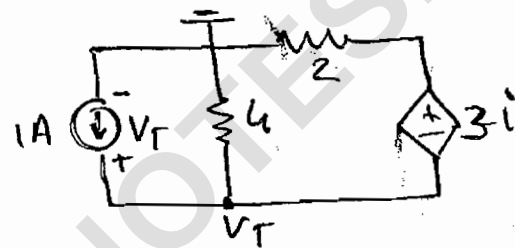
$$I_0 = 10 \text{ A (given)}$$

$$\tau = \frac{L}{R_{eq}}$$

$$= -\frac{3}{2} \times \frac{1}{2} = -\frac{3}{4} \text{ sec}$$

$$\therefore i(t) = I_0 e^{-t/-3/4}$$

$$i(t) = I_0 e^{\frac{4}{3}t} \text{ A}$$



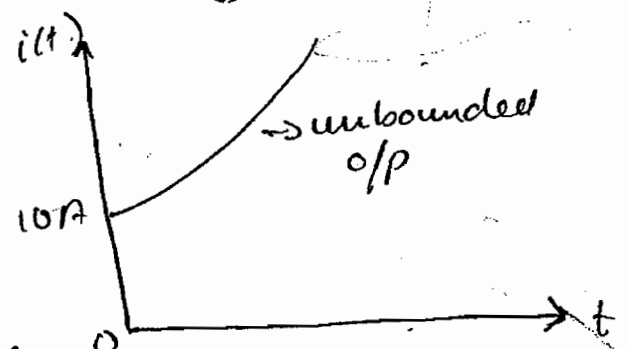
$$-1 + \frac{V_T}{4} + \frac{V_T+3}{2} = 0$$

$$\therefore -4 + V_T + 2V_T + 6 = 0$$

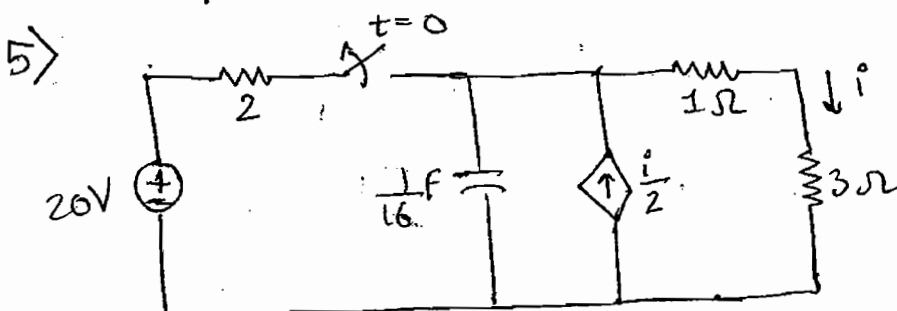
$$3V_T = -2$$

$$V_T = -2/3$$

$$R = -\frac{2}{3} \Omega$$



→ Such circuits are conceptual & not practical.



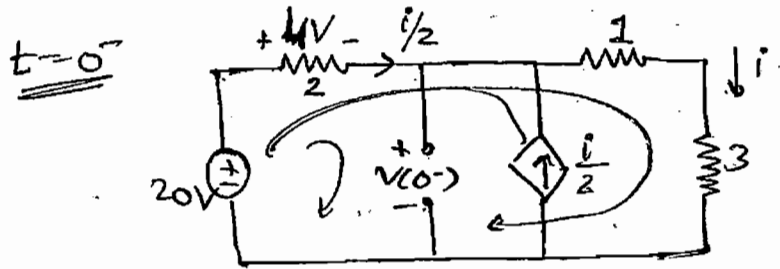
Find out complete expression for i

This is a 1st order, ~~source~~ free R-C circ.

S.V $\rightarrow$ 'V'

KVL
 $-20 + i + 4i = 0$
 $i = 4$

KVL $-20 + 4 + V(0^-) = 0$
 $\therefore V(0^-) = 16 \text{ V} = V_0$



T, t > 0

So, $i_T = \frac{1}{8}$

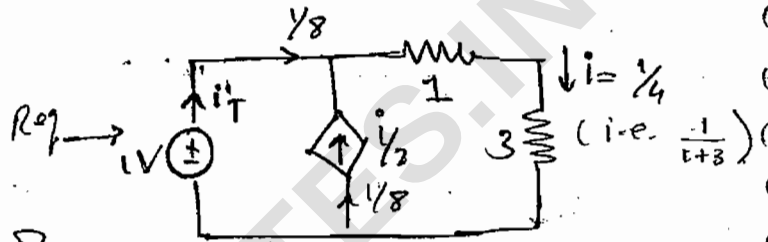
$R_{eq} = \frac{1}{i_T} = \frac{1}{1/8} = 8 \Omega$

$T = R_{eq} \cdot C = 8 \times \frac{1}{16} = \frac{1}{2} \text{ sec.}$

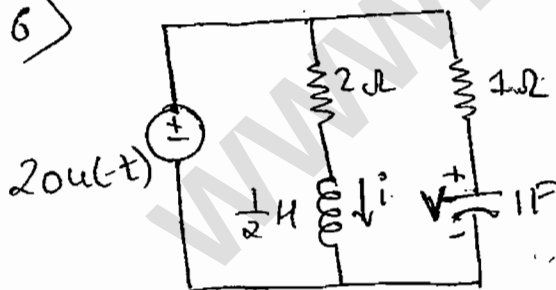
$V(t) = 16 e^{-2t} \text{ V.}$

Then

$i(t) = \frac{V(t)}{1+3} = 4 e^{-2t} \text{ A}$



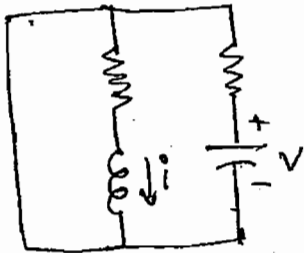
6)



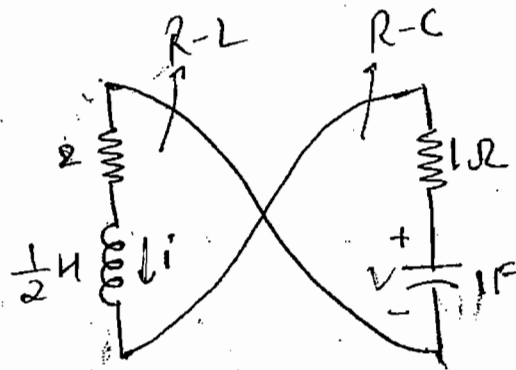
Find $\frac{V}{i}$

Find complete expression for V & i

t > 0



$\Rightarrow$

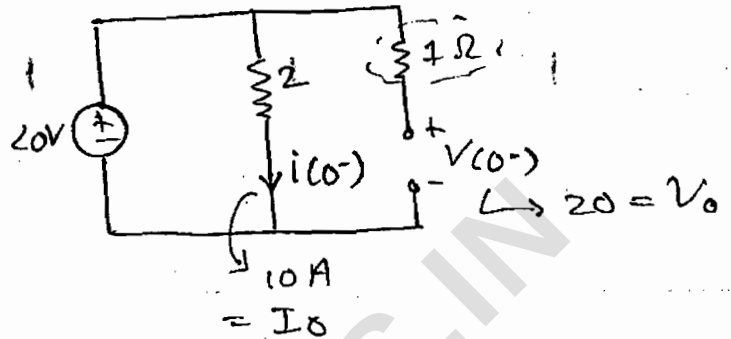


$$\tau_i = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4} \text{ sec}$$

$$\tau_v = RC = 1 \text{ sec}$$

$$(a) \frac{\tau_v}{\tau_i} = \frac{1}{1/4} = 4$$

$$(b) \begin{aligned} v(t) &= V_0 \cdot e^{-t/\tau} \\ i(t) &= I_0 e^{-t/\tau} \end{aligned}$$



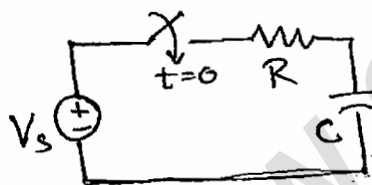
$$\therefore v(t) = 20 e^{-t/4} \text{ V}$$

$$i(t) = 10 e^{-4t} \text{ A}$$

Here current is decaying faster than vltg

[II] Step Response of 1st order ckt :-

(a) Series R-C ckt :-



$$V(t) = V_{ss}(t) + V_{tr}(t)$$

$\downarrow$
 Total response

$\downarrow$ steady state resp. $\downarrow$ Transient resp.

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

where, $V(0) \rightarrow$ vltg across capacitor before switch operation & steady state

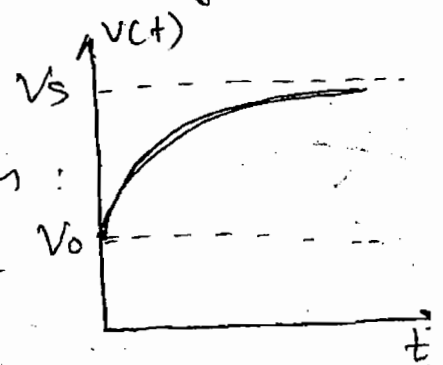
$V(\infty) \rightarrow$ vltg across capacitor after switch operation & steady state

$$\tau = RC$$

Case ① with initial condition given :

$$\text{Let } V(0) = V_0$$

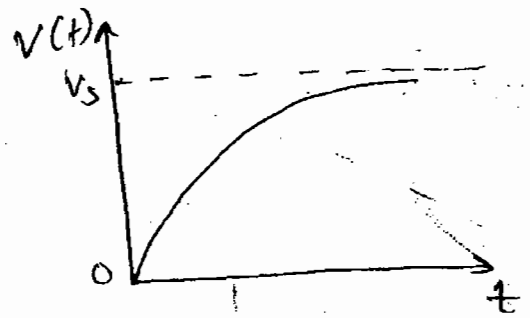
$$V(t) = V_s + [V_0 - V_s] e^{-t/\tau}$$



Case ② without initial condition.

Let $V(0) = 0$

$$\underline{V(t) = V_s [1 - e^{-t/\tau}]}$$

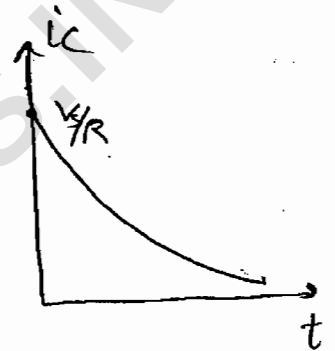


Expression for ' i_c ' :-

$$i_c(t) = C \frac{d}{dt} [V_s (1 - e^{-t/\tau})]$$

$$= C V_s [0 - e^{-t/\tau} \times (-1/\tau)]$$

$$\underline{i_c(t) = \frac{V_s}{R} e^{-t/\tau}}$$



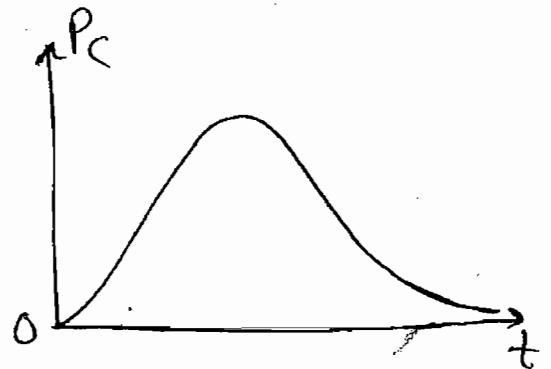
Expression for Power :-

$$P_c(t) = V_c(t) \cdot i_c(t)$$

$$= \frac{V_s^2}{R} [e^{-t/\tau} - e^{-2t/\tau}]$$

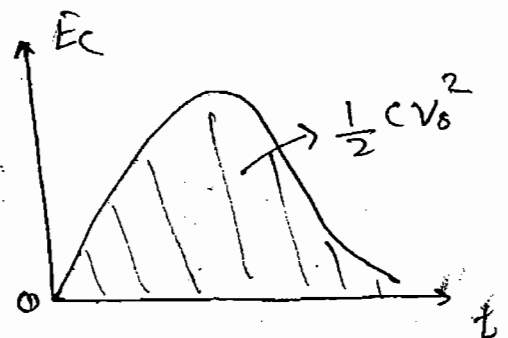
at $t=0 \Rightarrow P_c=0$

at $t \rightarrow \infty \Rightarrow P_c=0$



Expression for Energy

$$E_c = \int_0^{\infty} P_c dt = \frac{1}{2} C [V_s]^2$$



$$\rightarrow i_R(t) = \frac{V_s}{R} e^{-t/\tau}$$

KVL $-V_s + V_R + V_c = 0$

$$V_R = V_s - [V_s (1 - e^{-t/\tau})]$$

$$V_R(t) = V_s e^{-t/\tau}$$

$$\rightarrow P_R(t) = V_R(t) \cdot i_R(t)$$

$$= \frac{V_s^2}{R} \cdot e^{-2t/\tau} \quad \underline{\underline{w}}$$

$$\rightarrow E_R(t) = \int_0^{\infty} P_R dt = \frac{1}{2} C V_s^2 \quad \underline{\underline{J}}$$

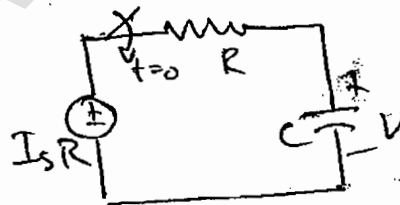
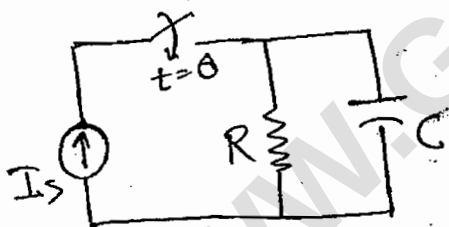
$$\% \text{ Efficiency} = \frac{O/P}{I/P} \times 100\%$$

$$= \frac{E_C}{E_R + E_C} \times 100\%$$

$$= \frac{\frac{1}{2} C V_s^2}{\frac{1}{2} C V_s^2 + \frac{1}{2} C V_s^2} \times 100\% = 50\%$$

$$\therefore \boxed{\eta = 50\%}$$

Note (Parallel R-C ckt)

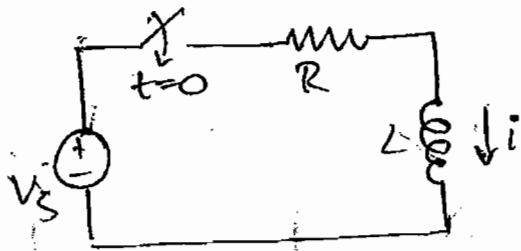


$$V(t) = I_s R + [V_0 - I_s R] e^{-t/\tau} \quad \Delta \tau = RC$$

$\Rightarrow$ Here we cannot connect a vltg source across capacitor as it will cause sudden change of vltg across capacitor ~~with~~ which will result in large current which may damage the ckt.

$\therefore$ The ckt behaviour cannot be determined at $t=0^+$

(b) R-L ckt :—



$$i(t) = i_{ss}(t) + i_{tr}(t)$$

$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$$

$I(0) \rightarrow$ current through inductor before switch operation & steady state.

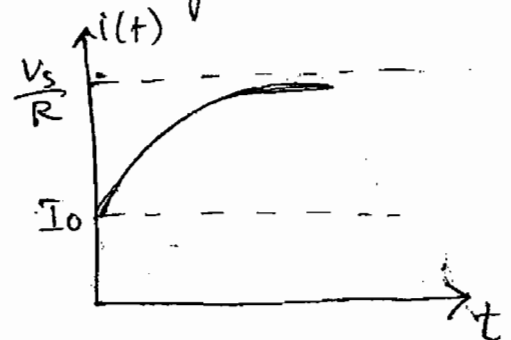
$I(\infty) \rightarrow$ current through inductor after switch operation & steady state.

$$\tau = \frac{L}{R}$$

Case ① with initial condition given:—

$$\text{Let } I(0) = I_0$$

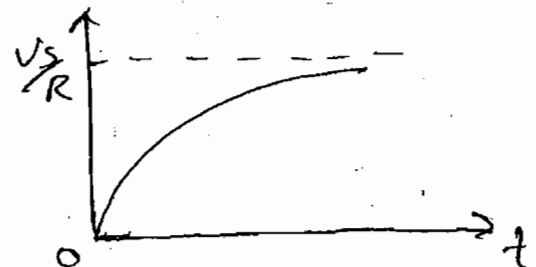
$$i_L(t) = \frac{V_s}{R} + \left[I_0 - \frac{V_s}{R} \right] e^{-t/\tau}$$



Case ② without initial conditions:—

$$I(0) = 0$$

$$i_L(t) = \frac{V_s}{R} [1 - e^{-t/\tau}]$$

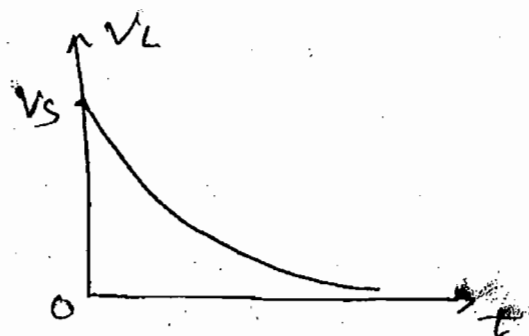


Expression for vltg across inductor:—

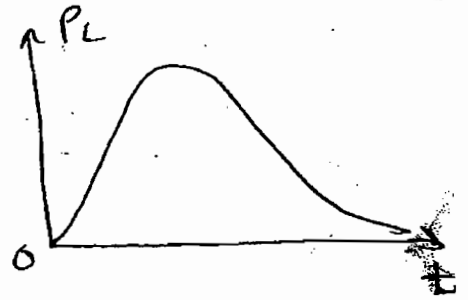
$$V_L = L \frac{d}{dt} \left[\frac{V_s}{R} (1 - e^{-t/\tau}) \right]$$

$$= \frac{L V_s}{R} \left[0 - e^{-t/\tau} \times -\frac{1}{\tau} \right]$$

$$\therefore V_L(t) = V_s e^{-t/\tau}$$



$$\begin{aligned}\Rightarrow P_L(t) &= v_L(t) \cdot i_L(t) \\ &= \frac{V_s^2}{R} [e^{-t/\tau} - e^{-2t/\tau}]\end{aligned}$$



$$\begin{aligned}\Rightarrow E_L(t) &= \int_0^{\infty} P_L(t) dt \\ &= \frac{1}{2} L [i(\infty)]^2 \\ &= \frac{1}{2} L \left[\frac{V_s}{R} \right]^2 \quad \Downarrow\end{aligned}$$

$$\Rightarrow i_R(t) = \frac{V_s}{R} [1 - e^{-t/\tau}] \quad \begin{array}{l} \text{KVL} \\ -V_s + V_R + V_L = 0 \end{array}$$

$$\begin{aligned}\Rightarrow V_R(t) &= V_s - V_L(t) = V_s - V_s e^{-t/\tau} \\ &= V_s [1 - e^{-t/\tau}]\end{aligned}$$

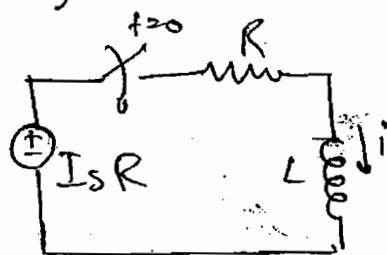
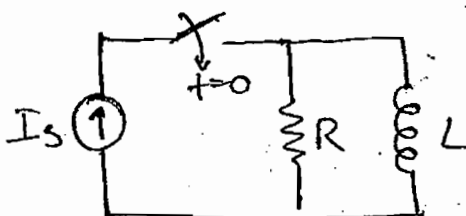
$$\Rightarrow P_R(t) = V_R \cdot i_R = \frac{V_s^2}{R} [1 - e^{-t/\tau}]^2 \quad \underline{\underline{w}}$$

$$\Rightarrow E_R(t) = \int_0^{\infty} P_R(t) dt = \infty \quad \Downarrow$$

$$\begin{aligned}\Rightarrow \% \eta &= \frac{E_L}{E_L + E_R} \times 100\% \\ &= \frac{\frac{1}{2} L \left(\frac{V_s}{R} \right)^2}{\frac{1}{2} L \left(\frac{V_s}{R} \right)^2 + \infty} \times 100\%\end{aligned}$$

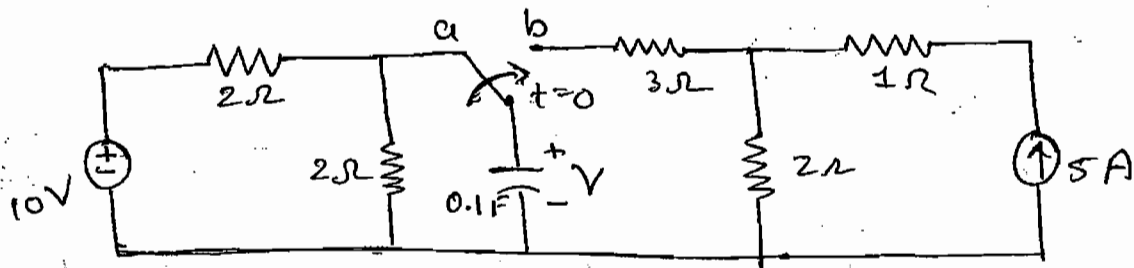
$$\boxed{\% \eta = 0\%}$$

Note (Parallel R-L ckt)



$$\begin{aligned}i(t) &= I_s + [I_0 - I_s] e^{-t/\tau} \\ \tau &= L/R\end{aligned}$$

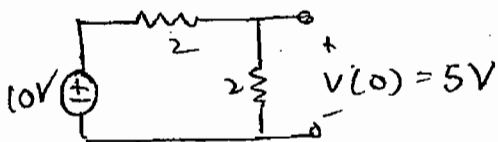
1) Find the complete expression for V .



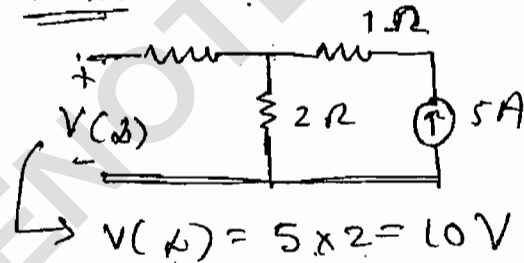
This is step response of 1st order R-C n/w
S.V $\rightarrow$ 'V'

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

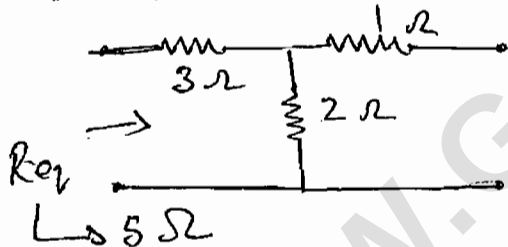
$\Rightarrow \underline{V(0) = ?}$



$\Rightarrow \underline{V(\infty) = ?}$

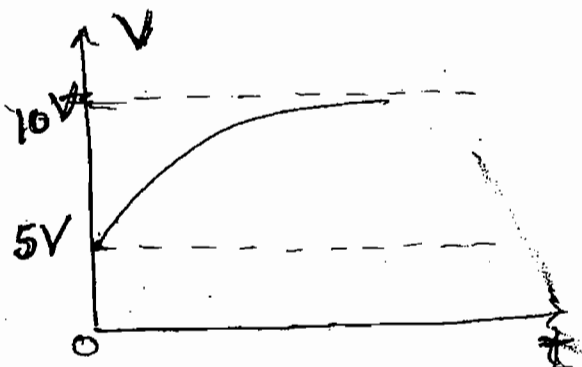


$\tau, t > 0$

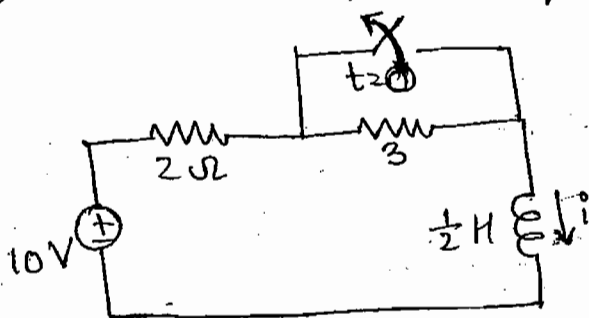


$$\begin{aligned} \tau &= R_{eq} \cdot C \\ &= 5 \times 0.1 \\ &= \frac{1}{2} \text{ sec} \end{aligned}$$

$$\begin{aligned} V(t) &= 10 + [5 - 10] e^{-t/1/2} \\ &= 10 - 5 e^{-2t} \text{ A} \end{aligned}$$



2) Find complete expression for I .



This is step response of
 1st order R-L n/w.
S.V $\rightarrow$ 'i'

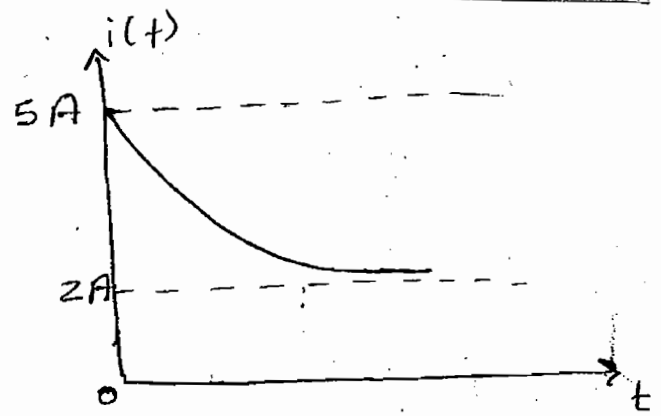
$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$$

$$\rightarrow I(0) = \frac{10}{2} = 5 \text{ A}$$

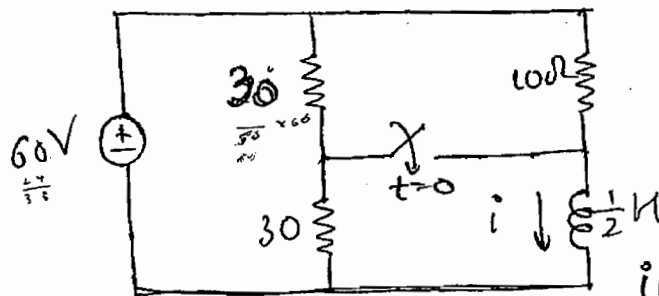
$$\rightarrow I(\infty) = \frac{10}{5} = 2 \text{ A}$$

$$\rightarrow \tau = \frac{L}{R} = \frac{1/2}{5} = \frac{1}{10} \text{ sec}$$

$$i(t) = 2 + 3e^{-10t} \text{ A}$$



3) Find the complete expression for i



This is a 1st order, step-response R-L ckt
S.V $\rightarrow I$

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau}$$

$$\underline{i(0)} = \frac{60}{30+10} \times \frac{60 \times (50+10)}{50 \times 10} = 6 \text{ A}$$

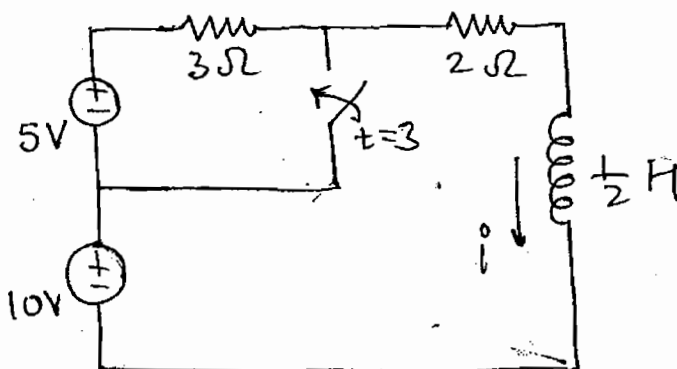
$$\underline{i(\infty)} = ? \quad i(\infty) = \frac{30 \times 40}{30 \times 40} = 8 \text{ A}$$

$$\underline{\tau} = \frac{1/2}{30 \parallel 30 \parallel 10} = \frac{1}{12} \text{ sec}$$

$$\therefore \boxed{i(t) = 8 - 2e^{-12t}} \text{ A}$$

4) Find the complete expression for i & the energy stored in the inductor.

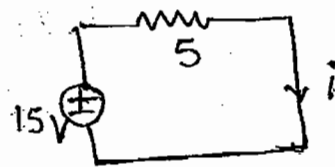
upto 3.25 sec.



This is 1st order, step response, R-L circuit. (delayed)
S.V $\rightarrow i$

$0 < t \leq 3$ $\rightarrow$ Steady State

$$i = 3 \text{ A}$$



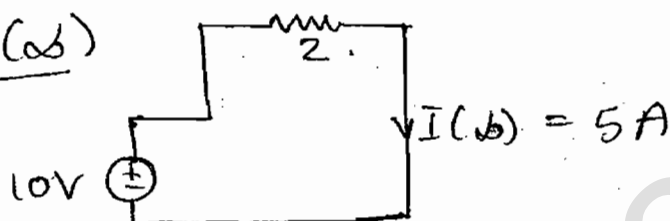
$t > 3$ $\rightarrow$ Transient Solution

$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$$

$\rightarrow$ Here $I(0) \rightarrow i(3^-) \rightarrow$ from previous state

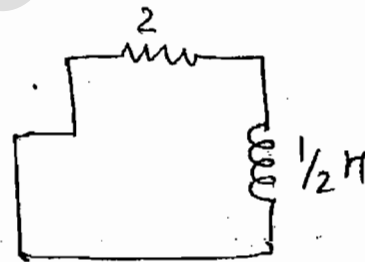
$$i(3^-) = 3 \text{ A}$$

$\Rightarrow \underline{I(\infty)}$



$\Rightarrow \underline{\tau, t > 3}$

$$\tau = \frac{L}{R} = \frac{1/2}{2} = \frac{1}{4} \text{ sec}$$



$$i(t) = 5 + [3 - 5] e^{-(t-3)/(1/4)}$$

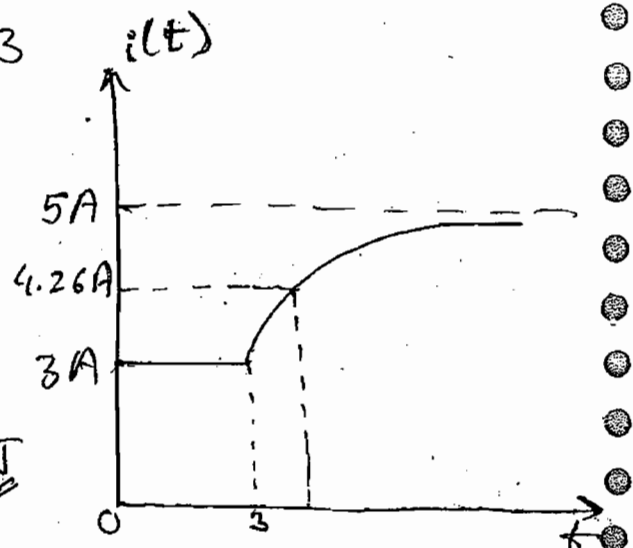
Complete expression

$$i(t) = \begin{cases} 3 \text{ A} , & ; 0 < t \leq 3 \\ 5 - 2 e^{-4(t-3)} & ; t > 3 \end{cases}$$

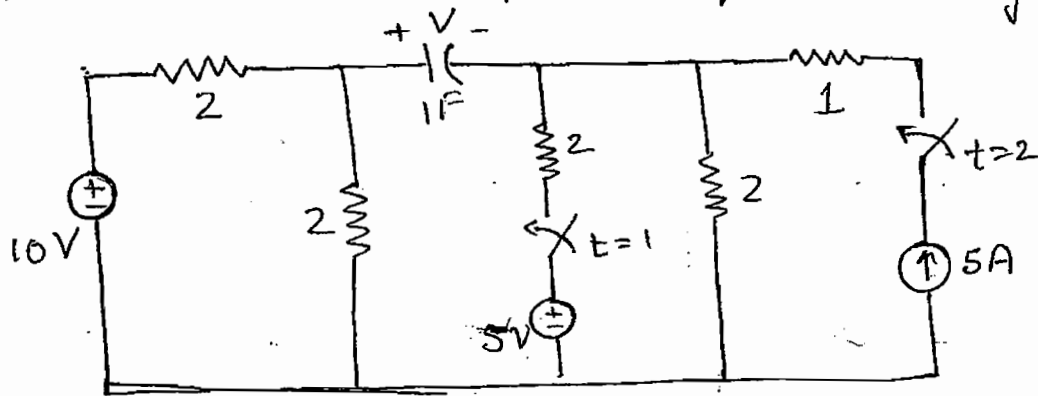
$$E_L(t=3.25) = \frac{1}{2} L i^2$$

$$i(t=3.25) = 5 - 2 e^{-4(3.25-3)} = 4.26 \text{ A}$$

$$E_L = \frac{1}{2} \times \frac{1}{2} \times (4.26)^2 = 4.545 \text{ J}$$



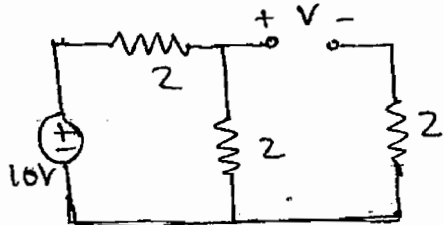
5) Find the complete expression of V



This is Step response, 1st order R-C ckt.

S.V $\rightarrow$ 'V'

$0 < t \leq 1$ (Steady state)



$$\therefore V = 5V$$

$1 < t \leq 2$ (1st part of T.R.)

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

Here $V(0) \rightarrow V(1^-) \rightarrow$ from previous state.

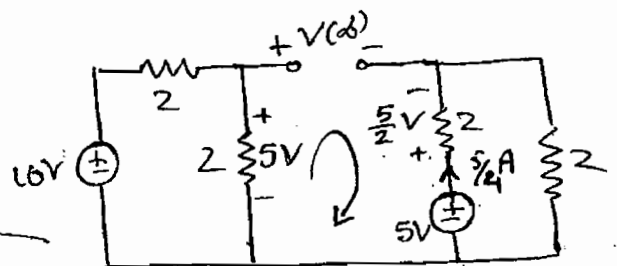
$$\therefore V(1^-) = 5V$$

$\Rightarrow V(\infty)$

KVL

$$-5 + V(\infty) + 5 - \frac{5}{2} = 0$$

$$\therefore V(\infty) = \frac{5}{2} V$$

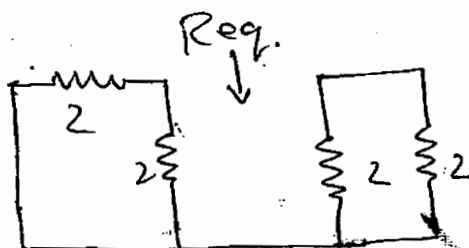


$\Rightarrow$

τ

$$\tau = RC$$

$$= 2 \times 1 = 2 \text{ sec}$$



$$V(t) = 2.5 \left[1 + e^{-\frac{(t-1)}{2}} \right]$$

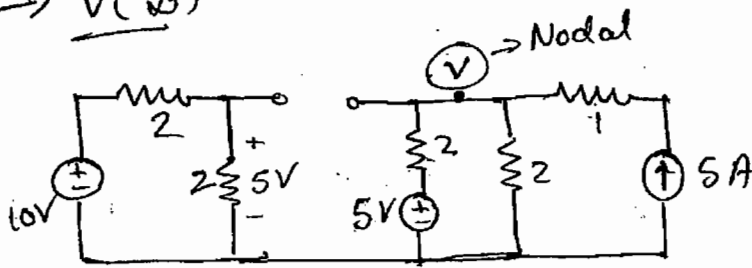
$t > 2$ (2nd part of T.R.)

$$V(t) = V(\infty) + [V(0) - V(\infty)] e^{-t/\tau}$$

Here $V(0) = V(2^-) \rightarrow$ from previous state

$$V(2^-) = 2.5 [1 + e^{-1/2}] = 4V$$

→ $V(\infty)$



$$\frac{(V-5)}{2} + \frac{V}{2} - 5 = 0$$

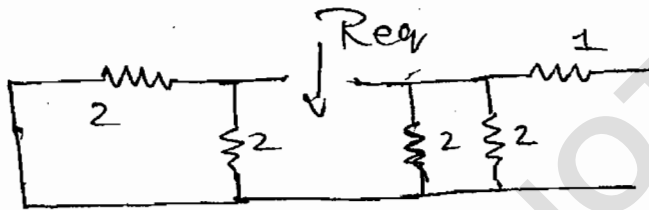
$$2V = 15$$

$$V = \frac{15}{2}$$

KVL $-5 + V(\infty) + \frac{15}{2} = 0$

$$V(\infty) = -\frac{5}{2} = -2.5V$$

→ τ



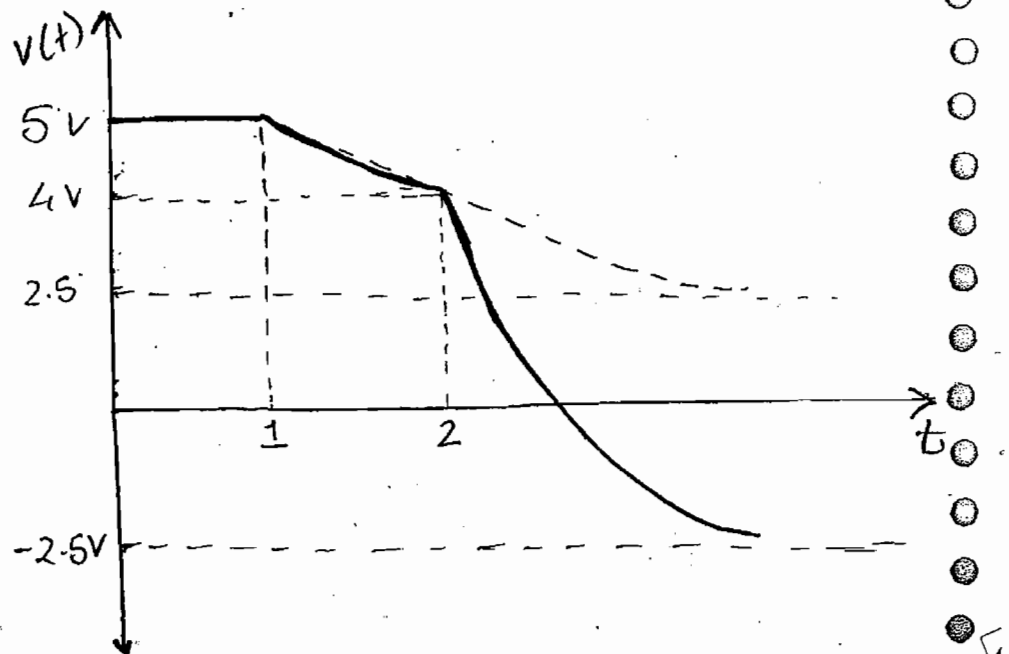
$$\tau = R-C$$

$$= 2 \cdot 1$$

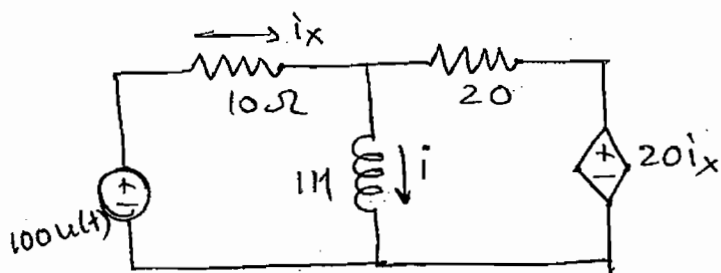
$$= 2 \text{ sec}$$

$$V(t) = -2.5 + 6.5 e^{\frac{-(t-2)}{2}} V$$

$$V(t) = \begin{cases} 5V & ; 0 \leq t \leq 1 \\ 2.5 \left[1 + e^{\frac{-(t-1)}{2}} \right] V & ; 1 < t \leq 2 \\ -2.5 + 6.5 e^{\frac{-(t-2)}{2}} V & ; 2 < t \end{cases}$$



6) Find the complete expression for 'i'

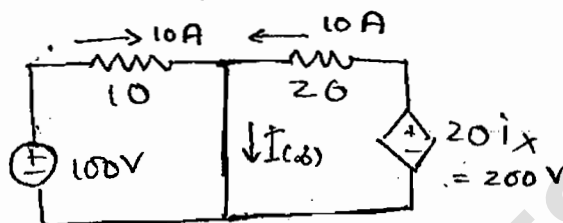


→ $I(0) = 0 \text{ A}$ (initially relaxed)

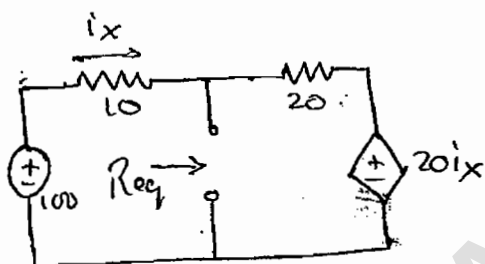
→ $I(\infty)$

KCL

$$I(\infty) = 20 \text{ A}$$



$$\Rightarrow T = \frac{L}{R_{eq}}$$

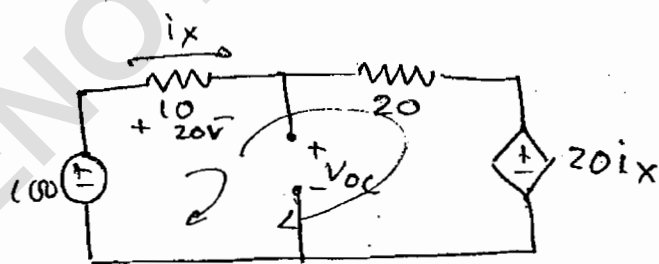


S-II

$$I_{sc} = I(\infty)$$

$$\therefore I_{sc} = 20 \text{ A}$$

S-I



KVL

$$-100 + 30i_x + 20i_x = 0$$

$$i_x = 2$$

KVL

$$-100 + 20 + V_{oc} = 0$$

$$V_{oc} = 80 \text{ V}$$

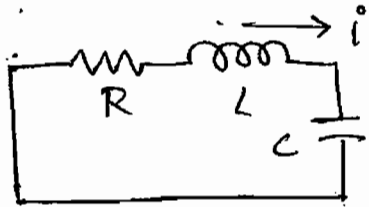
$$R_{eq} = \frac{V_{oc}}{I_{sc}} = \frac{80}{20} = 4 \Omega$$

$$T = \frac{L}{R} = \frac{1}{4} \text{ sec.}$$

$$i(t) = 20[1 - e^{-4t}] \text{ A}$$

III] Source free 2^{nd} order circuits :- (Canonical form)

(a) Series R-L-C



Dominant S.V. $\rightarrow i$

KVL

$$iR + L \frac{di}{dt} + \frac{1}{C} \int i dt = 0$$

$$R \frac{di}{dt} + L \frac{d^2 i}{dt^2} + \frac{1}{C} \cdot i = 0$$

$\rightarrow$ Use Laplace T/f (homogeneous)

$$L s^2 I(s) + R s I(s) + \frac{I(s)}{C} = 0$$

$$I(s) \left[L s^2 + R s + \frac{1}{C} \right] = 0$$

$$I(s) \left[s^2 + \frac{R}{L} s + \frac{1}{LC} \right] = 0 \quad (\text{As } I(s) \neq 0)$$

$$\therefore s^2 + \frac{R}{L} s + \frac{1}{LC} = 0$$

$$\text{The 2 roots are : } s_1, s_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2}$$

$$\therefore s_1, s_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{\sqrt{LC}}\right)^2}$$

Let $\alpha = \frac{R}{2L}$ } Damping factor

$\omega_0 = \frac{1}{\sqrt{LC}}$ } Undamped natural freq

$$\therefore s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Case ①: If $\alpha > \omega_0 \rightarrow$ overdamped

$$\frac{R}{2L} > \frac{1}{\sqrt{LC}}$$

$\rightarrow$ Two roots are: -ve, real & unequal
then,

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$\rightarrow A_1, A_2$ are arbitrary const. that can be determined from initial condition.

Case ②: If $\alpha = \omega_0 \rightarrow$ critically damped

$$\frac{R}{2L} = \frac{1}{\sqrt{LC}}$$

$\rightarrow$ The 2 roots are: -ve, real & equal

Then,

$$i(t) = e^{-\alpha t} [A_1 + A_2 t]$$

Case ③: If $\alpha < \omega_0 \rightarrow$ underdamped

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}}$$

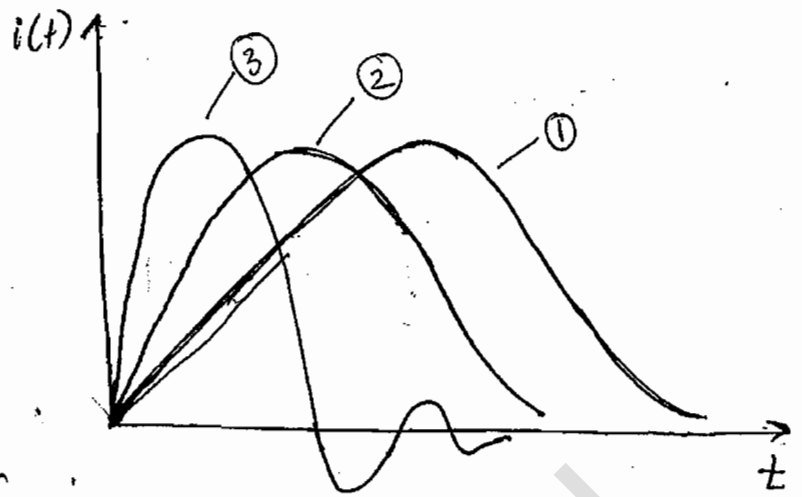
$\rightarrow$ The 2 roots are: complex conjugate with -ve, real

Then,

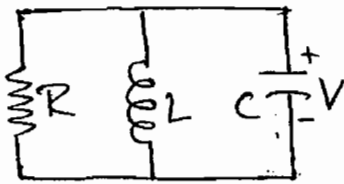
$$i(t) = e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t]$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

↳ damped freq.



(b) Parallel R-L-C :



Dominant S.V. $\rightarrow V$

KVL

$$\frac{V}{R} + \frac{1}{L} \int V dt + C \frac{dV}{dt} = 0$$

$$C \frac{d^2 V}{dt^2} + \frac{1}{R} \frac{dV}{dt} + \frac{V}{L} = 0$$

→ Use Laplace T/f (homogeneous)

$$C s^2 V(s) + \frac{1}{R} s V(s) + \frac{V(s)}{L} = 0$$

$$V(s) \left[s^2 + \frac{1}{RC} s + \frac{1}{LC} \right] = 0$$

$$\therefore s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$\text{The 2 roots are: } s_1, s_2 = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{LC}\right)^2}$$

$$\text{Let, } \alpha = \frac{1}{2RC} \left\{ \begin{array}{l} \text{damping} \\ \text{factor} \end{array} \right.$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \left\{ \begin{array}{l} \text{undamped natural freq.} \end{array} \right.$$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

Case ① If $\alpha > \omega_0 \rightarrow$ overdamped

$$\frac{1}{2RC} > \frac{1}{\sqrt{LC}}$$

$\rightarrow$ The two roots are -ve, real & unequal

Then,

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$\rightarrow A_1, A_2$ are arbitrary const. that can be determined from initial conditions.

Case ② If $\alpha = \omega_0 \rightarrow$ critically damped

$$\frac{1}{2RC} = \frac{1}{\sqrt{LC}}$$

$\rightarrow$ The two roots are -ve, real & equal

Then,

$$v(t) = e^{-\alpha t} [A_1 + A_2 t]$$

Case ③ If $\alpha < \omega_0 \rightarrow$ Underdamped

$$\frac{1}{2RC} < \frac{1}{\sqrt{LC}}$$

$\Rightarrow$ The 2 roots are : complex conjugate
with -ve, real

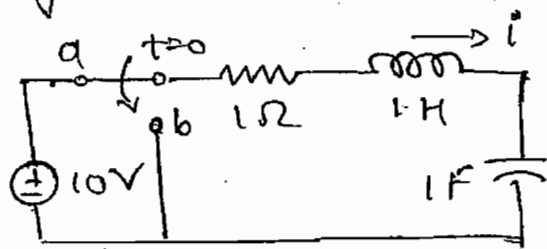
then,

$$v(t) = e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t]$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

1) The nature of response of $i(t)$, $t > 0$ is:

- ☒ (a) UD (c) OD
~~(b) CD~~ ~~(d) Simusoidal~~



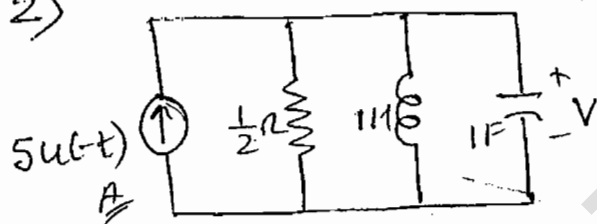
→ This is 2nd order, source free, series R-L-C circuit.

$$\alpha = \frac{R}{2L} = \frac{1}{2 \times 1} = \frac{1}{2}$$

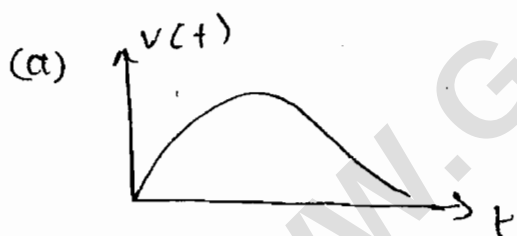
$$\Rightarrow \alpha < \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1}} = 1$$

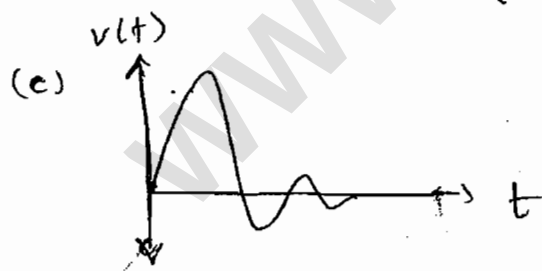
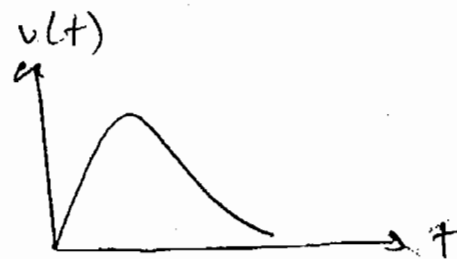
2)



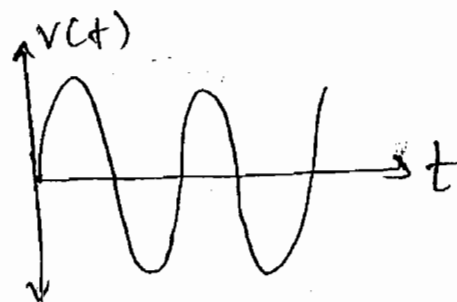
The nature of resp. of $V(t)$, $t > 0$ is —.



☒ (b)



(d)



→ This is 2nd order, source free, parallel R-L-C circuit.

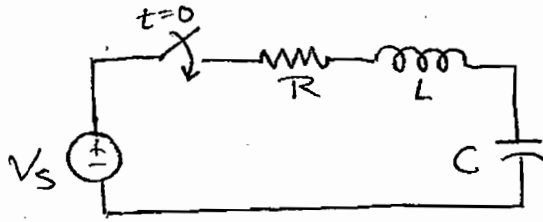
$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times \frac{1}{2} \times 1} = 1$$

$$\Rightarrow \alpha = \omega_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1}} = 1$$

IV] Step Response of 2^{nd} Order sys.

(a) Series R-L-C

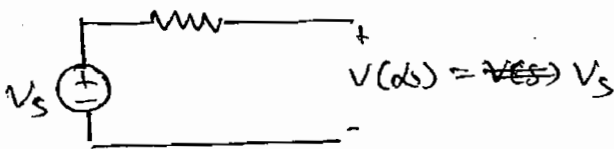


Dominant variable $\rightarrow$ 'V'
(i.e. vltg across capacitor in steady state condition).

$$V(t) = V_{ss}(t) + V_{tr}(t)$$

But

$V_{ss}(t)$, is as $t \rightarrow \infty$

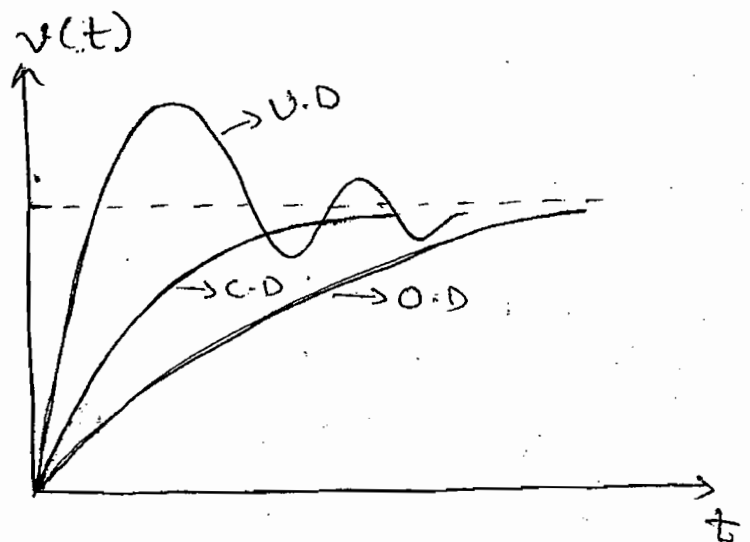


But $V_{tr}(t)$ depends upon the values of R-L-C

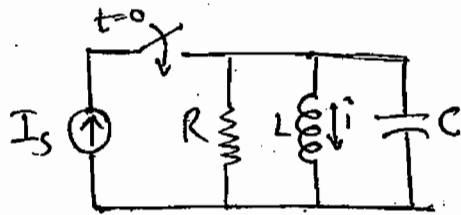
So here, $\alpha = \frac{R}{2L}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

So the complete solution is:

$$V(t) = \begin{cases} V_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \rightarrow \text{overdamped } (\alpha > \omega_0) \\ \text{(or)} \\ V_s + e^{-\alpha t} [A_1 + A_2 t] \rightarrow \text{critically damped } (\alpha = \omega_0) \\ \text{(or)} \\ V_s + e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t] \rightarrow \text{underdamped } (\alpha < \omega_0) \end{cases}$$



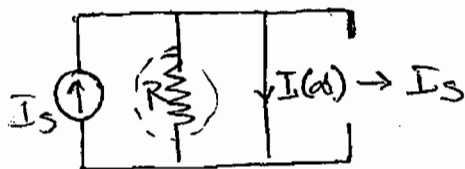
(b) Parallel R-L-C



Dominant S.V. $\rightarrow$ 'i'

$$i(t) = I_{ss}(t) + I_{tr}(t)$$

But $I_{ss}(t)$ is as $t \rightarrow \infty$

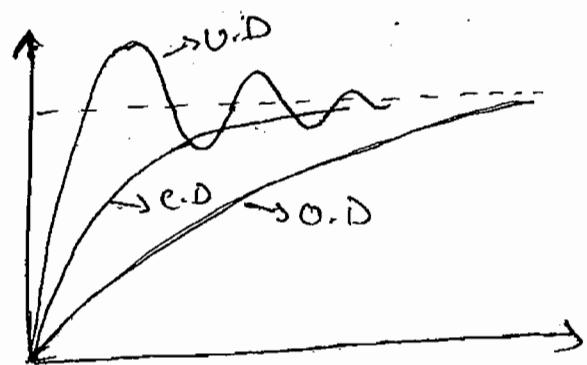


But $I_{tr}(t)$, depends upon the values of R-L-C.

So here, $\alpha = \frac{1}{2RC}$ $\omega_0 = \frac{1}{\sqrt{LC}}$

So, complete solution is,

$$i(t) = \begin{cases} I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \rightarrow \text{over damped } (\alpha > \omega_0) \\ I_s + e^{-\alpha t} [A_1 + A_2 t] \rightarrow \text{critically damped } (\alpha = \omega_0) \\ I_s + e^{-\alpha t} [A_1 \cos \omega_d t + A_2 \sin \omega_d t] \rightarrow \text{under damped } (\alpha < \omega_0) \end{cases}$$



Note:-

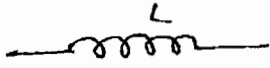
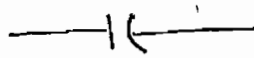
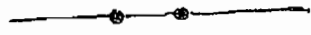
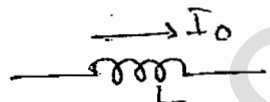
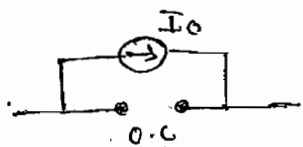
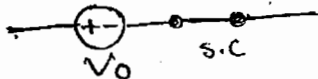
In determining the solution to 2nd order ckt, we need to find the correct values of const. A_1 & A_2 through initial conditions.

In 2nd order ckt, there are 2 const., so we require mini. 2 eq^s.

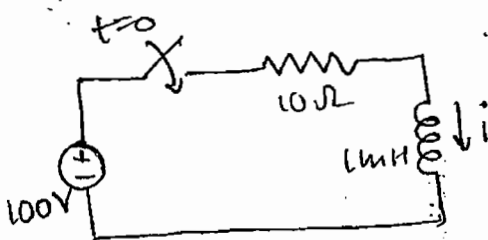
V] Initial Condition Problems :-

(Transient State Problems, i.e. problems at $t = 0^+$)

Equivalent rep^{ct} representation of passive elements during transient state (at $t = 0^+$) :-

Element	Equi. ckt ($t = 0^+$)
	
	
	
	
	

➤ Find $i(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{d^2i(0^+)}{dt^2}$



At $t = 0^-$, s/w was open
 $\therefore i(0^-) = 0 \text{ A} = i(0^+)$

($\because$ inductor cannot allow sudden change in current.)

KVL

$$-100 + 10i + 1\text{m} \frac{di}{dt} = 0 \rightarrow \text{exact form}$$

$$\text{At } t=0^+$$

$$-100 + 10 \left[\underset{\substack{\downarrow \\ 0 \text{ A}}}{i(0^+)} \right] + 1 \text{ m} \frac{di(0^+)}{dt} = 0$$

$$\therefore \frac{di(0^+)}{dt} = \underline{\underline{100 \text{ k A/sec}}}$$

1) Differentiating,

$$10 \frac{di}{dt} + 1 \text{ m} \frac{d^2 i}{dt^2} = 0$$

$$\text{At } t=0^+$$

$$10 \frac{di(0^+)}{dt} + 1 \text{ m} \frac{d^2 i}{dt^2}(0^+) = 0$$

$\downarrow$
100k

$$\frac{d^2 i(0^+)}{dt^2} = -1000 \text{ mA/sec}^2$$

→ Suppose in this problem find complete solution.

$$i(t) = I(\infty) + [I(0) - I(\infty)] e^{-t/\tau}$$

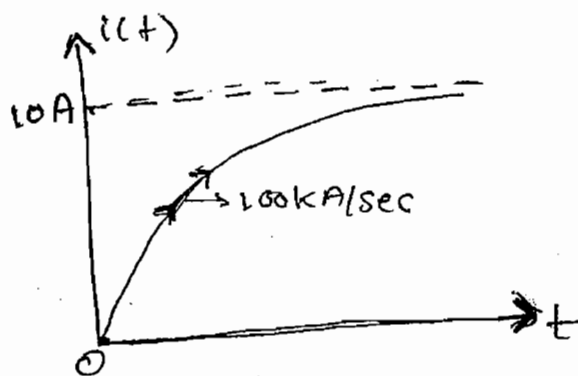
here,

$$I(0) = 0 \text{ A}$$

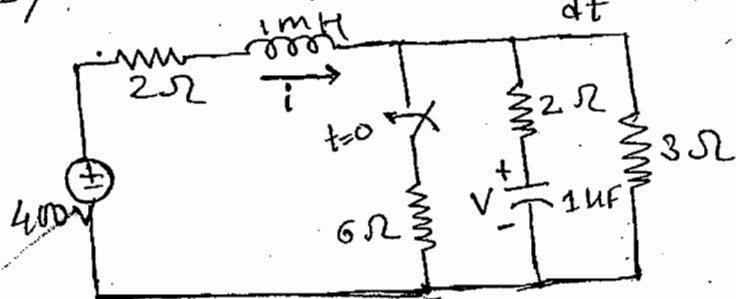
$$I(\infty) = \frac{100}{10} = 10 \text{ A}$$

$$\tau = \frac{L}{R} = \frac{1 \text{ m}}{10} = 10^{-4}$$

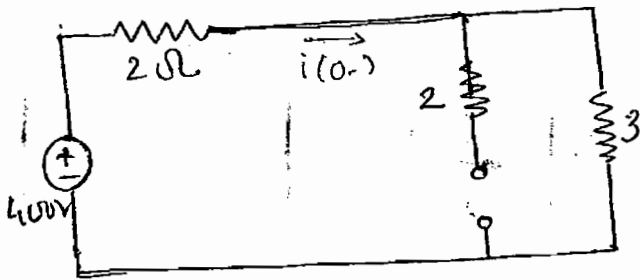
$$i(t) = 10 [1 - e^{-10000t}] \text{ A}$$



2) Find $i(0^+)$, $v(0^+)$, $\frac{di(0^+)}{dt}$, $\frac{dv(0^+)}{dt}$, $i(\infty)$, $v(\infty)$



① Past ($t=0^-$)

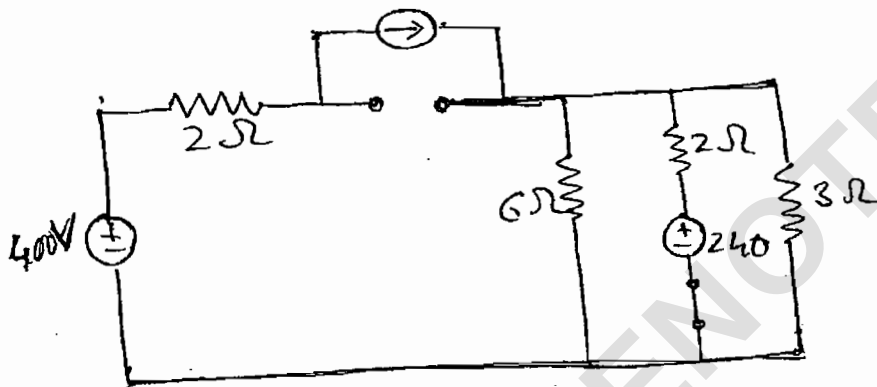


$$i(0^-) = \frac{400}{5} = 80 \text{ A}$$

$$i(0^+) = 80 \text{ A}$$

$$V(0^-) = 240 \text{ V} \rightarrow V(0^+)$$

② Present State ($t=0^+$)



$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

$$\frac{dV(0^+)}{dt} = \frac{i_C(0^+)}{C}$$

Nodal

$$-80 + \frac{V}{6} + \frac{(V-240)}{2} + \frac{V}{3} = 0$$

$$-480 + V + 3V - 720 + 2V = 0$$

$$\therefore \boxed{V = 200 \text{ V}}$$

KVL

$$-400 + 160 + V_L(0^+) + 200 = 0$$

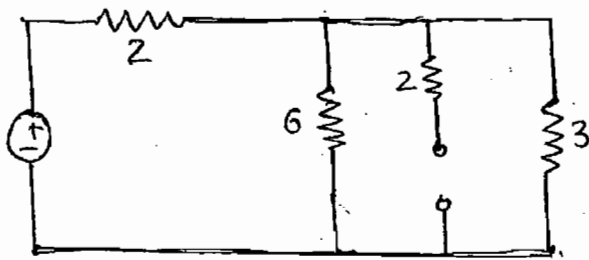
$$V_L(0^+) = 40$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{40}{1\text{m}} = 40 \text{ kA/sec}$$

$$i_C(0^+) = \frac{(V-240)}{2} = \frac{(200-240)}{2} = -20$$

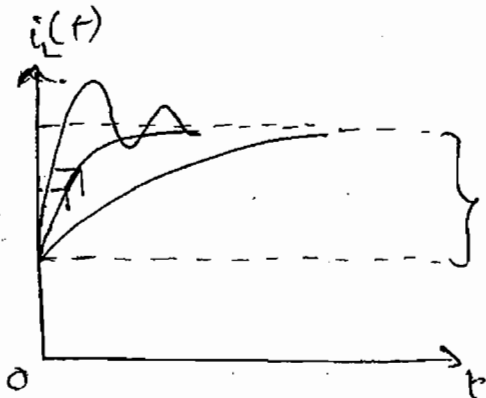
$$\frac{dV(0^+)}{dt} = \frac{i_C(0^+)}{C} = \frac{-20}{1\mu} = -20 \text{ MV/sec}$$

③ Future ($t \rightarrow \infty$)

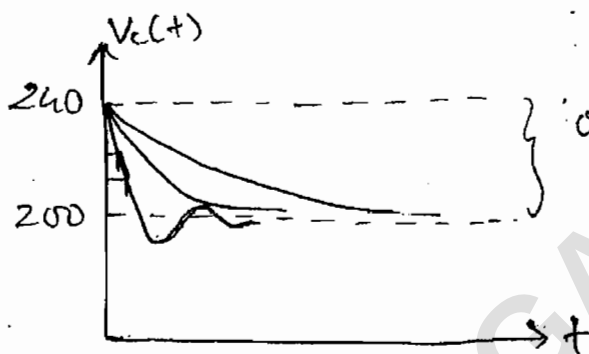


$$I(\infty) = \frac{400}{4} = 100 \text{ A}$$

$$V(\infty) = 200 \text{ V}$$

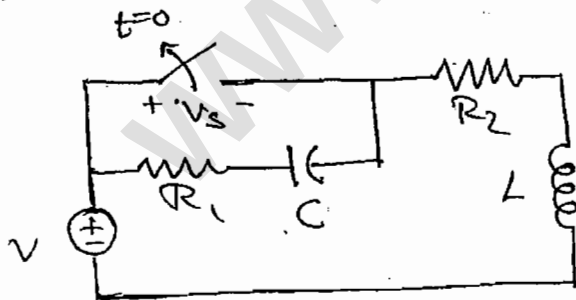


One of this is correct solution



One of this is correct solution

3) Find $V_S(0^+)$



(a) $V \left[\frac{R_1}{R_1 + R_2} \right]$

(b) $V \left[\frac{R_2}{R_1 + R_2} \right]$

(c) $V \left[\frac{R_1}{R_2} \right]$

(d) $V \left[\frac{R_2}{R_1} \right]$

① At $t=0^-$

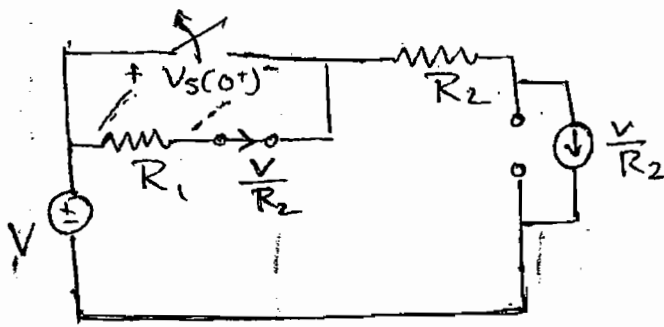
$$i(0^-) = \frac{V}{R_2} \text{ A}$$

$$= i(0^+)$$

$$V(0^-) = 0 \text{ V} \quad , \quad V_S(0^-) = 0 \text{ V}$$

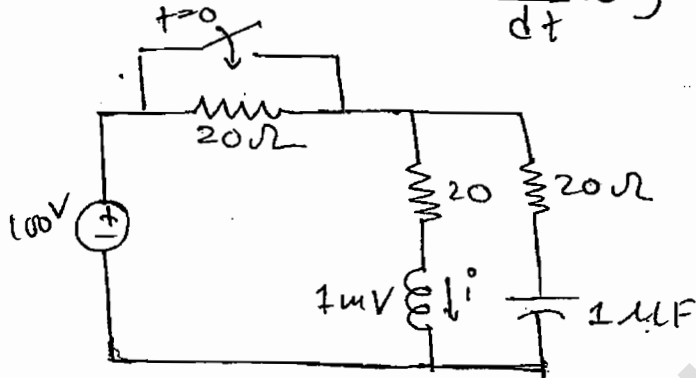
$$= V(0^+)$$

② At $t=0^+$



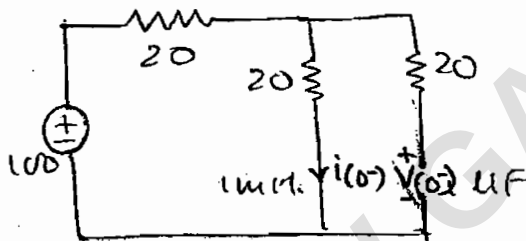
$$V_s(0^+) = \frac{V}{R_2} \times R_1$$

2) Find $V_L(0^+)$ $\frac{di(0^+)}{dt}$



① at $t=0^-$

$$i(0^-) = \frac{100}{40} = \frac{5}{2} \text{ A}$$



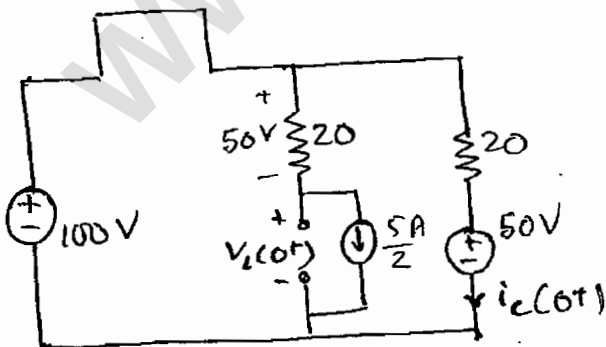
$$V(0^-) = \frac{20}{40} \times 100 = 50 \text{ V}$$

② at $t=0^+$

KVL

$$-100 + 50 + V_L(0^+) = 0$$

$$V_L(0^+) = 50$$

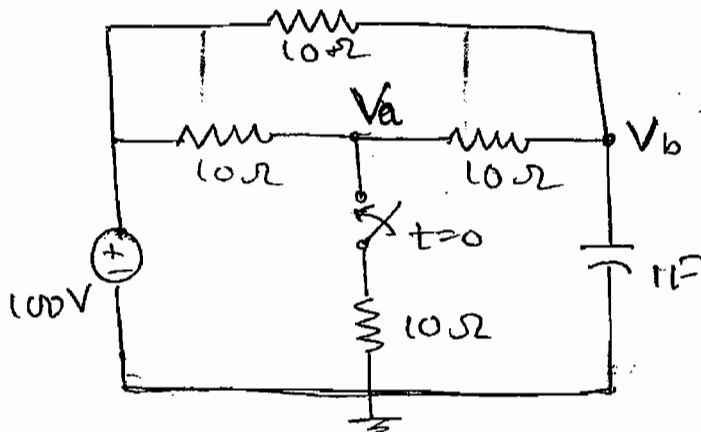


$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$

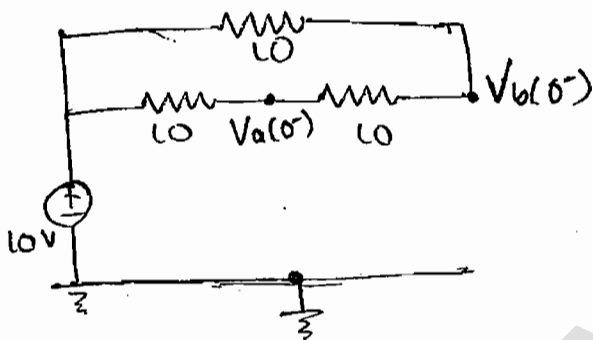
$$= \frac{50}{1 \text{ m}}$$

$$= 50 \text{ kA/sec}$$

3) $V_a(0^-), V_b(0^-)$
 $V_a(0^+), V_b(0^+)$



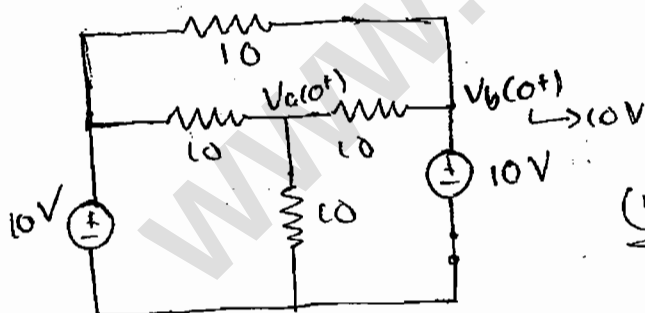
① At $t = 0^-$



$$V_a(0^-) = 10V$$

$$V_b(0^-) = 10V$$

② At $t = 0^+$

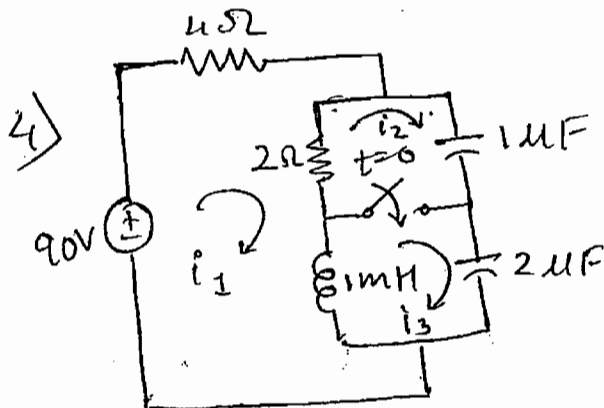


$$V_b(0^+) = 10V$$

Nodal

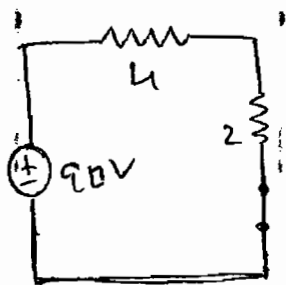
$$\frac{(V_a - 10)}{10} + \frac{V_a}{10} + \frac{(V_a - 10)}{10} = 0$$

$$V_a(0^+) = \frac{20}{3} V$$



Find $i_1(0^+), i_2(0^+), i_3(0^+)$

① At $t = 0^-$

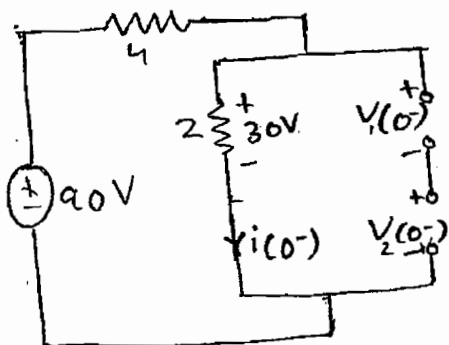


$$i_1(0^-) = \frac{90}{6} = 15 \text{ A}$$

$$i_2(0^-) = 0 \text{ A}$$

$$i_3(0^-) = 0 \text{ A}$$

② At $t = 0^+$

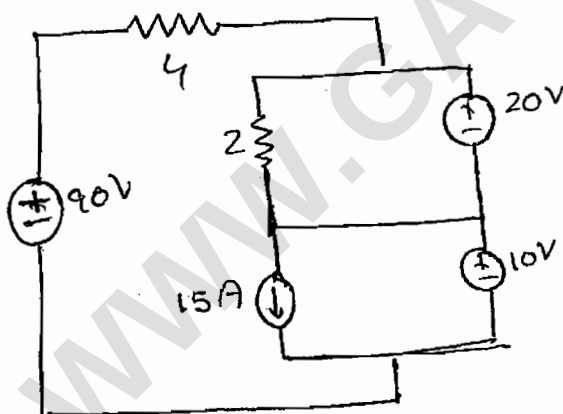


$$i_1(0^+) = 15 \text{ A}$$

$$V_1(0^-) = 30 \left[\frac{2}{3} \right] = 20 \text{ V}$$

$$V_2(0^-) = 30 \left[\frac{1}{3} \right] = 10 \text{ V}$$

③ At $t = 0^+$



Mesh

$$-90 + 4i_1 + 2[i_1 - i_2] + 10 = 0 \quad \text{--- (1)}$$

$$2[i_2 - i_1] + 20 = 0 \quad \text{--- (2)}$$

$$i_1 - i_3 = 15 \quad \text{--- (3)}$$

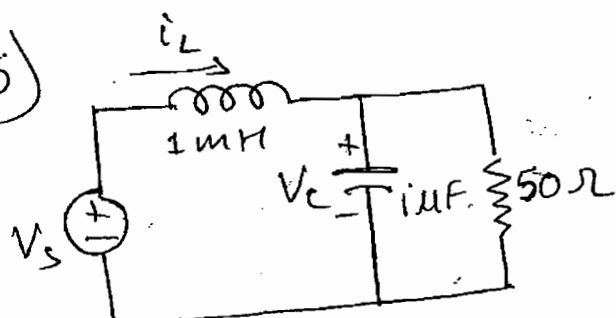
Solve

$$i_1(0^+) = 15 \text{ A}$$

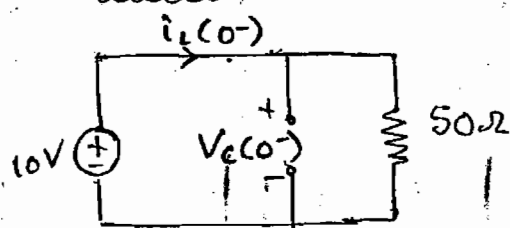
$$i_2(0^+) = 5 \text{ A}$$

$$i_3(0^+) = 0 \text{ A}$$

5)



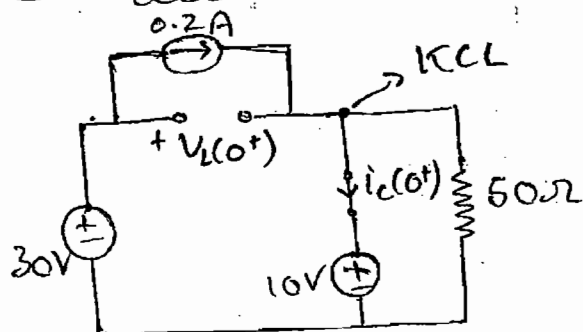
① At $t=0^-$



$$i(0^-) = \frac{10}{50} = 0.2 \text{ A}$$

$$V(0^-) = 10 \text{ V}$$

② At $t=0^+$



KVL

$$-30 + V_L(0^+) + 10 = 0$$

$$V_L(0^+) = 20$$

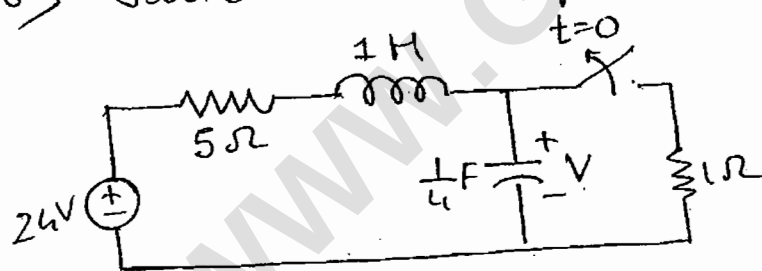
KCL

$$i_L(0^+) = 0 \text{ A}$$

$$\rightarrow \frac{dV_C(0^+)}{dt} = \frac{0}{C} = 0 \text{ V/sec}$$

$$\rightarrow \frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{20}{1\text{m}} = 20 \text{ kA/sec}$$

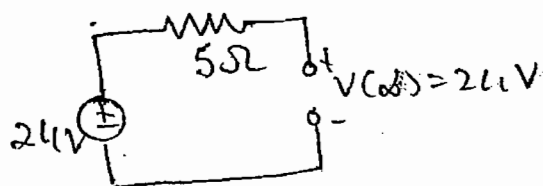
6) Find the complete expression for V



This is a Step resp., 2nd order, series RLC. (canonical form)

$$V(t) = V_{ss}(t) + V_{tr}(t)$$

$$\rightarrow V_{ss}(t) \text{ as } t \rightarrow \infty$$



But

$V_{tr}(t)$ depends upon values of R, L, C

$$\alpha = \frac{R}{2L} = \frac{5}{2 \times 1} = 2.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times \frac{1}{4}}} = 2$$

$\alpha > \omega_0 \Rightarrow$ over damped 'R'

$\rightarrow$ So complete solution is of the form

$$V(t) = 24 + A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

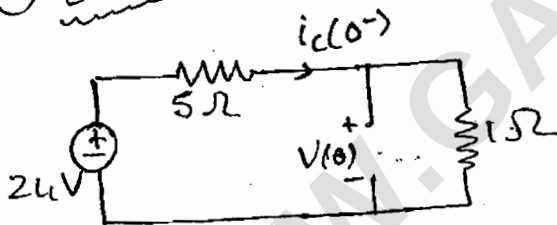
$$= -2.5 \pm \sqrt{(2.5)^2 - (2)^2}$$

$$= -1, -1$$

So,

$$V(t) = 24 + A_1 e^{-4t} + A_2 e^{-t} \quad \text{--- (A)}$$

① $t=0^-$



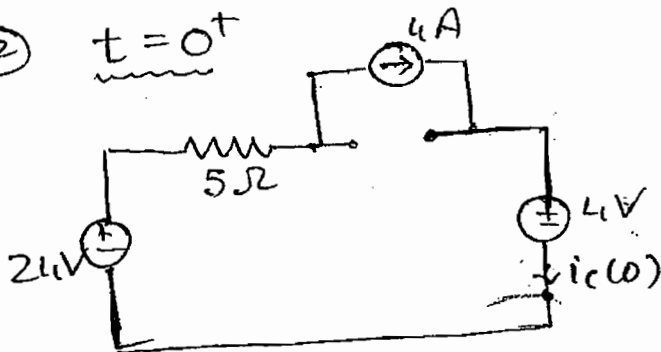
$$i(0^-) = \frac{24}{6} = 4A$$

$$V(0^-) = 4 \times 1 = 4V$$

Substituting in (A) ; $4 = 24 + A_1 + A_2$

$$\therefore A_1 + A_2 = -20 \quad \text{--- (1)}$$

② $t=0^+$



$$\frac{dV(0)}{dt} = \frac{i_c(0)}{C} = \frac{4}{1/4}$$

$$= 16 \text{ V/sec}$$

$$\frac{dV}{dt} = -4A_1 e^{-4t} - A_2 e^{-t}$$

At $t=0$

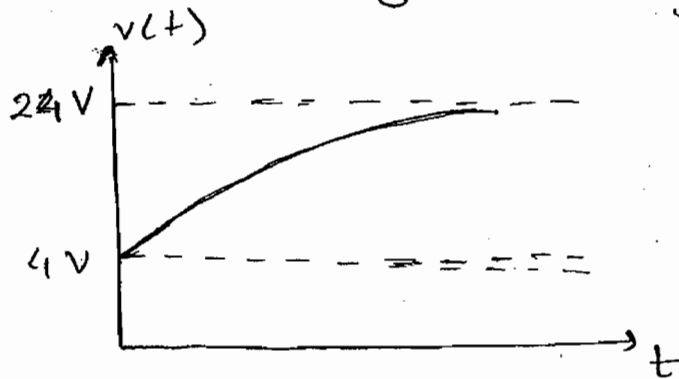
$$16 = -4A_1 - A_2 \quad \text{--- (2)}$$

Solving ① & ②

$$A_1 = \frac{4}{3} \quad A_2 = -\frac{64}{3}$$

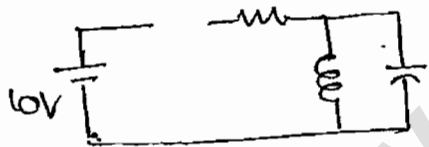
Then complete solⁿ / expression for $v(t)$ is,

$$v(t) = 24 + \frac{4}{3} e^{-4t} - \frac{64}{3} e^{-t}$$



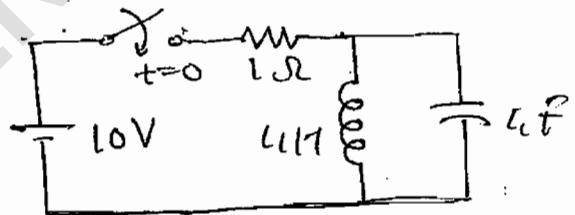
→ Find current through battery at $t=0^+$ and $t \rightarrow \infty$

① $t=0^-$



$$i_L(0^-) = 0V$$

$$V_C(0^-) = 0V$$



⇒ It is initially relaxed (i.e. zero state)

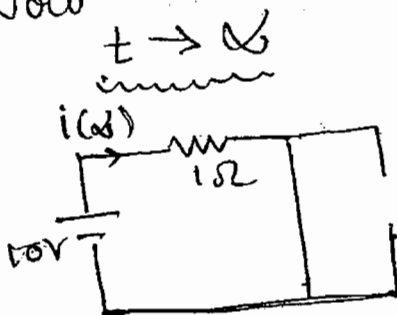
② $t=0^+$



$$\therefore i(0^+) = \frac{10}{1}$$

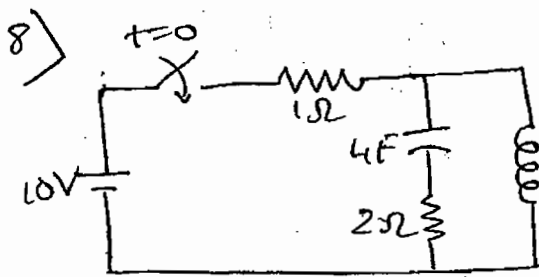
$$= 10A$$

Now



$$i(\infty) = \frac{10}{1}$$

$$= 10A$$



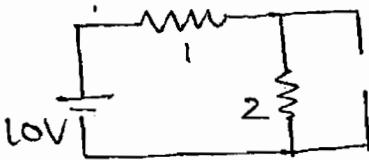
Find current through the battery at $t=0^+$ & $t \rightarrow \infty$

$t=0^-$ (initially relaxed)

$$V(0^-) = 0 \text{ V}$$

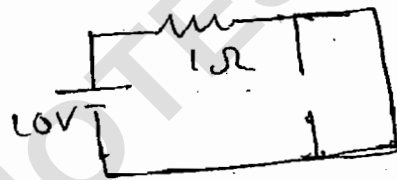
$$i(0^-) = 0 \text{ A}$$

① $t=0^+$



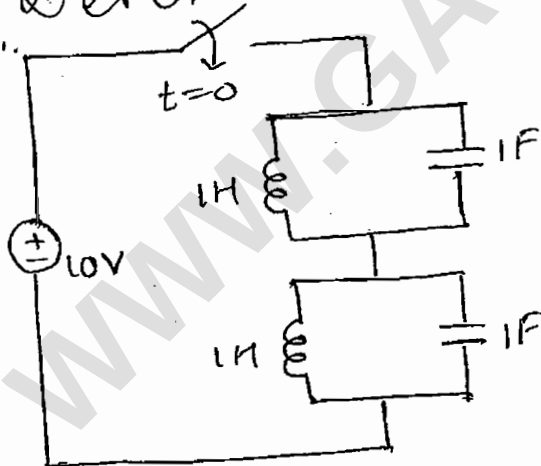
$$i(0^+) = \frac{10}{3} \text{ A}$$

② $t \rightarrow \infty$



$$i(\infty) = 10 \text{ A}$$

a) Deter



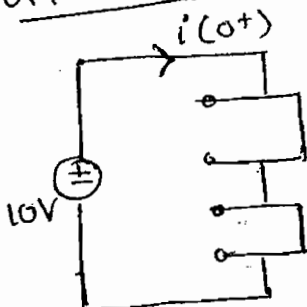
Find current through the battery at $t=0^+$ & $t \rightarrow \infty$

At $t=0^-$

$$i(0^-) = 0$$

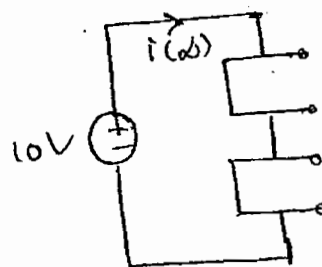
$$v(0^-) = 0$$

At $t=0^+$



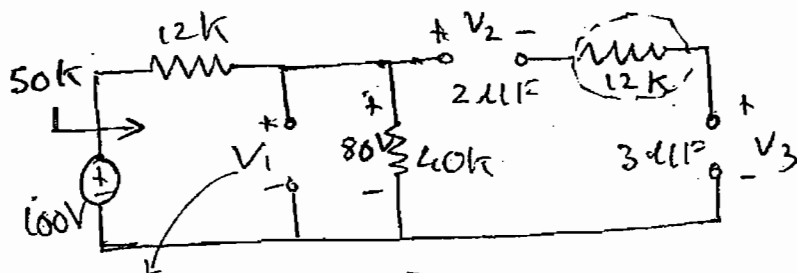
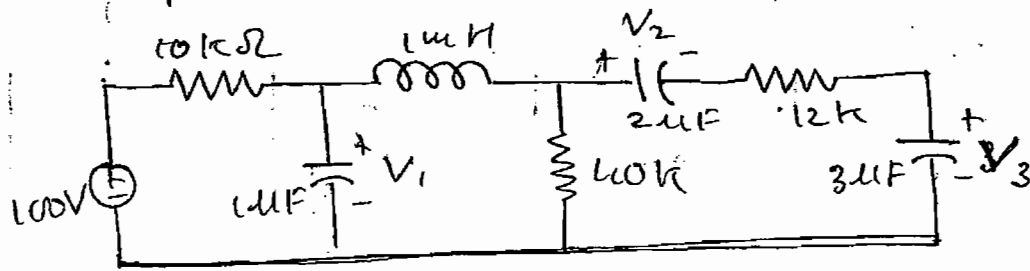
$i(0^+) \rightarrow$ very high practically

At $t = \infty$



$i(\infty) \rightarrow$ very high practically

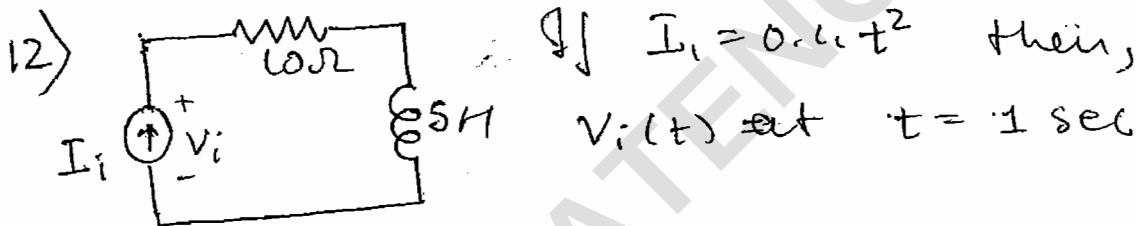
10) Determine steady state voltages across capacitor.



$$V_2 = 80 \left(\frac{3}{5} \right) = 48V$$

$$V_3 = 80 \left(\frac{2}{5} \right) = 32V$$

$$V_1 = 100 \left[\frac{40k}{80k} \right] = 8V$$



KVL $-V_1 + 10 I_i + 5 \frac{dI_i}{dt} = 0$

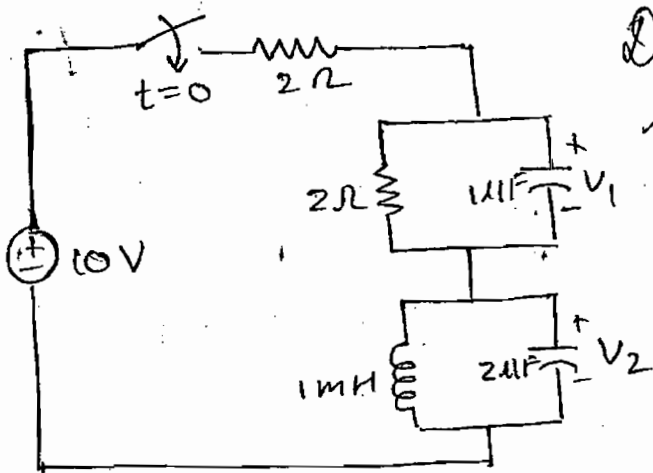
$$V_1 = 10(0.4t^2) + 5 \frac{d}{dt}(0.4t^2)$$

$$V_1(t) = 4t^2 + 4t$$

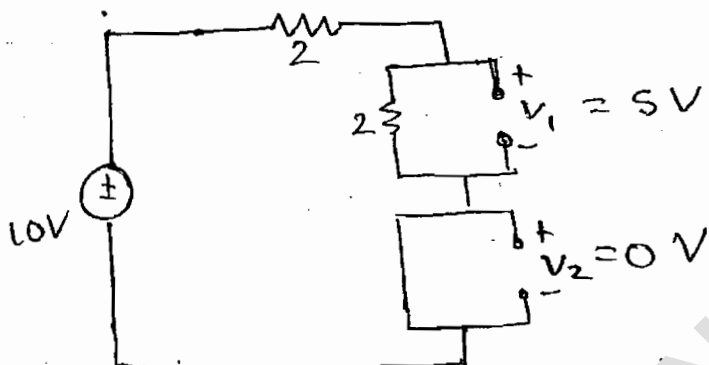
At $t = 1$

$$\therefore V_1 = 8 \text{ Volt}$$

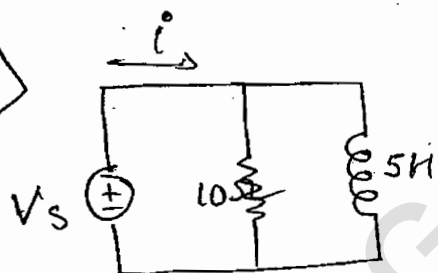
11)



Determine steady state voltages across capacitors.



13)



$$V_s = 40t, t \geq 0$$

$$i_L(0) = 5A \text{ then}$$

$$i(t) \text{ at } t = 2\text{sec} = ?$$

$$i = i_R + i_L$$

$$i = \frac{V}{R} + \frac{1}{L} \int_{-\infty}^t V dt$$

$$i = \frac{40t}{10} + \frac{1}{5} \left[\int_{-\infty}^0 V dt + \int_0^t V dt \right]$$

$$= 4t + \underbrace{i(0)}_{\text{initial current}} + \frac{1}{5} \int_0^t 40t dt$$

$$= 4t + 5 + \frac{8t^2}{2}$$

$$i(t) = 4t^2 + 4t + 5$$

At $t=2$

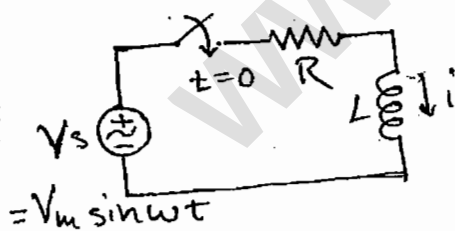
$$i = 4(2)^2 + 4(2) + 5 = 29A$$

VI] AC Transients

AC Transients are less effective than DC Transients because:

- ① Once the equipment in AC is designed for peak rated values, operating at any point other than peak value, the equipment or n/w is safe.
- ② Even the surges that occur can hardly travel half a cycle & get naturally suppressed.
- ③ Since there are natural zero vltg or current instances, we can avoid transients completely if we can operate the switch exactly at those instants of time when current or vltg is zero.

(a) R-L circuit



$$i(t) = i_{tr}(t) + i_{ss}(t)$$

KVL

$$-V_s + iR + L \frac{di}{dt} = 0$$

$$L \frac{di}{dt} + Ri = V_m \sin \omega t$$

→ The nature of solution for $i(t)$ is:

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t - \phi) + A e^{-t/\tau}$$

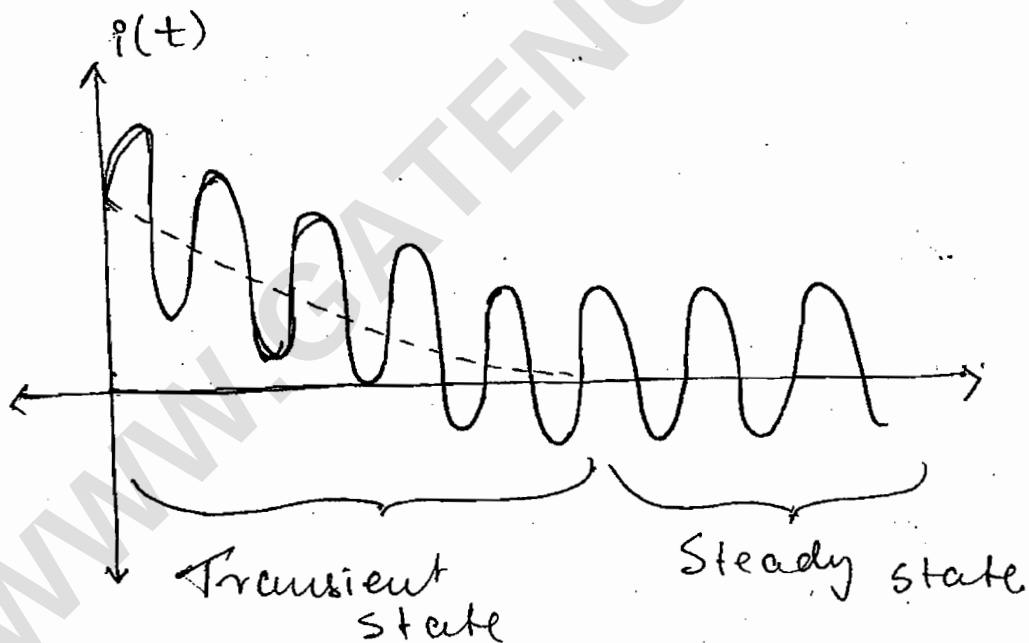
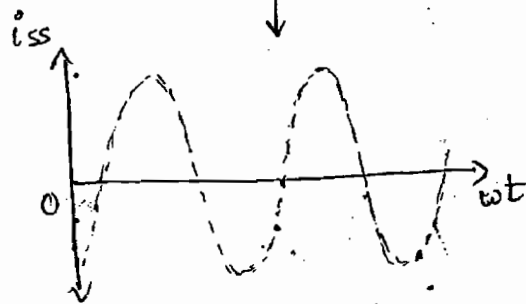
Here, $|Z| = \sqrt{R^2 + (\omega L)^2}$ $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

$$\tau = \frac{L}{R}$$

But at $t=0$, $i=0$

$$0 = \frac{V_m}{|Z|} \sin(-\phi) + A \Rightarrow A = \frac{V_m}{|Z|} \sin \phi$$

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t - \phi) + \frac{V_m}{|Z|} \sin \phi e^{-t/\tau}$$



NOTE:-

① Let $V_s = V_m \sin(\omega t + \theta)$

Then, the solution for $i(t)$ is like,

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi) + A e^{-t/\tau}$$

But at $t=0$, $i=0$

$$0 = \frac{V_m}{|Z|} \sin(\theta - \phi) + A$$

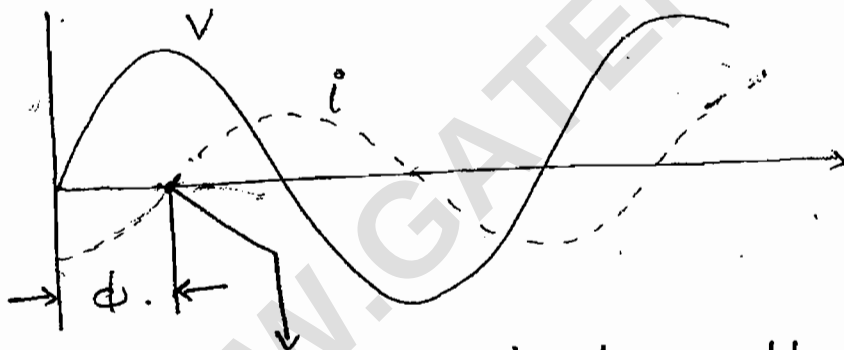
$$\therefore A = -\frac{V_m}{|Z|} \sin(\theta - \phi)$$

So, complete solⁿ is of the form

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta - \phi) - \frac{V_m}{|Z|} \sin(\theta - \phi) e^{-t/\tau}$$

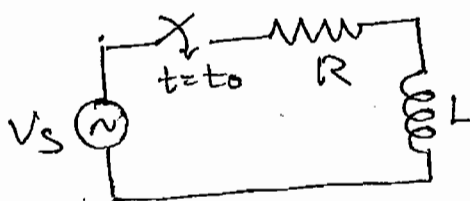
② If i/p excitation is in cosine terms, then express the o/p solⁿ for $i(t)$ also in cosine terms.

③ For general R-L load



This is the instant exactly when $i=0$
 so we can operate the sw exactly at
 this instant of time, we can avoid.

TRANSIENTS

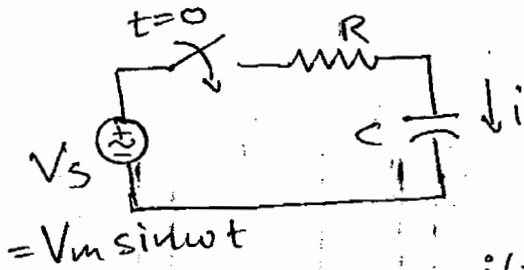


At $\omega t_0 = \phi$

$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

→ This is switch time,
 when we can avoid
 Transients.

(b) Series R-C ckt:-



$$i(t) = i_{ss}(t) + i_{tr}(t)$$

The natural solⁿ of $i(t)$ is

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \phi) + A e^{-t/\tau}$$

where, $|Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$ $\phi = \tan^{-1}\left(\frac{1}{\omega RC}\right)$

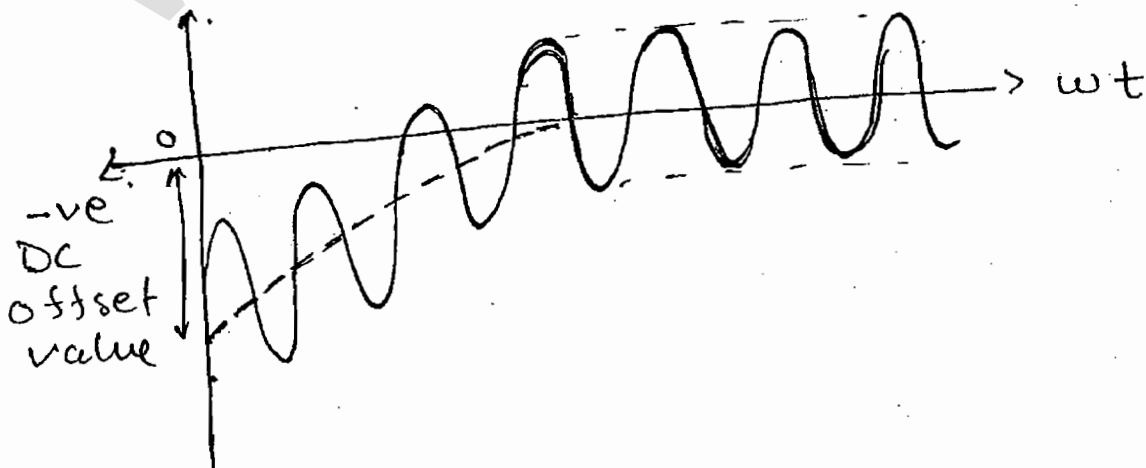
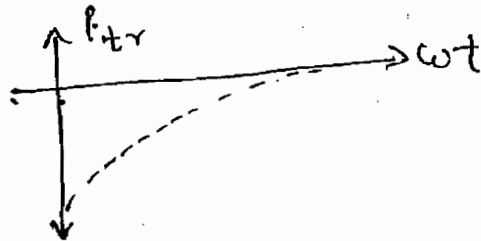
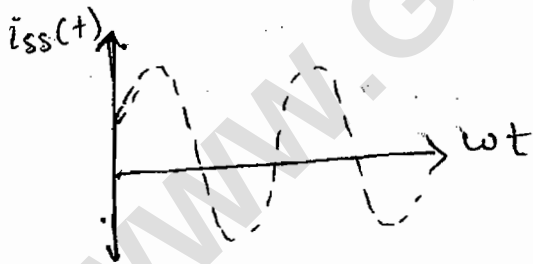
$$\tau = RC$$

At $t=0$, $i=0$

$$0 = \frac{V_m}{|Z|} \sin \phi + A \Rightarrow A = -\frac{V_m}{|Z|} \sin \phi$$

→ So complete solⁿ is:

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \phi) - \frac{V_m}{|Z|} \sin \phi e^{-t/\tau}$$



NOTE:-

① Here if $V_s = V_m \sin(\omega t + \theta)$

Then, $i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) + A e^{-t/\tau}$

at $t=0$, $i=0$;

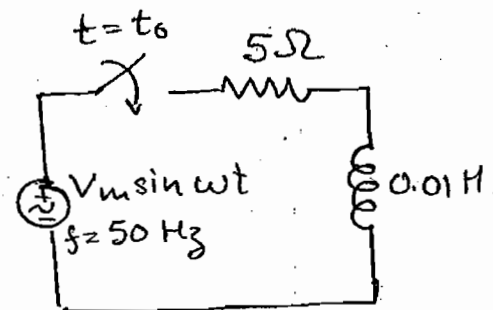
$$0 = \frac{V_m}{|Z|} \sin(\theta + \phi) + A \Rightarrow A = -\frac{V_m}{|Z|} \sin(\theta + \phi)$$

→ The complete solⁿ is:

$$i(t) = \frac{V_m}{|Z|} \sin(\omega t + \theta + \phi) - \frac{V_m}{|Z|} \sin(\theta + \phi) e^{-t/\tau}$$

② If i/p excitation is in cosine terms then express the o/p solⁿ for $i(t)$ also in cosine terms

1) At what switching instant time ' t_0 ', the current in the circuit has transient free resp.



$$\omega t_0 = \phi$$

$$\omega t_0 = \tan^{-1} \left(\frac{\omega L}{R} \right)$$

$$2\pi(50)t_0 = \underbrace{\tan^{-1} \left[\frac{2\pi(50)(0.01)}{5} \right]}_{\text{radians}}$$

$$100\pi t_0 = 0.56$$

$$\therefore t_0 = 1.78 \text{ msec}$$

So, here if we can operate the switch exactly at 1.78 ms from the instant where vltg became zero, it can completely avoid transients.

2) In the above problem if $V_s = V_m \sin(\omega t - 10^\circ)$ then determine the value of t_0 which results into transient free resp.

$$\text{So, } \omega t_0 \mp 10^\circ = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$2\pi(50)t_0 \mp 10 \times \frac{\pi}{180} = \tan^{-1}\left[\frac{2\pi(50)(0.01)}{5}\right]$$

$$\therefore 100\pi t_0 = 0.73$$

$$\therefore t_0 = 2.33 \text{ msec.}$$

VII] Laplace Transform & its

Application to ckt Analysis:—

When to use L.T. methods in ckt analysis?

- 1) If determination of τ is difficult
- 2) If order of ckt ≥ 2
- 3) Non-canonical form of ckt
- 4) Non-standard excitation
(i.e. impulse, pulse, ramp, parabolic, step, exponential, etc)

$$\mathcal{L}[f(t)] \longleftrightarrow F(s)$$

$$s = j\omega$$

$\hookrightarrow$ complex freq

$$v(t) \longleftrightarrow V(s)$$

$$i(t) \longleftrightarrow I(s)$$

$$V \longleftrightarrow \frac{V}{s}$$

$$I \longleftrightarrow \frac{I}{s}$$

$$V_m \sin \omega t \longleftrightarrow \frac{V_m \omega}{s^2 + \omega^2}$$

$$V_m \cos \omega t \longleftrightarrow \frac{I_m s}{s^2 + \omega^2}$$

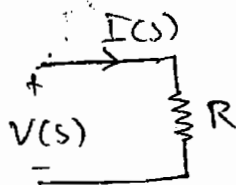
$$R \longleftrightarrow R$$

$$L \longleftrightarrow sL$$

$$C \longleftrightarrow \frac{1}{Cs}$$

Equivalent ckt representation of passive elements in Laplace domain:-

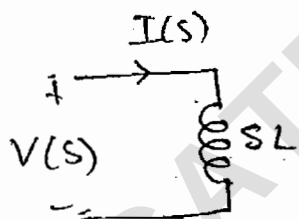
(a) Resistor



$$V(s) = I(s) \cdot R$$

$$I(s) = \frac{V(s)}{R}$$

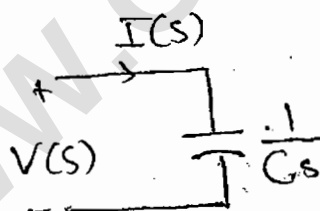
(b) Inductor



$$V(s) = I(s) \cdot sL$$

$$I(s) = \frac{V(s)}{sL}$$

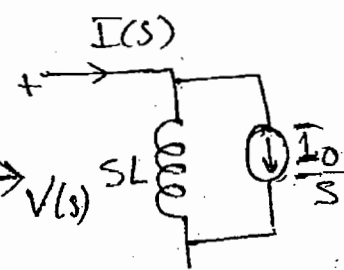
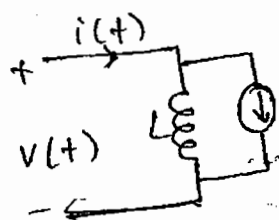
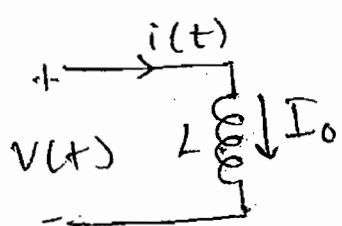
(c) Capacitor



$$V(s) = \frac{I(s)}{Cs}$$

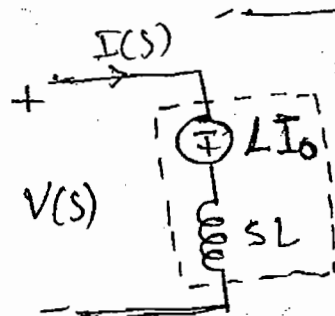
$$I(s) = V(s) \cdot Cs$$

(d) Inductor with I_0

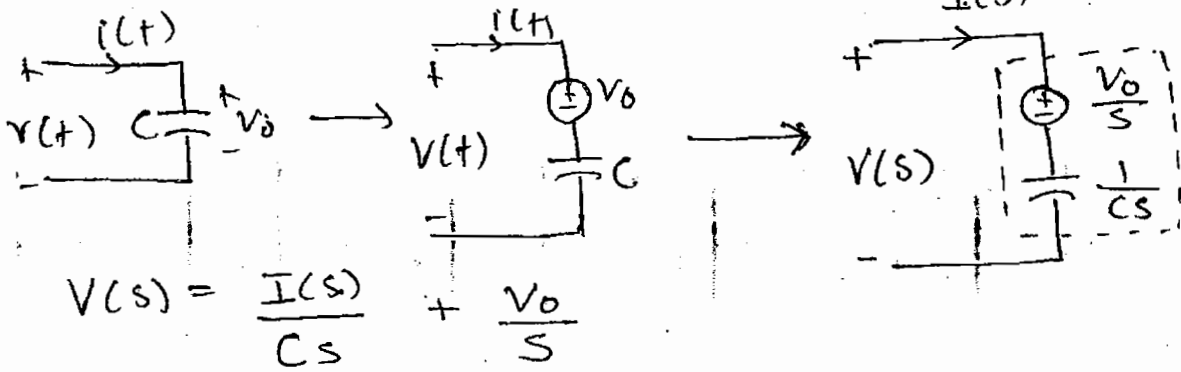


$$V(s) = sLI(s) - LI_0$$

$$I(s) = \frac{V(s)}{sL} + \frac{I_0}{s}$$



(c) Capacitor with ' V_0 '



$$I(s) = Cs V(s) - CV_0$$

1) If $I_0 = 10 \text{ A}$, find the complete expression of I

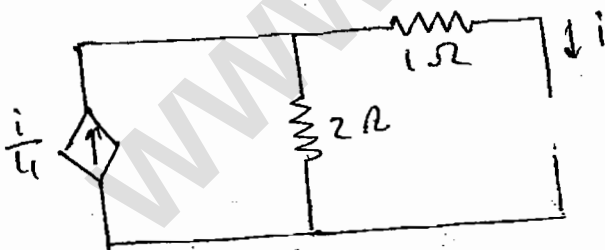
(a) in Time domain

(b) in Laplace domain

(a) $i(t) = I_0 e^{-t/\tau}$

$I_0 = 10 \text{ A}$ given.

$$\tau = \frac{L}{R_{eq}}$$



KVL

$$\frac{3}{2} + 1 - V_T = 0$$

$$V_T = \frac{5}{2}$$

$$R_{eq} = \frac{V_T}{1} = \frac{5}{2} \Omega$$

$$\tau = \frac{1/2}{5/2} = \frac{1}{5} \text{ sec}$$

So, $i(t) = 10 e^{-5t}$

(b) KVL

$$\frac{3}{2} I(s) + I(s) - 5 + I(s) \cdot \frac{5}{2} = 0$$

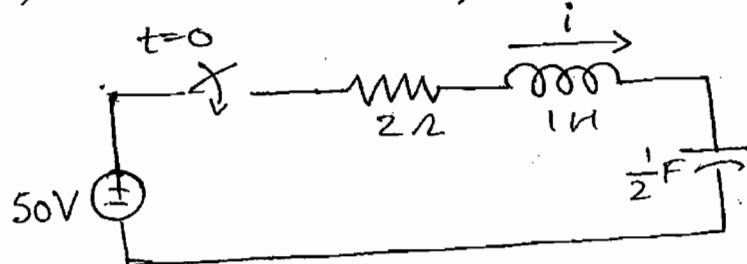
$$I(s) \left[\frac{s}{2} + \frac{s}{2} \right] = 5$$

$$I(s) = \frac{10}{s+5}$$

$$i(t) = \mathcal{L}^{-1} \left[\frac{10}{s+5} \right] = 10 e^{-5t}$$

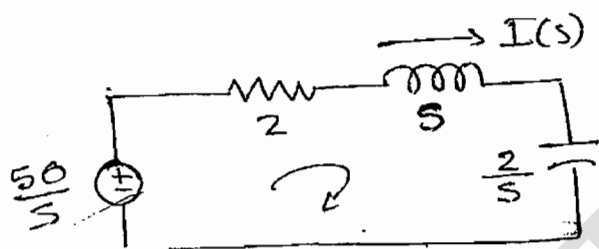
2) Find $i(t)$, $t > 0$

→ This circuit is initially relaxed.



KVL

$$-\frac{50}{s} + I(s) \left[2 + s + \frac{2}{s} \right] = 0$$



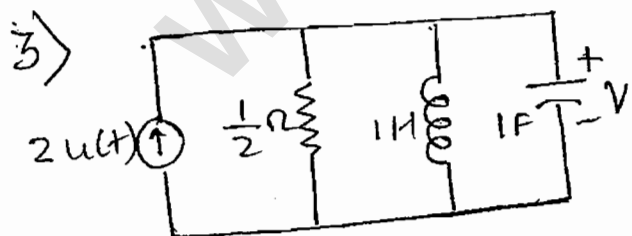
$$I(s) \left[\frac{s^2 + 2s + 2}{s} \right] = \frac{50}{s}$$

$$I(s) = \frac{50}{s^2 + 2s + 2}$$

$$I(s) = \frac{50}{(s+1)^2 + 1^2}$$

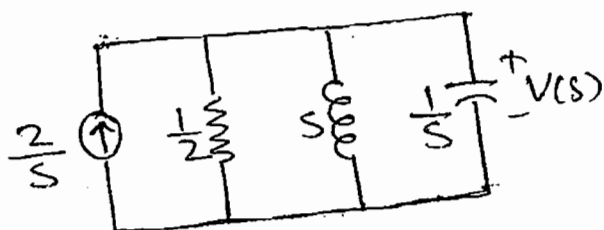
$$i(t) = \mathcal{L}^{-1} [I(s)] = 50 e^{-t} \sin t \text{ A}$$

→ This circuit is initially relaxed.



Find $v(t)$, $t > 0$

→ This is initially relaxed circuit



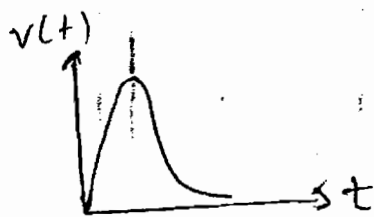
KCL

$$-\frac{2}{s} + \frac{V(s)}{1/2} + \frac{V(s)}{s} + \frac{V(s)}{1/s} = 0$$

$$V(s) \left[2 + \frac{1}{s} + s \right] = \frac{2}{s}$$

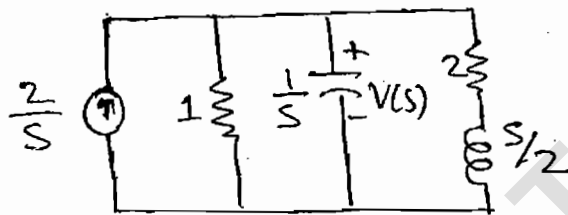
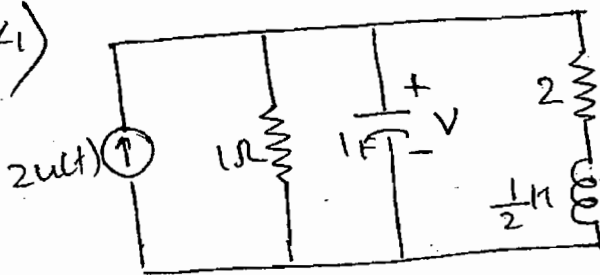
$$V(s) = \left[\frac{s^2 + 2s + 1}{s} \right] = \frac{2}{s}$$

$$V(s) = \frac{2}{(s+1)^2} \Rightarrow v(t) = \underline{2te^{-t}}$$



→ critically damped

4)



KCL

$$-\frac{2}{s} + \frac{V(s)}{1} + \frac{V(s)}{1/s} + \frac{V(s)}{(s+4)/2} = 0$$

$$\therefore V(s) \left[1 + s + \frac{s}{s+4} \right] = \frac{2}{s}$$

$$V(s) = \frac{2}{s} \frac{(s+4)}{s^2 + 5s + 6}$$

$$V(s) = \frac{2(s+4)}{s(s+2)(s+3)}$$

$$= \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$\text{where, } A = \frac{2(0+4)}{(0+2)(0+3)} = \frac{8}{6} = \frac{4}{3}$$

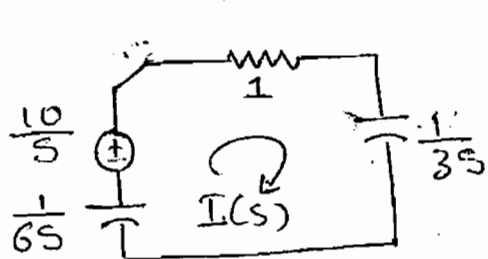
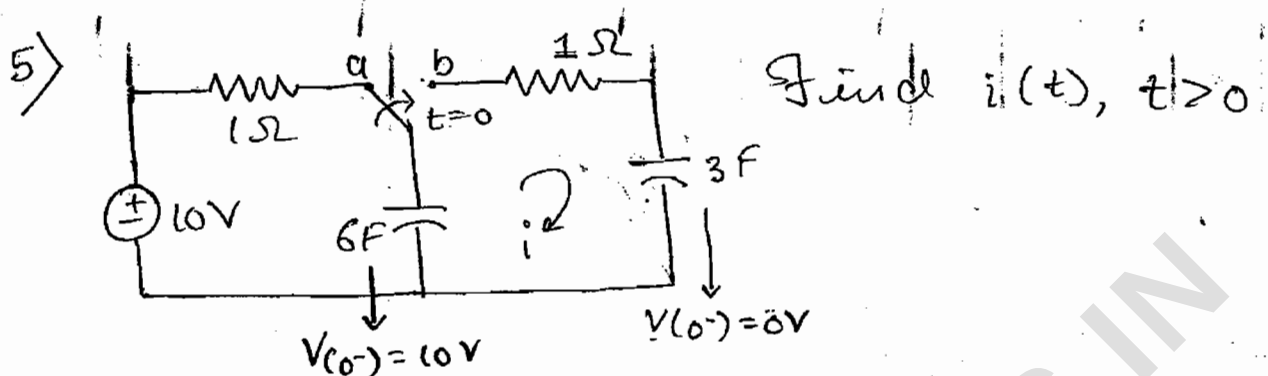
$$B = \frac{2(-2+4)}{(-2)(-2+3)} = \frac{4}{-2} = -2$$

$$C = \frac{2(-3+4)}{(-3)(-3+2)} = \frac{+2}{3} = \frac{2}{3}$$

$$\therefore V(s) = \frac{4/3}{s} - \frac{2}{s+2} + \frac{2/3}{s+3}$$

$$\therefore v(t) = \frac{4}{3} u(t) - 2e^{-2t} + \frac{2}{3} e^{-3t}$$

⇒ over damped step resp.



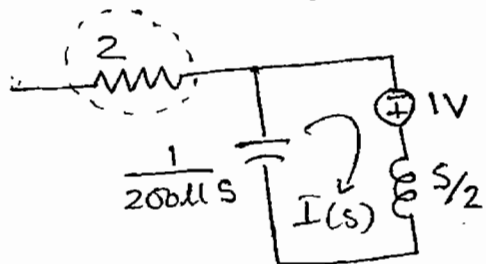
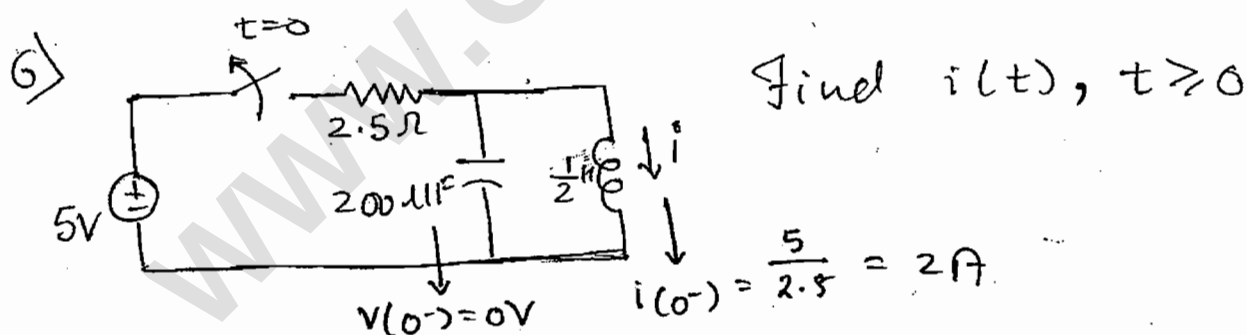
KVL

$$-\frac{10}{s} + I(s) \left[1 + \frac{1}{6s} + \frac{1}{3s} \right] = 0$$

$$I(s) \left[\frac{6s+3}{6s} \right] = \frac{10}{s}$$

$$\therefore I(s) = \frac{60}{6s+3} = \frac{20}{2s+1} = \frac{10}{s+1/2}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \underline{10 e^{-t/2} \text{ A}}$$



KVL

$$I(s) \left[\frac{s}{2} + \frac{1}{200\mu s} \right] = 1$$

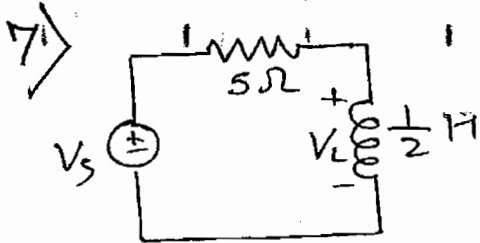
$$I(s) \left[s + \frac{10^6}{100s} \right] = 2$$

$$I(s) \left[s + \frac{10^4}{s} \right] = 2 \Rightarrow I(s) \left[\frac{s^2 + 10^4}{s} \right] = 2$$

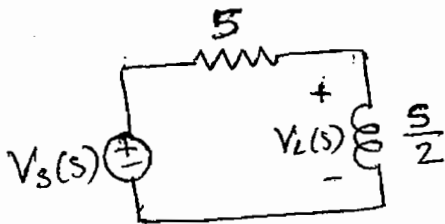
$$I(s) = \frac{2 \cdot s}{s^2 + (100)^2}$$

$$i(t) = 2 \cos 100t$$

sinusoidal o/p.



Find V_L if (a) $V_s(t) = \delta(t)$
(b) $V_s(t) = e^{-t} u(t)$



$$V_L(s) = V_s(s) \left[\frac{s/2}{s/2 + 5} \right]$$

$$V_L(s) = V_s(s) \left[\frac{s}{s+10} \right]$$

$$(a) \quad V_s(t) = \delta(t)$$

$$V_s(s) = 1$$

$$\text{So, } V_L(s) = 1 \times \frac{s}{s+10} = \frac{s+10-10}{s+10} = 1 - \frac{10}{s+10}$$

$$\therefore V_L(t) = \delta(t) - 10e^{-10t} \quad \underline{V}$$

$$(b) \quad V_s(t) = e^{-t} u(t)$$

$$V_s(s) = \frac{1}{s+1}$$

$$\text{So, } V_L(s) = \frac{1}{(s+1)} \cdot \frac{s}{(s+10)} = \frac{A}{s+1} + \frac{B}{s+10}$$

$$\text{where, } A = \frac{-1}{-1+10} = \frac{-1}{9}$$

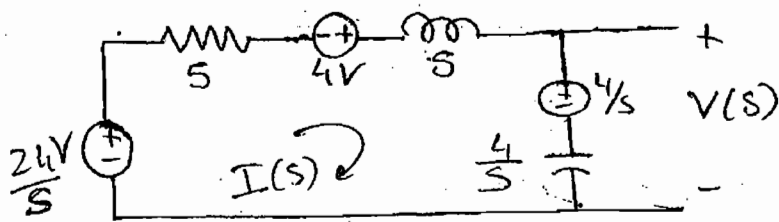
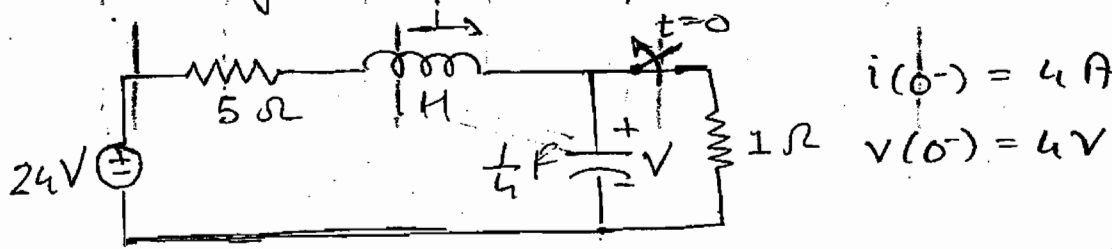
$$B = \frac{-10}{-10+1} = \frac{10}{9}$$

So,

$$V_L(s) = \frac{10}{9(s+10)} - \frac{1}{9(s+1)}$$

$$V_1(t) = \frac{10}{9} e^{-10t} - \frac{1}{9} e^{-t}$$

8) Find the complete expression for 'V' using L-T method.



$$-\frac{24}{s} - 4 + \frac{4}{s} + I(s) \left[5 + s + \frac{4}{s} \right] = 0$$

$$I(s) \left[\frac{s^2 + 5s + 4}{s} \right] = \frac{4s + 20}{s}$$

$$I(s) = \frac{4s + 20}{s^2 + 5s + 4}$$

$$V(s) = \frac{4}{s} + I(s) \times \frac{4}{s}$$

$$= \frac{4}{s} + \frac{4}{s} \frac{(4s + 20)}{(s^2 + 5s + 4)}$$

$$= \frac{4(s^2 + 5s + 4) + 16s + 80}{s(s+1)(s+4)}$$

$$= \frac{4s^2 + 36s + 96}{s(s+1)(s+4)}$$

$$V(s) = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

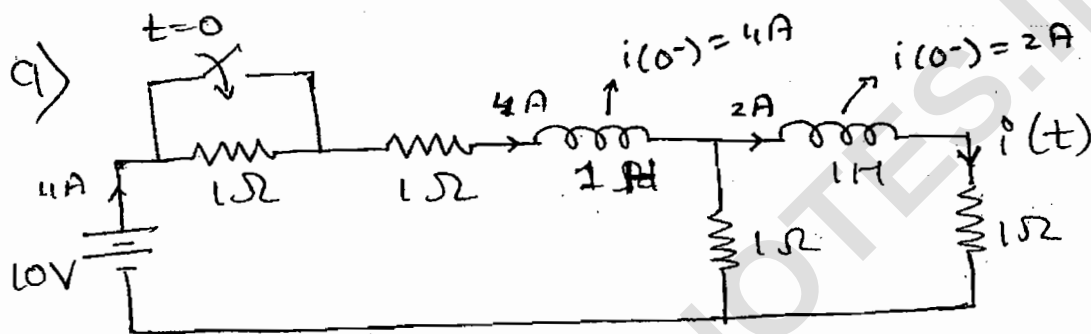
when, $A = \frac{96}{(1)(4)} = 24$

$$B = \frac{64 - 144 + 96}{-12} = \frac{4}{3}$$

$$C = \frac{4 - 36 + 96}{-3} = -\frac{64}{3}$$

$$V(s) = \frac{24}{s} + \frac{4}{3(s+4)} + \frac{-64}{3(s+1)}$$

$$V(t) = 24 + \frac{4}{3}e^{-4t} - \frac{64}{3}e^{-t}$$

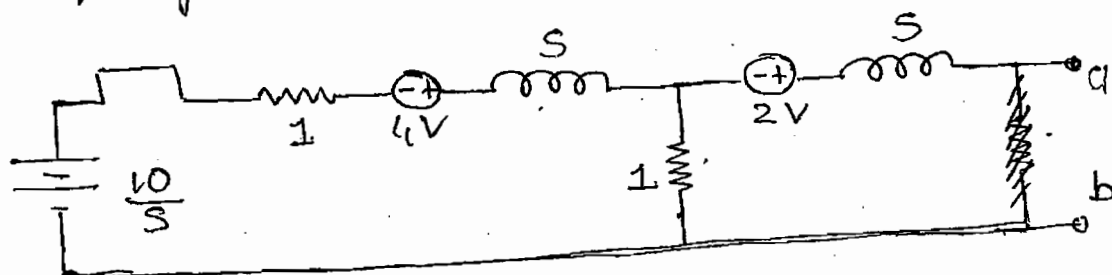


Apply Thevenin's theo. to obtain the expression of current $i(t)$ in 1Ω resis.

→ Thevenin's theo. can be applied to simplify the circuit in steady state.

But, this is a transient problem.

So, we can convert this problem in steady state freq. domain & then apply Thevenin's theo.



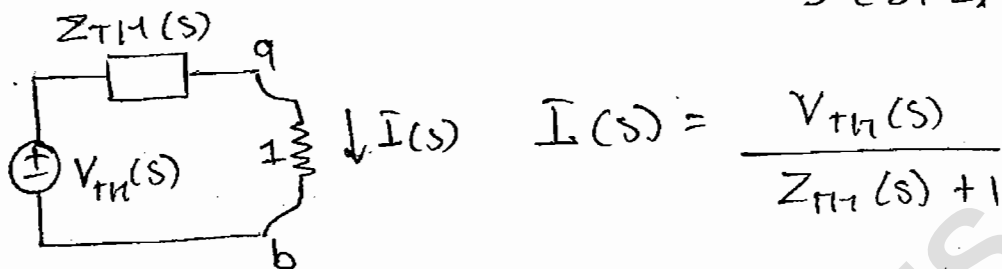
$$Z_{TH} = s + [1 \parallel (s+1)]$$

$$= s + \frac{s+1}{s+2} = \frac{s^2 + 3s + 1}{s+2}$$

$$V_{TH} = 2 + \left(\frac{10}{s} + 4 \right) \left[\frac{1}{s+2} \right]$$

$$= 2 + \left(\frac{10+4s}{s} \right) \left(\frac{1}{s+2} \right)$$

$$= 2 + \frac{4s+10}{s(s+2)} = \frac{2s^2+8s+10}{s(s+2)}$$



$$\therefore I(s) = \frac{(2s^2+8s+10)(s+2)}{(s+2)s[(s^2+3s+1)+s+2]}$$

$$= \frac{2s^3+8s^2+10s+4s^2+8s+20}{s(s^2+4s+3)}$$

$$= \frac{2s^3+8s^2+10s}{s(s+3)(s+1)}$$

$$I(s) = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+1}$$

where,

$$A = \frac{10}{3 \times 1} = \frac{10}{3}$$

$$B = \frac{18-24+10}{(-3)(-2)} = \frac{4}{6} = \frac{2}{3}$$

$$C = \frac{2-8+10}{(-1)(2)} = \frac{4}{-2} = -2$$

$$I(s) = \frac{10}{3} \cdot \frac{1}{s} + \frac{2}{3} \cdot \frac{1}{s+3} - 2 \cdot \frac{1}{s+1}$$

$$\therefore i(t) = \frac{10}{3} + \frac{2}{3} e^{-3t} - 2 e^{-t}$$

NETWORK FUNCTIONS

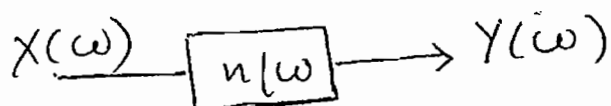
FILTER CONCEPTS

If the magnitude of i/p supply is kept const. but freq. is varied, then the o/p is defined as the complete freq. resp. of the ckt or n/w.

eg:- Resonance

Freq. resp. of any n/w gives its complete steady state performance which is useful in design & analysis. & synthesis of filters, antennas, SONARS, radars, etc in communication engg.

To obtain the complete freq. resp. of the n/w, we need to build the n/w's transfer function.



$$\text{N/w T.F.} = H(w) = \frac{Y(w)}{X(w)}$$

$$s = j\omega$$

↳ complex

In n/w's, there are only 4 types of T.F that can be defined.

(a) Voltage Gain T.F.

$$G(s) = \frac{V_o(s)}{V_i(s)}$$

(b) Current Gain T.F.

$$\alpha(s) = \frac{I_o(s)}{I_i(s)}$$

(c) Transfer Impedance

$$Z(s) = \frac{V_o(s)}{I_i(s)}$$

(d) Transfer Admittance

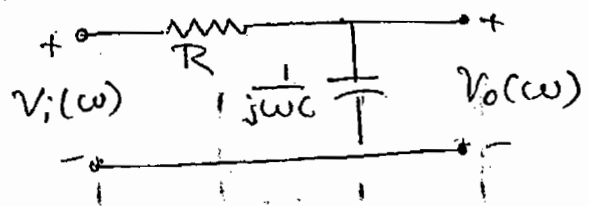
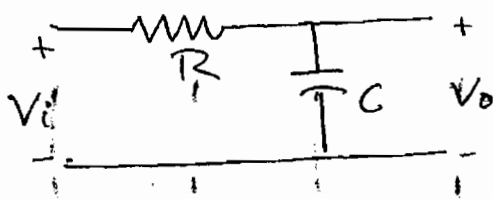
$$Y(s) = \frac{I_o(s)}{V_i(s)}$$

→ In general, impedance & admittance together are called as Immitance for then, in analysing filter ckts, we generally consider V.G.T.F : $H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$

→ The n/w func. $H(\omega)$ is complex quantity so, it has magnitude modulus $|H(\omega)|$ & phase of $\angle \phi(\omega)$

To obtain complete freq. resp. of n/w T.F., we need to plot both mag. & phase as ω is varied from 0 to ∞ .

1) Obtain the complete freq. resp. of the voltage gain T.F. ckt shown below.



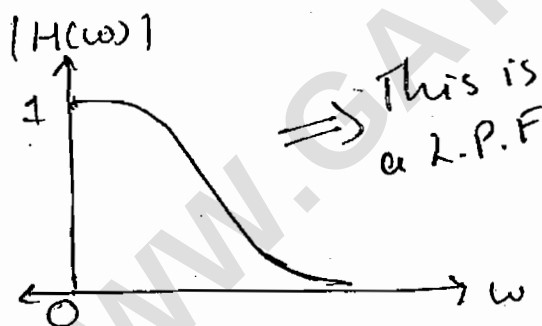
$$V_o(\omega) = V_i(\omega) \times \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}}$$

$$\therefore H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{1 + j\omega RC}$$

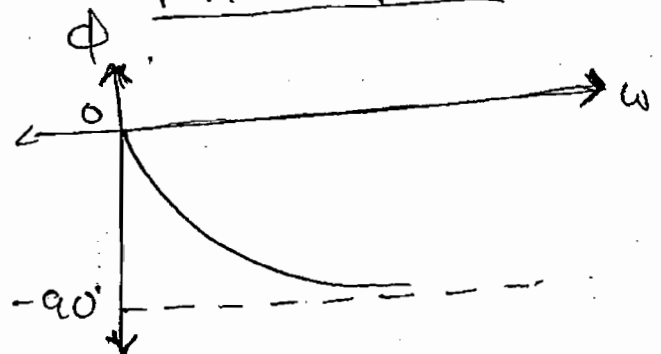
→ Magnitude → $|H(\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$

→ Phase → $\phi(\omega) = -\tan^{-1}(\omega RC)$

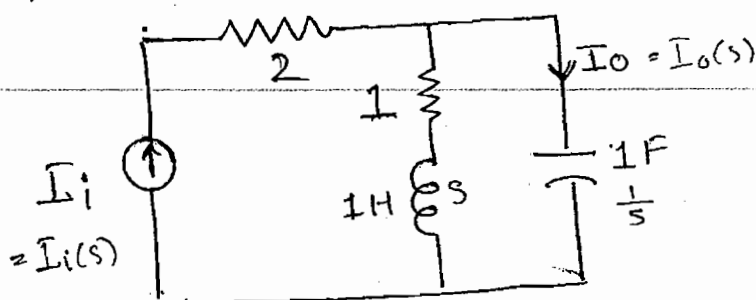
→ Magnitude plot



Phase plot

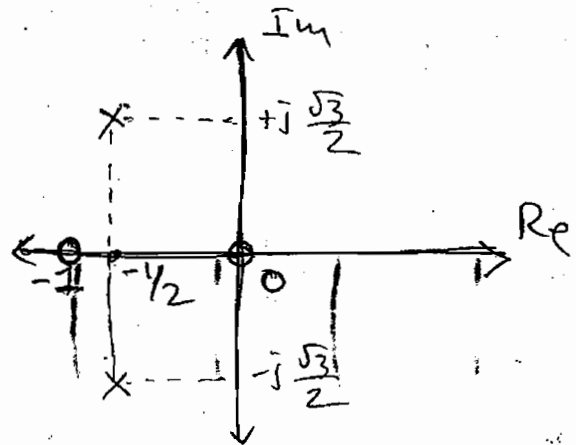


2) Obtain the current gain T.F. of the ckt shown and plot the location of poles & zero in s-plane.

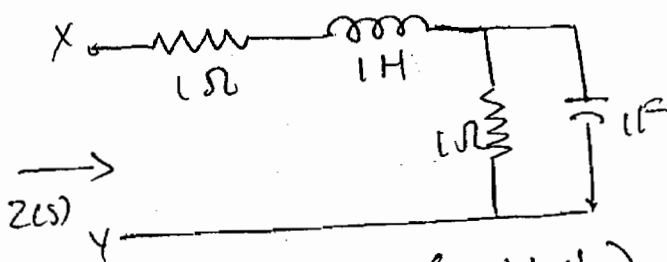


$$I_o(s) = I_i(s) \left[\frac{s+1}{s+1+\frac{1}{s}} \right]$$

$$X(s) = \frac{I_o(s)}{I_i(s)} = \frac{s(s+1)}{s^2+s+1}$$

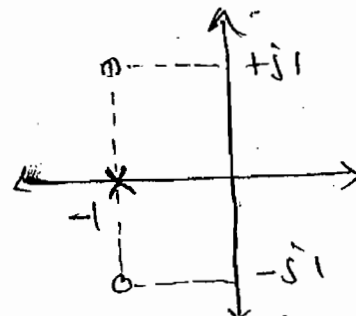


3) Obtain the driving pt. impedance of the n/w shown below.

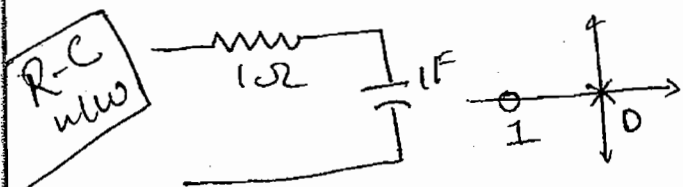


$$Z(s) = (1+s) \parallel (1 \parallel \frac{1}{s})$$

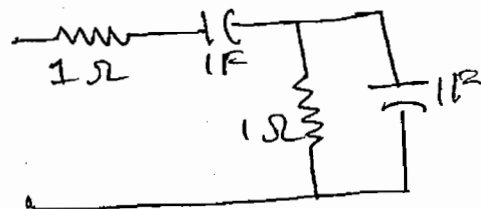
$$Z(s) = (1+s) + \left(1 \parallel \frac{1}{s}\right) = \frac{s^2 + 2s + 2}{s+1}$$



4) Obtain the driving pt. impedance of n/w shown below & plot the location of poles & zero in S-plane

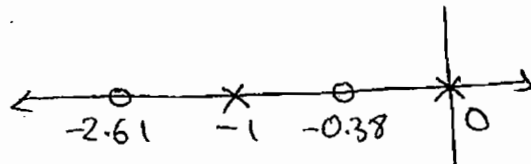


$$Z(s) = 1 + \frac{1}{s} = \frac{s+1}{s}$$

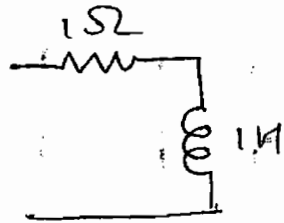


$$Z(s) = 1 + \frac{1}{s} + \frac{1/s}{s+1} = \frac{s^2 + s + s + 1 + s}{s(s+1)}$$

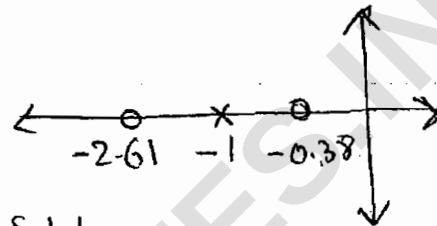
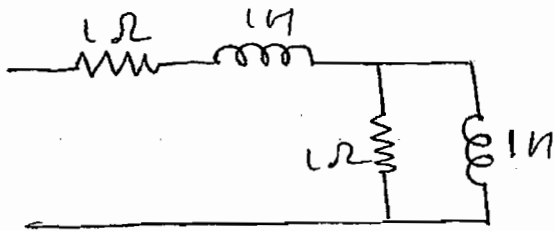
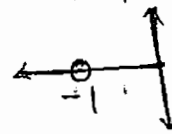
$$Z(s) = \frac{s^2 + 3s + 1}{s(s+1)}$$



R-L
n/w

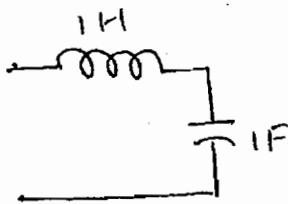


$$Z(s) = \frac{s+1}{1}$$



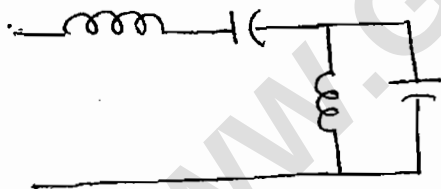
$$Z(s) = s+1 + \frac{s}{s+1} = \frac{s^2 + 3s + 1}{s+1}$$

L-C
n/w



$$Z(s) = s + \frac{1}{s}$$

$$= \frac{s^2 + 1}{s}$$



$$Z(s) = s + \frac{1}{s} + \frac{s \times \frac{1}{s}}{s + \frac{1}{s}}$$

$$= s + \frac{1}{s} + \frac{s}{s^2 + 1}$$

→ For any R-L-C n/w & its driving pt. impedances the poles and zeroes on left hand side of s-plane & scattered throughout the plane.

→ For any R-C n/w & its driving pt. impedance poles & zeros are located on the -ve real axis only. & are alternately placed.

And the nearest critical freq. to origin is a pole.

→ For any R-L n/w & its driving pt. impedance all the poles & zeros are located on the -ve real axis only & are alternately placed & the nearest critical freq. to origin is a zero.

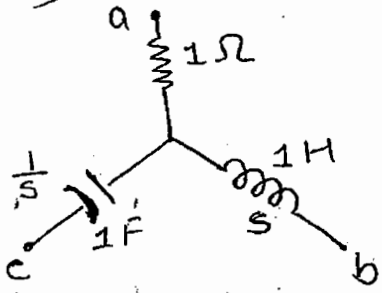
→ For any pure L-C n/w & its driving pt. impedance all poles & zeros are located on the imaginary axis only. & are alternately placed.

→ The pole-zero pattern of R-L driving pt. impedance is similar to R-C driving pt. admittance.

→ Ugly, the pole-zero pattern of R-C driving pt. impedance func. is similar to R-L driving pt. admittance func.

→ In general, these driving pt. impedance & admittance func. are together called as Imittance func.

1) Convert into Δ delta.

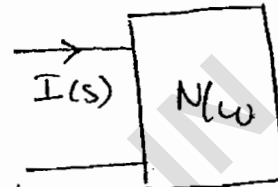


$$Z_{ab} = 1 + s + \frac{s}{1/s} = s^2 + s + 1 \Omega$$

$$Z_{ac} = 1 + \frac{1}{s} + \frac{1/s}{s} = \frac{s^2 + s + 1}{s^2} \Omega$$

$$Z_{bc} = s + \frac{1}{s} + \frac{1}{1} = \frac{s^2 + s + 1}{s} \Omega$$

2) If $I(s) = \frac{s+4}{(s+2)(s+3)}$



Find the initial value of current.

$I(s)$ From initial value theo. :

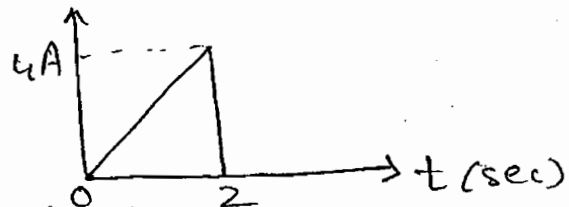
$$i(0) = \lim_{s \rightarrow \infty} \frac{s(s+4)}{(s+2)(s+3)} = \lim_{s \rightarrow \infty} \frac{1 + 4/s}{(1 + 2/s)(1 + 3/s)}$$

$$= \frac{1+0}{(1)(1)} = 1 \text{ A}$$

3) Find L.T. of

$$i(t) = 2t [u(t) - u(t-2)]$$

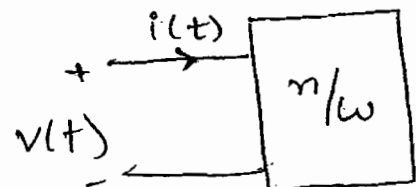
$$= 2tu(t) - 2(t-2+2)u(t-2)$$



$$\therefore I(s) = \frac{2}{s^2} - \frac{2 \cdot e^{-2s}}{s^2} - \frac{4e^{-2s}}{s}$$

4) If $i(t) = e^{-3t}$ A for $v(t) = u(t) \forall, t \geq 0$.

Determine the n/w elements



$$\frac{v(t)}{i(t)} = \frac{u(t)}{e^{-3t}}$$

$$\frac{V(s)}{I(s)} = \frac{1/s}{1/(s+3)} = \frac{s+3}{s} = 1 + \frac{3}{s}$$

$$Z(s) = 1 + \frac{1}{\frac{1}{3}s}$$

$$\Rightarrow \left. \begin{matrix} R = 1 \Omega \\ C = \frac{1}{3} F \end{matrix} \right\} \text{Series RC n/w}$$

Note :-

$$Z(s) = k_0 + k_1 s + \frac{k_2}{s}$$

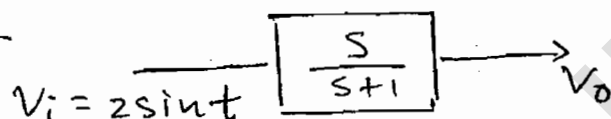
Here,

$k_0 \rightarrow$ resistance $\rightarrow$ value $\rightarrow k_0 \Omega$

$k_1 \rightarrow$ inductance $\rightarrow$ value $\rightarrow k_1 H$

$k_2 \rightarrow$ capacitance $\rightarrow$ value $\rightarrow \frac{1}{k_2} F$

5) Find $V_o =$ —



$$V_o = V_i \left(\frac{s}{s+1} \right)$$

But $s = j\omega$ & here $\omega = 1 \Rightarrow s = j$

$$\therefore V_o = V_i \left(\frac{j}{j+1} \right) = 2 \sin t \left[\frac{1 \angle 90^\circ}{\sqrt{2} \angle 45^\circ} \right]$$

$$\therefore V_o = \sqrt{2} \sin(t + 45^\circ) \underline{\underline{V}}$$

Passive Filters :-

Filters are ckt's which operate for a particular range of frequencies & attenuate other frequencies.

Passive filters are ckt's which are designed based on passive elements : R, L, C .

Active filters are ckt's at electronic / signal level based on OP Amp. & digital filters also perform signal processing & they are based on DSP.

Passive filters are still used at power level to control harmonics & stabilize the power fed to the load. eg: laptop charges,