5

260

(260)

-: HAND WRITTEN NOTES:-

OF

(1)

Electrical ENGG

-: SUBJECT:-

NETWORK THEORY

2

Network Theory

1. Basics
2. Steady State AC Circuits (Resonance).
3. Theorems (obj & conv).
4. Transients (")
5. Two port (")
6. Graph theory and magnetic circuits
7. Filters
8. Synthesis (obj)

Books

1. Fundamentals of Electric Circuits - (PDF)

   By Alexander Sadiku.

2. Engg Circuit Analysis

   By, Hayt kemmerly.

3. Networks and Systems

   By, Roy Choudhary.

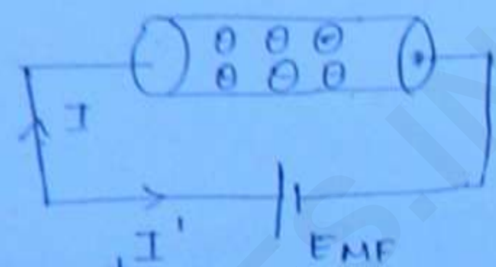
4. Network Analysis — Van Valkenburg.

e basic quantity in the circuit is charge, The charge of the e^- is given by -1.602×10^{-19} C. (4)

e flow of electrons is called as current (or)
The time rate of charge is also called as current.

$$I = \frac{dQ}{dt} \; C/s \text{ (or) } A$$

$I \rightarrow$ conventional current.



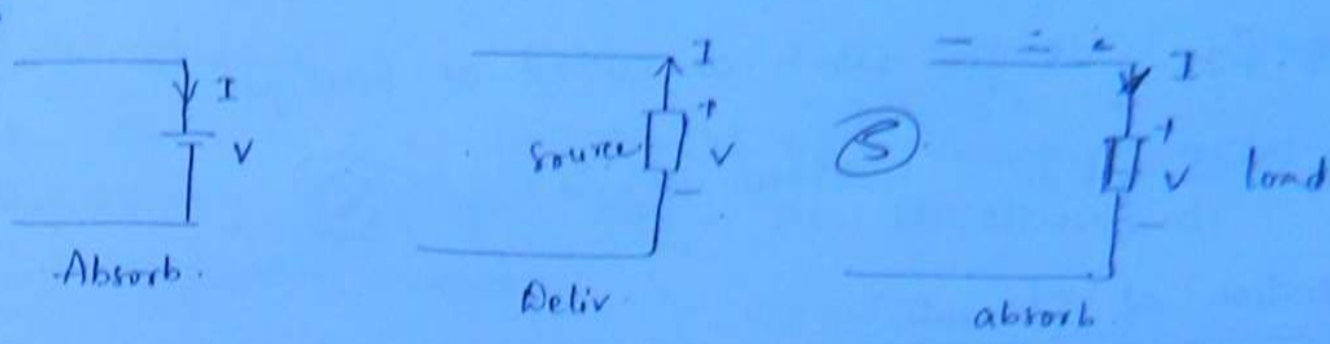
In the network theory, while developing KVL & KCL equation conventional current is used.
To move the e^- from one point to other point in particular direction, external force is required,
In the circuit, external force is provided by EMF and it is given by.

$$V = \frac{dW}{dQ} \; J/C \text{ (or) volts.}$$

The time rate of energy is called as power

$$P = \frac{dW}{dt} \; J/s \text{ (or) Watts.}$$

$$P = \frac{dW}{dQ} \frac{dQ}{dt} \Rightarrow \boxed{P = VI}$$



Note :

1. when current is entering at positive terminal element is absorbing power.
2. when current is leaving from the +ve terminal, element is delivering the power.

Q. Find power of each element, of the circuit shown.

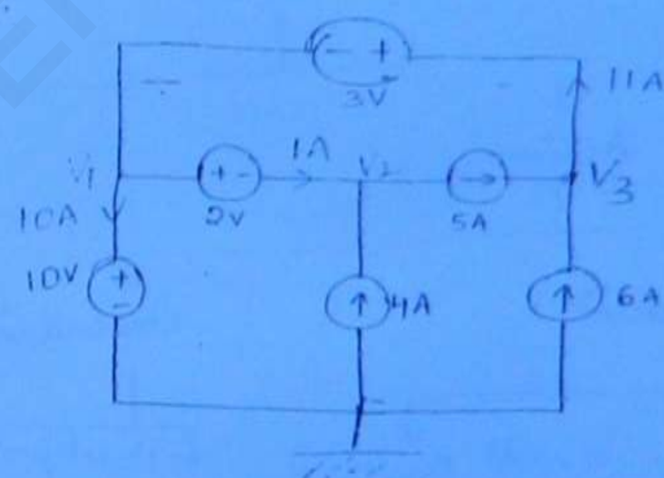
$V_1 - V_2 = 2V$

$V_2 = 10 - 2 = 8V$

$V_2 = 8V$

$V_3 - V_1 = 3$

$V_3 = 13V$



$P_4 = 8 \times 4 = 32W$ (Del)

$P_6 = 13 \times 6 = 78W$ (Del)

$P_5 = 5 \times 5 = 25W$ (Del)

$P_3 = 11 \times 3 = 33W$ (abs.)

$P_{10V} = 10 \times 10 = 100W$ (abs)

$P_2 = 2 \times 1 = 2W$ (abs)

$(P_T)_{Abs} = (P_T)_{Del} = 135W$

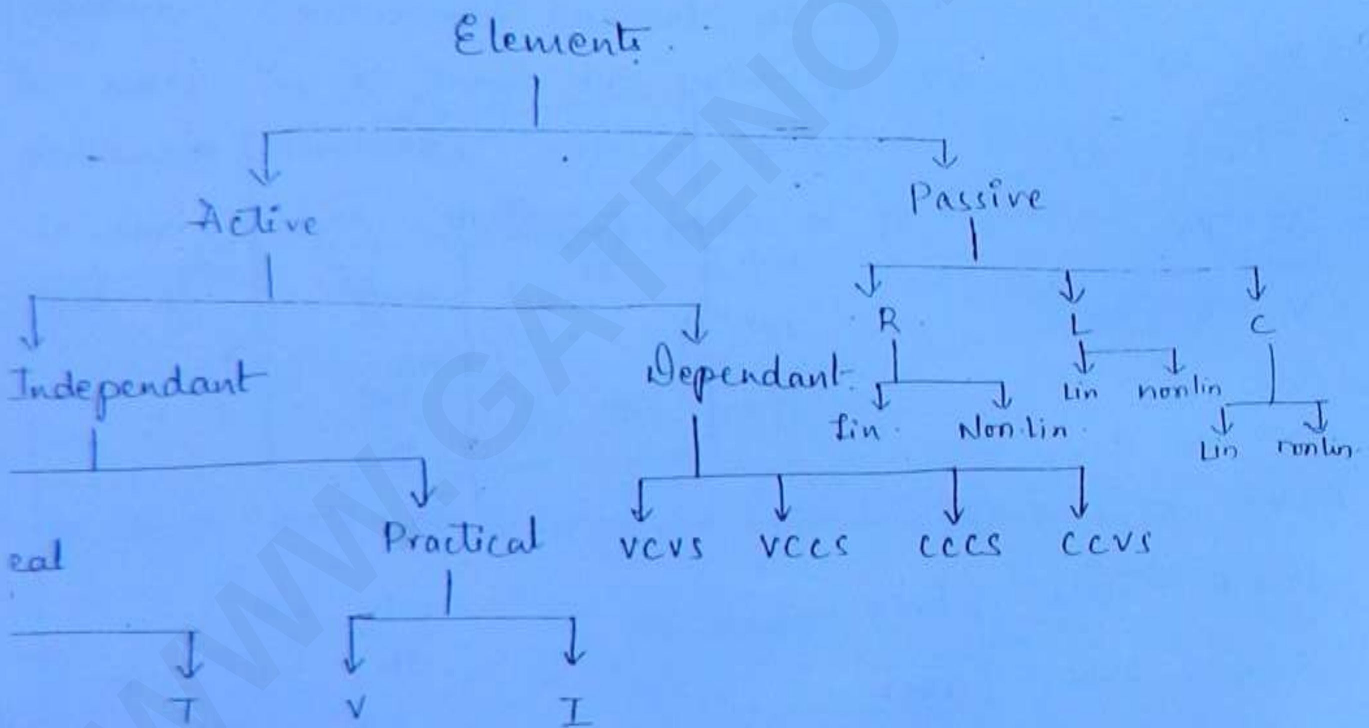
ie capacity to do the work is called as Energy.

$$W = \int_0^t P\,dt \quad \text{Watt-sec (or) J.}$$

6

Classification of Elements.

1. Active and Passive Elements.
2. Unidirectional & Bidirectional elements.
- linear and non linear elements.
- Time variant and invariant elements.
- lumped and distributed Elements.



Active Elements :- when element is capable of delivering Energy independantly for infinite time (or) when the element is having property of internal amplification then the element is called as active element.

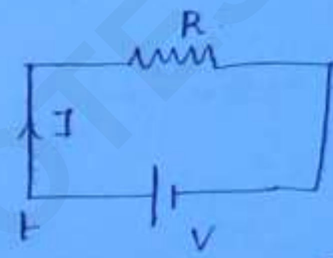
Ex V Independent ... Transistor ...

Passive Elements

When the element is not capable of delivering energy independantly for infinite time, then the element is called as passive element.

Ex. R, L, & C, bulb, transformer.

Resistance is a property of the resistor. It opposes flow of current. By doing so, it converts electrical energy to heat energy.



$P = I^2R$

$W = I^2Rt$
↓
(heat)

$$\boxed{R = \frac{\rho L}{a}} \quad \Omega \qquad \rho \rightarrow \Omega m$$

ρ → conductors
→ semi conductors
→ Insulators

When resistivity (ρ) of a conductor, $\rho = 0$, then it is called super conductor

Ex. At 4.15K Mercury acts as a superconductor.

$$\boxed{R_1 = R_0(1 + \alpha_0 t)}$$

Where,

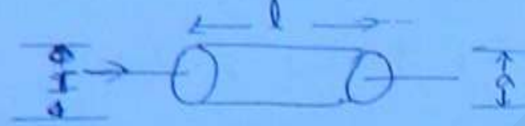
R_0 = resistance of the material at $0^\circ C$

α_0 : temperature coefficient per $^\circ C$.

t : change in temperature

Ohm's law.

→ Ohm's law states that, at constant temperature, current density is directly proportional to electric field intensity.



$$J \propto E$$

$$J = \sigma E$$

$$\frac{I}{a} = \frac{1}{\rho} \cdot \frac{V}{l}$$

$$\frac{V}{I} = \frac{\rho L}{A} = R.$$

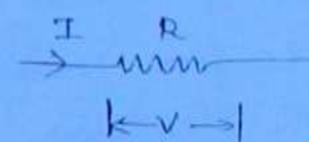
$$R = \frac{\rho L}{A}$$

$$J = \frac{I}{A}$$

$$E = \frac{V}{l}$$

$$\sigma = \frac{1}{\rho} \quad \text{mho/m}.$$

→ At constant temperature, potential difference across an element is directly proportional to the current flowing across the element



$$V \propto i$$

$$V = RI.$$

$$R = V/I = \text{constant}.$$

Ohm's law can be applied when temperature and conductivity of the material are constant.

Ohm's law

1st form ← $J = \sigma E$

2nd form ← $V = iR$

3rd form ← $i = GV$

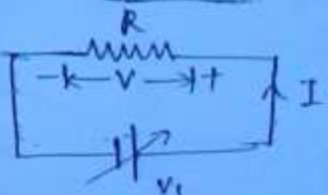
4th form ← $V = \frac{dQ}{dt} R$

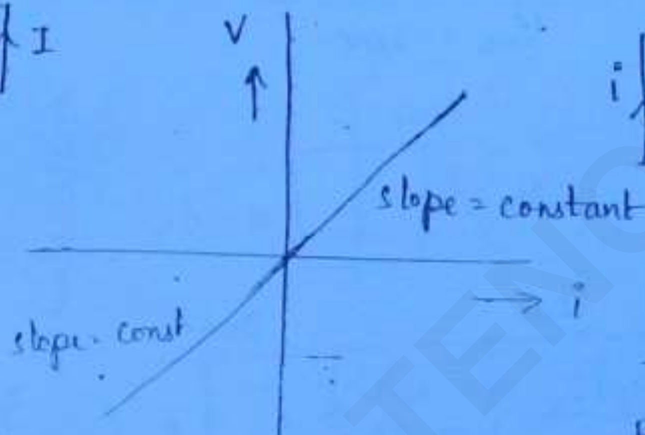
$$G = \frac{1}{R} \quad \text{mho (or) Si}$$

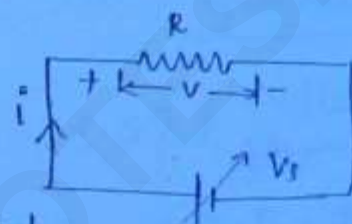
→ When element properties and characteristics are independant on the direction of the current, then the element is called as bi directional element.

→ When element obeys the ohms law, then the element is called as linear resistor.

Every linear element should obey the bi-directional property. But not vice versa.







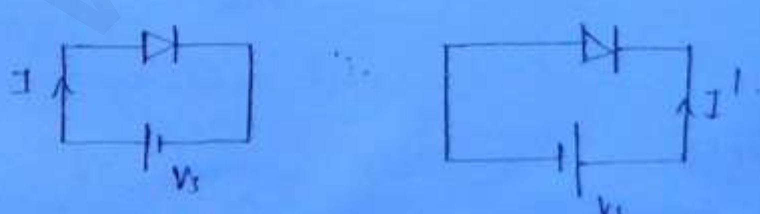
$i \uparrow 10\% \Rightarrow V \uparrow 10\%$

$i \uparrow 90\% \rightarrow V \uparrow 90\%$

$R = \frac{V}{I} = \text{constant}$

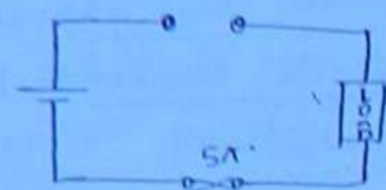
→ When element properties and characteristic depends on direction of the current, then the element is called as uni directional element.

→ When element doesnot obey the Ohms Law, then the element is called as non linear resistor.



$|I| \neq |I'|$

Open circuit O.C.

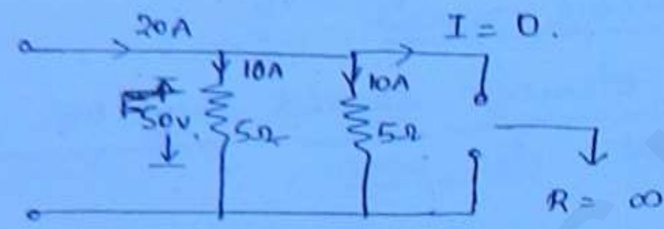


Properties of O.C. (Ideal).

$R = \infty$

$I = 0$

$V = \infty$ (max. possible).



$V_{oc} = 50V.$

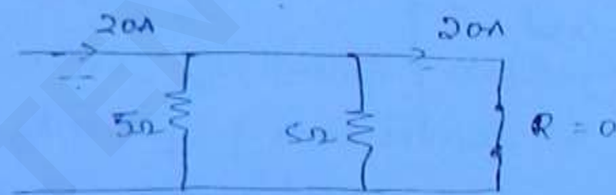
Short circuit. (S.C.)

Properties of S.C. (IDEAL)

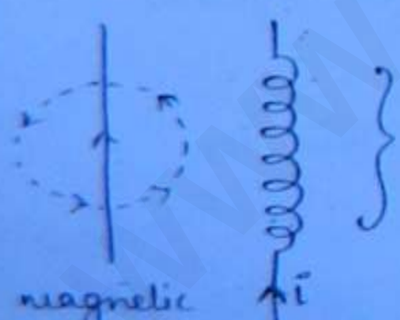
$R = 0$

$I = \infty$

$V = 0$.



Inductor



group of flux lines

flux direction — right hand thumb rule.

k magnetic field.

∴ strong magnetic field.

Faradys 1st law.

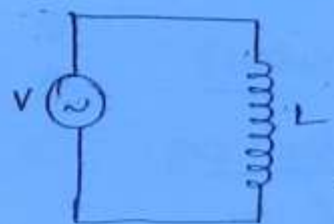
when conductor cuts a magnetic lines of the force, an emf is induced in the conductor.

The emf induced ...

1. emf is induced (e).
2. $e \propto \frac{d\phi}{dt}$

$$\boxed{e = -N \frac{d\phi}{dt}}$$

This is dynamically induced emf, Ex. - generator.



$e \propto \frac{d\phi}{dt}$ $\Rightarrow$ This is Statically induced emf.
Ex. Transformer (T/F)

$\boxed{e = -N \frac{d\phi}{dt}}$ → lenz law

where -ve sign indicates, based on lenz law, induced voltage opposes its cause of existance.

$\psi = N\phi$
$V = \frac{d\psi}{dt}$
$\phi \propto i$
$\psi \propto i$

$\psi = N\phi$
↳ flux linkage.

$V = \frac{d\psi}{dt}$

$\left.\begin{array}{l} \phi \propto i \\ \psi \propto \phi \end{array}\right\}$ $\psi \propto i \rightarrow \boxed{\psi = L i}$

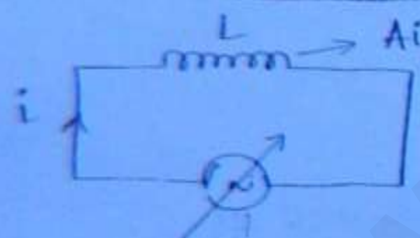
$$V = L\frac{di}{dt}$$

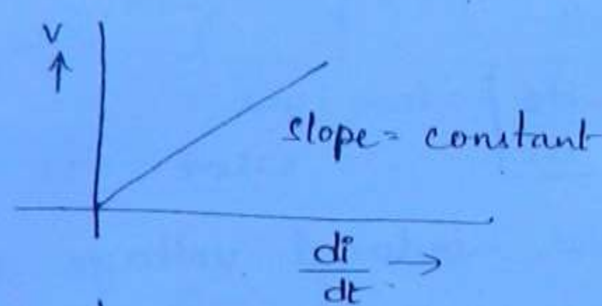
$$L = \frac{V}{(di/dt)}$$

$$\left.\begin{aligned}\psi &= N\phi \\ \psi &= Li\end{aligned}\right\}$$

$$\boxed{L = \frac{N\phi}{I}} \text{ H.}$$

→ when inductance of the inductor independant on current magnitude then the inductor is called as linear inductor. Ex: Air core inductor.

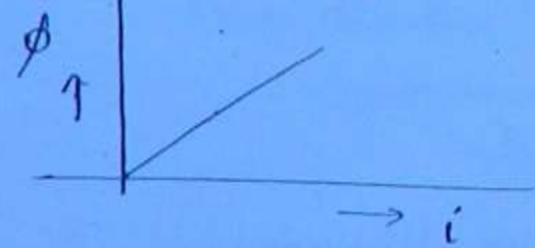




$$L = \frac{N\phi}{I} = \text{constant}$$

$i \uparrow 10\%$, $\phi \uparrow 10\%$

$i \uparrow 90\%$, $\phi \uparrow 90\%$



$$V = \frac{L\,di}{dt} \Rightarrow \therefore L = \frac{V}{(di/dt)}$$

when inductance of the inductor depends on current magnitude, Then the inductor is called as non linear inductor Ex: IRON CORE inductor

38. $V_{La} = \dfrac{e_1(t) + e_2(t)}{3}$ (77)

$= \frac{1}{3}\left(\sqrt{3}\cos(\omega t + 30^\circ) + \sqrt{3}\sin(\omega t + 60^\circ) \right)$

$= \frac{1}{3} \cdot \sqrt{3}\left[\cos(\omega t + 30^\circ) + \cos(\omega t + 60 - 90^\circ)\right]$

convert it to either cos or sine.

$= \frac{1}{\sqrt{3}}\left[\cos(\omega t + 30^\circ) + \cos(\omega t - 30^\circ)\right]$

$= \frac{1}{\sqrt{3}}\left[1\angle 30^\circ + 1\angle -30^\circ\right] = 1V \cdot \; 1\angle 0^\circ$

$= 1\cos(\omega t + 0^\circ) = \underline{\underline{\cos \omega t}}$.

43. $V = \sqrt{V_R^2 + V_L^2}$

$250 = \sqrt{V_R^2 + 150V} \Rightarrow V_R = 200V$.

$i = \frac{V_R}{R} = \frac{200}{100} = 2A \Rightarrow i_{rms} = 2A$.

$X_L = \frac{V_L}{I} = \frac{150}{2} = 75. \Rightarrow \omega L = 75$.

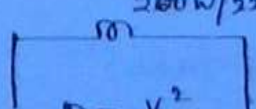
$L = \frac{75}{300} = 0.25H$.

45. let current thru $60V$, $= i'$

$i + i' = 12 \Rightarrow i = 12 - i' = < 12A$

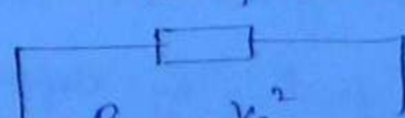
$\therefore$ Ans = 10A

46. 200W/220V



$R = \frac{V^2}{P}$

100W/220V



$R_{eq} = \frac{V_{eq}^2}{P_{eq}}$

$R_{eq} = nR$

$n = 2$

n = no. of bulbs

$= \frac{(220)^2}{}$ ... $= \frac{(220)^2}{}$

$$L = \frac{N^2}{l/a\mu_0\mu_r}$$

(14)

$S = \frac{l}{\mu_r\mu_0 a}$

$$\boxed{L = \frac{N^2}{S}}$$

$$\boxed{S = \frac{MMF}{\phi} = \frac{NI}{\phi} \quad AT/weber}$$

$L = \frac{N^2}{S}$

$S = \frac{MMF}{\phi} = \frac{NI}{\phi}$

law form → $\boxed{V = L\frac{di}{dt}}$

law form → $\boxed{i = \frac{1}{L}\int_{-\infty}^{t} V\,dt}$

$P = Vi$

$$\boxed{P = L\frac{di}{dt}\, i}$$

$W = \int P\,dt$

$$\boxed{W = \frac{1}{2}Li^2}$$

Power dissipation in an ideal inductor = 0.

Inductor stores Energy in the form of magnetic field. (Kinetic Energy)

Conclusions:

For dc supply at steady state, inductor acts as a short circuit.

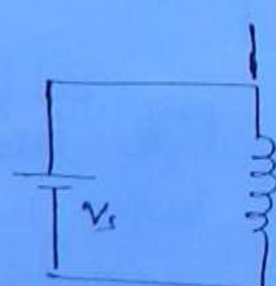
$V = L\frac{di}{dt}$

Steady state → $di/dt = 0$

$V = 0$

↓

S.C.

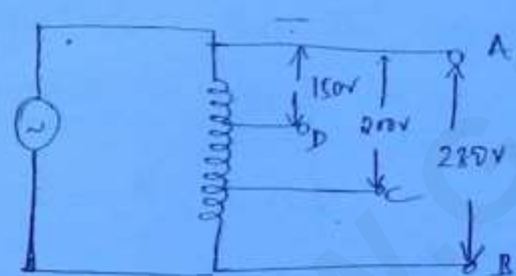


- Inductor doesnot allow sudden change of current. since to allow sudden change of current infinite voltage is required and time constant of the circuit should be equal to zero.

i) $V = L\frac{di}{dt} = \infty \quad \Big|_{dt \to 0}$

ii) Time constant = L/R.

r = internal resistance.



C_c = inter turn capacitance

(or)

Self capacitance

$X_c = 1/2\pi f C$

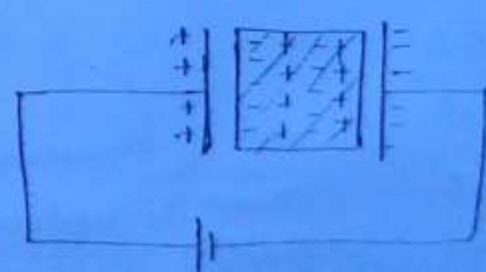
* Inter turn capacitance is present when inductor is operated either at high frequency or high voltage.

Capacitor

$Q \propto V$

$Q = CV$

$C = \frac{Q}{V}$ C/V (or) F



(16)

$Q = CV$

$\frac{dQ}{dt} = C\frac{dv}{dt}$

$i = C\frac{dv}{dt}$

$C = i/(dv/dt)$

$$\boxed{i = C\frac{dv}{dt}}$$

ohm's law 7th form.

$P = vi$

$P = C\frac{dv}{dt}V$.

$W = \int P\,dt$

$$\boxed{W = \frac{1}{2}CV^2}$$

energy stored in the form of electric field.

$$\boxed{V = \frac{1}{C}\int_{-\infty}^{t} i\,dt}$$

8th form.

Power dissipation in ideal capacitor = 0.

> Capacitor stores energy in the form of electric field (potential energy).

$C = \frac{\epsilon A}{d}$

$\epsilon = \epsilon_0\epsilon_r$

$\epsilon_0 = 8.85\ pF/m$

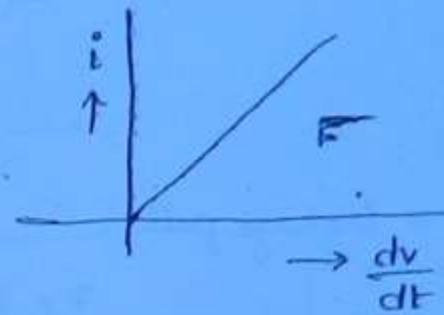
permitivity is the property of the medium in which electric field exists.

$A \Rightarrow$ area of cross section of each conducting plate

$d \Rightarrow$ distance b/w 2 conducting plates

→ when capacitance of the capacitor independant on the voltage magnitude, then the capacitor is called as linear capacitor.

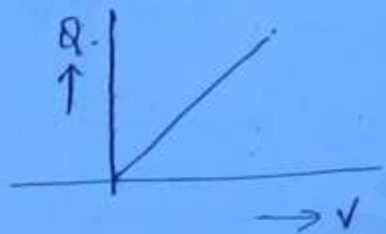
$$C = \frac{Q}{V} = \text{constant.}$$



$V \uparrow 10\%$, $Q \uparrow 10\%$.

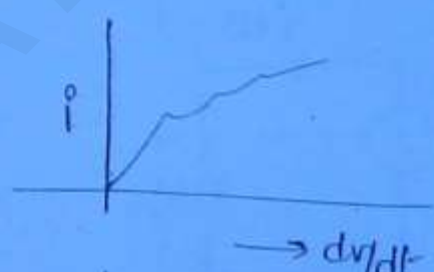
$V \uparrow 90\%$, $Q \uparrow 90\%$.

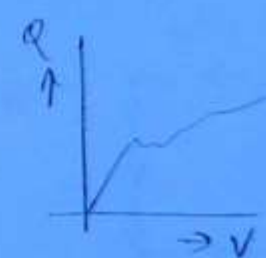
$$i = C\frac{dv}{dt} \Rightarrow C = \frac{i}{(dv/dt)}$$



→ when capacitance of the capacitor depends on voltage magnitude, then the capacitor is called as non-linear capacitor. Ex. Varactor diode.

$$C = \frac{Q}{V} = \text{variable.}$$





Conclusions.

→ 1. For dc supply, at steady state, capacitor acts as O.C.

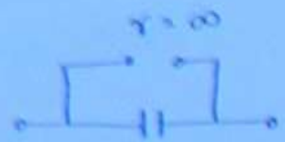
$i = C\frac{dv}{dt}$ | steady state $\frac{dv}{dt} = 0 \Rightarrow$ | $i = 0$ ∴ O.C.

2. (i) $i = C\frac{dv}{dt}$

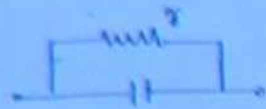
Capacitor doesnot allow sudden change of voltages

∵ for sudden change of voltages ∞ current is required

But practically it is not possible.



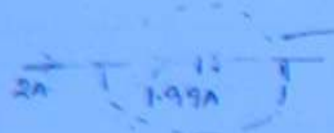
← ideal capacitor.



← practical.

$r \approx M\Omega$

in inductor $r = m\Omega$.



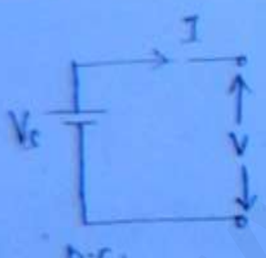
→ leakage path.

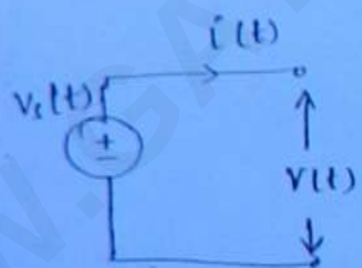
r = leakage path resistance.

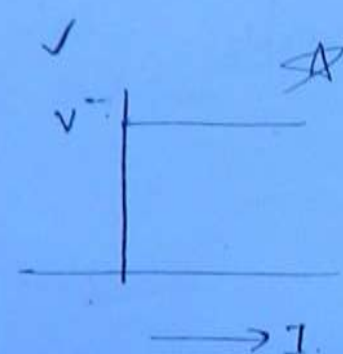
Active Elements

Voltage Source

Ideal voltage source delivers energy at the specified voltage (V), which is independant on current delivered by this source.

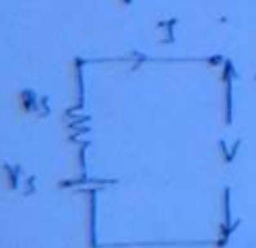


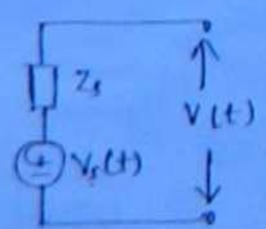


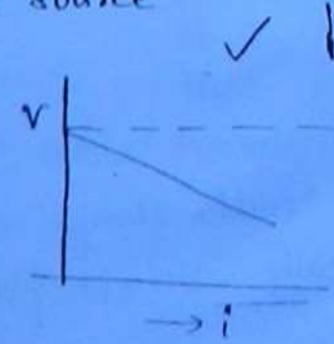


$R_s = 0$.

Practical voltage source delivers energy at specified voltage (V), which depends on current delivered by the source.







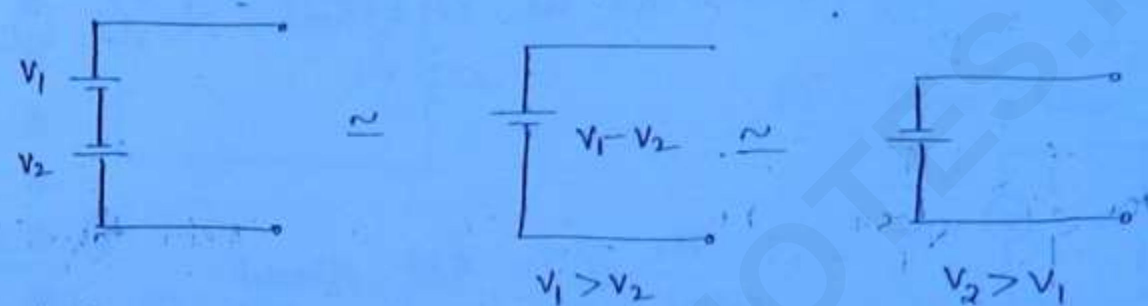
$V_s = V + IR_s \Rightarrow V = V_s - IR_s$

Note:

Independant voltage and current source doesnot obey the ohm's law since voltage and current characteristic is non-linear.

1.

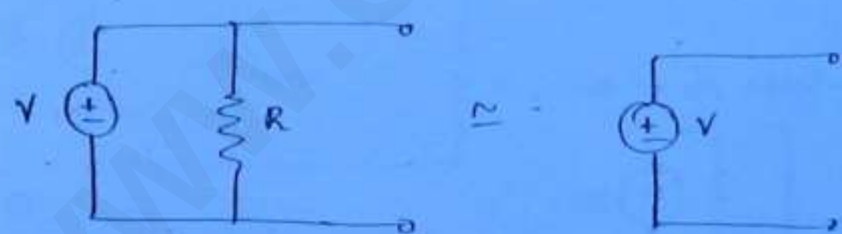
2.



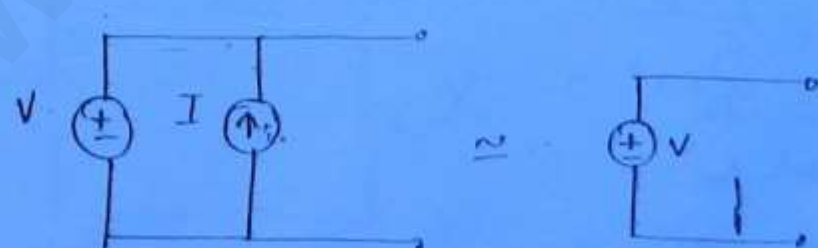
Ideal source.

3)

4) *

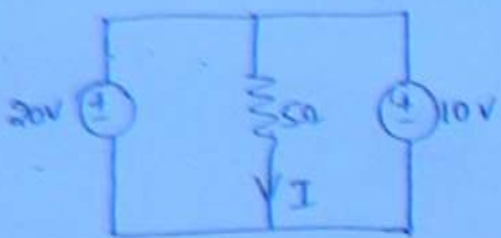


5) *



Find the value of I for this ckt. shown.

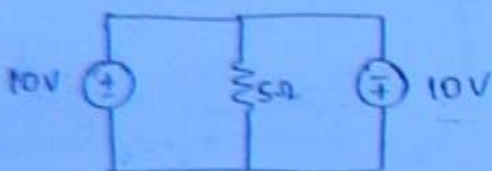
(20)



(a) 2A (b) 4A (c) 6A

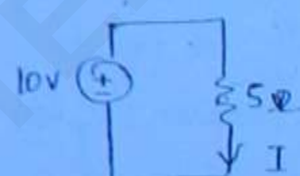
(d) none.

Sol: w.r.t KVL voltage across all the // branches should be equal.



→ The given ckt doesn't satisfy KVL.

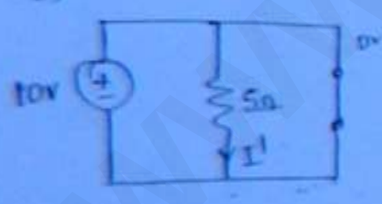
→



$\Rightarrow I = 10/5 = 2A$

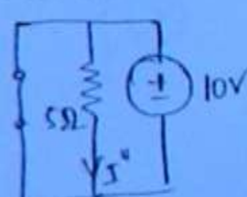
using superposition

case (i)



$I' = 0$

Case (ii)



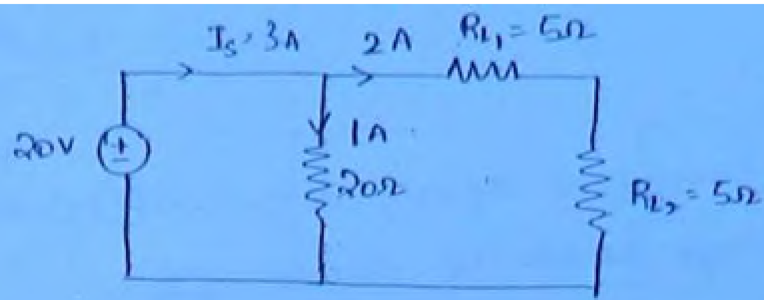
$I'' = 0$.

$I = I' + I'' = 0$. ✗

∴ 10V & 0V ~~can't be~~

for the above circuit superposition theorem cannot be applied since, case 1 & case 2 circuits are not satisfying KVL.

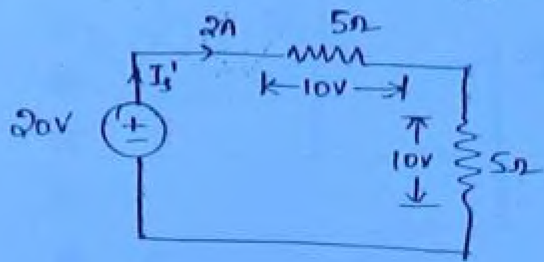
Q.



$I_s = 3$

$P_s = 20 \times 3 = 60W$

as per eq. ckt (4) neglect $R = 20\Omega$



$I_s' = 2$

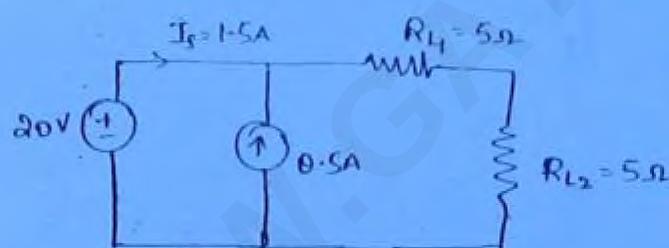
$P_s' = 20 \times 2 = 40W$.

Note:

(i) In the above circuit 20Ω resistance can be neglected while calculating either load current or load voltage.

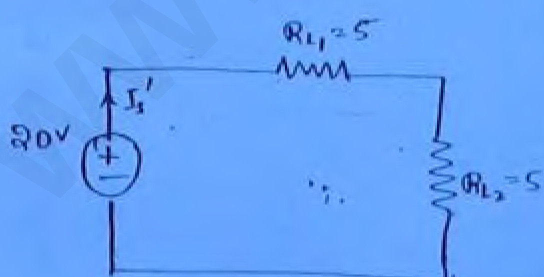
(ii) In the above circuit 20Ω resistance cannot be neglected while calculating either source current or power.

Q.



$I_s = 1.5$

$P_s = 20 \times 1.5 = 30W$



$I_s' = 2$

$P_s' = 20 \times 2 = 40W$.

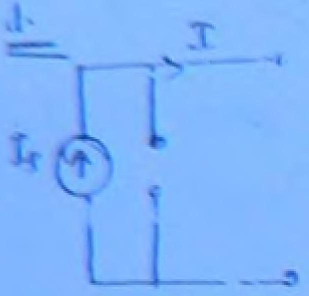
Note: (i) In the above ckt, current source can be neglected while calculating either load current or load voltage.

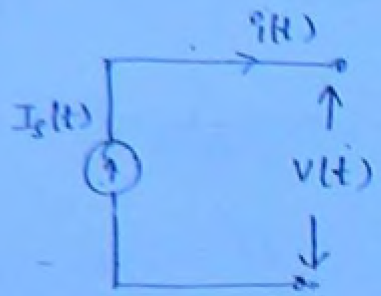
(ii) In the above circuit, current source cannot be neglected while calculating either voltage source current or power

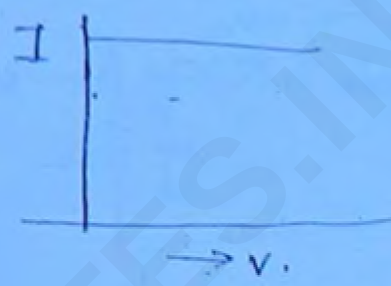
Current Sources.

Ideal current source delivers energy at specified current (I) which is independant on voltage across the source. 22

Internal resistance of ideal current source $= \infty$.

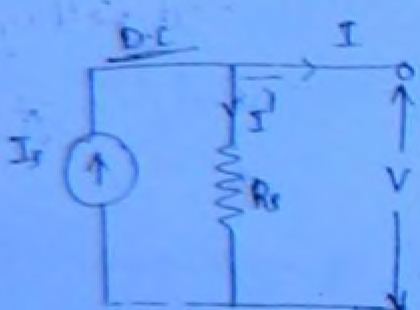


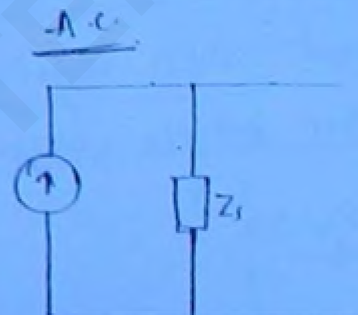




D.C. $R_s = \infty$ A.C.

Ideal current source delivers energy at specified current (I) which depends on voltage across the source.

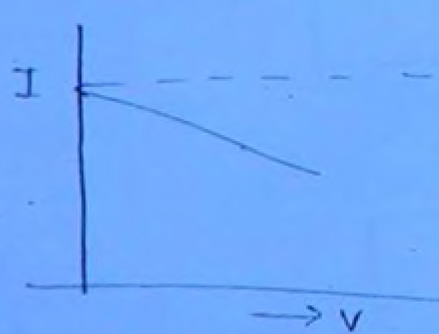




$$I_s = I' + I$$

$$I = I_s - I'$$

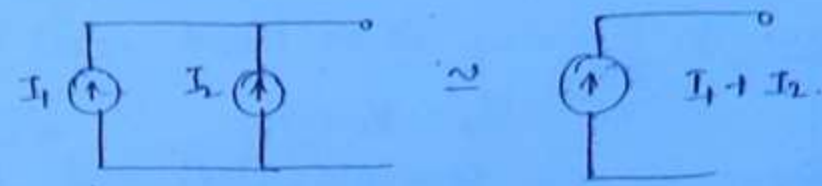
$$I = I_s - \frac{V}{R_s}.$$



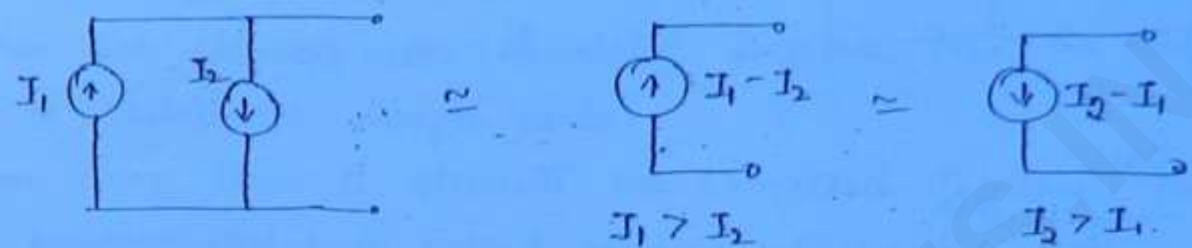
In the real time system no independant current source exist.

Equivalent circuits:

1.



2.

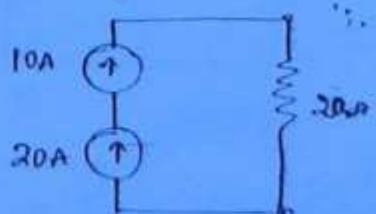


3.

4.

5.

Ex 1.



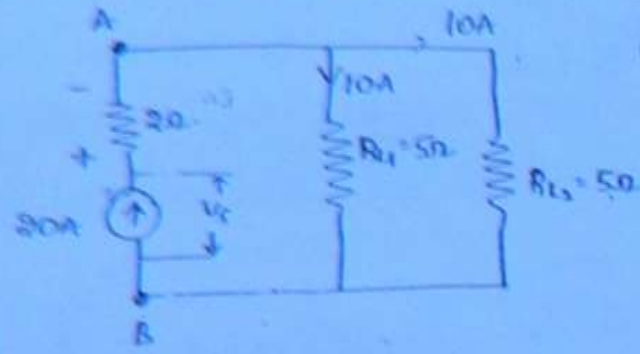
(A) 10A. (b) 20A (c) 30A ~~(d)~~ none

not satisfying KCl.

Ex.

not satisfying kcl

Ex.



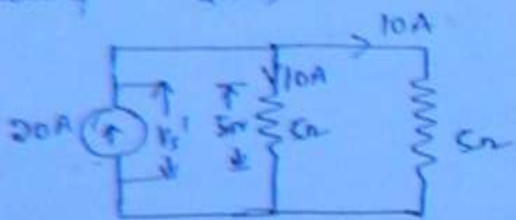
$V_{AB} = V_s - 40$

$50 = V_s - 40$

$V_s = 90$

$P_s = 90 \times 20$.

neglecting 2Ω,



$V_s' = 50V$

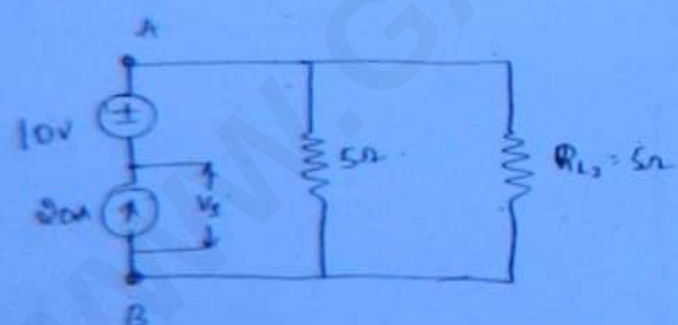
$P_s' = 50 \times 20$.

50

L

In the above circuit 2Ω resistance can be neglected while calculating either load current or load voltage.

In the above circuit 2Ω resistance cannot be neglected while calculating either source voltage or source power.

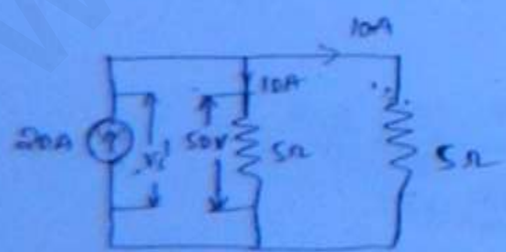


$V_{AB} = V_s + 10$

$50 = V_s + 10$

$V_s = 40V$.

$P_s = 40 \times 20$.



$V_s' = 50V$

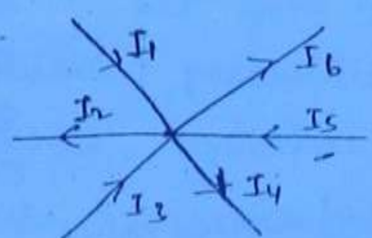
$P_s' = 50 \times 20$.

L: 1) In the above circuit voltage source can be neglected while calculating either load current or load voltage.

In the above ckt voltage source cannot be neglected

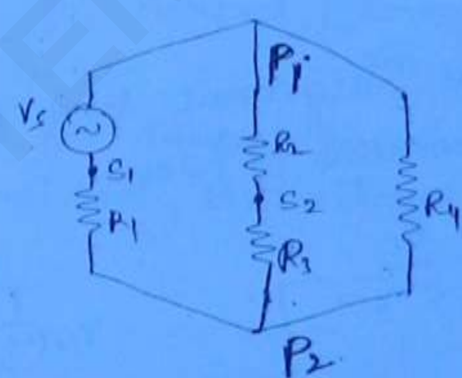
KCL.

1. KCL works based on the principle of law of conservation of charge.
2. KCL states that algebraic sum of currents meeting at a point is equal to zero.
3. When two elements are connected together then the common point is called as simple node.
4. When more than 2 elements are connected together, then the common point is called as principle node.

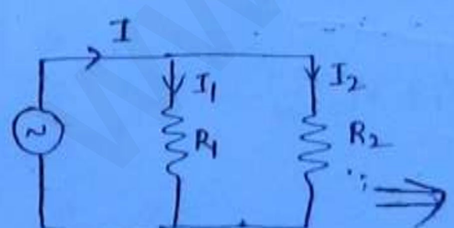


$$I_1 - I_2 + I_3 - I_4 + I_5 - I_6 = 0$$

Node < simple, principle

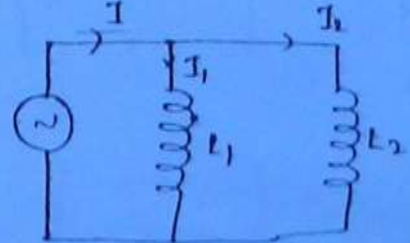


Current dividing rules



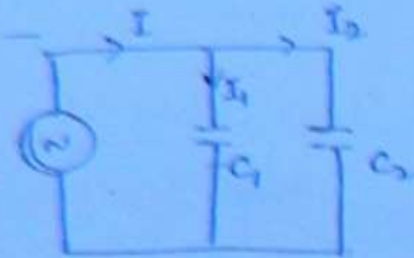
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\Rightarrow \quad I_1 = \frac{I \cdot R_2}{R_1 + R_2} \quad ; \quad I_2 = \frac{I \, R_1}{R_1 + R_2}$$



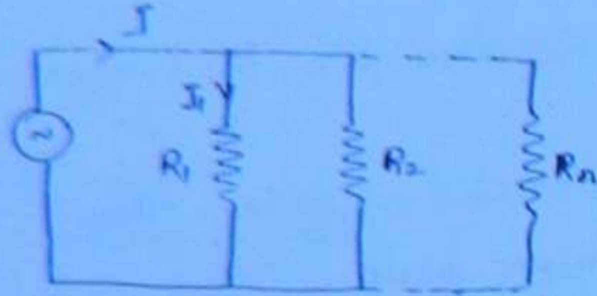
$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$I_1 = \frac{I \, L_2}{L_1 + L_2} \quad ; \quad I_2 = I \cdot \frac{L_1}{L_1 + L_2}$$



$C_{eq} = C_1 + C_2$

$\longrightarrow \quad I_1 = I \cdot \dfrac{C_1}{C_1 + C_2}$; $I_2 = I \cdot \dfrac{C_2}{C_1 + C_2}$.

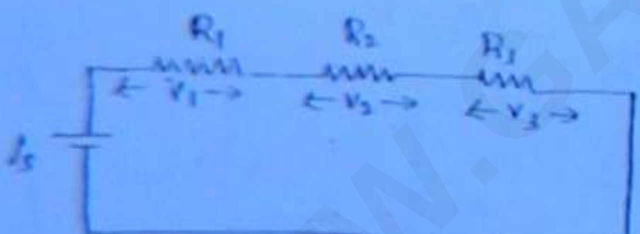


$\longrightarrow \quad I_1 = I \cdot \dfrac{1/R_1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}}$.

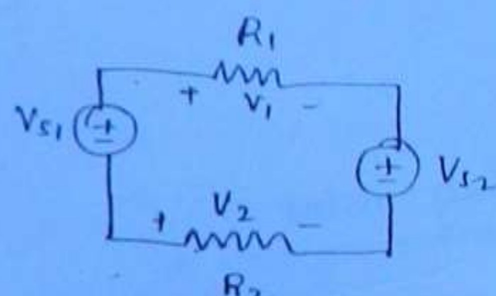
KVL

1. KVL works based on the principle of law of conservation of energy.

KVL states that the algebraic sum of the voltages in a closed loop is equal to zero.

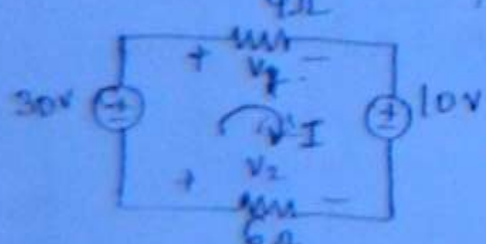


$V_1 + V_2 + V_3 - V_S = 0$.



$V_2 + V_{S_1} = V_1 + V_{S_2}$

Find V_1 & V_2 of the ckt shown.



$30 - V_1 - 10 + V_2 = 0$

$V_1 - V_2 = 20 \longrightarrow (1)$

$V_1 = 4I$, $V_2 = -6I$ ↗ $\quad 4i + 6i = 20$

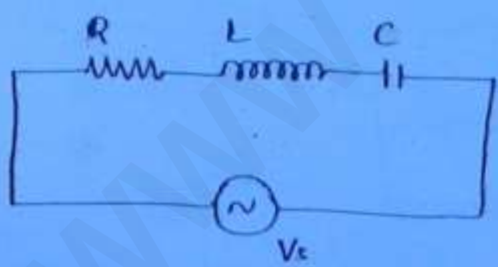
Conclusions.

1. Field theory can be applied either for low frequency or high frequency circuits. (accurate results are obtained) (27)
2. Network theory can be applied only for low frequency circuits.
3. KVL & KCL fails for high frequency circuits.

Lumped & Distributed parameters.

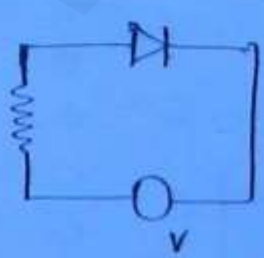
∴ KVL & KCL fails for distributed parameters since in distributed parameters electrically, it is not possible to separate resistance, inductance and capacitance effect.

2. Ohm's law can be applied for lumped (linear) and distributed parameters.
3. KVL & KCL equations used for lumped parameters circuit. (linear, non linear, unidirectional, bidirectional, time variant, and invariant elements).



$$V_s = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

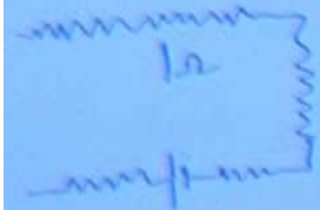
lumped (linear)

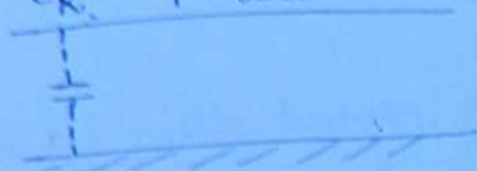


→ lumped (non linear).

→ lumped

Distributed



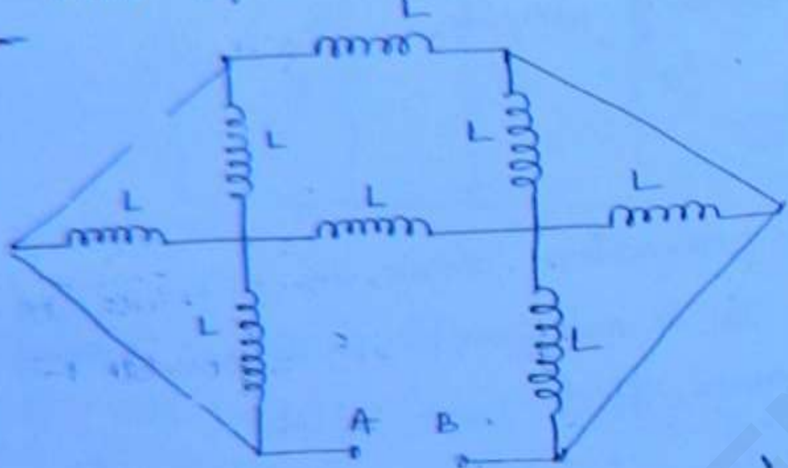


$J = \sigma E$

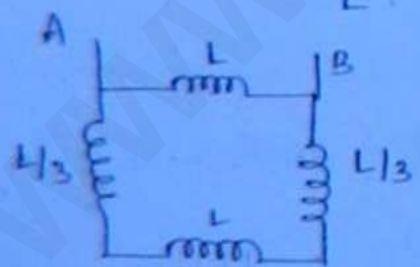
Distributed:

(28)

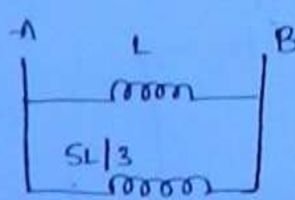
Find equivalent inductance wrt A & B.



$\frac{2L \times L}{3L}$ $\frac{2L}{3}$ $2L$



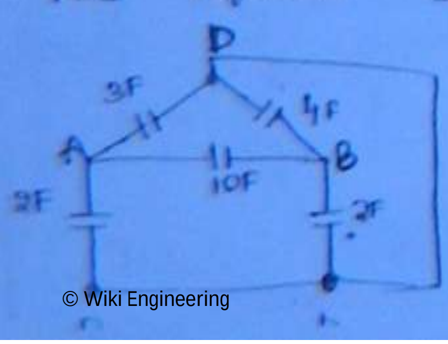
=

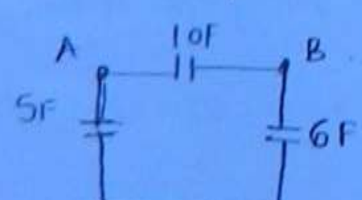


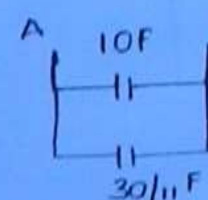
$L_{eq} = \frac{5L}{8}$

$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$

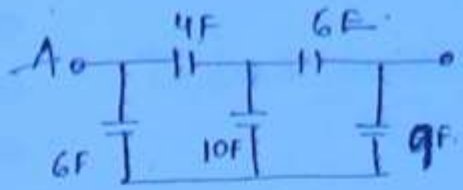
Find equivalent capacitance wrt A & B.

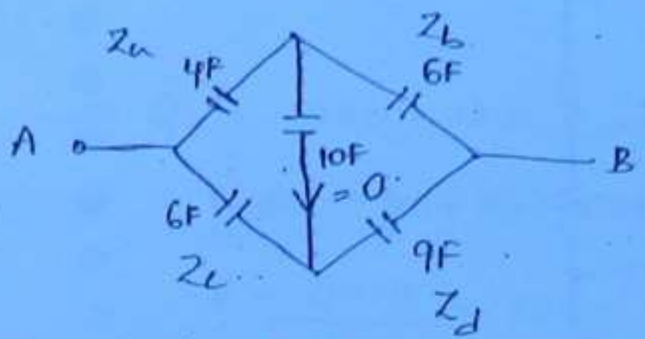




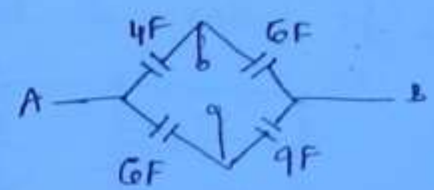


Q. Find C_{eq} w.r.t A & B.





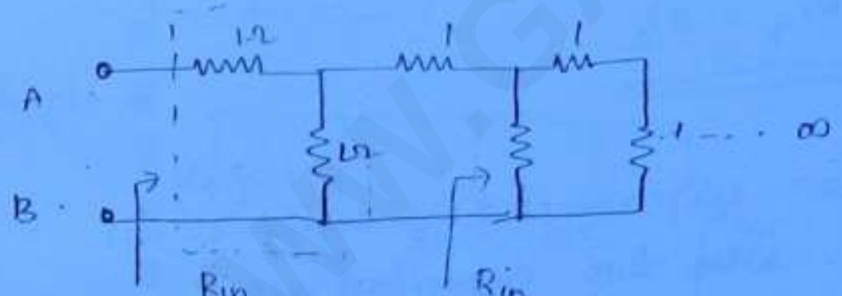
Balanced bridge ⇒ $4 \times 9 = 6 \times 6$.

→ 

→ 

$C_{eq} = 6.0F$

Q. Find eq resistance w.r.t A & B.



$$R_{eq} = 1 + \frac{R_{eq}}{1+R_{eq}}$$

$$R_{eq} = \frac{1+2R_{eq}+}{1+R_{eq}}$$

$$R_{eq} + R^2 = 1 + 2R_{eq}$$

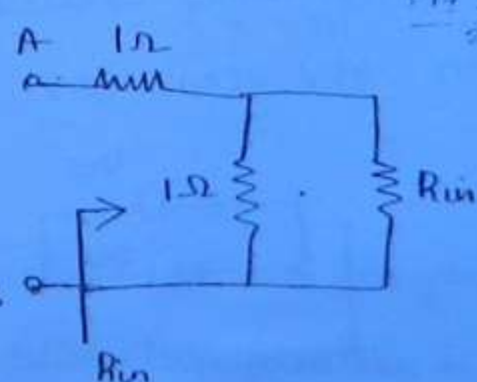
$$R^2 - R - 1 = 0$$

$$\frac{+1 \pm \sqrt{1+4}}{2} = \frac{1+\sqrt{5}}{2}$$

$$R_{in} = 1 + \frac{R_{in}}{1+R_{in}}$$

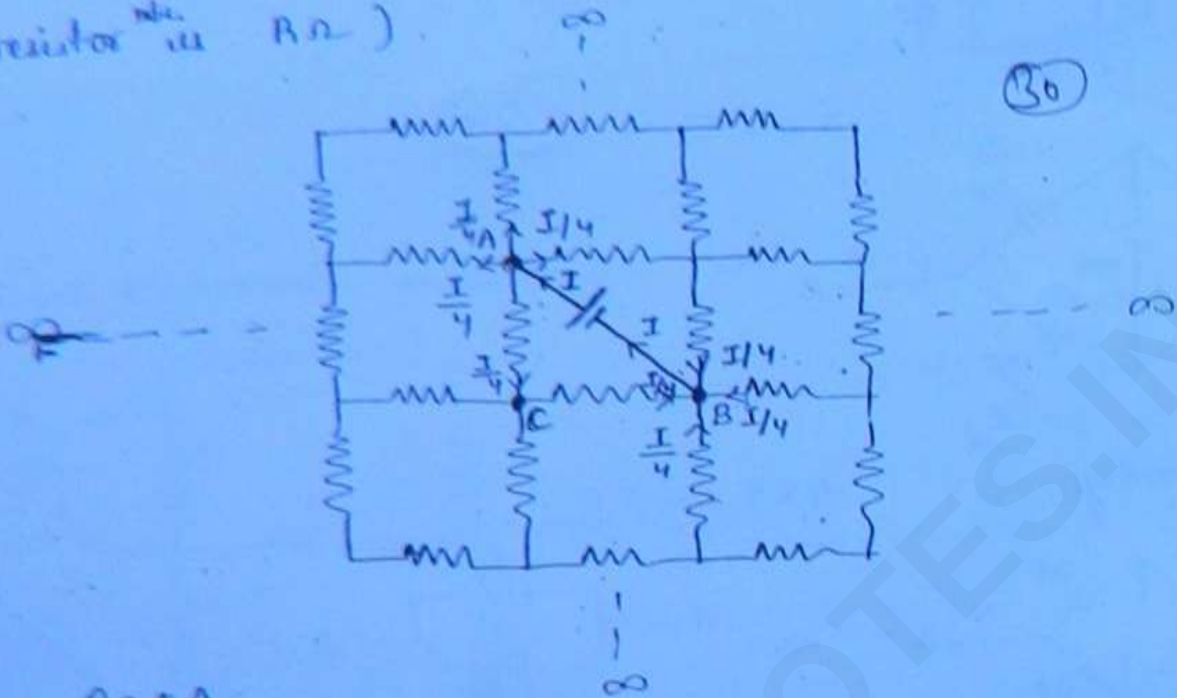
$$R_{in}^2 - R_{in} - 1 = 0$$

$$R_{in} = \frac{1 \pm \sqrt{5}}{2}$$



Find eq. resistance wrt. A & B. (assume each resistor value is $R\Omega$).

(30)



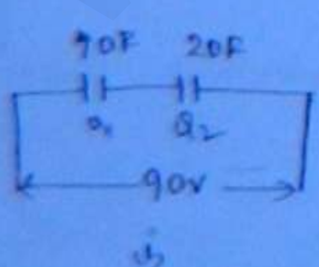
ACBA

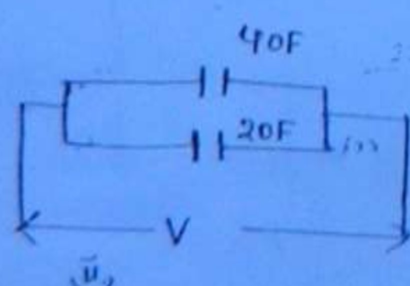
$$V = R\cdot\frac{I}{4} + I\cdot\frac{R}{4} = \frac{R\cdot I}{2}$$

$$\Rightarrow \frac{V}{I} = \frac{R}{2} \Rightarrow \underline{\underline{R_{eq} = R/2}}$$

Two capacitors of 40F & 20F are connected in series to a source voltage of 90V. When two capacitors are charged fully, they are connected in parallel. Find voltage across capacitors in parallel connection.

(a) 40V (b) 45V (c) 30V (d) 60V





$V_1 = \frac{90\times 20}{60} = 30V$ $V_2 = \frac{90\times 40}{}$

(i) $C_{eq} = \dfrac{40 \times 20}{40+20} = \dfrac{40}{3}$

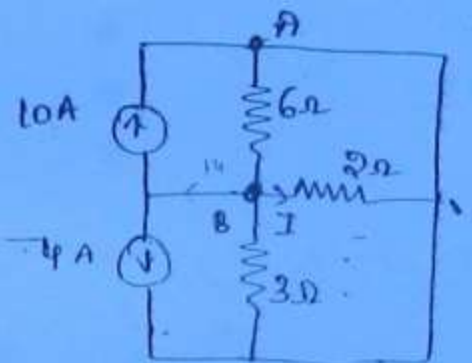
$Q = C_{eq} \cdot V \Rightarrow Q = \dfrac{40}{3} \times 90 = 1200\,C$

(ii) $\rightarrow Q_T = 1200 + 1200 = 2400\,C$ ★

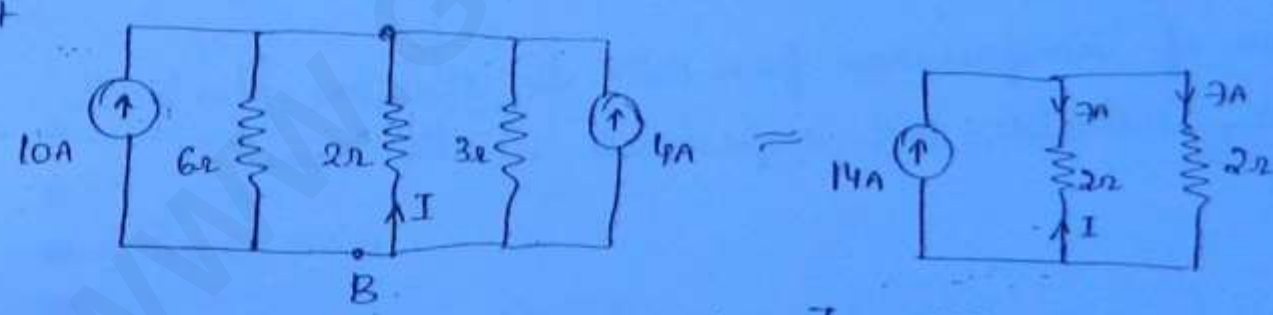
$C_{eq} = 20 + 40 = 60F$

$V = \dfrac{2400}{60} = 40V$

Q. ★ Find the value of I for the ckt shown.



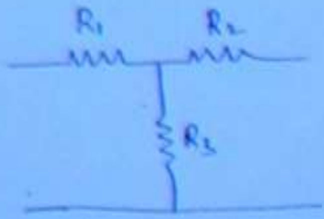
Eq. ckt



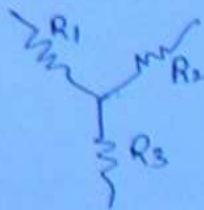
$\therefore I = -7A$

Note :-

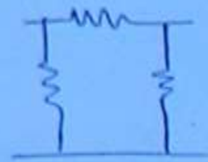
When elements are connected neither in series nor in parallel, to reduce the network, Star delta transformation is used.



T.



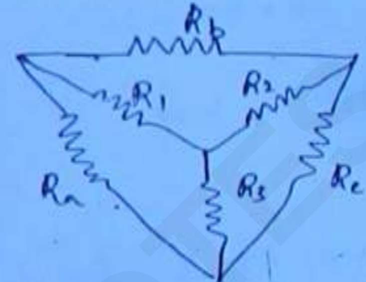
Star



π

delta

32



$\frac{R_a R_b}{R_a+R_b+R_c}$

SD1

Delta to star

$$R_1 = \frac{R_a R_b}{R_a+R_b+R_c}, \quad R_2 = \frac{R_b R_c}{''}$$

R_3 ..

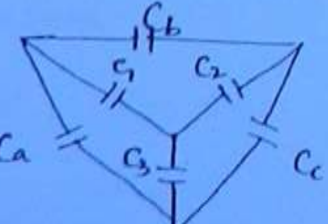
Star to delta

$$R_a = \frac{R_1R_2+R_2R_3+R_3R_1}{R_2}, \quad R_b = \frac{''}{R_3}, \quad R_c = \frac{''}{R_1}$$

procedure of Transformation from delta to star (or) star to delta resistors, inductors and impedances is the same.

Capacitors

Delta to star



$$\frac{1}{C_1} = \frac{1/C_a \cdot 1/C_b}{1/C_a + 1/C_b + 1/C_c}$$

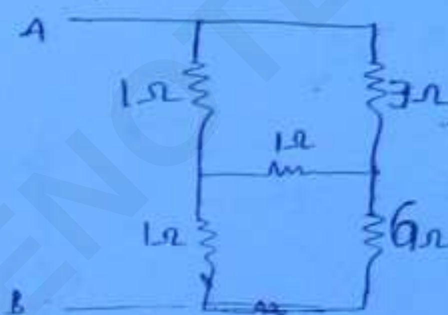
$$\frac{1}{C_2} = \frac{1/C_b \cdot 1/C_c}{1/C_a + 1/C_b + 1/C_c}, \quad \frac{1}{C_3} = \frac{1/C_a \cdot 1/C_c}{1/C_a + 1/C_b + 1/C_c}$$

Star to delta.

$$\frac{1}{C_a} = \frac{\frac{1}{C_1}\frac{1}{C_2} + \frac{1}{C_2}\frac{1}{C_3} + \frac{1}{C_3}\frac{1}{C_1}}{\frac{1}{C_2}} \quad (33)$$

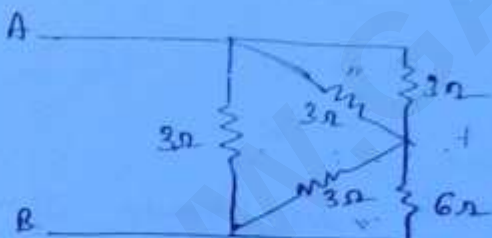
$$\frac{1}{C_b} = \frac{\text{"}}{\frac{1}{C_3}}, \quad \frac{1}{C_c} = \frac{\text{"}}{\frac{1}{C_1}}$$

Q. Find eq resistance wrt A & B.



Convert 1Ω, 1Ω, 1Ω Y→Δ

Sol



$$R_a = \frac{(1\times1)+(1\times1)+(1\times1)}{1}$$

$R_a = 3\Omega$

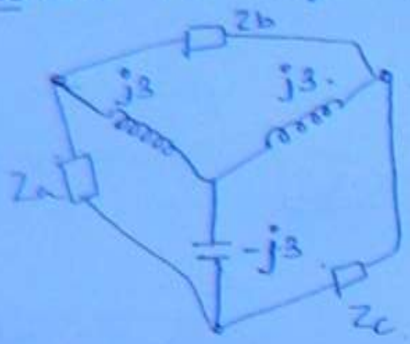
IIly $R_b = R_c = 3\Omega$.

$R_{ab} = 1.6\ \Omega$.

Note.

1. when resistors of equal value are transformed from Y → Δ, resistance is increased by 3 times (SDI)
2. when capacitors of equal value are transformed from Y → Δ Capacitance decreases by 3 times (DSI)

Draw the eq. Δ network for the network shown.



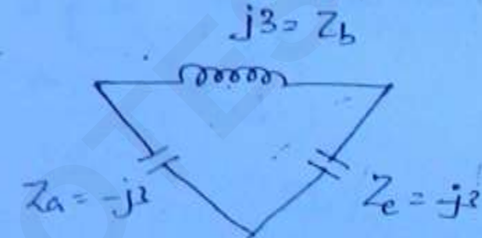
B.4

$$Z_a = \frac{(j3)(j3) + (j3)(-j3) + (j3)(-j3)}{(j3)} = -j3$$

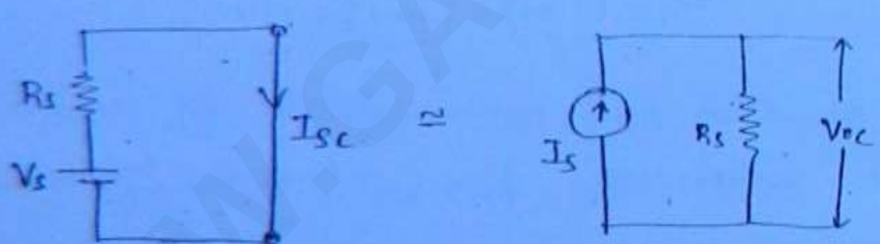
$$Z_b = \frac{(j3)(-j3)}{-j3} = j3$$

$$Z_c = \frac{(j3)(-j3)}{j3} = -j3$$

→



Source Transformation



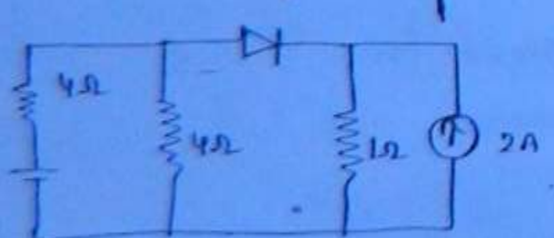
$I_s = I_{sc} = \frac{V_s}{R_s}$

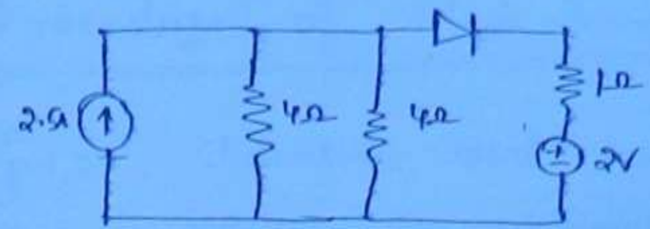
$R_s = R_s$.

$V_s = V_{oc} = I_s R_s$

$R_s = R_s$.

ind current of ideal diode of the ckt shown





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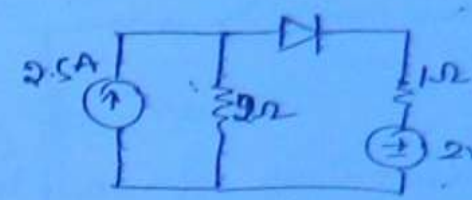
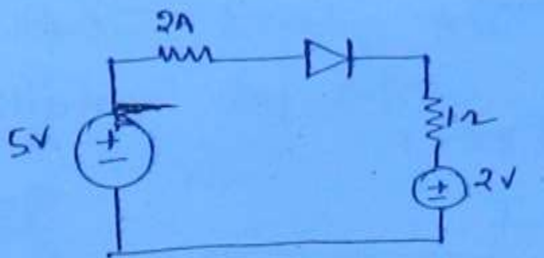
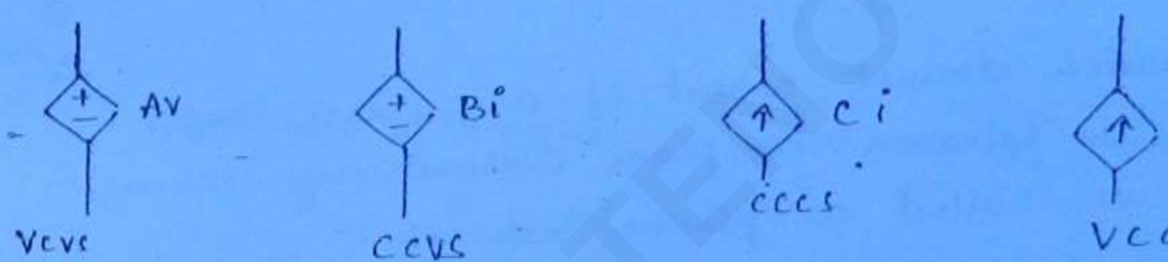




$$\Rightarrow \quad i = \frac{5-2}{2+1} = 1A$$

Dependant Sources.



A → no unit

B → Ω

V = iR.

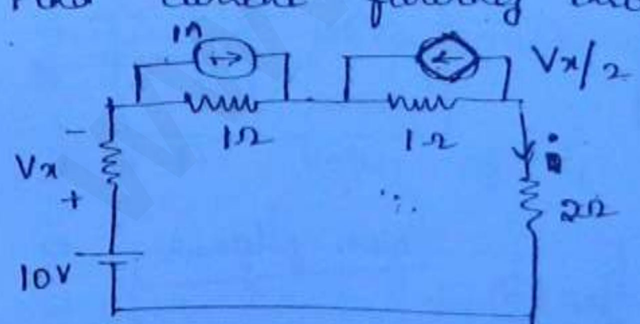
C → no unit

D → mho (or) S.

$i = V/R = GV$

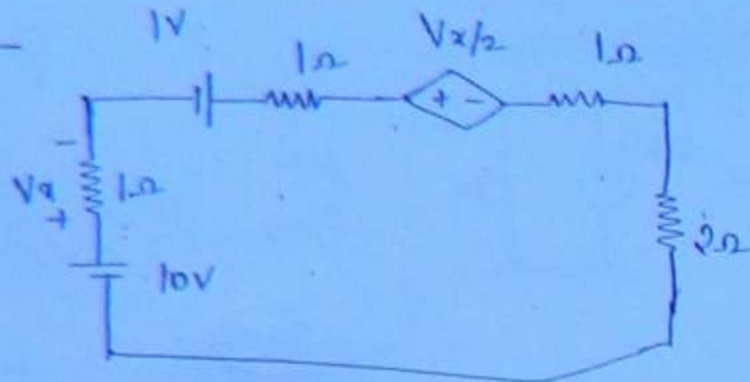
→ All the above are linear dependant sources.

Q. Find current flowing through 2Ω resistor



Note: ★★

while applying source transformation for dependant source, whenever dependant source magnitude depends without disturbing ...



(36)

$$I = \frac{10+1-V_x/2}{1+1+1+2}$$

$$V_x = (1 \times I)$$

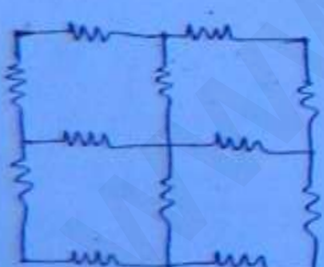
$$V_x = I$$

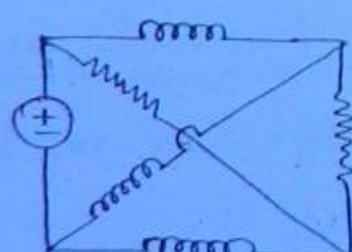
$$I = 2A$$

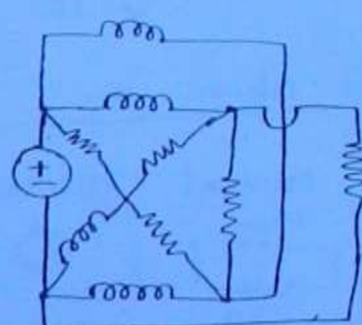
Mesh Analysis

Mesh is a loop which doesnot consist of any inner loop. When the network is drawn on plane without any crossover, then the network is called as planar network.

Mesh analysis can be applied only for planar networks.

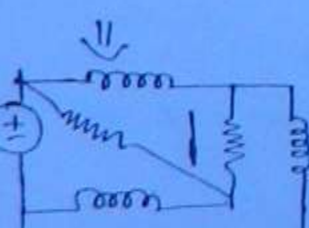






planar network

non planar

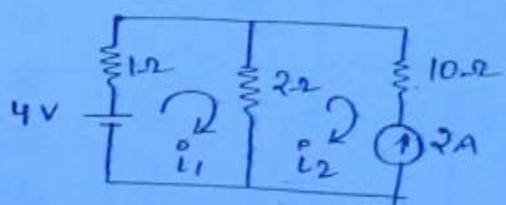


non planar

Procedure of Mesh Analysis :

Step 1: Identify total number of meshes.
Step 2: Assign the current direction for each mesh.
Step 3: Develop KVL equation for each mesh. (37)
Step 4: By solving KVL equations, find loop currents.

Ex.



$$-4+3i_1-2i_2=0$$
$$i_2=-2.$$
$$i_1=0.$$

Note.

Total no. of eqns = Total no. of meshes.

$e = M = 2.$

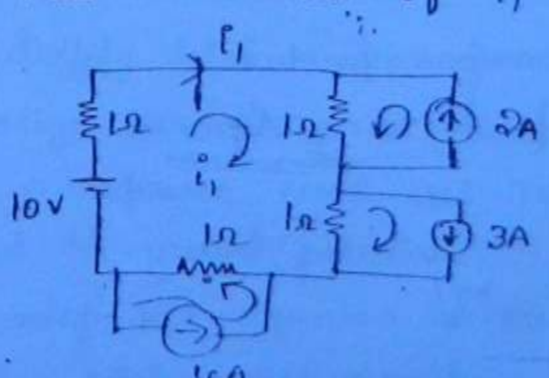
b = Total number of branches

N = Total no. of nodes.

$$\boxed{e = b-(N-1)} \quad **$$

* In the above circuit, to find the loop current, minimum one equation is required.

Q. Find the value of i_1 of this circuit shown.



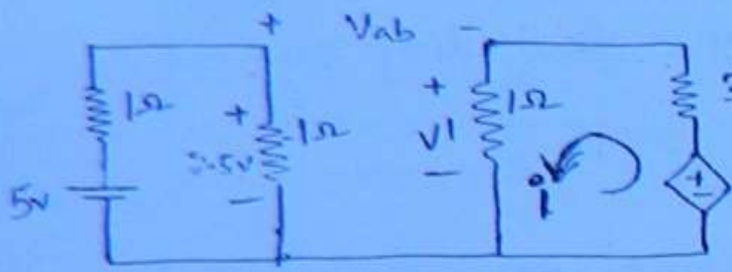
$$-10+i_1+i_1+2+i_1+3+i_1+4=0$$
$$10+4i_1+3=0$$
$$i_1=7/4$$

$0 = -10 + 4I_1 + (1\times 2) - (1\times 3) + 4\times 1$

$i_1 = 7/4$ A

38

Find the value of i of the circuit shown.



$i = \frac{4V_{ab}}{3+1} = V_{ab}$.

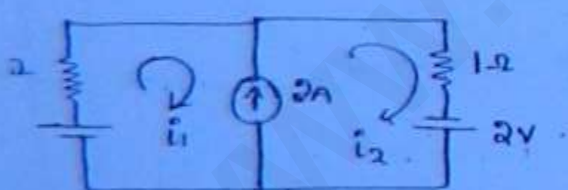
$V' = (1\times I) \Rightarrow V' = I = V_{ab}$.

$-2.5 + V_{ab} + V' = 0$.

$-2.5 + V_{ab} + V_{ab} = 0 \Rightarrow V_{ab} = 1.25$ V.

$i = 1.25$A.

Find i_1 & i_2 of the circuit shown:



When current source branch is common for two meshes, it is possible to find solution, by using super mesh technique

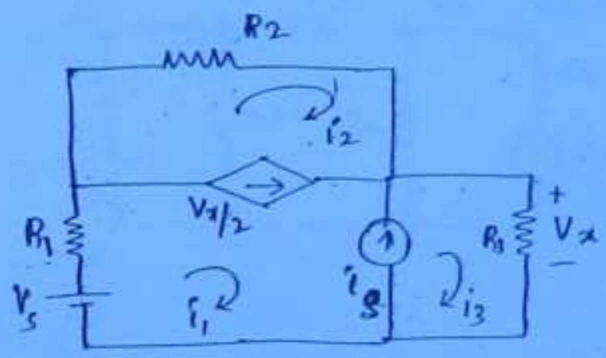
$-5 + (1\times I_1) + (1\times I_2) - 2 = 0 \rightarrow$ KVL.

$I_2 - I_1 = 2 \longrightarrow$ KCL.

mesh $\longrightarrow$ KVL + ohm's law

** Super mesh $\longrightarrow$ KVL + KCL + ohm's law.

Q. Develop mesh equations of the circuit shown.



$$-V_S + i_1R_1 + i_2R_2 + i_3R_3 = 0 \longrightarrow KVL$$

$$i_1 - i_2 = \frac{V_x}{2} \longrightarrow KCL$$

$$i_3 - i_1 = i_s \longrightarrow KCL$$

Nodal Analysis

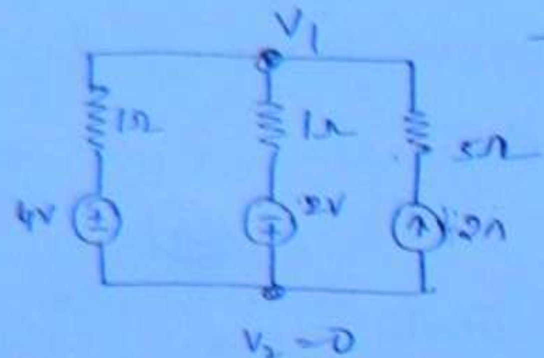
→ Nodal analysis can be applied for planar and non-planar networks.

mesh only for planar

Procedure of nodal analysis.

1. Identify total number of nodes.
2. Assign the voltage at each node, one of the nodes is taken as a reference node and reference node potential should be equal to ground potential.
3. develop KCL equation at each non reference node.
4. By solving KCL equations find node voltages.

Ex.

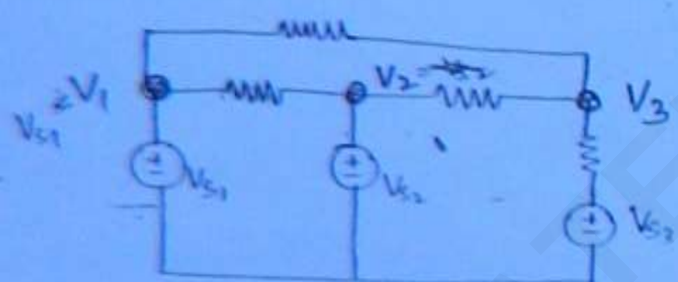


(40)

$$\frac{V_1 - 4}{1} + \frac{V_1 + 2}{1} = 2 \Rightarrow V_1 = 2V.$$

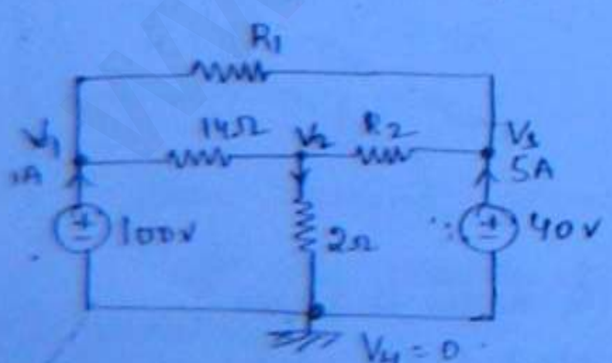
Total no. of eqns $\boxed{e = N - 1}$

N = no. of nodes.



To find node voltages in the above circuit, minimum one equation is required.

Find R_1 & R_2 of the circuit shown.



node 4 → $I = 5 + 10 = 15A$

$V_2 = 15 \times 2 = 30V$.

$V_1 = 100V$

$V_3 = 40V$

node 1 → $10 = \frac{V_1 - V_2}{14} + \frac{V_1 - V_3}{R_1}$

$R_1 = 12\Omega$.

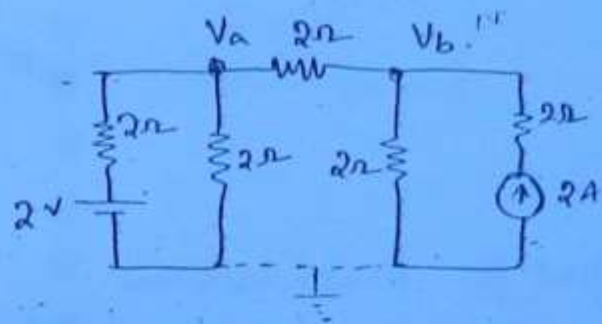
node 3 $\longrightarrow$

$$5 = \frac{V_3 - V_2}{R_2} + \frac{V_3 - V_1}{R_1}$$

$R_2 = 1\Omega$

(4)

Q. Find V_a & V_b of the circuit shown. [DRDO]



$$\frac{V_a - 2}{2} + \frac{V_a}{2} + \frac{V_a - V_b}{2} = 0$$

$$V_a + \frac{V_a - V_b}{2} - 1 = 0$$

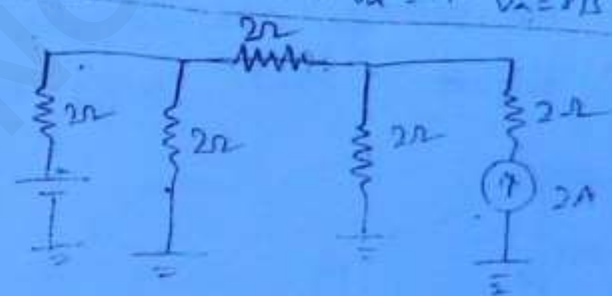
$3V_a - V_b = 2$

$$3V_a - 4 - \frac{V_a}{2} \cdot \frac{V_b}{2} - 2 + \frac{V_b - V_a}{2} = 0$$

$6V_a - 4 - V_a = 4$

$5V_a = 8$

$V_a = 8/5$



$$\frac{V_a - 2}{2} + \frac{V_a}{2} + \frac{V_a - V_b}{2} = 0$$

$$\frac{V_b}{2} + \frac{V_b - V_a}{2} = 2$$

$V_a = 8/5\ V$

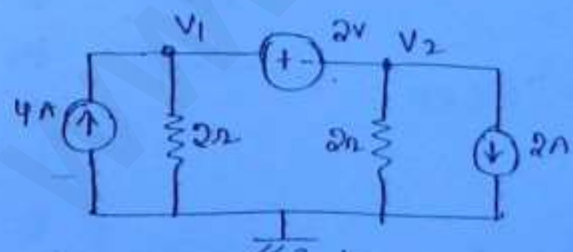
$V_b = 14/5\ V$

$$V_b - \frac{V_a}{2} = 2$$

$2V_b - V_a = 4$

$$V_b = \frac{4 + V_a}{2}$$

Q. Find V_1 & V_2 of the circuit shown.



Note :-

when ideal voltage source is connected b/n two non reference nodes, it is possible to find solution by using super node technique

$V_1 + V_2 = 4$

$V_1 - V_2 = 2$

$2V_1 = 6$

(42)

$4 = \frac{V_1}{2} + \frac{V_2}{2} + 2 \longrightarrow$ KCL.

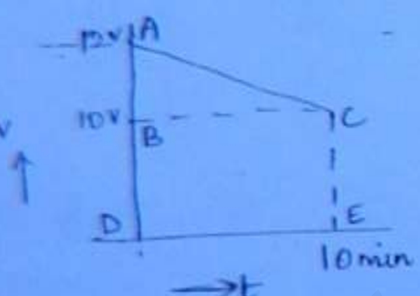
$V_1 - V_2 = 2 \longrightarrow$ KVL.

$V_1 = 3V, \quad V_2 = 1V$

nodal $\longrightarrow$ KCL + Ohm's law

Super node $\longrightarrow$ KVL + KCL + Ohm's law.

A fully charged mobile phone is good for 10 min talk time. During talk time, battery delivers a constant current 2A. The voltage characteristic of battery is as shown in the figure. Find energy of the battery during talk time.



10 min = (600 sec)

$Vt = \Delta ABC + \square BCED$

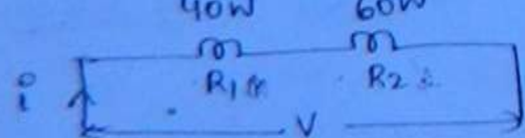
$= \frac{1}{2} \times 2 \times 600 + 600 \times 10 = 6600$

$W = Vit$

Energy $= 6600 \times 2A = \underline{13.2 kJ = W}$

which bulb gives more brightness.

(i)



$\overset{\to 230V}{V}, P, \overset{\to 50Hz}{f}$

$P = \frac{V^2}{R} \Rightarrow P \propto \frac{1}{R}$

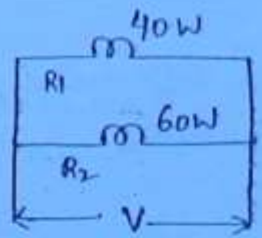
$\therefore P \propto R_1$

$$P_1 = i^2 R_1 \quad ; \quad P_2 = i^2 R_2 .$$

$\therefore \; P_1 > P_2 . \qquad \therefore \; B1.$

(43)

ii,



$R_1 > R_2 .$

$$P_1 = \frac{V^2}{R_1}$$

$$P_2 = \frac{V^2}{R_2} \Rightarrow P_1 < P_2 .$$

$\therefore$ B2

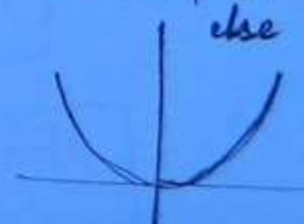
Steady state ★★*

1) Bi-directional
2) Linear
3) Passive

1. Non linear
2. Uni-directional
3. Passive

Bi direc — shud be identical in 1st & 3rd quad. else uni-direc.

passive — $\frac{V}{I}$ shud be → +ve in 1st & 3rd quad else active



1. non linear
2. Bi-direc
3. Passive

1. Uni-direc.
2. non-lin
3. active

1. uni direc.
2. non lin.
3. active.

Steady state A.C. Circuits.

Advantages of sine wave.

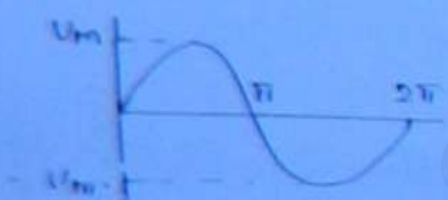
(44)

1. It is easy to handle mathematically. (differential and integral of the sine func. can be re-written in terms of sine function.

- The natural phenomena like the motion of the simple pendulum & response of undamped system, shows sinusoidal character.

- Any periodic waveform can be expressed in terms of sine function by using fourier analysis.

- It is easy to generate in the laboratory.



$V(t) = V_m \sin \omega t$

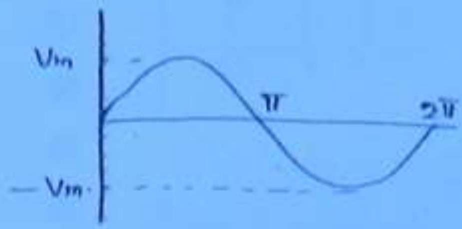
V_m = peak (or) max. value

ω = Angular frequency $\rightarrow$ rad/sec.

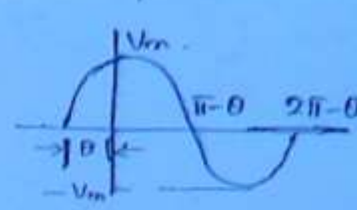
ωt = argument $\rightarrow$ rad.

$\omega T = 2\pi \Rightarrow \boxed{T = \frac{2\pi}{\omega}}$ sec.

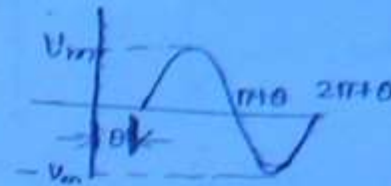
$f = \frac{1}{T} \Rightarrow \boxed{f = \frac{\omega}{2\pi}}$ Hz (or) cycles/sec.



(1) $V(t) = V_m \sin(\omega t)$



$V(t) = V_m \sin(\omega t + \theta)$



$V(t) = V_m \sin(\omega t - \theta)$.

→ wrt 1st waveform, 2nd waveform is leading by an angle θ.

→ wrt 1st waveform, 3rd waveform is lagging by an angle θ.

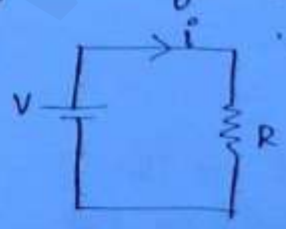
→ wrt second waveform, 3rd waveform is lagging by an angle 2θ.

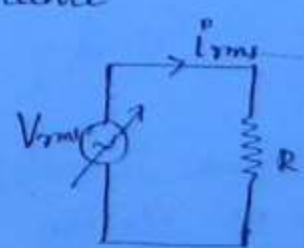
23/6/11

RMS Value

→ RMS value is defined based on heating effect of the waveform.

→ The voltage at which heat dissipation in AC circuit is equal to heat dissipation in DC circuit is called as V_{rms}, provided both ac & dc circuit have equal value of resistance and operated for same time





$W_{A.C} = W_{D.C}$

$P = i^2 R$

$W = i^2 R t$

heat D.C

$P = i^2 R$

$W = i^2 R t$

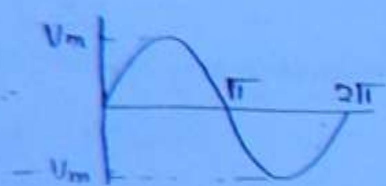
General Expressions

$$V_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2 \, d\omega t}$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^{T} v^2 \, dt}$$

- Find RMS value of the following waveforms.

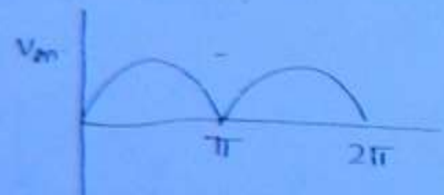
1)



$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2 \sin^2\theta \, d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(V_m \sin \omega t)^2 dt}$$

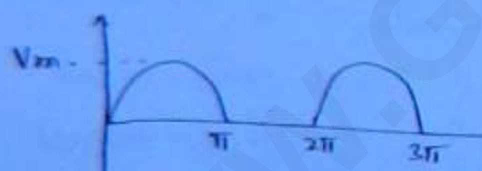
$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} V_m^2\left(\frac{1-\cos 2\omega t}{2}\right)dt} = \boxed{\frac{V_m}{\sqrt{2}} = V_{rms}}$$

2)



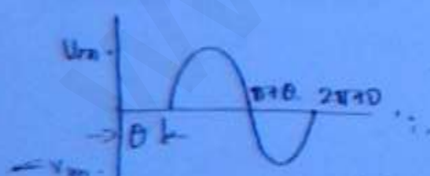
= Full wave rectified $= \frac{V_m}{\sqrt{2}}$

3)



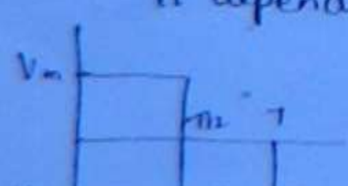
$=$ ½ wave $= \frac{V_m}{2}$

4)

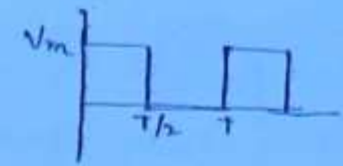


$V_{rms} = \frac{V_m}{\sqrt{2}}$

Note: RMS value is independant on the position of starting of waveform. But it depends on shape of the waveform.



$V_{rms} = V_m$

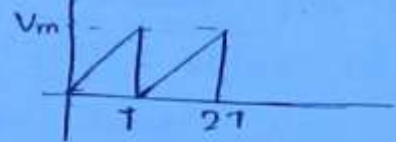
6) 

$= V_{rms} = \frac{V_m}{\sqrt{2}}$ $\qquad \frac{V_m}{\sqrt{2}}$

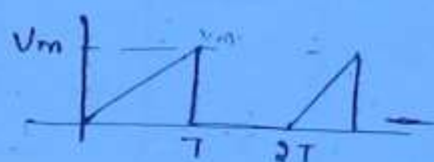
7) 

(47)

$= V_m/\sqrt{3}$

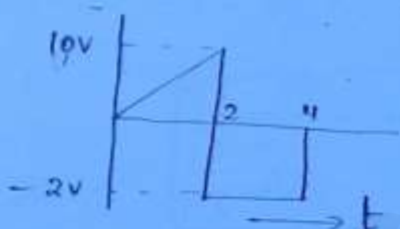
8) 

$= V_m/\sqrt{3}$

(9) 

$V_{rms} = \frac{V_m}{\sqrt{6}}$

$\frac{V_m^2}{T^2 \cdot 2T}\int_0^T \frac{t^3}{3} = \frac{V_m}{2T^2}\times\frac{T^2}{2}$ $\qquad \frac{V_m}{\sqrt{6}}$ $\qquad \frac{V_m^2}{6}$

(10) 

$0 \le t \le 2$

$y = mx$

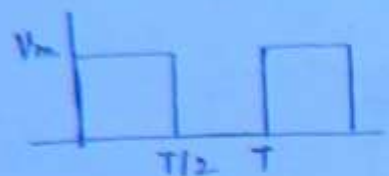
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{2 - 0} = 5.$

$V = 5t$

$$V_{rms} = \sqrt{\frac{1}{4}\left[\int_0^2 v^2 dt + \int_2^4 v^2 dt\right]}$$

$$= \frac{1}{4}\left[\int_0^2 (5t)^2 dt + \int_2^4 (-2)^2 dt\right] =$$

★ Find power dissipation in the resistor for a given waveform.



(a) $P_{av} = P_{peak}/2$. (48)

(b) $P_{av} = P_P/\sqrt{2}$

(c) $P_{av} = P_P/\sqrt{3}$.

(d) $P_{av} = P_{peak}$.

$$P_{av} = \frac{V_{rms}^2}{R}$$

$$P_{av} = \frac{(V_m/\sqrt{2})^2}{R} = \frac{V_m^2}{2R}$$

$$P_{av} = \frac{V_m^2}{2R} . \qquad P_{peak} = \frac{V_m^2}{R}$$

$$P_{av} = \frac{P_{peak}}{2}$$

$$\boxed{\frac{P_{D.C}}{P_{A.C}} = \frac{I_{av}^2 R}{I_{RMS}^2 R}}$$ ★

$$\boxed{\text{Form factor} = \frac{V_{RMS}}{V_{avg}}}$$

Find RMS value of the following function

$$V(t) = 3 + \sin t + \sin 3t + \cos t .$$

$$V_{rms} = \sqrt{V_{rms1}^2 + V_{rms2}^2 + \cdots + V_{rms\,n}^2}$$

$$= \sqrt{3^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

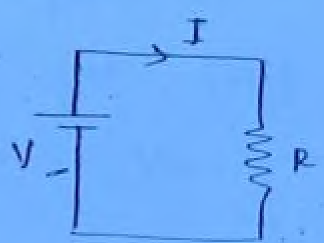
$$V_{rms} = \sqrt{21/2}$$

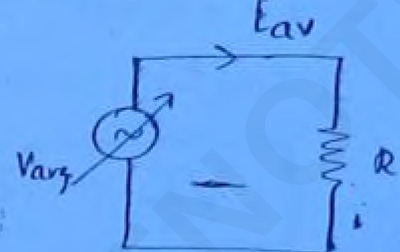
E :- RMS value is independant on frequency of the waveform.

Average Value :

→ Average value is defined based on charge transfer in the circuit. (49)

⇒ The voltage at which the charge transfer in A.C circuit is equal to charge transfer in D.C. circuit is called as V_{avg}, provided both A.C. & D.C circuits consist of equal value of resistance and operated for same time.





$I = V/R$

$I = Q/t$

$Q = it$

D.C.

$i = V/R$

$i = Q/t$

$Q = it$

A.C.

$$\boxed{Q_{A.C} = Q_{D.C}}$$

→ Avg. value of complete cycle of symmetrical wave = 0.

⇒ For analysis while finding avg. value of symmetrical wave only the 1/2 cycle is considered.

→ while finding avg. value of unsymmetrical wave angle made by complete cycle is considered.

★ Symmetrical wave

$V_{av} = \frac{1}{\pi}\int^{\pi} v\,d\omega t$

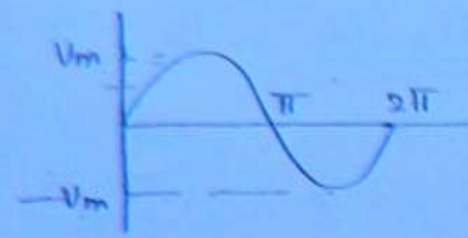
★ unsymmetrical wave

$V_{av} = \frac{1}{\;}\left[\int^{\pi} V d\omega t + \int^{2\pi} V d\omega t\right]$

Find avg value of the following waveforms :

50

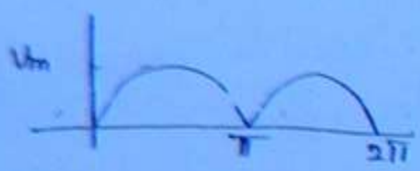
1.



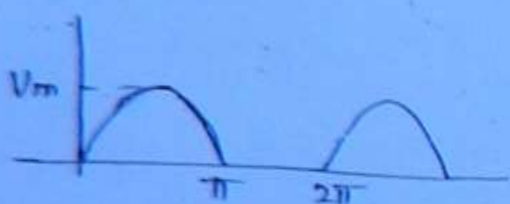
$$V_{av} = \frac{1}{\pi}\int_0^{\pi} v \, d\omega t$$

$$= \frac{1}{\pi}\int_0^{\pi} V_m \sin \omega t \, dt$$

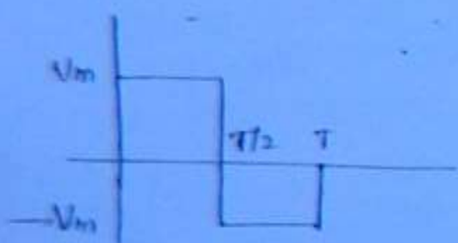
$$\boxed{V_{av} = \frac{2V_m}{\pi}}$$



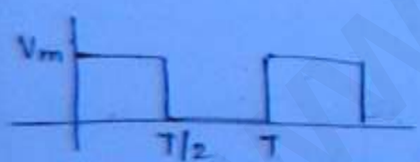
$$\boxed{V_{av} = \frac{2V_m}{\pi}}$$



$$\boxed{V_{av} = \frac{V_m}{\pi}}$$



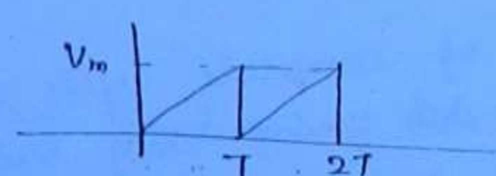
$$\boxed{V_{av} = V_{rms} = V_m}$$



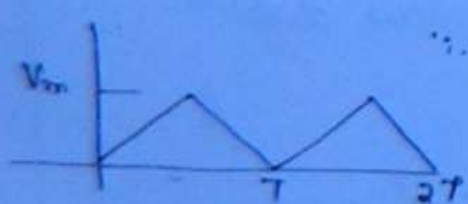
$$\boxed{V_{av} = V_m/2}$$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

6.

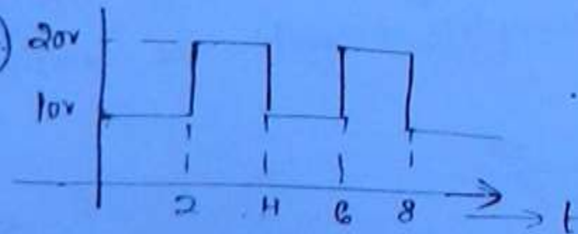


$$\boxed{V_{av} = V_m/2}$$



$$\boxed{V_{av} = V_m/2}$$

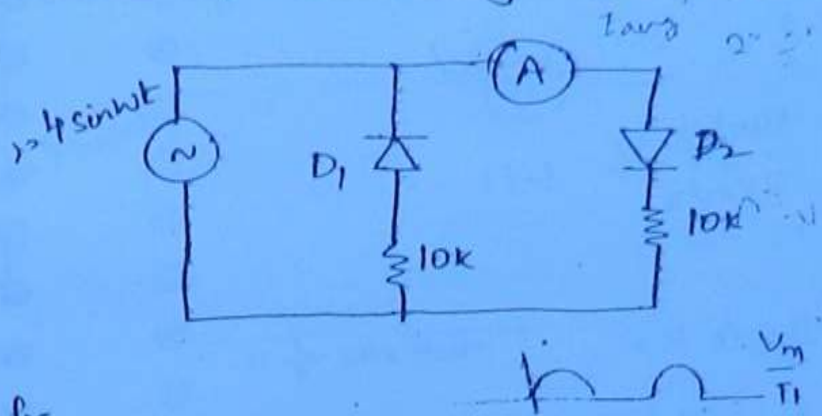
(8)



$$V_{avg} = \frac{1}{4}\left[\int_0^2 10\,dt + \int_2^4 20\,dt\right]$$

$V_{avg} = 15$

When circuit is having ideal diodes and average value of indicating ammeter. Find the value of ammeter.



(a) $\frac{0.4}{\pi}$ mA (57)

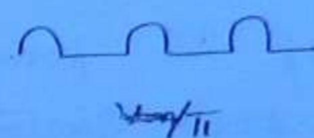
(b) $\frac{0.8}{\pi}$ mA

(c) $\frac{0.2}{\pi}$ mA

(d) $\frac{1.6}{\pi}$ mA

current flows in D_2 branch only for +ve ½ cyle.

∴ we get half wave output.



$$O/P = V_{av} = \frac{V_m}{\pi} = \frac{4}{\pi}$$

$$I_{av} = \frac{V_{av}}{R} = \frac{4/\pi}{10k} = \frac{0.4}{\pi}\ mA$$

FORM FACTOR

Form factor is the ratio of RMS value of the waveform to average value of the waveform.

$$\boxed{\text{form factor} = \frac{V_{Rms}}{V_{avg}}}$$

PEAK FACTOR

Peak factor is the ratio of max. value of the wave form to RMS value of the waveform.

$$\boxed{\text{Peak factor} = \frac{V_m}{V_{Rms}}}$$

To justify abt shape of the waveform, form factor and peak factor are introduced.

For sine wave, form factor = $\frac{V_m/\sqrt{2}}{2V_m/\pi} = 1.11$.

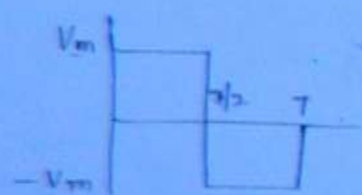
Power System,

11 kV, 33 kV, 66 kV, 132 kV, 220 kV. 3 multiples of 11.

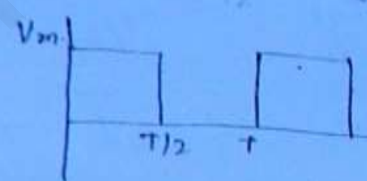
These power systems are chosen based on the form factor = 1.11.

: which of the following waveforms have form factor equal to peak factor.

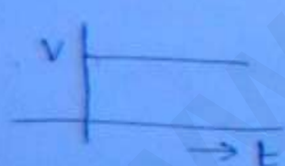
a)



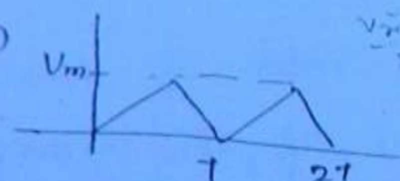
(b)



c)



(d)



a) $V_{Rms} = V_{avg} = V_m$.

form factor = 1.

peak factor = 1.

✓(c) $V_{rms} = V_{av} = V$

form factor = peak factor = 1

b) $V_{rms} = \frac{V_m}{\sqrt{2}}$, $V_{av} = \frac{V_m}{2}$

form factor = $\frac{V_{rms}}{V_{avg}} = \sqrt{2}$

peak factor. $\frac{V_m}{V_{rms}} = \sqrt{2}$

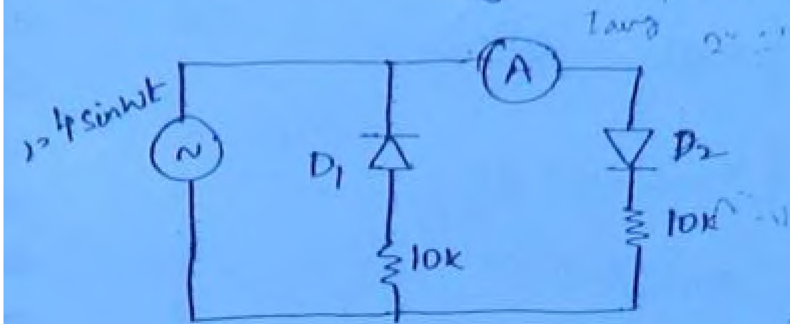
✗(d) $V_{rms} = V_m/\sqrt{3}$

$V_{avg} = V_m/2$

form factor = $2/\sqrt{3}$

Peak factor = $\sqrt{3}$

When circuit is having ideal diodes and average value of indicating ammeter. Find the value of ammeter.



(a) $\frac{0.4}{\pi}$ mA (S3)

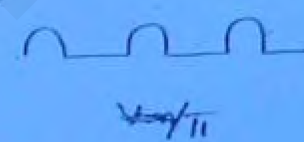
(b) $\frac{0.8}{\pi}$ mA

(c) $\frac{0.2}{\pi}$ mA

(d) $\frac{1.6}{\pi}$ mA

current flows in D2 branch only for +ve ½ cycle.

∴ we get half wave output.



$$o/p = V_{av} = \frac{V_m}{\pi} = \frac{4}{\pi}$$

$$I_{av} = \frac{V_{av}}{R} = \frac{4/\pi}{10k} = \frac{0.4}{\pi} \text{ mA}$$

FORM FACTOR

Form factor is the ratio of RMS value of the waveform to average value of the waveform.

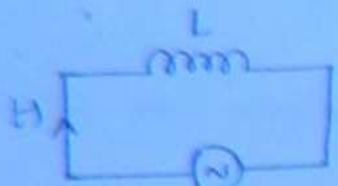
$$\boxed{\text{form factor} = \frac{V_{Rms}}{V_{avg}}}$$

PEAK FACTOR

Peak factor is the ratio of max. value of the waveform to RMS value of the waveform.

$$\boxed{\text{Peak factor} = \frac{V_m}{V_{Rms}}}$$

AC source across inductor —



$V(t) = V_m \sin \omega t$

$$i_L = \frac{1}{L}\int v\,dt$$ (54)

$$i_L = \frac{1}{L}\int V_m \sin \omega t\,dt$$

$$i_L = -\frac{V_m}{\omega L}\cos \omega t$$

$$= \frac{V_m}{\omega L}\sin(\omega t - 90) \qquad (X_L = \omega L).$$

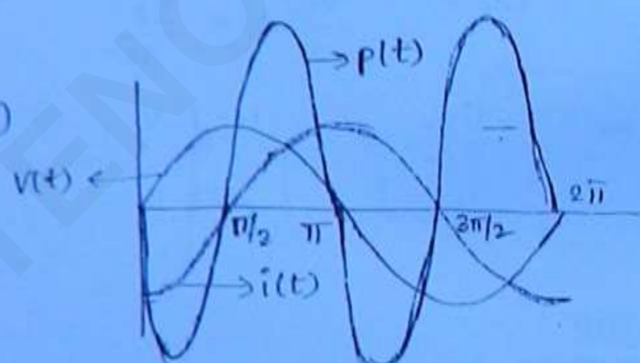
$$i_L = I_m \sin(\omega t - 90) \qquad \therefore X_L = \omega L;\ I = \frac{V}{X_L}$$

$$p = v(t)\, i(t)$$

$$= V_m \sin \omega t \cdot I_m \sin(\omega t - 90^\circ)$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} p(t)\, d\omega t$$

$$\boxed{P_{avg} = 0.}$$



p(t) freq = double v(t)

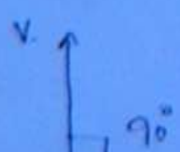
:-

...ing the +ve 1/2 cycle of the power, inductor takes energy from the source & in -ve 1/2 cycle of the power, inductor delivers energy to the source.

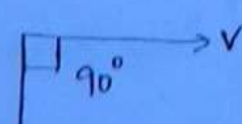
Thereby, net power taken from the source = 0.

$f = 50Hz$; $f_p = 100Hz$

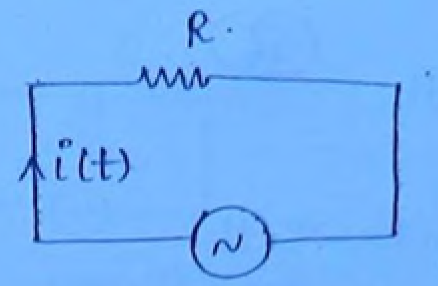
...sor diagram :



(or)



AC source across resistance



$v(t) = V_m \sin \omega t$

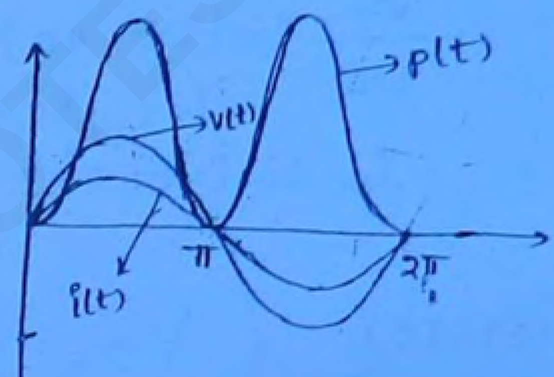
$i(t) = v(t)/R$

$i(t) = \dfrac{V_m \sin \omega t}{R}$

$i(t) = I_m \sin \omega t$

$P(t) = v(t)\, i(t)$

$= (V_m \sin \omega t)(I_m \sin \omega t)$



$$P(t) = \frac{V_m I_m}{2}\left[1 - \cos 2\omega t\right]$$

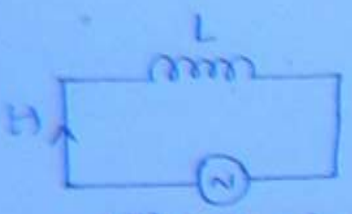
$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} P(t)\, d\omega t$$

$$\boxed{P_{avg} = {}^{*}\ \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}}$$

$f = 50\,Hz \Rightarrow f_P$ (power frequency) $= 100\,Hz$

∵ when voltage completes one cycle from $0 - 2\pi$, power completes 2 cycles ∴ it has double the freq. of voltage.

AC source across inductor –



$V(t) = V_m \sin \omega t$

$$i_L = \frac{1}{L}\int v\,dt$$

(56)

$$i_L = \frac{1}{L}\int V_m \sin \omega t\,dt$$

$$i_L = -\frac{V_m}{\omega L}\cos \omega t$$

$$= \frac{V_m}{\omega L}\sin(\omega t - 90)$$

$(X_L = \omega L)$.

$$i_L = I_m \sin(\omega t - 90)$$

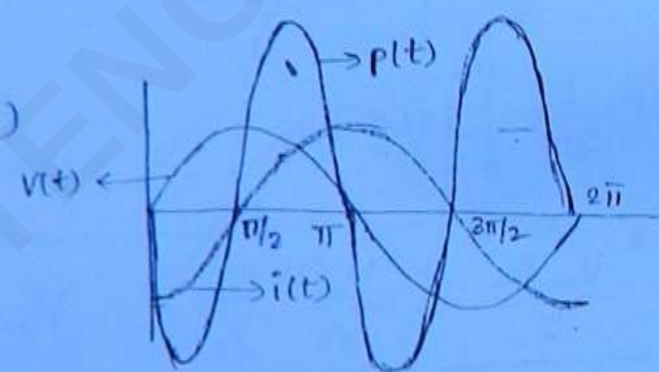
$\therefore X_L = \omega L;\ I = \frac{V}{X_L}$

$) = V(t)\,i(t)$

$= V_m \sin \omega t \cdot I_m \sin(\omega t - 90^\circ)$

$$av = \frac{1}{2\pi}\int_0^{2\pi} p(t)\,d\omega t$$

$$\boxed{P_{avg} = 0.}$$



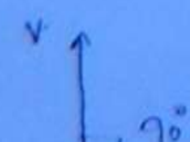
p(t) freq = double v(t)

:-

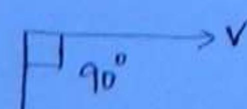
ing the ½ cycle of the power, inductor takes energy
om the source & in –ve ½ cycle of the power,
nductor delivers energy to the source.
hereby, net power taken from the source = 0.

$f = 50\,Hz$ & $f_p = 100\,Hz$

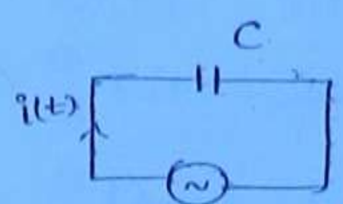
usor diagram :



(or)



AC source across Capacitor



$v(t) = V_m \sin\omega t$

$$i = C\frac{dv}{dt}$$

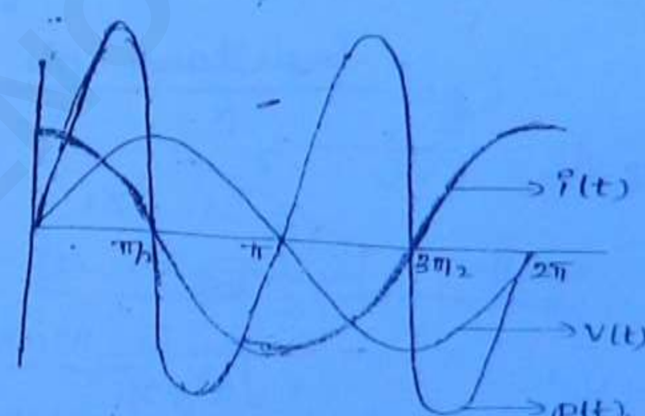
$$= C\frac{d}{dt}(V_m \sin\omega t)$$

$$i = \omega C\, V_m \cos\omega t$$

$$i(t) = \omega C\, V_m \cos\omega t$$

$$i(t) = \frac{V_m}{1/\omega C}\sin(\omega t + 90) \qquad (\because X_C = 1/\omega C)$$

$$i(t) = I_m \sin(\omega t + 90^\circ)$$



$$p(t) = v(t)\, i(t)$$

$$p(t) = V_m \sin\omega t \cdot I_m \cos\omega t$$

$$p(t) = \frac{V_m I_m}{2}\sin 2\omega t$$

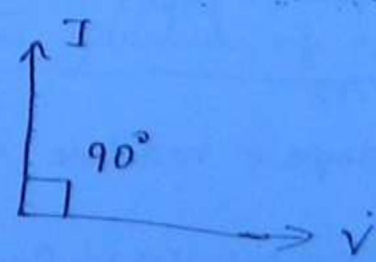
$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} p(t)\, d\omega t$$

$$\boxed{P_{avg} = 0}$$

$f = 50\,Hz$ — voltage

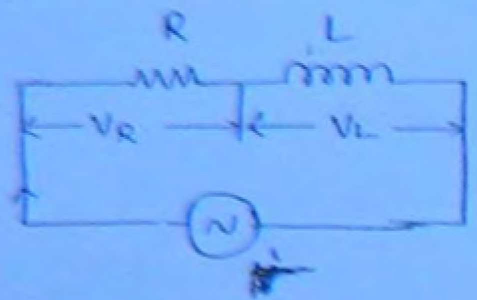
$f_P = 100\,Hz \longrightarrow$ power

phasor diagram :

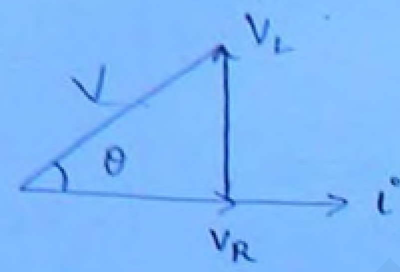


R-L Series Circuit

(58)



phasor



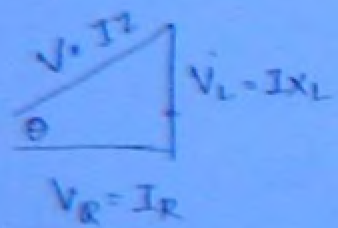
By KVL,

$$V = V_R\angle 0^\circ + V_L\angle 90^\circ$$

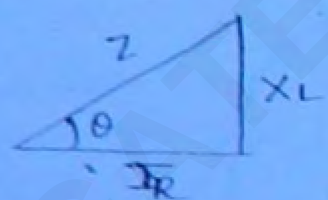
$$IZ = IR\angle 0^\circ + IX_L\angle 90^\circ$$

$$IZ = I(R + jX_L) \Rightarrow \boxed{Z = R + jX_L}$$

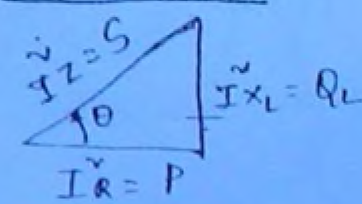
age Δle



impedance Δle



power Δle



$$V = \sqrt{V_R^2 + V_L^2} \qquad Z = \sqrt{R^2 + X_L^2} \qquad S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right) \qquad \theta = \tan^{-1}\left(\frac{X_L}{R}\right) \qquad \theta = \tan^{-1}\left(\frac{Q_L}{P}\right)$$

$\Rightarrow$ P $\rightarrow$ active power (or)
(W). True " (or)
real "
avg. "
effective power

$Q_L \rightarrow$ Inductive reactive power

$Q_L \rightarrow$ unit $\rightarrow$ VAR
(volt. ampere reactive) $\leftarrow$

$S \rightarrow$ Apparent power (or) Complex power

$\rightarrow$ $S \rightarrow$ VA (volt ampere).

Power Expressions

59

$$p(t) = v(t)\, i(t)$$

$$p(t) = I_m \sin\omega t \cdot V_m \sin(\omega t + \theta)$$

$$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} p(t)\, d\omega t$$

$$\boxed{\begin{aligned} P_{avg} &= \frac{V_m I_m}{2}\cos\theta \\ P_{avg} &= \frac{V_m}{\sqrt{2}}\cdot\frac{I_m}{\sqrt{2}}\cos\theta = V I \cos\theta \end{aligned}}$$

V & I are Rms values in any ac ckt.

power factor $= \cos\theta = \frac{V_R}{V_T} = \frac{R}{Z} = \frac{P}{S}$ (lag)

wrt V i lags

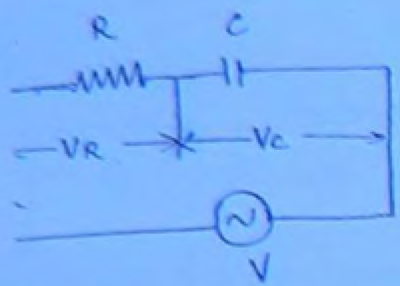
→ While defining power factor for any circuit, voltage phasor is taken as a reference. Since,

i) In the real time systems, only independant voltage source exists. (source voltage constant)

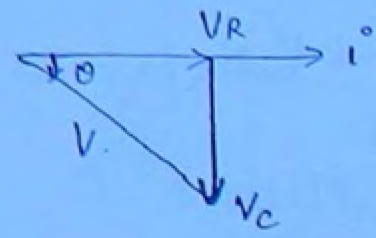
ii) In the real time system, loads are connected in parallel. (voltage is constant).

⇒ Power factor angle indicates position of current phasor wrt voltage phasor.

C Series Circuit



By phasor



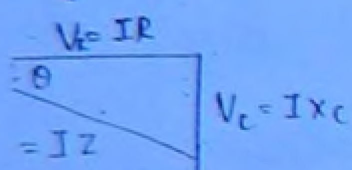
KVL,

$= V_R + V_C \angle -90^\circ$

$Z = IR\angle 0^\circ + IX_C \angle -90^\circ$

$IZ = I(R - jX_C) \Rightarrow \boxed{Z = R - jX_C}$

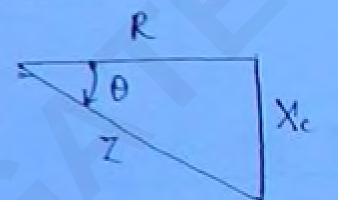
Voltage Δle



$\sqrt{V_R^2 + V_C^2}$

$\tan^{-1}\left(\frac{-V_C}{V_R}\right)$

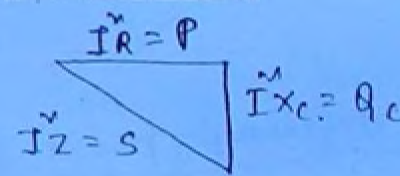
Impedance Δle



$Z = \sqrt{R^2 + X_C^2}$

$\theta = \tan^{-1}\left(\frac{-X_C}{R}\right)$

Power Δle



$S = \sqrt{P^2 + Q_C^2}$

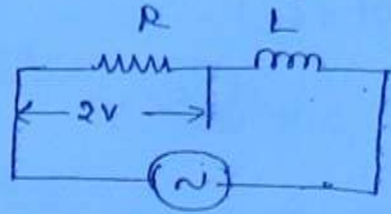
$\theta = \tan^{-1}\left(\frac{-Q_C}{P}\right)$

-ve ∴ CW

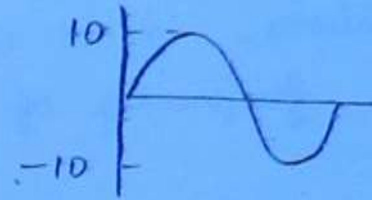
Powerfactor $= \cos\theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$ (lead) (w.r.t V)

Q. Find voltage across the inductor of the ckt shown :

G1



20 V p-p



$\frac{10\,rms}{\sqrt{2}}$ = rms

$V_m = 10$

$V_{rms} = 10/\sqrt{2} = V$

$V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2}$

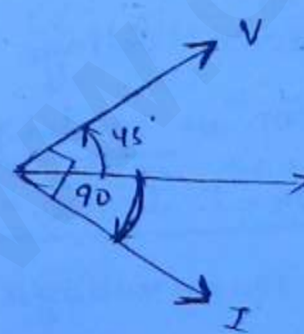
$\left(\frac{10}{\sqrt{2}}\right)^2 - 2^2$

$V_L = \sqrt{46}$

24/6/2011

Q. Find the circuit elements for a given current and voltage eqn

$v(t) = 9\sin(t + 45)$ $\quad$ $i(t) = 3\sin(t - 45^\circ)$



wrt I , V is leading by 90

∴ it is an inductor .

$X_L = \frac{V}{I}$

$= \frac{9/\sqrt{2}}{3/\sqrt{2}} = 3.$

$X_L = 3 \Rightarrow \omega L = 3$

$\omega = 1$ $\quad$ $L = 3H.$

1. Find circuit elements for a given voltage and current eqns.

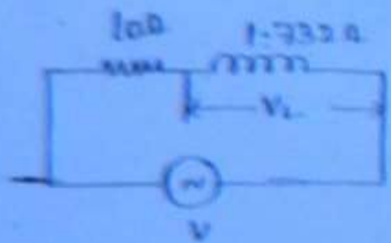
$v(t) = 9\sin(t+30^\circ)$ $\quad$ $i(t) = 3\sin(2t+60^\circ)$

(D)

Note :-

For the above equations, it is not possible to design the circuit. since, frequency of voltage and current are unequal.

2. Find angle made by source voltage wrt V_L.



(a) 30, (b) 60

(c) −30 (d) −60

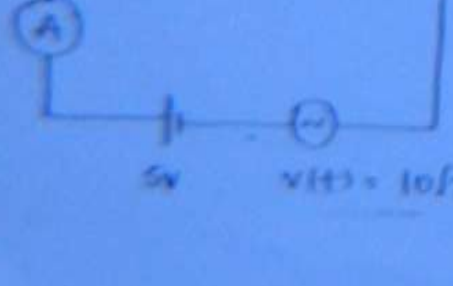
$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{17.32}{10}\right) = \tan^{-1}(\sqrt{3})$

$\theta = 60^\circ$

wrt. V_L angle made by source voltage is 30°. The angle vector is rotating in clock wise direction. ∴ it is −ve

Thus the angle made by source voltage wrt to V_L is −30°

3. Find ammeter reading of the circuit shown.



Apply Superposition theorem.

D.C.

$C \longrightarrow O.C \qquad \therefore \ i_{DC} = 0$.

A.C.

$X_C = \frac{1}{\omega C} = 1$.

$Z = \sqrt{R^2 + X_C^2} = \sqrt{1+1} = \sqrt{2}$.

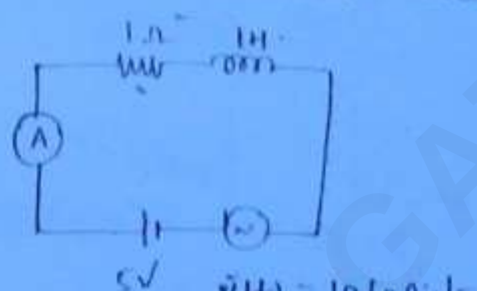
$I_{AC} = \frac{V}{Z} = \frac{10}{\sqrt{2}}$

$V = V_{rms} = \frac{10\sqrt{2}}{\sqrt{2}}$

$V = 10$

ammeter reading $= \frac{10}{\sqrt{2}} + 0 = \frac{10}{\sqrt{2}}$ A.

Q. Find the ammeter reading of the circuit shown.



Sol.

$I_{DC} = \frac{5}{1} = 5A$. $\quad L \to S.C. \qquad$ Superposition

$X_L = \omega L = 1\Omega$.

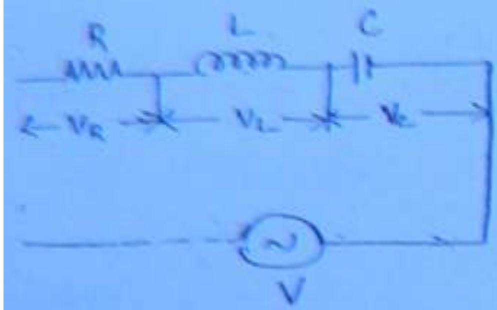
$Z = \sqrt{R^2 + X_L^2} = \sqrt{1+1} = \sqrt{2}$.

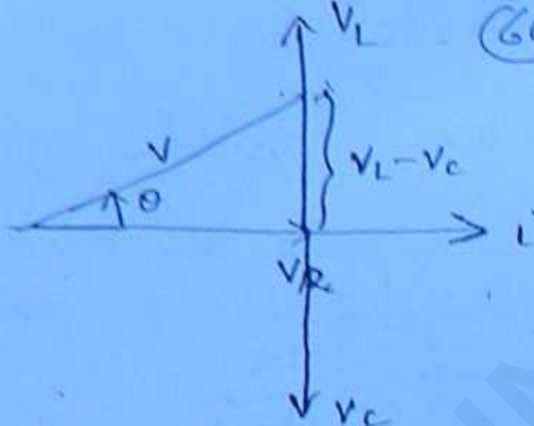
$I_{AC} = \frac{V}{Z} = \frac{10}{\sqrt{2}}$

$\because$ Two different frequencies are present we cannot add i_{DC} & i_{AC} for total current

$\therefore \ I = \sqrt{I_{DC}^2 + I_{AC}^2} = \sqrt{5^2 + (10/\sqrt{2})^2}$

RLC Series Circuit





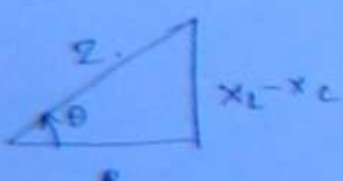
By KVL,

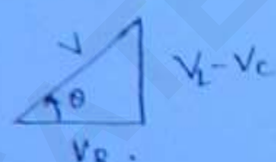
$$V = V_R\angle 0 + V_L\angle 90^\circ + V_C\angle -90^\circ$$

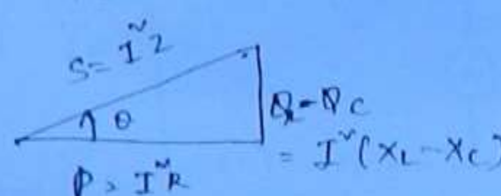
$$IZ = I[R + j(X_L - X_C)]$$

$$Z = R + j(X_L - X_C).$$

Impedance Δle | Voltage Δle | Power Δle



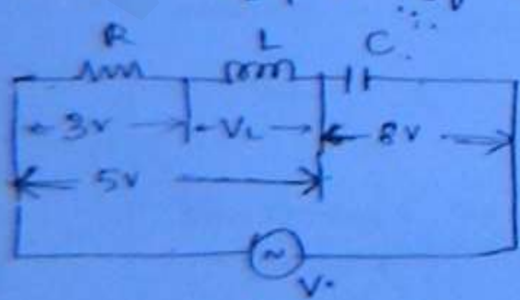




$$Z = \sqrt{R^2 + (X_L - X_C)^2} \rightarrow V = \sqrt{V_R^2 + (V_L - V_C)^2} \rightarrow S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad \theta = \tan^{-1}\left(\frac{V_L - V_C}{R}\right). \qquad \theta = \tan^{-1}\left(\frac{Q_L - Q_C}{P}\right).$$

Find V_L & V of the circuit shown.



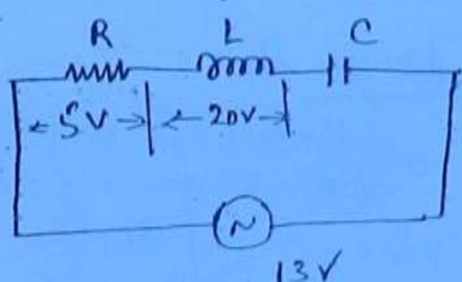
$5 = \sqrt{V_L^2 + 3^2} \Rightarrow V_L = 4.$

$\cancel{V = \sqrt{3^2 + V_L^2 + 5^2} = \sqrt{3}}$

$\rightarrow V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{3^2 + (4-8)^2}$

$V = 5V$

Q. Find voltage across capacitor.



(a) 8V (b) 32V

(c) 8 or 32V (d) 10V

$13^2 = 5^2 + (20 - V_C)^2$

$V^2 = V_R^2 + (V_L - V_C)^2.$

$(V_L - V_C)^2 = V^2 - V_R^2$

$(20 - V_C)^2 = 13^2 - 5^2$

$20 - V_C = \pm 12.$

for

$20 - V_C = 12$

$V_C = 8V.$

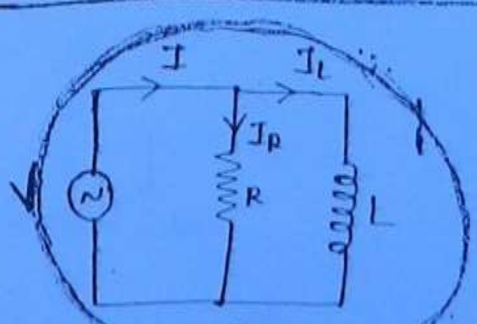
&

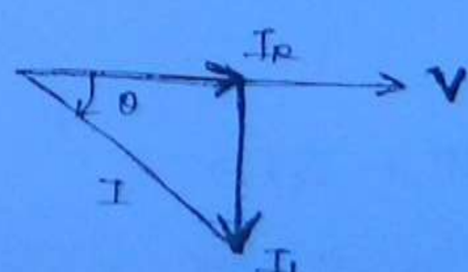
$20 - V_C = -12$

$32 = V_C.$

$\therefore V_C = 8 \text{ (or) } 32V.$

RL parallel circuit





By kcl

$$I = I_R \angle 0 + I_L \angle -90^\circ$$

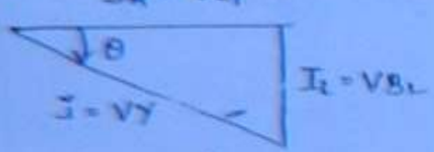
$$\frac{V}{Z} = \frac{V}{R} \angle 0^\circ + \frac{V}{X_L} \angle -90^\circ$$

$$VY = VG - jVB_L \Rightarrow \boxed{Y = G - jB_L}$$

℧ mho (or) S.

mho (or) S

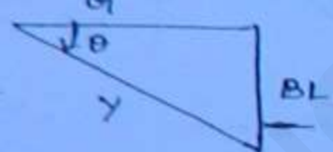
Current Δle.



$$I = \sqrt{I_R^2 + I_L^2}$$

$$\theta = \tan^{-1}\left(\frac{-I_L}{I_R}\right)$$

admittance Δle



$$Y = \sqrt{G^2 + B_L^2}$$

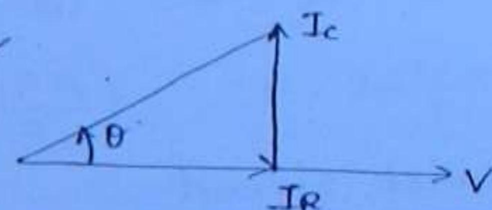
$$\theta = \tan^{-1}\left(\frac{-B_L}{G}\right)$$

power Δle.

$$S = \sqrt{P^2 + Q_L^2}$$

$$\theta = \tan^{-1}\left(\frac{-Q_L}{P}\right)$$

RC - parallel circuit



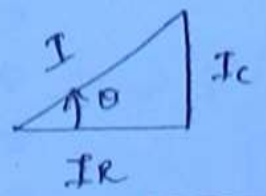
By kcl,

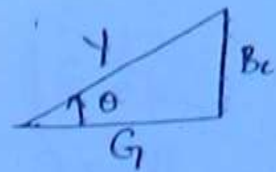
$$I = I_R \angle 0^\circ + I_C \angle 90^\circ$$

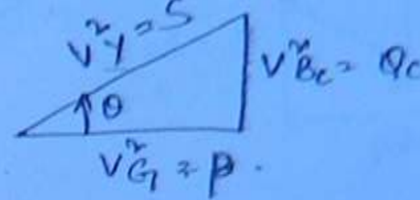
$$\frac{V}{Z} = \frac{V}{R} \angle 0^\circ + \frac{V}{X_C} \angle 90^\circ$$

$$VY = VG + jVB_C$$

current $\triangle$le.   admittance $\triangle$le.   Power $\triangle$le



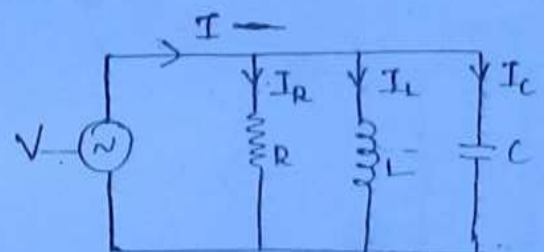


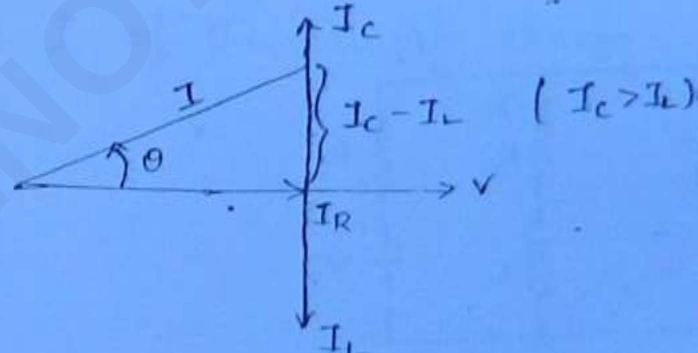


$I = \sqrt{I_R^2 + I_C^2}$ $Y = \sqrt{G^2 + B_C^2}$ $S = \sqrt{P^2 + Q_C^2}$

$\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right)$. $\theta = \tan^{-1}\left(\frac{B_C}{G}\right)$. $\theta = \tan^{-1}\left(\frac{Q_C}{P}\right)$.

RLC parallel Circuit





$(I_C > I_L)$

By KCL.

$I = I_R\angle 0^\circ + I_L\angle -90^\circ + I_C\angle 90^\circ$

$\frac{V}{Z} = \frac{V}{R}\angle 0^\circ + \frac{V}{X_L}\angle -90^\circ + \frac{V}{X_C}\angle 90^\circ$

$VY = VG - jVB_L + jVB_C$

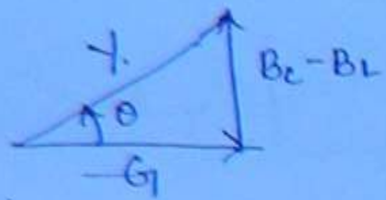
$$\boxed{Y = G + j(B_C - B_L)} \Rightarrow$$

current $\triangle$le.

$I = \sqrt{I_R^2 + (I_C - I_L)^2}$

$\theta = \tan^{-1}\left(\frac{I_C - I_L}{\cdots}\right)$

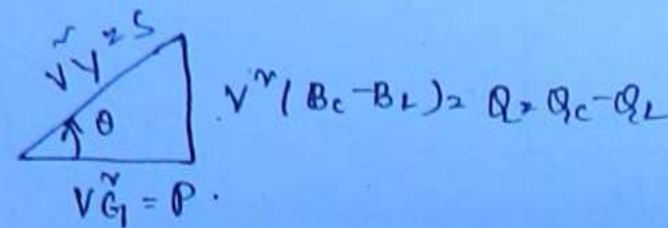
admittance Δle.



$$Y = \sqrt{G + (B_C - B_L)^2}$$

$$\theta = \tan^{-1}\left(\frac{B_C - B_L}{G}\right).$$

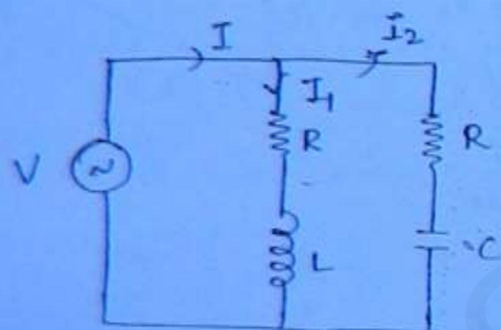
power Δle.



$$S = \sqrt{P^2 + Q^2}$$

$$\theta = \tan^{-1}\left(\frac{Q}{P}\right).$$

RC parallel.



$$I_1 = \frac{V}{R + jX_L} \times \frac{R - jX_L}{R - jX_L}$$

$$\frac{V}{Z_1} = V\left[\frac{R_1}{R_1^2 + X_L^2} - \frac{jX_L}{R_1^2 + X_L^2}\right] \Leftarrow$$

** **

$$VY_1 = V\left[G - jB_L\right]$$

$$Y_1 = G_1 - jB_L$$

$$I_2 = \frac{V}{R_2 - jX_C} \cdot \frac{R_2 + jX_C}{R_2 + jX_C}$$

$$\frac{V}{Z_2} = V\left[\frac{R_2}{R_2^2+X_C^2} + j\frac{X_C}{R_2^2+X_C^2}\right].$$

$$VY_2 = V\,(G_2 + jB_C)$$

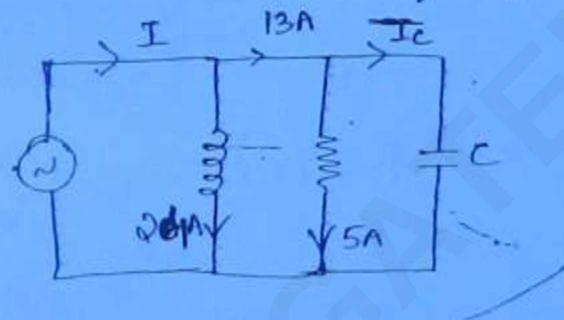
$$\boxed{Y_2 = G_2 + jB_C}$$

$$I = I_1 + I_2$$

$$V\,Y_{eq} = VY_1 + VY_2$$

$$\boxed{Y_{eq} = Y_1 + Y_2}$$

Q. Find the value of I_C & I of the circuit shown.



Soln.

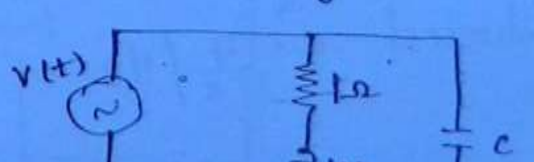
$$13 = \sqrt{I_R^2 + I_C^2} = \sqrt{5^2 + I_C^2}$$

$$I_C = 12A$$

$$\Rightarrow I = \sqrt{I_R^2 + (I_C - I_L)^2} = \sqrt{5^2 + (24-12)^2}$$

$$I = 13A.$$

Q. Find the value of C when power factor of the ckt is 0.8 lag.



$X_L = \omega L$

$X_L = 1$

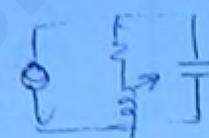
$$\tan\theta = \frac{B_C - B_L}{G}$$

$$G_1 = \frac{R_1}{R_1^2 + X_L^2} = \frac{1}{1^2 + 1^2} = 1/2$$

$$B_L = \frac{X_L}{R_1^2 + X_L^2} = \frac{1}{1^2+1^2} = 1/2$$

$Y_1 = G_1 - j B_L$

$Y_1 = 1/2 - j\,1/2$



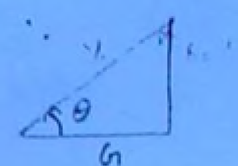
$I = I_1 + j I_2$

$Y_{eq} = Y_1 + Y_2$

$Y_2 = j B_C = j\omega C = jC$

$Y_2 = jC$

$Y_{eq} = Y_1 + Y_2 \Rightarrow Y_{eq} = \frac{1}{2} + j\left(C - 1/2\right)$



$$\Rightarrow \cos\theta = \frac{G}{\sqrt{G^2 + (B_C - B_L)^2}}$$

$$0.8 = \frac{1/2}{\sqrt{\left(\frac{1}{2}\right)^2 + \left(C - 1/2\right)^2}} \Rightarrow C = \frac{7}{8}, \frac{1}{8}$$

But power factor is 0.8 LAG. Which means the ckt is inductive in nature. $\therefore$ $B_L > B_C$

$B_L = 1/2$ $\therefore$ for $C = 1/8$ satisfies $B_L > B_C$.

if 0.8 lead $\Rightarrow C = 7/8$

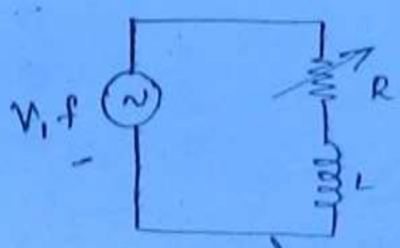
if neither lead nor lag is mentioned $C = \frac{7}{8} / \frac{1}{8}$

Locus Diagram

→ Locus diagrams are useful for analysis and designing of the circuits. Ex. filters.

→ The path traced by terminals of the current vectors by varying anyone of the circuit elements (or) by varying frequency is called as current locus.

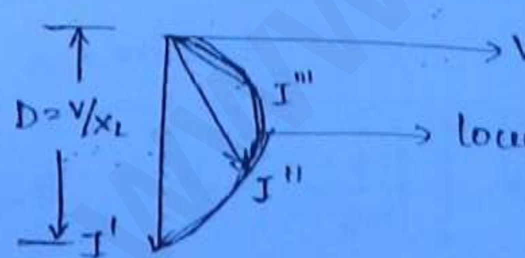
Q. Draw the current locus of the circuit shown.



$R = 0$ $Z = X_L$ $I = V/X_L$ $\theta = 90°$

$R \uparrow$ $Z \uparrow$ $I \downarrow$ $\theta = \tan^{-1}\left(\frac{X_L}{R}\right) \downarrow$

$R = \infty$ $I = 0$

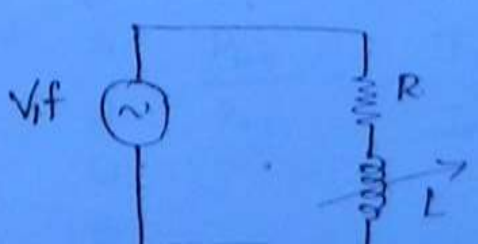


→ locus of I

$D \rightarrow$ diameter of locus.

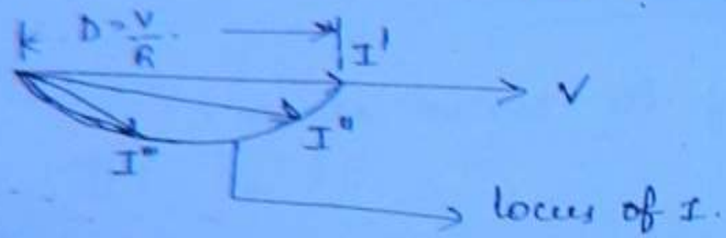
radius $r = \frac{V}{2X_L}$

Q. Draw the current locus of the ckt shown.



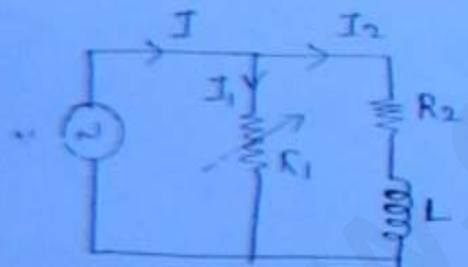
$X_L = 2\pi f L$.

$X_L = 0$	$X_L \uparrow$	$X_L = \infty$
$Z = R$.	$Z \uparrow$	$I = 0$.
$I = \frac{V}{R}$.	$I \downarrow$	
$\theta = 0$.	$\theta = \tan^{-1}\left(\frac{X_L}{R}\right) \uparrow$	



For the above ckt if R & L are kept constant & freq f is varied, then also the locus is the same.

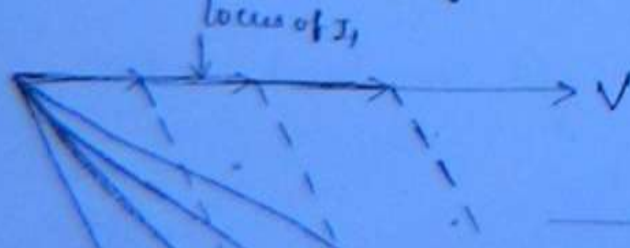
) Draw the current locus of i_1 and i of the ckt shown.



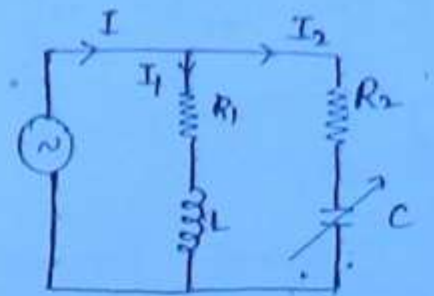
$I_2 = \frac{V}{R_2 + jX_L}$: const.

let $R = 0$-1Ω

$I_1 = \frac{V}{R_1}$	$R_1 \uparrow$	$R_1 = \infty$
$\theta_1 = 0$	$I_1 \downarrow$	$I_1 = 0$
$I = I_1 + I_2$	$\theta_1 = 0$	$I = I_2$
	$\therefore I_2 =$ Const.	
	$I \downarrow$	



Q. Draw the current locus of I_2 & I of the ckt shown.



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$I_1 = \frac{V}{R_L + jX_L}$

$I_2 = \frac{V}{R_2 - jX_C}$ ↑

$\tan^{-1}\left(\frac{X_C}{R_2}\right)$

$X_C = \frac{1}{2\pi f C}$

$X_C \simeq 0$

$I_2 = \frac{V}{R_2}$

$\theta_2 = 0$

$I = I_1 + I_2$

$X_C \uparrow$

$I_2 \downarrow$

$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$

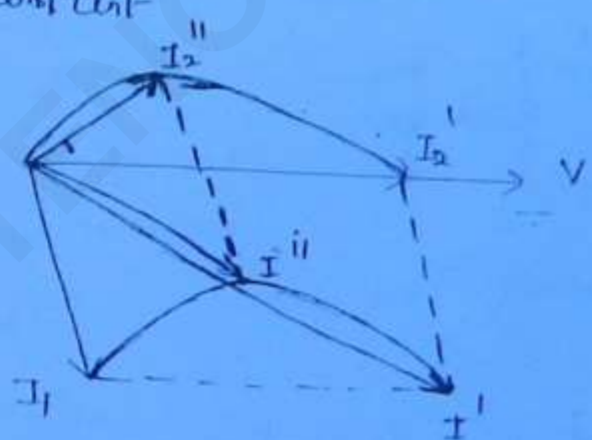
I_1 = constant

$I \downarrow$

$X_C = \infty$

$I_2 = 0$

$I = I_1$.



?/2011

Workbook

1. $V_{av} = \frac{1}{2\pi}\left[\int_0^{\pi} V\sin(\omega t + \phi)\,d\omega t + \int_{\pi}^{2\pi} 0\right]$ V

$= \frac{V}{\pi}\cos\phi$

2. $\frac{P_{D.C}}{P_{A.C}} = \frac{I_{av}^2 R}{I_{rms}^2 R} = \frac{(2I_m/\pi)^2}{(I_m/\sqrt{2})^2} = \frac{8}{\pi^2}$

5. $X_L = 20$

$X_C = 20$

$R = 10$, $V = 200$.

(74)

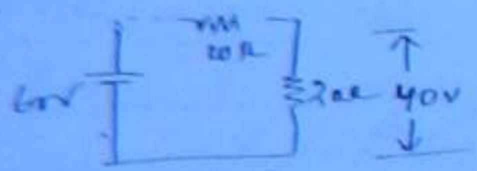
$Z = R + j(X_L - X_C) = 10$.

$I = \frac{V}{Z} = \frac{200}{10} = 20$ A.

$V_C = -jIX_C = (20)(20)\angle{-90} = 400\angle{-90^\circ}$ V.

$\therefore$ $I = \sqrt{I_R^2 + I_L^2}$

1. $V_1 = 40V$.



So, the current source has no effect on 20Ω across 60V.

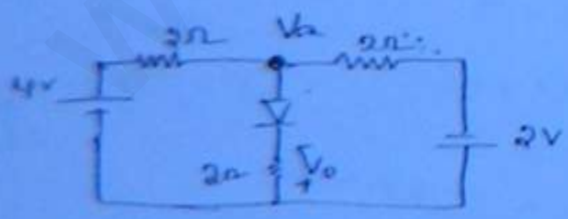
$\therefore$ whatever be the value of R, The voltage across 20Ω = 40V $\therefore$ $i_R = 0$ $\Rightarrow$ $V_1 = V_2 = 40V$

3. Current mag. is const

Voltage doubles for double resistor

$I_L = e^{at} + e^{bt}$ $\Rightarrow$ $V = L\frac{di}{dt}$ $\Rightarrow$ $V = ae^{at} + be^{bt}$

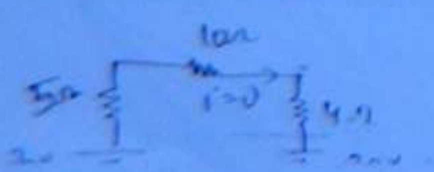
$10 + 5 + E + 1 + 0 = 0$ $\Rightarrow$ $E = -16V$.



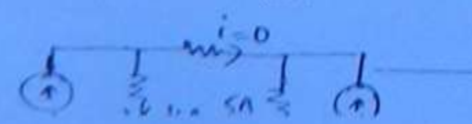
$\Rightarrow$ $\frac{Va - 4}{2} + \frac{Va}{2} + \frac{Va + 2}{2} = 0$

$Va = 2/3$.

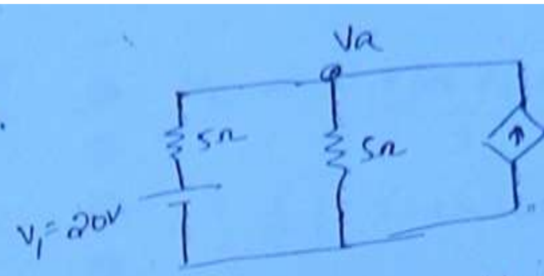
$V_0 = -Va = -2/3$.



$\Rightarrow$



22.

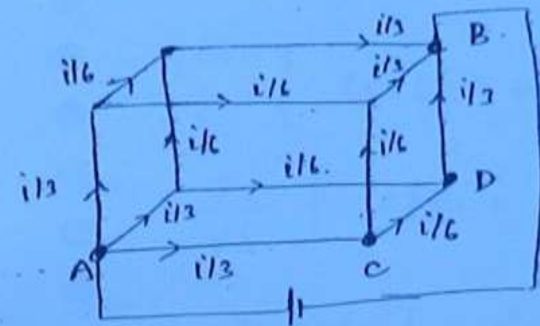


$\frac{V_1}{5} = \frac{20}{5} = 4$ $\frac{Va-20}{5} + \frac{Va}{5} = 4$

$Va = 20$.

current source ⇒ $20 \times 4 =$ 80 W deliver.

24.



Considers equal resistors of value = R.

loop ACDB.

$V = \frac{I}{3}R + \frac{i}{6}R + \frac{i}{3}R$

$\frac{V}{I} = \frac{5}{6}R = R_{AB}$

For resistors, $R_{AB} = \frac{5}{6}R$.

inductors $L_{AB} = \frac{5}{6}L$

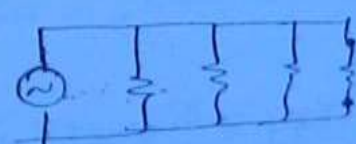
capacitors $C_{AB} = \frac{6}{5}C$

25.

variable angle → semicircle

const. " → st. line.

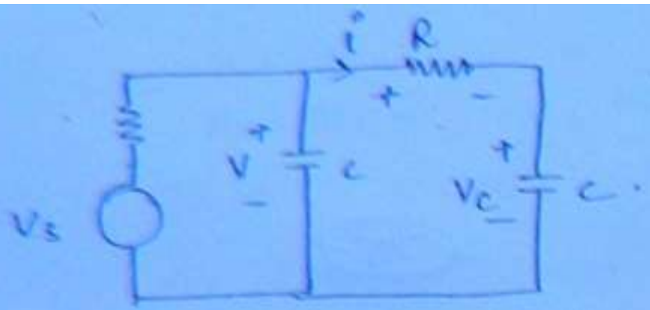
28. consider,



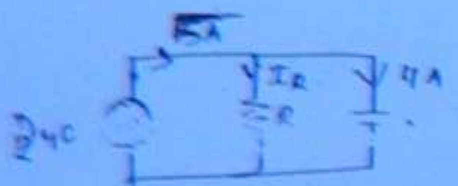
29. $I = VY = (3-2j+6j)\sin 2t = 5\sin(2t+53.1)$

30. $V_0 = V_i \frac{1/j\omega C}{R + 1/j\omega C} = \frac{V_i}{1+jRC\omega}$.

$\omega = 10^3$, $RC = 10^{-3}$ ⇒ $\frac{V_i}{1+j} = \frac{\sqrt{2}\sin 10^3 t}{1+j}$

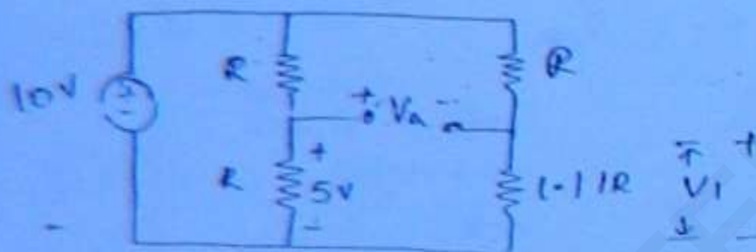


$V = iR + V_c \Rightarrow V = c\frac{dV_c}{dt} + V_c$



$I_R = \sqrt{5^2 - 4^2} = 3A$

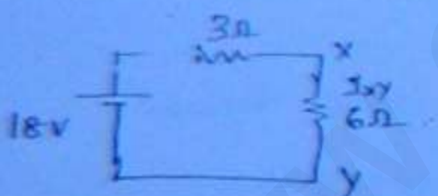
$R = \frac{240V}{3A} = 80\Omega$

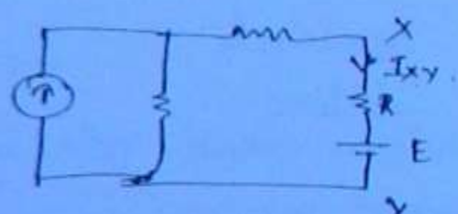


$V_1 = \frac{10 \times 1.1R}{1.1R + R}$

$V_1 = 5.238V$

$V_a = 5 - 5.238 = -0.238V$





$I_{xy} = \frac{18}{9} = 2A$

$V_{xy} = R I_{xy} + E$

$V_{xy} = 2 \times 6 = 12V$

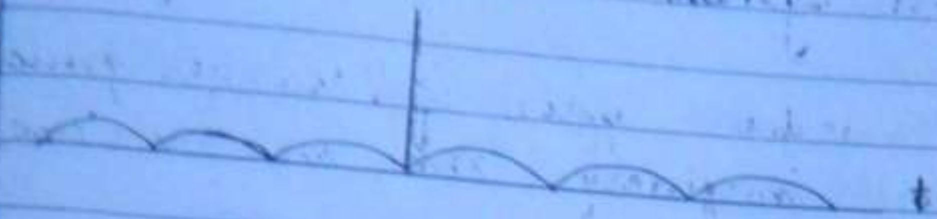
$\boxed{12 = R_2 + E}$

Substitute the given set of values and check.

For eq. ckt, load current should be equal.

equal load current $\Rightarrow$ equal load voltage.

3) $h(t) = |\sin(t)|$



Condition for LTI system to be stable

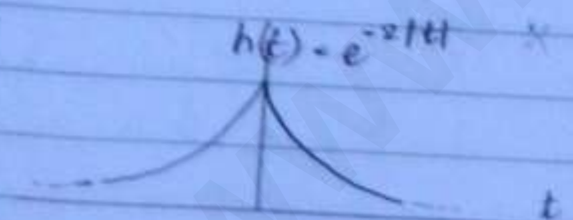
$$\boxed{\int_{-\infty}^{\infty} |h(t)|\, dt < \infty}$$

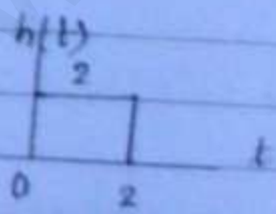
ie impulse response should be absolutely integrable

If impulse response of system is represented by an energy signal then system will be stable.

Impulse response & transfer function terms are used only for LTI systems

eg





$h(t) = Sa(t)$

The bridge is unbalanced becoz, the angles j & $-j$ are different for inductor & capacitor.

$\therefore$ Go for $\nabla \rightarrow Y$ transformation.

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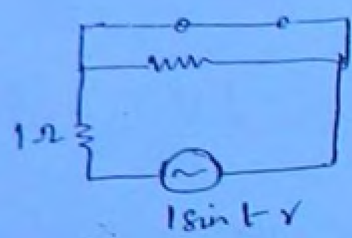
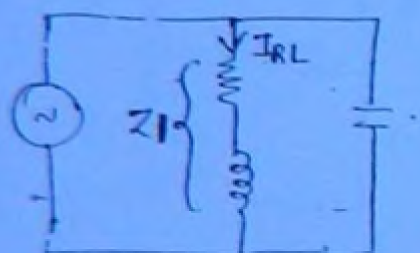
$\omega = 1$

$X_L = \omega L = 1$

$X_C = \frac{1}{\omega C} = 1 \Rightarrow j(X_L - X_C) = 0 \rightarrow$ S.C.



$\sqrt{2} \cdot \frac{1+j}{\sqrt{2}} (1+j) + \frac{1-j}{2}$



$$Z_1 = \frac{V_s}{I_{RL}}$$

$$Z_1 = \frac{1\angle 0}{\sqrt{2}\angle -45} = \frac{1}{\sqrt{2}} \angle 45$$

$$Z_1 = \frac{\cos 45}{\sqrt{2}} + j\frac{\sin 45}{\sqrt{2}} = \frac{1}{2} + j\frac{1}{2} = R + jX_L$$

$\therefore R = 1/2 \Rightarrow P = I_{rms}^2 R_L = (\sqrt{2})^2 (1/2) = \underline{1W}$

phasor sum is done with rms values.

$I_s = I_{RL} + I_c$ } $\rightarrow$ phasor sum $\rightarrow$ RMS

$i_{rms} = \sqrt{2}$ — by default

I_c

Note: In real time systems, the voltage or the current values are given in rms values.

$\therefore$ if nothing is specified, by default we take it as rms value.

Conventional Ques.

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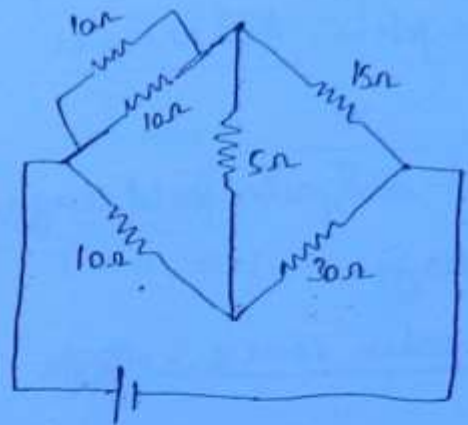
1.



Balanced bridge.

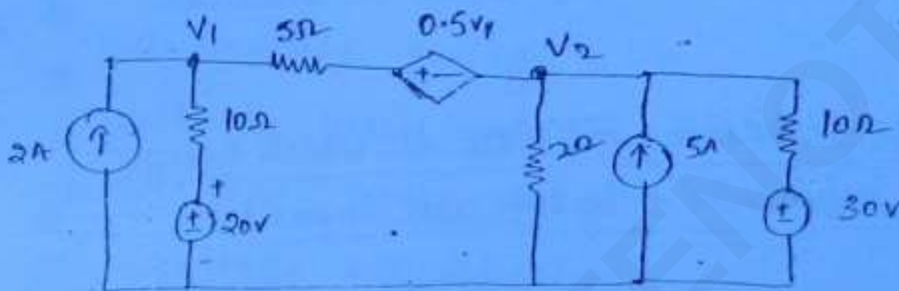
$5 \times 30 = 10 \times 15$.

$150 = 150$ V

$\therefore \; i_{5\Omega} = 0$.

2.

IES 2009



By kcl, at node 1

$$2 = \frac{V_1 - 20}{10} + \frac{V_1 - 0.5V_1 - V_2}{5} \rightarrow ①$$

at node 2.

$$5 = \frac{V_2}{2} + \frac{V_2 - 30}{10} + \frac{V_2 + 0.5V_1 - V_1}{5} \rightarrow ②$$

$2 = 0.1V_1 - 2 + 0.1V_1 - 0.2V_2$

$4 = 0.2V_1 - 0.2V_2 \Rightarrow V_1 - V_2 = 20$

$5 = 0.5V_2 + 0.1V_2 - 3 + 0.2V_2 - 0.1V_1$

$8 = 0.8V_2 - 0.1V_1 \Rightarrow 8V_2 - V_1 = 80$

$8V_2 - 20 + V_2 = 80$

Resonance.

For occurrance of resonance in any system, two energies are required.

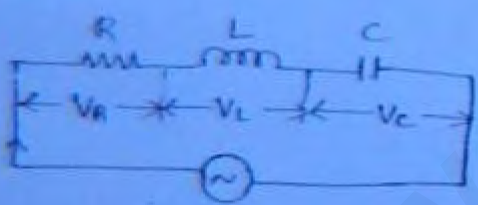
In RLC circuit, inductor consists of magnetic field energy, and capacitor consists of electric field energy.

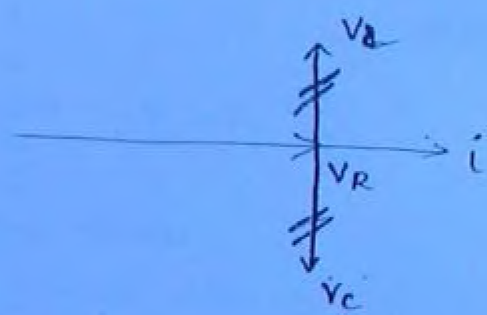
The circuit is said to be resonant, when source voltage and source current are in-phase.

The frequency at which $X_L = X_C$ is called as resonant frequency.

The resonant frequency indicates the rate at which energy transformation is done between inductor and capacitor.

series resonance





$$V = V_R\angle 0 + V_L\angle 90 + V_C\angle -90$$

for resonance,

$$V_L = V_C$$

$$IX_L = IX_C$$

$$\omega L = \frac{1}{\omega C} \Rightarrow$$

$$\boxed{\begin{array}{l} \omega_0 = \frac{1}{\sqrt{LC}}\ \text{rad/sec} \\ f_0 = \frac{1}{2\pi\sqrt{LC}}\ \text{Hz} \end{array}}$$

resonant freq, $f_0 = \frac{1}{2\pi\sqrt{LC}}$ Hz.

(81)

1. $Z = R + j(X_L - X_C)$

$$\boxed{Z_{min} = R}$$

2. $$I_{max} = \frac{V}{Z_{min}} = \frac{V}{R}$$

3. $\cos\theta = 1$ (∴ power factor = 1 ∵ in phase).

4. $V_R = V$.
(active component of the voltage)

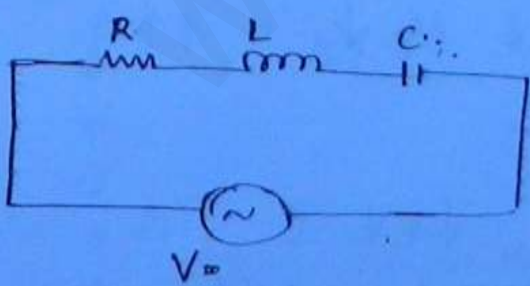
$V_R = V \rightarrow i$

5. Net reactive voltage = 0.

6. Voltage across inductor and voltage across capacitor greater than source voltage. This phenomena is called as voltage magnification

Applications

1. Oscillators
2. Filters (Band pass & Band elimination filter).
3. Tuning circuits.
4. Induction heating.



$V_C = I X_C$

$X_L = 2\pi f L$

$X_C = \dfrac{1}{2\pi f C}$

* The graph & derivation for f_1 & f_2 are [illegible]

let $f \to 0$ & f is varied (increased).

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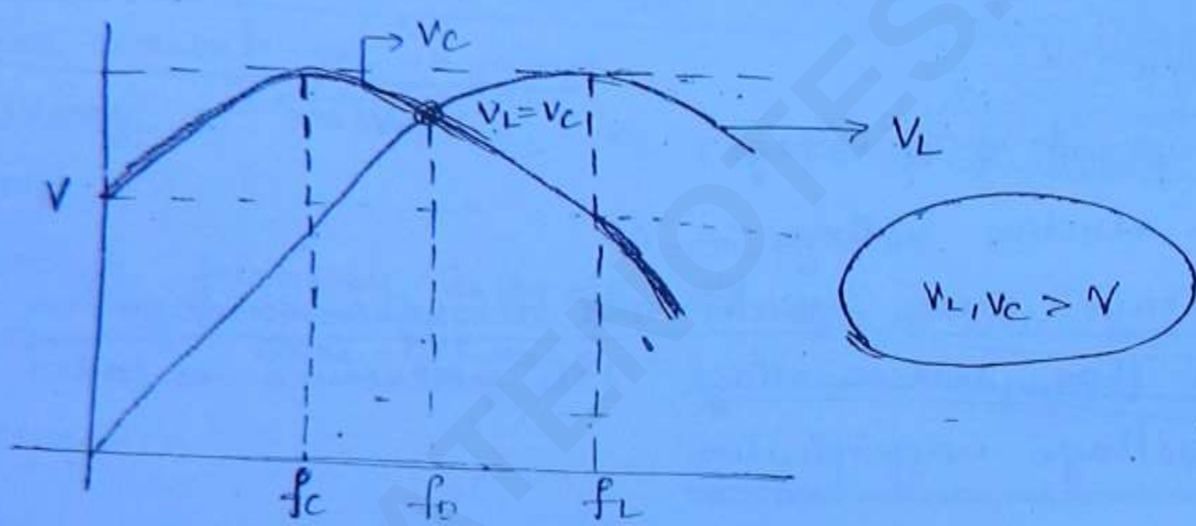
$X_L = 0$ $\quad |X_L - X_C| = \infty = |X|$.

$X_C = \infty$ $\quad \hookrightarrow Z = \infty, \quad I = 0$.

$\therefore \boxed{V_C = V}$ at $f = 0$.



$f\uparrow$, $X_L \uparrow$ $X_C \downarrow$ $|X| = |X_L - X_C| \downarrow\downarrow$

(10Ω) (100Ω) 90Ω

$Z \downarrow\downarrow \to \uparrow\uparrow I = \dfrac{V}{Z\downarrow\downarrow} \Rightarrow V_C \uparrow$

higher frequencies,

$f\uparrow\uparrow$ $X_L \uparrow\uparrow$ $X_C \downarrow\downarrow$ $Z\uparrow\uparrow$ $I\downarrow\downarrow$ $V_C\downarrow$

$\because$ (very low)

$V_C = I X_C$.

$$V_C = \frac{V X_C}{Z = \sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow V_C = \frac{V \cdot 1/\omega C}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \to (C)$$

differentiate eq ① wrt ω. & equate it to zero.

we obtain,

Series resonance

$$\boxed{f_c = \frac{1}{2\pi\sqrt{LC}} \sqrt{1-\left(\frac{R^2C}{2L}\right)}}$$

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for V_L:

$\frac{1}{2\pi\sqrt{LC}} \sqrt{1-\frac{R^2C}{2L}}$

$V_L = IX_L$

$X_L = 2\pi f L$

$X_C = \frac{1}{2\pi f C}$

for $f = 0$;

$X_L = 0$

$X_C = \infty$

$|X| = |X_L - X_C|$

$= \infty$.

$Z = \infty$, $I = 0$.

$\rightarrow \therefore \boxed{V_L = 0}$ at $f=0$.

for low freq.

$f\uparrow$, $X_L\uparrow$ $X_C\downarrow$ $|X| = |X_L - X_C| \downarrow\downarrow$

(10Ω) (100Ω) 90Ω

$Z\downarrow\downarrow \rightarrow \uparrow\uparrow I = \frac{V}{Z\downarrow\downarrow} \Rightarrow V_L\uparrow$

for high freq.

$f\uparrow\uparrow$, $X_L\uparrow\uparrow$, $X_C\downarrow\downarrow$, $Z = R+j(X_L - X_C)\uparrow\uparrow\uparrow$

(very low)

$\therefore$ $I\downarrow\downarrow\downarrow$ & $V_L\downarrow$

$$V_L = I X_L$$

(84)

$$V_L = \frac{V X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$V_L = \frac{V \omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \rightarrow (2).$$

differentiate (2) wrt ω & equating it to zero, we get,

$$\boxed{f_L = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{1}{1-(R^2C/2L)}}}$$

Quality factor - Q-factor

$$Q = 2\pi \frac{\text{Max. energy stored in the ckt}}{\text{power dissipation per cycle.}}$$

$$i(t) = i_m \sin\omega t$$

$$v_C = \frac{1}{C}\int i(t)\,dt = \frac{1}{C}\int i_m \sin\omega t \, dt.$$

$$\boxed{v_C(t) = \frac{1}{\omega C} I_m \cos\omega t}$$

$\frac{I_m \cos\omega t}{\omega C}$

$$\omega^2 = \frac{1}{LC} \Rightarrow L = \frac{1}{\omega^2 C}.$$

$$W_g = \frac{1}{2} L i^2 + \frac{1}{2} C V_c^2$$

$$W_g = \frac{1}{2} L \left(I_m \sin \omega t \right)^2 + \frac{1}{2} C \left(\frac{-1}{\omega C} I_m \cos \omega t \right)^2$$

$$W_g = \frac{1}{2} L I_{max}^2 \sin^2 \omega t + \frac{1}{2} \cdot \cancel{C} \cdot \frac{1}{\omega^2 C^{\cancel{2}}} I_m^2 \cos^2 \omega t$$

$$W_g = \frac{1}{2} L I_{max}^2 \sin^2 \omega t + \frac{1}{2} L I_{max}^2 \cos^2 \omega t \qquad \because \frac{1}{\omega^2 C} = L$$

$$\boxed{\begin{aligned} W_g &= \frac{1}{2} L I_{max}^2 \\ W_g &= \frac{1}{2} C V_{c\,max}^2 \end{aligned}}$$

where $\boxed{V_{C max} = \frac{I_{max}}{\omega C}}$

$$Q = 2\pi \frac{\frac{1}{2} L I_{max}^2}{\left(\frac{I_{max}}{\sqrt{2}}\right)^2 R \cdot 1/f}$$

$$Q = \frac{2\pi f L}{R} = \frac{\omega L}{R}$$

$$Q = \frac{\omega L}{R} \qquad \left(\omega = \frac{1}{\sqrt{LC}} \right)$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$Q = \frac{X_L}{R} = X_C$

$\because X_L = \omega L$

$$Q = \frac{X_L}{R} = \frac{I X_L}{I R} = \frac{V_L}{V_R} = \frac{V_L}{V}$$

But $V_L = V_C$

$$Q = \frac{V_C}{V} = \frac{I X_C}{V_R} = \frac{I X_C}{I R} = \frac{X_C}{R} = \frac{1}{\ldots}$$

$$\boxed{Q = \frac{I^2 X_L}{I^2 R} = \frac{Q_L}{P}}$$

(86)

6/11

f_0

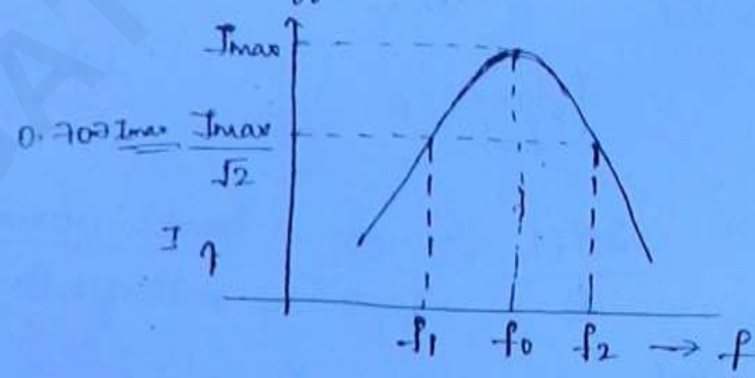
Band width

> when the curve is drawn between current vs and frequency then the curve is called as resonance curve.

Band width is a range of frequencies on either side of the resonant frequency where the current falls from maximum value to 70.7% of the max. value. It is given by,

$BW = f_2 - f_1 \rightarrow$ lower cut off

f_2 ↓ upper cut off



$f_0 \rightarrow I_{max}$, $Z = R$

$f_1, f_2 \rightarrow \frac{I_{max}}{\sqrt{2}}$, $Z = \sqrt{2} R$

$f_0 \rightarrow \cos\theta = 1$

$f_1 f_2 \longrightarrow Z = R \pm jX$

$Z = \sqrt{R^2 + X^2} = \sqrt{2} R$

$\Rightarrow X = R$

from the curve,

$X = X_L - X_C$ (87)

$X_L = 2\pi f L$, $X_C = \frac{1}{2\pi f C}$.

$f_1 \rightarrow X_C > X_L \Rightarrow X \rightarrow -ve$.

$f_2 \rightarrow X_L > X_C \Rightarrow X \rightarrow +ve$.

For f_1:

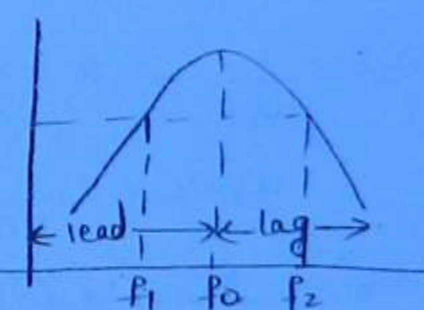
$f_1 \rightarrow Z = R - jX$, $X = R$.

Impedance angle $= \tan^{-1}\left(\frac{-X}{R}\right) = -45^\circ$

$I = \frac{V\angle 0}{Z\angle -45^\circ} = \frac{V}{Z}\angle +45^\circ$

P.f angle $= +45^\circ$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$ lead



for f_2:

$f_2 \rightarrow Z = R + jX$, $X = R$.

Impedance angle, $\tan^{-1}\left(\frac{X}{R}\right) = 45^\circ$

P.f angle $= -45^\circ$

$\cos(-45^\circ) = \frac{1}{\sqrt{2}}$ lag.

Note: Impedance angle & Pf angle always have same magnitude but have opposite sign.

$f_0 \longrightarrow I_{max}$, $P = I_{max}^2 R$

$f_1, f_2 \longrightarrow \frac{I_{max}}{\sqrt{2}}$, $P' = \left(\frac{I_{max}}{\sqrt{2}}\right)^2 R$. (88)

$\therefore \quad P' = \frac{I_{max}^2 R}{2} = \frac{P}{2}$.

$$\boxed{P' = \frac{P}{2}}$$

$\therefore$ Power at f_1 f_2 = $\frac{1}{2}$ power of f_0.

$\therefore \quad f_1 f_2 \longrightarrow$ half power frequencies

f_1 : $X_c > X_L$, $X = R$.

$$\frac{1}{\omega_1 C} - \omega_1 L = R \quad — (1)$$

$f_2 \rightarrow X_L > X_C$, $X = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R \longrightarrow (2)$$

(1) = (2)

$$\omega_1 \omega_2 = \frac{1}{LC} \longrightarrow (3)$$

$$\omega_0^2 = \frac{1}{LC} \longrightarrow (4)$$

(4) = (3)

$$\omega_0^2 = \omega_1 \omega_2$$

$$\boxed{\omega_0 = \sqrt{\omega_1 . \omega_2}} \; **$$

$$\boxed{f_0 = \sqrt{f_1 f_2}} \; **$$

dd eqn (1) & (2)

$$\frac{1}{C}\left[\frac{1}{\omega_1} - \frac{1}{\omega_2}\right] + L\left[\omega_2 - \omega_1\right] = 2R$$

$$\frac{1}{C}\left[\frac{\omega_2-\omega_1}{\omega_1\omega_2}\right]+L\left[\omega_2-\omega_1\right]=2R$$

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$$\left(\begin{array}{l}\omega_1\omega_2=\frac{1}{LC}\\ L=\frac{1}{\omega_1\omega_2 C}\end{array}\right)$$

$$L(\omega_2-\omega_1)+L(\omega_2-\omega_1)=2R$$

**

$$\boxed{\begin{array}{l}B.W = \omega_2-\omega_1=\frac{R}{L}\ \text{rad/s}\\ B.W = f_2-f_1=\frac{R}{2\pi L}\ Hz.\end{array}}$$

R/L

R/2πL

5.)

$$Q=\frac{\omega_0 L}{R}$$

$$=\frac{\omega_0}{(R/L)} \Rightarrow \boxed{\begin{array}{l}Q=\frac{\omega_0}{\omega_2-\omega_1}\\ Q=\frac{f_0}{f_2-f_1}\end{array}}$$ **

Objectives.

1. $R\uparrow$ $B.W\uparrow$ $f_1\downarrow$ $f_2\uparrow$, $f_0 =$ const.

2. $Q=\frac{\omega L}{R}=\frac{X_L}{R}=\frac{V_L}{V}$, $\boxed{Q>1}$ $\boxed{\begin{array}{l}X_L>R\\ X_C>R\end{array}}$

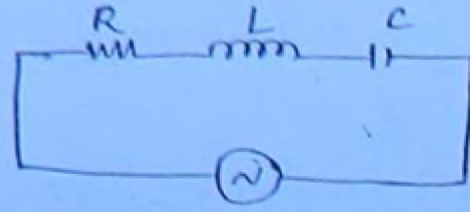
In series resonance, always $V_L > V$ $\Rightarrow$ $X_L > R$ & $X_C > R$.

$\because X_L = X_C$ at f_0

$$f_1=\frac{1}{2\pi\sqrt{LC}}\left(\sqrt{1-\frac{R^2C}{2L}}\right)$$

$$f_2=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{\left(1-\frac{R^2C}{2L}\right)}}$$

By KVL,



$$V = Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

diff. wrt t.

$$V' = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C}$$

$\div L \Rightarrow$

$$\boxed{\frac{d^2i}{dt^2} + \underbrace{\frac{R}{L}}_{\omega_2-\omega_1}\frac{di}{dt} + \underbrace{\frac{i}{LC}}_{\omega_0^2 = 1/LC} = \frac{V'}{L}}$$

If a given diff. eqn is of the above form, then we can obtain BW & ω_0^2

coeff of $\frac{di}{dt} = \omega_2 - \omega_1 = \frac{R}{L} = BW$.

coeff of $i = \frac{1}{LC} = \omega_0^2$.

$$\Rightarrow \quad Q = \frac{\omega_0}{\omega_2 - \omega_1}$$

$$\left.\begin{aligned} &\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 \\ &s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \end{aligned}\right\} \quad \frac{d}{dt} = D$$

$$\Rightarrow \quad \omega_n^2 = \frac{1}{LC} \quad \Rightarrow \quad \omega_n = \frac{1}{\sqrt{LC}}$$

$$2\xi\omega_n = \frac{R}{L}$$

$$\frac{R}{2}\sqrt{\frac{C}{L}} = \frac{1}{2Q}$$

Damping ratio $= \xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\frac{R}{2}\sqrt{\frac{C}{L}} \cdot \frac{1}{2Q}$

But $$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

⇒ ** $$\boxed{\xi_1 = \frac{1}{2Q}}$$

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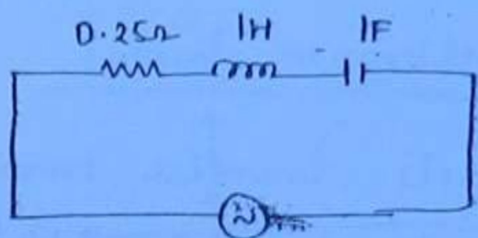
Q.



$V(t) = 10\sin\omega t$

find

(i) $f_0 = 1/2\pi\sqrt{LC} = \frac{1}{2\pi\sqrt{1\times 1}} = \frac{1}{2\pi}$

(ii) $Q = \frac{1}{R}\sqrt{\frac{C}{L}} = \frac{1}{0.25}\sqrt{\frac{1}{1}} = 4$

(iii) $BW_{(f_2 - f_1)} = \frac{f_0}{Q} = \frac{1}{8\pi}$

(iv) $\xi_1 = \frac{1}{2Q} = \frac{1}{8}$

(v) $f_2 - f_1 = 1/8\pi$ & $f_0^2 = f_1 f_2$

⇒ $f_1 f_2 = 1/4\pi^2$

$(f_1 + f_2)^2 - (f_2 - f_1)^2 = 4 f_1 f_2$

$(f_1 + f_2)^2 = \frac{1}{\pi^2} + \frac{1}{64\pi^2} =$

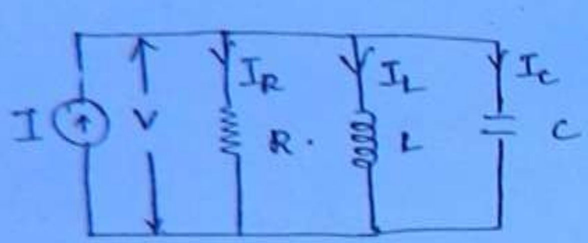
f_1 ?

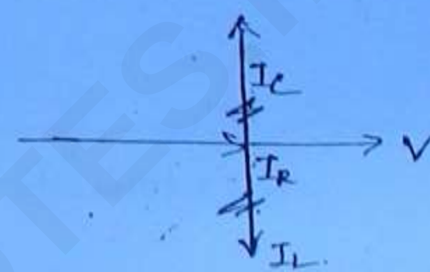
6. $I = \frac{V}{Z} = \frac{V}{R}$.

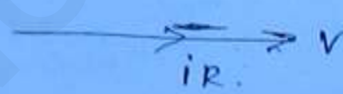
$$I = \frac{10/\sqrt{2}}{0.25} = 20\sqrt{2}.$$

Parallel Resonance

ckt. 1







$$I = I_R\angle 0° + I_L\angle -90° + I_C\angle 90°$$

$$I_L = I_C$$

$$\frac{V}{X_L} = \frac{V}{X_C} \Rightarrow \quad ** \ \boxed{B_L = B_C}$$

$$\frac{1}{\omega L} = \omega C \Rightarrow \boxed{\begin{array}{l} \omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec} \\ \omega_0 = \frac{1}{2\pi\sqrt{LC}} \text{ Hz} \end{array}}$$

$$Y = G + j(B_C - B_L)$$

$$Y_{min} = G$$

$$Z_{max} = \frac{1}{Y_{min}}$$

$$I_{min} = \frac{V}{Z_{max}}$$

5. $I_R = I$

(active component of the current).

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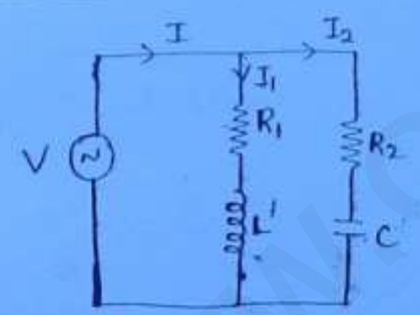
6. Net reactive current = 0.

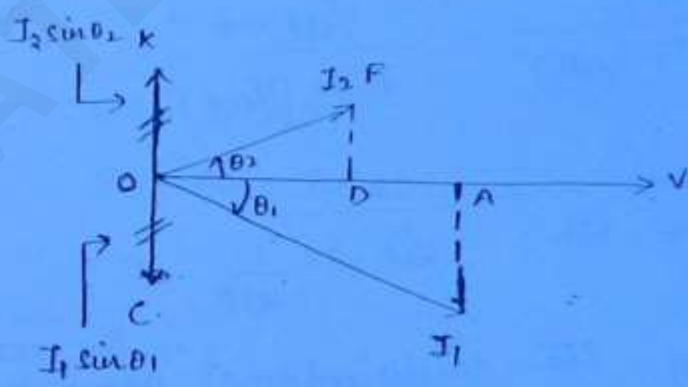
7. Current flowing through inductor and capacitor greater than total current. This phenomena is called as <u>current magnification</u>.

8. Parallel resonant circuit is also called as <u>Anti-resonant circuit</u>.

9. In the parallel circuit at resonance if <u>$G = 0$</u>, circuit behaves as <u>Open Circuit</u>.

<u>Case 2:</u>





$\longrightarrow V$

$I = I_1 \cos\theta_1 + I_2 \cos\theta_2$

$OA = I_1 \cos\theta_1$

$OC = AB = I_1 \sin\theta_1$

$OD = I_2 \cos\theta_2$

$OK = DF = I_2 \sin\theta_2$

$\Rightarrow \quad B_L = B_C$

$$\frac{X_L}{R^2 + X_L^2} = \frac{X_C}{R^2 + X_C^2}$$

$$\frac{\omega L}{R^2 + (\omega L)^2} = \frac{1/\omega C}{R_2^2 + (1/\omega C)^2} \rightarrow (1)$$

$\frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}$

$$** \quad \boxed{f_0 = \frac{1}{2\pi\sqrt{LC}} \sqrt{\frac{R_1^2 - L/C}{R_2^2 - L/C}}}$$

$R_1 = R_2 = \sqrt{\frac{L}{C}}$

Resonance Condition for all frequencies

from eqn (1) above.

$$\frac{\omega L}{R^2 + (\omega L)^2} = \frac{1/\omega C}{R_2^2 + (1/\omega C)^2}$$

$$\frac{1}{\frac{R_1^2}{\omega L} + \omega L} = \frac{1}{R_2^2 C \omega + \frac{1}{\omega C}}$$

Comparing the coefficients, ($\because$ for resonance condition in the above eqn $\forall f \Rightarrow$ it shud be independant of ω).

$\omega \longrightarrow \quad L = R_2^2 C \quad \Rightarrow \quad R_2 = \sqrt{\frac{L}{C}}$

$\frac{1}{\omega} \longrightarrow \quad \frac{R_1^2}{L} = \frac{1}{C} \quad \Rightarrow \quad R_1 = \sqrt{\frac{L}{C}}$

$$** \quad \boxed{R_1 = R_2 = \sqrt{\frac{L}{C}}}$$

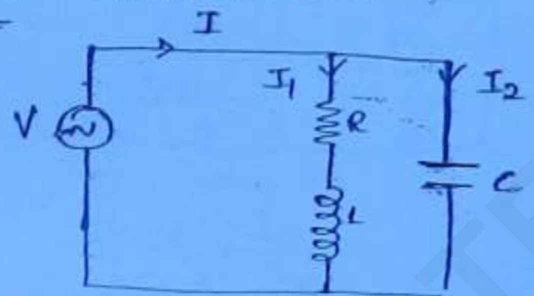
$$Y = (G_1 + G_2) + j(B_c - B_L).$$

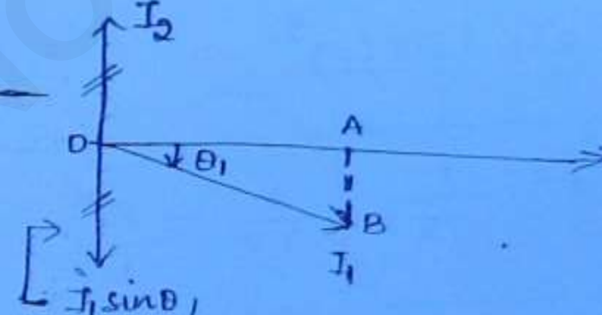
$$Y = \frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_c^2}$$

$$I = VY.$$

$$\boxed{I = V\left[\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_c^2}\right]}$$

Case 3: TANK Ckt

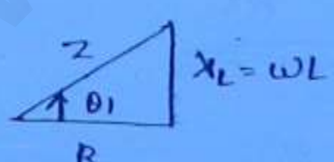




$I = I_1 \cos\theta$

$OA = I_1 \cos\theta_1$

$OC = AB = I_1 \sin\theta_1$



$$\sin\theta_1 = \frac{X_L}{Z_1}$$

$$\cos\theta_1 = \frac{R}{Z_1}$$

$$I_2 = I_1 \sin\theta_1$$

$$\frac{V}{X_c} = \frac{V}{Z_1}\frac{X_L}{Z_1}$$

$$Z_1^2 = X_L X_c = \frac{\omega L}{\omega C} = L/C$$

$$** \boxed{Z_1 = \sqrt{\frac{L}{C}}}\ \Omega$$

$$I = I_1 \cos\theta_1$$

$$I = \frac{V}{Z_1}\frac{R}{Z_1}$$

(96)

$$I = \frac{VR}{L/C}$$

** $$\boxed{I = \frac{V}{(L/RC)}}$$

$$\boxed{Z_{DY} = \frac{L}{RC}} \ \Omega$$

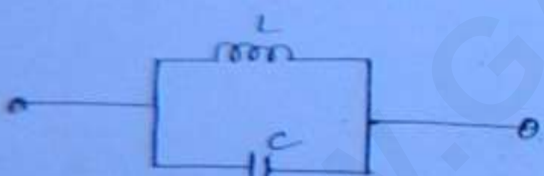
Dynamic impedance.

$$Z_1^2 = R^2 + X_L^2$$

$$\frac{L}{C} = R^2 + (2\pi f_L)^2$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} \cdot \frac{R^2}{L^2}}$$

** $$\boxed{f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$



$$Z_{DY} = \frac{L}{RC} = \infty$$

<u>Ideal Tank ckt</u> i.e. $R = 0$

<u>Q - factor</u>

$$Q = \frac{V_L \text{ (or) } V_C}{V} \quad \text{for series.}$$

<u>for parallel resonant ckt</u>

$$Q = \frac{I_L \text{ (or) } I_C}{I}$$

**

$$Q = \frac{I_L}{I} = \frac{I_L}{I_R} = \boxed{\frac{\text{Reactive component of current}}{\text{Active component of current.}}}$$

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$$\boxed{Q = \frac{I_L}{R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}}$$ for Case is // resonance

$$\omega = \frac{1}{\sqrt{LC}}$$

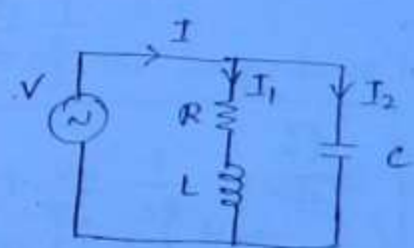
$$\boxed{Q = \frac{R}{\cancel{R}}\sqrt{\frac{C}{L}}}$$

$$\boxed{Q = \frac{1/X_L}{1/R} = \frac{B_L}{G}.}$$

$\longrightarrow$ $Q = \frac{I_C}{I} = \frac{I_C}{I_R}.$

$\longrightarrow$ $Q = \frac{V/X_C}{V/R} = \frac{R}{X_C} = \underline{\underline{R\omega C}}$

For tank ckt:



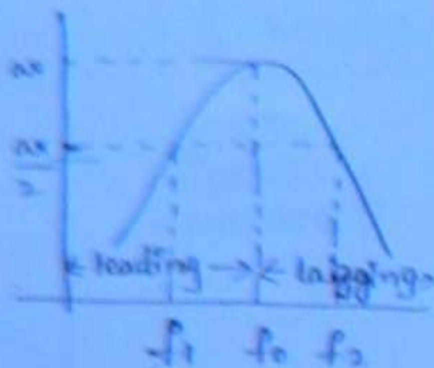
$$Q = \frac{I_2}{\cancel{I}}$$

$$Q = \frac{V/X_C}{V/(L/RC)} = \frac{1/(1/\omega C)}{1/(L/RC)}$$

**

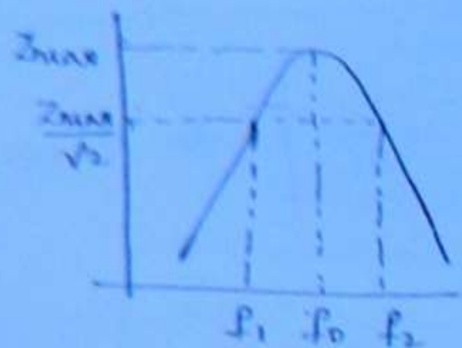
$$\boxed{Q = \frac{\omega L}{R.}}$$ for tank circuit $\rightarrow$ series

Series

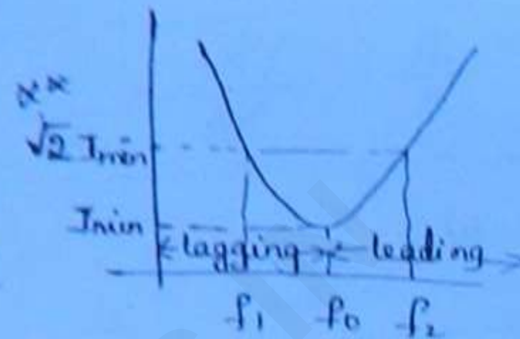


$BW = f_2 - f_1$

Parallel



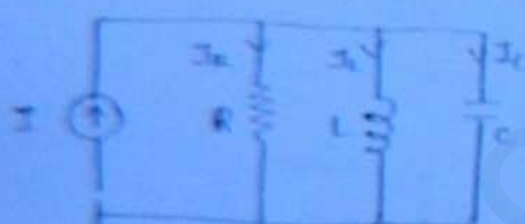
$BW = f_2 - f_1$



$B_L = \frac{1}{2\pi f L}$

$B_C = 2\pi f C$

Relation between damping factor and Q-factor



$$I = \frac{V}{R} + C\frac{dv}{dt} + \frac{1}{L}\int v\,dt$$

differentiate wrt t

$$I' = \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{V}{L}$$

$\div C$

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{V}{LC} = \frac{I'}{C}}$$

$\omega_2 - \omega_1 = \frac{1}{RC}$ $\qquad$ $\omega_0^2 = \frac{1}{LC}$

$$\boxed{Q = \frac{\omega_0}{\omega_2 - \omega_1}}$$

$\frac{1}{R}\sqrt{\frac{L}{C}}$ $\quad$ $R\sqrt{\frac{C}{L}}$

$$\left(D^2 + \frac{1}{RC}D + \frac{1}{LC}\right)V = 0$$

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$\frac{d}{dt} = D$

(99)

** $\boxed{\text{damping ratio} = \xi = \frac{1}{2R}\sqrt{\frac{L}{C}}}$

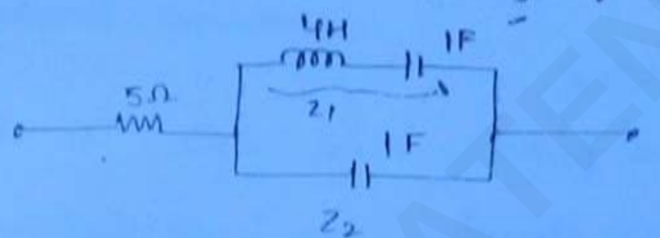
$\to \omega_n = \frac{1}{\sqrt{LC}}$

** $\boxed{\xi = \frac{1}{2Q}}$

$\to 2\xi\omega_n = \frac{1}{RC}$

where $Q = R\sqrt{\frac{C}{L}}$ for parallel resonance

** Q. Find resonant frequency of the circuit shown.



(a) 1/2 rad/s (b) 2 rad/s

(c) 1 rad/s (d) 4 rad/sec

Sol.

Note :- To find resonant frequency for any circuit,

→ i) find equivalent impedance

→ ii) Equate imaginary part of the impedance to zero.

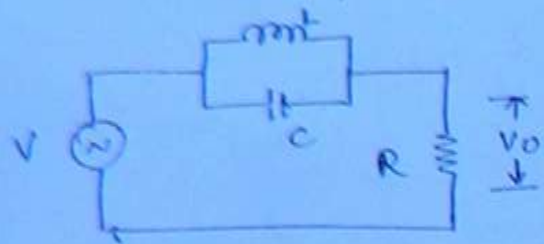
$$Z_1 = j(X_L - X_C) = j\left(\omega L - \frac{1}{\omega C}\right) = j\left(4\omega - \frac{1}{\omega}\right)$$

$$Z_2 = \frac{-j}{\omega}$$

$$Z_1 // Z_2 \Rightarrow Z_{eq} = \frac{j\left(4\omega - \frac{1}{\omega}\right)\left(\frac{-j}{\omega}\right)}{j\left(4\omega - \frac{1}{\omega}\right) - \frac{j}{\omega}} = \frac{\frac{1}{\omega}\left(\frac{1}{\omega} - 4\omega\right)}{\frac{1}{\omega}\left(4\omega^2 - 1 - 1\right)} = 0$$

$$= \left(4\omega - \frac{1}{\omega}\right)\frac{1}{\omega} = 0 \Rightarrow \omega = 0.5 \text{ rad/sec}$$

Q. Find the value of V_0 at resonance.



(a) V (b) 0

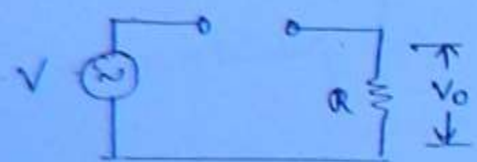
(c) V/2 (d) ∞.

100

Sol.

The given LC ckt is an ideal tank ckt.

$\therefore$ Dynamic impedance $Z_{DY} = \infty$.



$\Rightarrow$ current in the ckt = 0

$i = 0$

$\therefore \Rightarrow V_0 = 0$

- A parallel RLC circuit have

BW = 1kHz & C = 0.1 μF

Find effective resistance of the circuit

$$BW = \omega_2 - \omega_1 = \frac{1}{RC} \text{ rad/s}$$

BW in Hz convert to 'f'

$$\Rightarrow f_2 - f_1 = \frac{1}{2\pi RC}$$

$$R = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 10^{3}} = \frac{10^4}{2\pi}$$

$\therefore$

* For conventional, Thevenin's & MPT are very important —

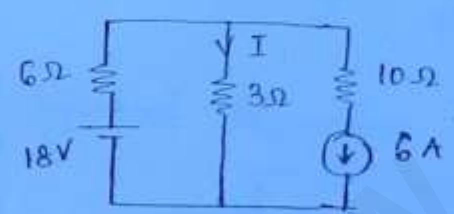
THEOREMS.

→ when network is having several nodes, meshes & sources, the response in any element can be obtained by using theorems

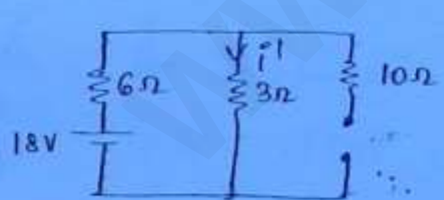
Superposition theorem

In any linear bidirectional circuit having more number of sources the response in anyone of the elements is equal to algebraic sum of the responses caused by individual sources, while the rest of the sources are replaced by its internal resistance.

Q. Find the value of 'i' using superposition theorem

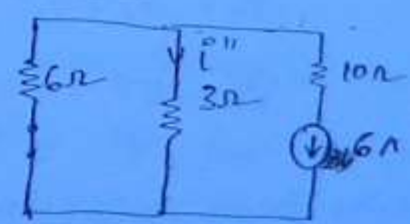


case (i) (18V)



$$i' = \frac{18}{9} = 2A$$

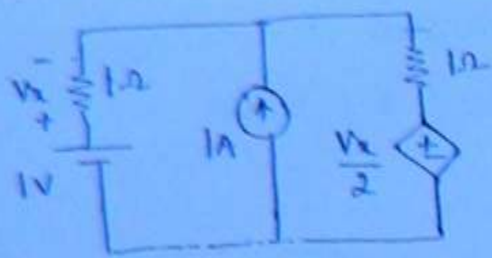
case (ii) (6A)



$$i'' = \frac{-6 \times 6}{9} = -4A$$

$$i = i' + i'' = 2 - 4 = -2A$$

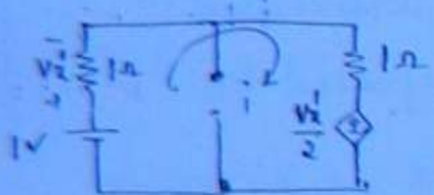
Q. Find the value of V_x by using superposition theorem.



(102)

Note: While applying superposition theorem, dependant source remains same as the original circuit.

Case i) 1V acting



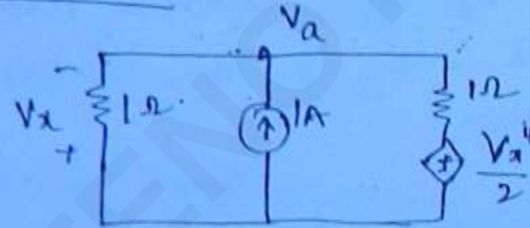
$$i = \frac{1 - V_x'/2}{1+1}$$

$$2i = 1 - \frac{V_x'}{2}$$

$$V_x' = 1 \times I = I.$$

$$V_x = 2/5\ V$$

Case ii)



$$V_a + V_a - \frac{V_x''}{2} = 1.$$

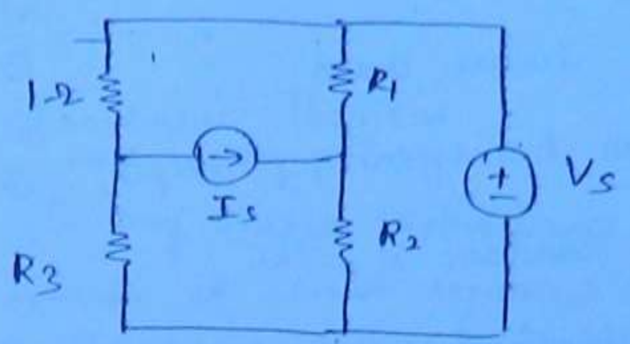
$$V_a = -V_x''.$$

$$V_x'' = -2/5\ V$$

$$V_x = V_x' + V_x''$$

$$V_x = 0V.$$

In the circuit shown, power dissipation in the 1Ω resistor is 5.76 W when voltage source is acting alone & power dissipation in 1Ω resistor is 1W when current source is acting alone. Find total power dissipation in 1Ω resistor.



Sol:

$P_1 = I_1^2 R$, $P_2 = I_2^2 R$

$I_1 = \sqrt{\frac{P_1}{R}}$ $\quad I_2 = \sqrt{\frac{P_2}{R}}$

total $I = I_1 + I_2$ & total power dissipation $= I^2 R$

$\because R = 1\Omega$

$I = \sqrt{P_1} + \sqrt{P_2}$

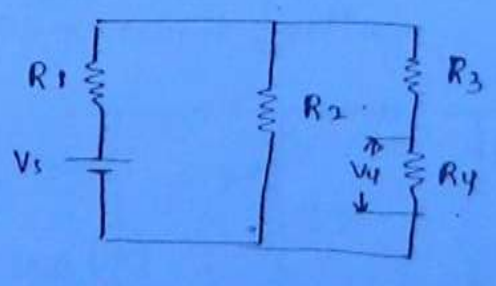
$P = (\sqrt{P_1} + \sqrt{P_2})^2 \cdot 1\Omega = (\sqrt{576} + \sqrt{1})^2$

$P = 625W$

General Expression for $R = 1\Omega$ total power dissipation

$$\boxed{P = (\pm\sqrt{P_1} \pm \sqrt{P_2})^2}$$

Q. In the circuit shown, if source voltage is increased by 10%. Find change in power of R_4 resistor.



(a) 10% (b) 20%

(c) 21% (d) 30%

$\frac{V R_4}{R_3 + R_4}$ $\quad V_1 + 0.1 = 1.1 V_1$ $\quad 1.21$

Note :- When circuit is having linear and bidirectional elements, based on homogenity principle, if excitation is multiplied with constant K, the response of each element is also multiplied with const. K.

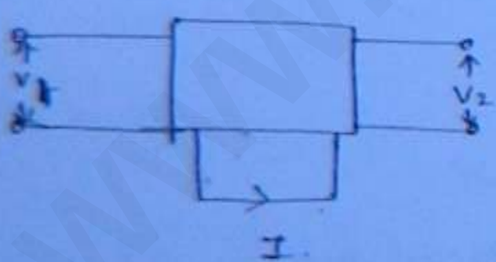
$$P_4 = \frac{V_4^2}{R_4}$$

$$P_4 = \frac{(1.1 V_4)^2}{R_4} \qquad \because \uparrow 10\% = 1 + 0.1 = 1.1$$

$$= 1.21 \frac{V_4^2}{R_4}$$

$\therefore$ increase = 21 %.

Find the value of I when $V_1 = 10V$ & $V_2 = -4V$



V_1	V_2	I
2	0	3
0	4	-2

A-

$V_1 = 2V \qquad I = 3A$

$V_1 = 10V = 2\times5 \qquad \therefore I' = 3 \times 5 = 15A$

$V_2 = 4 \qquad I = -2$

$V_2 = -4 \qquad I = 2A$

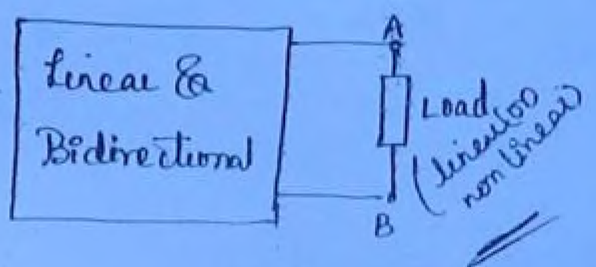
$$\begin{array}{r} 15 \\ 2 \\ \hline 17 \end{array}$$

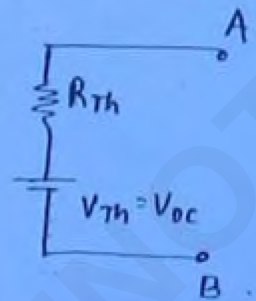
A $I = 15 + 2 = 17A$

Thevenin's Theorem

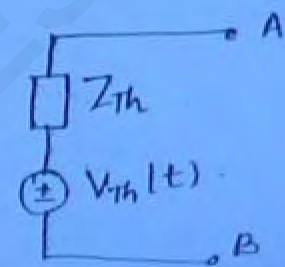
→ In any linear bidirectional circuit having more number of elements it can be replaced by single equivalent circuit consisting of equivalent voltage (V_{Th}) in series with eq. resistance. (R_{Th}).

⟹ By using Thevenin's theorem load current can be calculated either in linear or non linear load.



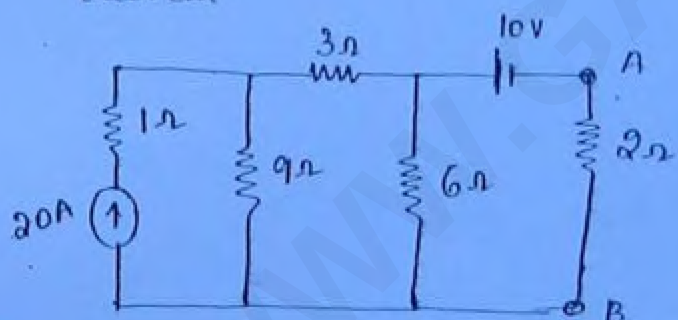


D.C

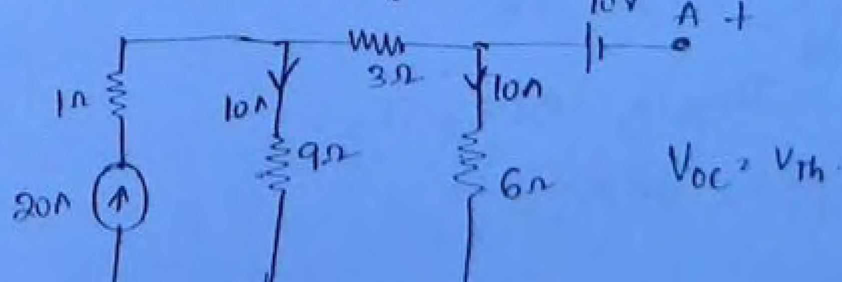


A.C

Q. Find current flowing through 2Ω resistor using Thevenin's Theorem.



Case i, (V_{Th}) — ∴ disconnect the load resistor and find open circuit voltage.



$V_{OC} = V_{Th}$

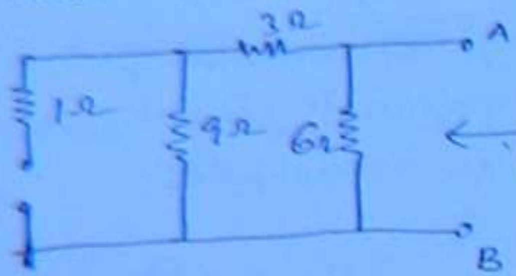
KVL,

$-60 + 10 + V_{Th} = 0$

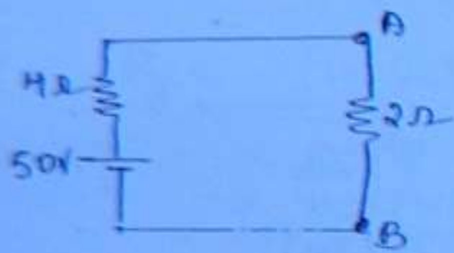
$V_{Th} = 50$

Case (2) (R_{Th}).

Deactivate all independant sources and find equivalent resistance w.r.t load terminals.

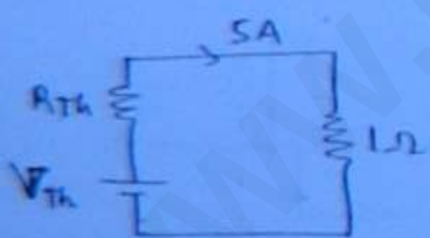


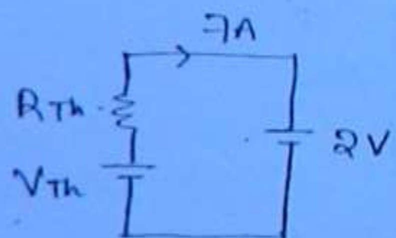
$$R_{Th} = \frac{12 \times 6}{12+6} = 4\Omega.$$



$$\Rightarrow \quad I = \frac{50}{4+2} = 8.3A.$$

A battery charger drives a current of 5A when it is connected to load resistance of 1Ω. when the same battery charger is used for charging of ideal 2V battery at 7A rate find V_{Th} & R_{Th}.





$$V_{Th} = 5(R_{Th}+1) \rightarrow (1)$$

$$7 = \frac{V_{Th}-2}{R_{Th}}.$$

$$7R_{Th} = V_{Th}-2$$

$$7R_{Th} = 5R_{Th}+5-2.$$

$$\therefore R_{Th} = 1.5\Omega$$

$$V_{Th} = 5(1.5+1)$$

$$V_{Th} = 12.5V$$

Q. A complex network of N is connected to load resistor of R Ω. power dissipation in the resistor is P watt. When two identical complex networks are connected to load resistor. Find power dissipation in the load.

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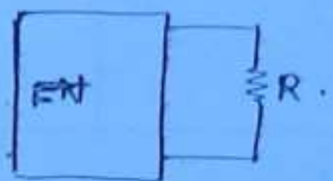
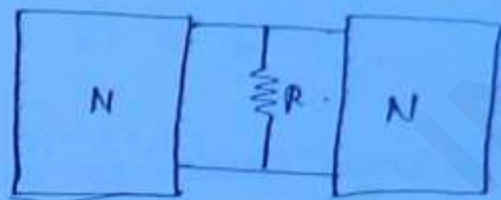
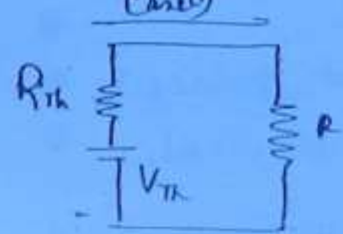


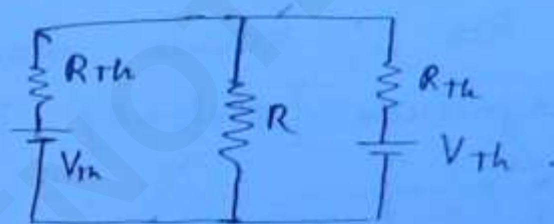


(a) 2P (b) 4P (c) P to 4P (d) 3P

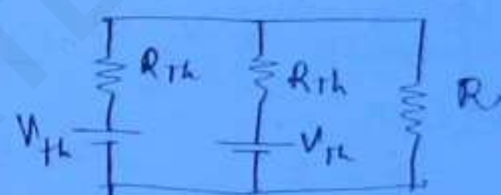
Sol:

Case 1)





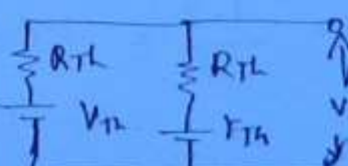
$I = \dfrac{V_{Th}}{R_{Th}+R}$



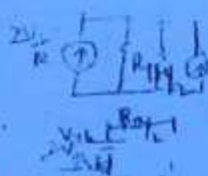
$P = I^2 R$

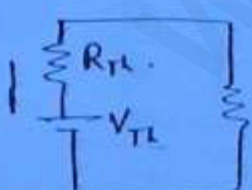
now apply thevenins

$P = \left(\dfrac{V_{Th}}{R_{Th}+R}\right)^2 R \rightarrow (1)$

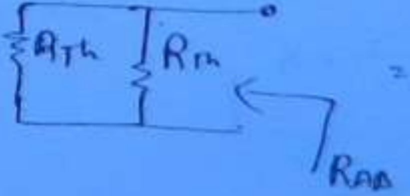


$\Rightarrow V_{Th}' = V_{Th}$





$R_{Th}' = R_{Th}/2$

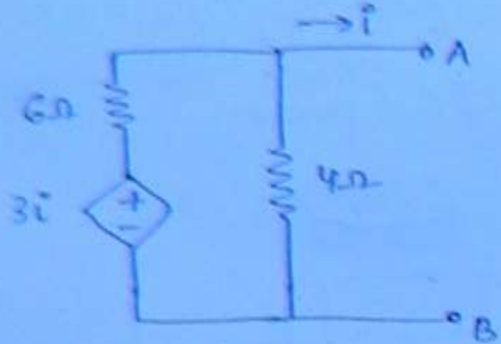


$= R_{Th}' = \dfrac{R_{Th}}{2}$

$P = I^2 R = \left(\dfrac{2V_{Th}}{R_{Th}+2R}\right)^2 R \rightarrow (2)$

$I' = \dfrac{V_{Th}}{\frac{R_{Th}}{2}+R}$

Find V_{Th} & R_{Th} wrt A & B.



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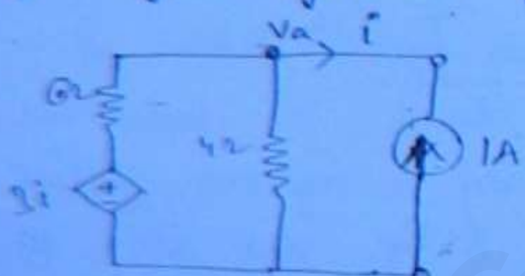
Sol:- In the above circuit no independant source is present. Hence $V_{Th} = 0$

For finding R_{Th}, assume either voltage/current source of any magnitude [preferably 1] across load terminals.



$i = -1A$

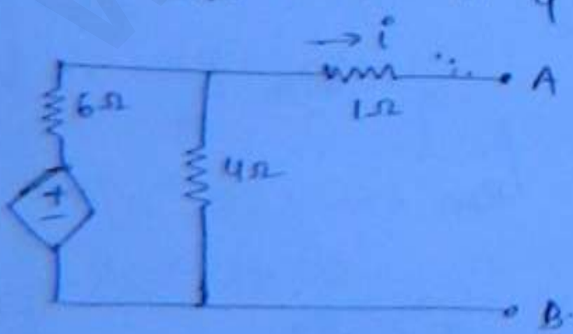
$$\frac{V_a - 3i}{6} + \frac{V_a}{4\Omega} = 1$$

$V_a = 1.2V$

$V_{ab} = V_a = 1.2V$

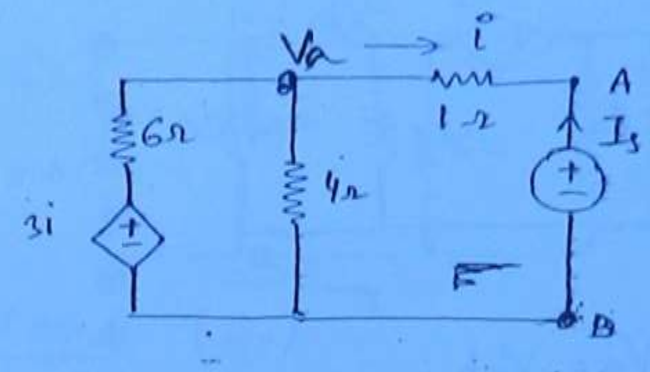
$$R_{Th} = \frac{V_{ab}}{I_s} = \frac{1.2}{1} = 1.2\,\Omega$$

Find R_{Th} wrt A & B.



Here it is easy to solve if we connect 1v voltage source across terminal.

(109)



$$0 = \frac{V_1 - 3i}{6} + \frac{V_1}{4} + \frac{V_1 - 1}{1}$$

$$i = V_1 - 1$$

$$\frac{V_1 - 3V_1 + 3}{6} + \frac{V_1}{4} + \frac{V_1 - 1}{1} = 0$$

$V_1 = 6/11 \Rightarrow i = \frac{-5}{11}$

$I_S \rightarrow$ source current

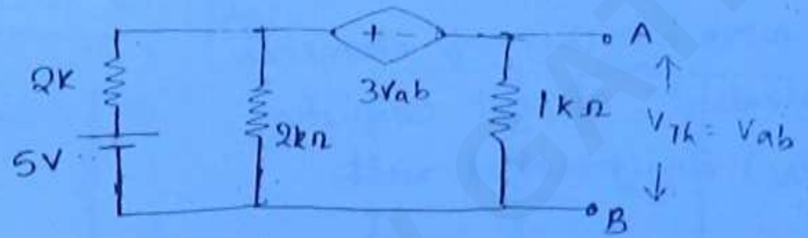
$$I_S = -i = -\left(\frac{-5}{11}\right) = 5/11$$

$$R_{Th} = \frac{V_{ab}}{I_s} = \frac{1}{5/11} = 2.2\,\Omega$$

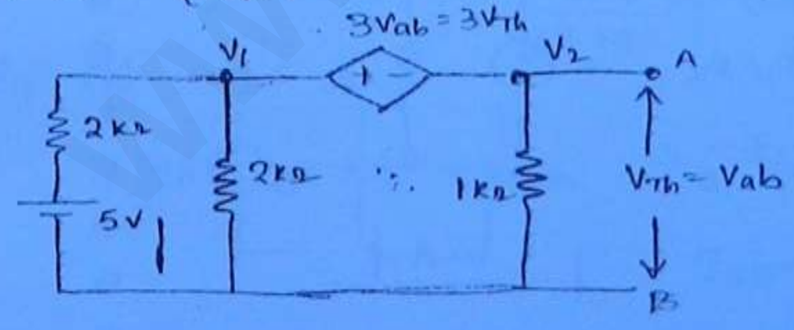
Q. Find V_{Th} & R_{Th} wrt A & B.

Gate. Common data ques & linked data

* Thevenins & max power th. is imp.



Case i, (V_{Th}).



$$\frac{V_1 - 5}{2k} + \frac{V_1}{2k} + \frac{V_2}{1k} = 0 \quad \& \quad V_1 - V_2 = 3V_{Th}$$

$$\& \; V_2 = V_{Th}$$

$$4V_{Th} - 5 + 4V_{Th} + V_{Th} = 0$$

ide 12 (R_{Th}) —

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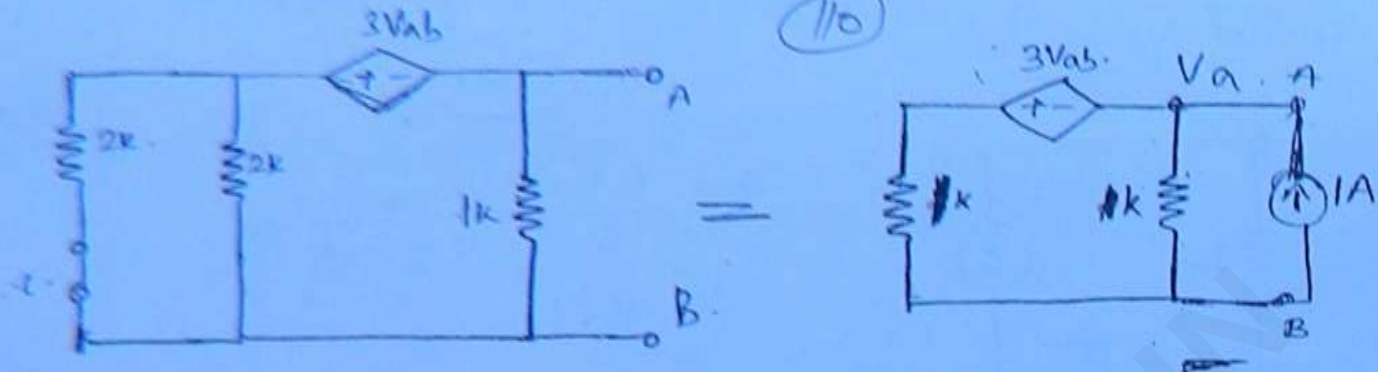


$$\frac{V_a - 3V_a}{1k} + \frac{V_a}{1k} = 1$$

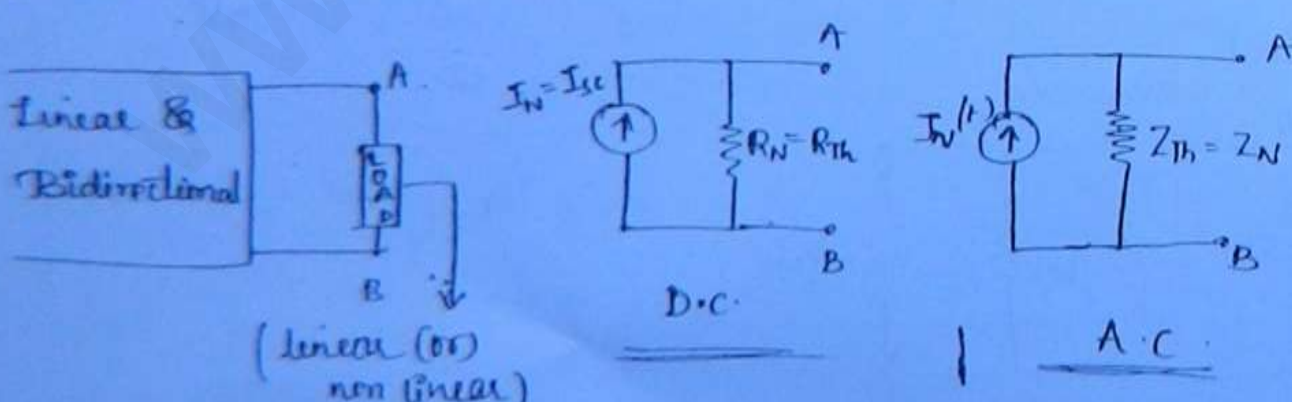
$V_a = V_{ab}$

$V_{ab} = 200V$.

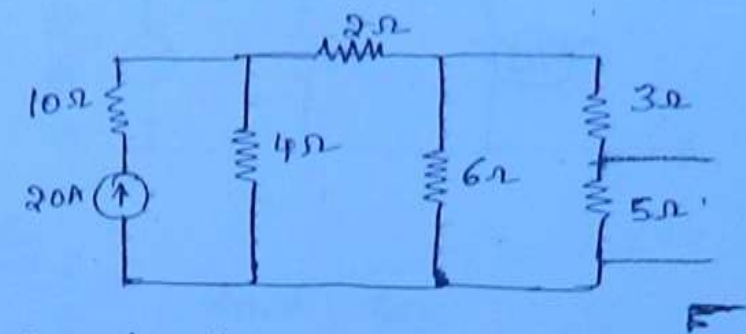
$$R_{Th} = \frac{V_{ab}}{I_s} = \frac{200}{1} = \underline{\underline{200\Omega}}.$$

orton's Theorem

In any complex network having more number of elements, can be replaced by single equivalent circuit consisting of equivalent current source (I_N) in parallel with eq. resistance (R_N).

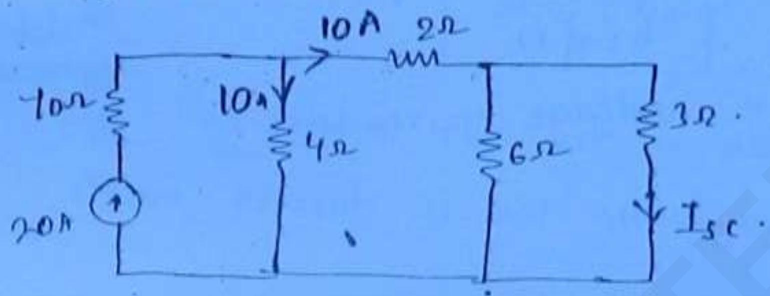


Q. Find current flowing through 5Ω resistor using Norton's theorem.



Case i, (I_{sc}).

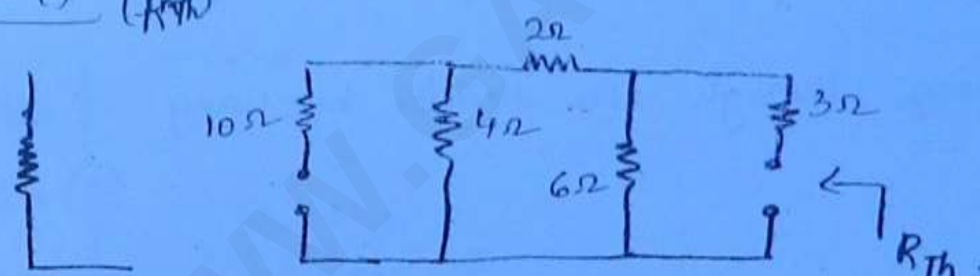
Replace the load resistor by short circuit and find short circuit current.



$$I_{sc} = 10 \cdot \frac{6}{6+3}$$

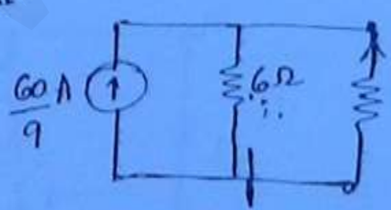
$$I_{sc} = \frac{60}{9} = \frac{20}{3}\text{ A}$$

Case ii, (R_{Th})



$$R_{Th} = 3 + \frac{6 \times 6}{6+6} = 6\,\Omega$$

Eq ckt

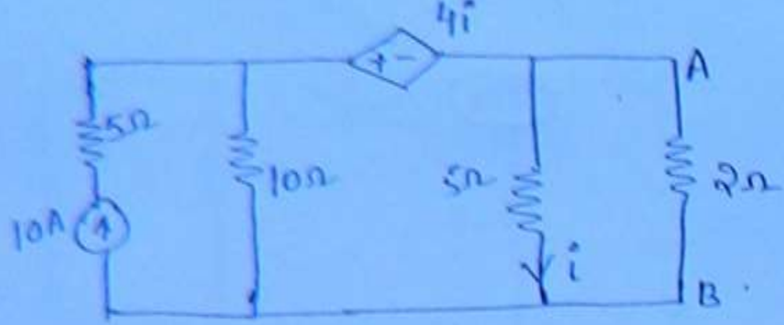


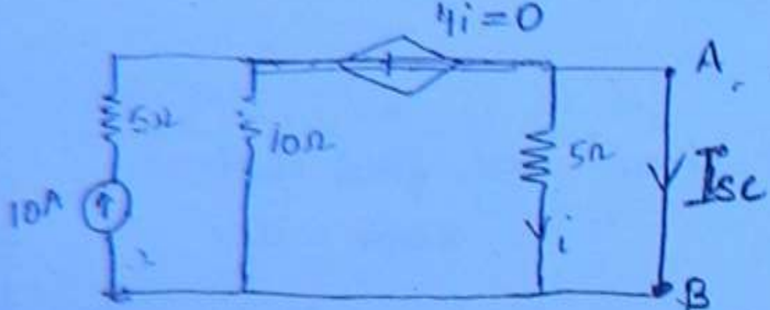
$$\Rightarrow I_{5\Omega} = \frac{60}{9} \cdot \frac{6}{6+5}$$

$$I_{5\Omega} = \frac{40}{11}\text{ A}$$

Q. Find short ckt current (I_{sc}) w.r.t A & B.

(112)



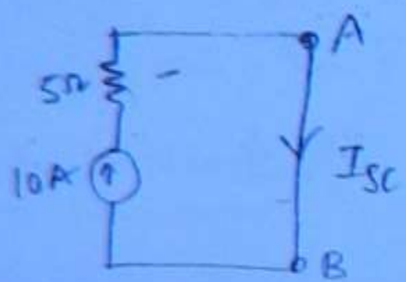


∵ AB is short ckted,

$i = 0$.

∴ $4i = 0$

⇒ voltage source is S.C.



$I_{sc} = \overline{10A}$

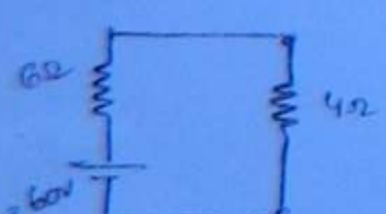
Find current flowing through 4Ω resistor by using the following data.

V	0	60
I	10	0

$I_{sc} = 10A$

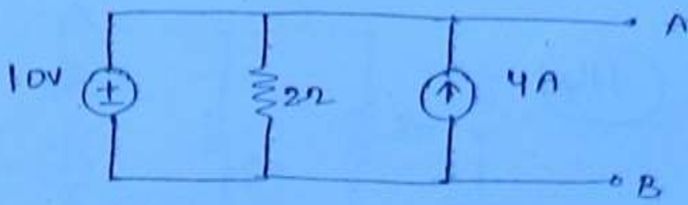
$V_{oc} = 60V$

$$R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{60}{10} = 6\Omega$$



$$\Rightarrow \quad i_{4\Omega} = \frac{60}{6+4} = 6A$$

Q. Obtain thevenins & nortons eq. ckt wrt. A & B.



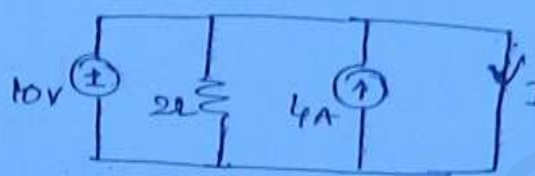
(113)

Soln.

$R_{Th} = 2\Omega$.

$V_{Th} = 10V$.

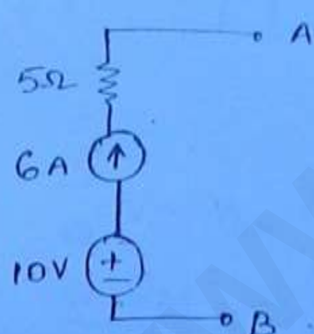
For I_{sc}.



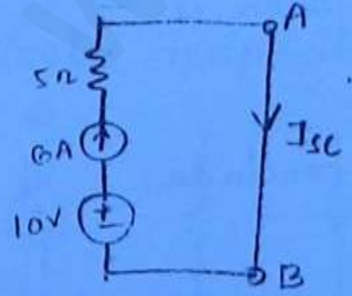
Note :

In the above ckt, it is not possible to find I_N, since above circuit is not satisfying KVL.

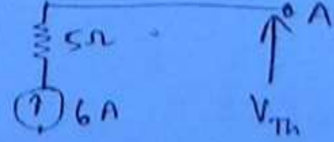
Q. Develop thevenin's & nortons eq. wrt A & B.



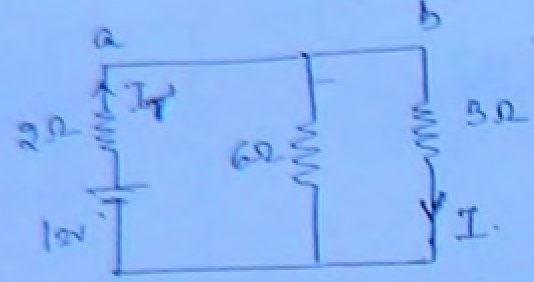
Soln.



$\therefore$ $I_{sc} = 6A$



Note : For the 2^{nd} ckt, it isnt possible to develop thevenin's eq ckt since the ckt is not satisfying ecipıocity theorem.



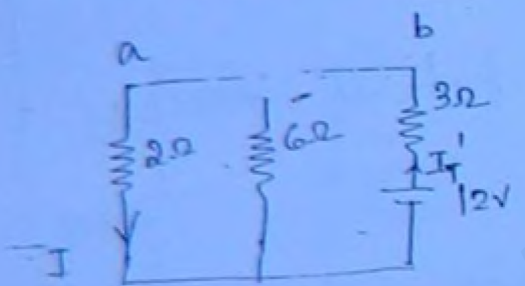
(114)

$$R_{eq} = 2 + \frac{6\times 3}{9} = 4\Omega$$

$$I_T = \frac{12}{4} = 3A$$

$$I = 3\times\frac{6}{6+3} = 2A$$

$$\therefore \frac{\text{Response}}{\text{Excitation}} = \frac{2}{12}$$



$$R_{eq}' = 3 + \frac{6\times 2}{6+2}$$

$$I_T' = \frac{12}{R_{eq}'}$$

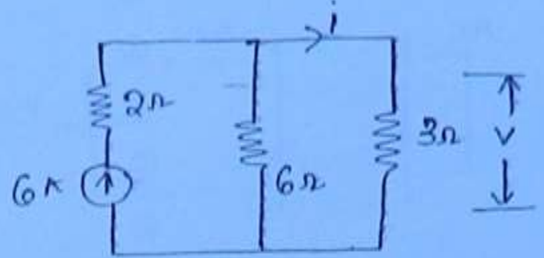
$$I = I_T' \frac{6}{6+2}$$

$$I = 2A \Rightarrow \frac{\text{Response}}{\text{Excitation}} = \frac{2}{12}$$

In the above ckt after interchanging position of response and excitation, the ratio of response to excitation is const. Hence, above network satisfies reciprocity.

Using Reciprocity theorem, it is possible to conclude whether network is linear or non linear.

Q. — Verify reciprocity th. for the circuit shown.

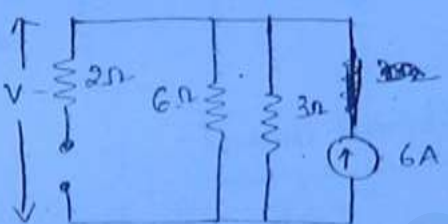


Soln.

$$i = \frac{6 \times 6}{6+3} = 4A$$

$$V = 4 \times 3 = 12V.$$

$$\frac{Res}{Exci} = \frac{12V}{6A} = 2.$$



Current source when interchanged, should be connected in parallel.

$$i_{6\Omega} = \frac{6 \times 3}{9} = 2A$$

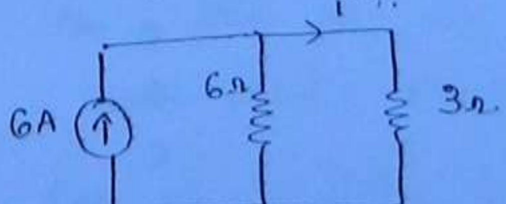
$$V_{6\Omega} = 2 \times 6 = 12V$$

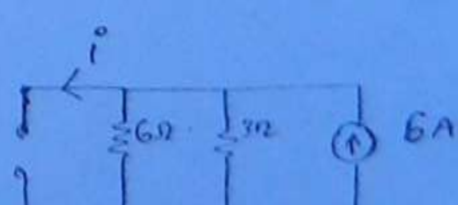
$$V = V_{6\Omega} = 12V$$

$$\frac{Res}{Exci} = \frac{12}{6} = 2.$$

Q. Verify reciprocity theorem of the circuit shown.

$$\frac{6 \times 6}{9} = 4A$$





$$i = 6 \cdot \frac{6}{6+3}$$

In the above 2 problems.

$$\frac{Res.}{Exci} = \frac{i}{V_s}, \quad \text{mho}$$

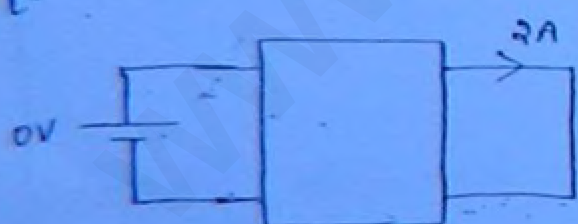
$$\frac{Res}{Exci} = \frac{V}{I_s} \rightarrow \Omega$$

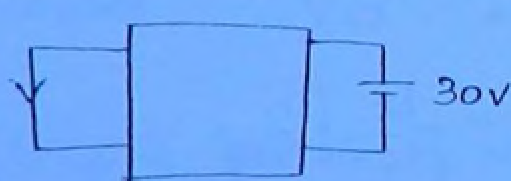
$$\left.\begin{array}{l} \dfrac{Res}{Exci} = \dfrac{I}{I_s} \\[2ex] \dfrac{Res}{Exci} = \dfrac{V}{V_s} \end{array}\right\} \rightarrow \text{No unit.}$$

To apply the reciprocity theorem, unit of response excitation should be either mho (or) Ω.

while applying reciprocity theorem circuit should consist of only one independant source.

When given network satisfies reciprocity find the value of i.





$$\text{const.} = \frac{Res}{Exci} = \frac{2}{10} = \frac{I}{30}$$

$$I = 6A$$

Q. when given n/w satisfies reciprocity find the value of i.

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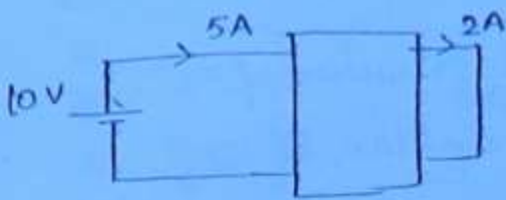
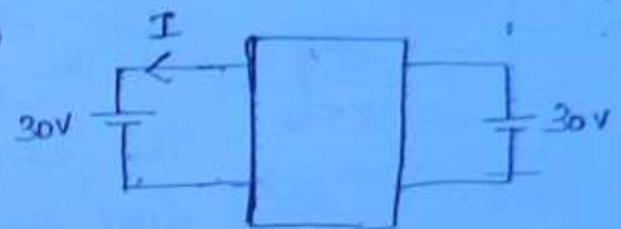
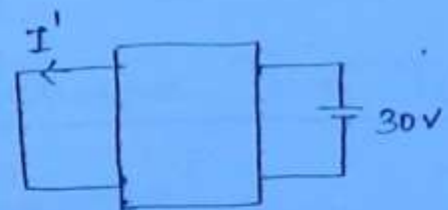




Case (i) (30V)



$$\frac{Res}{Exci} = \frac{2}{10} = \frac{I'}{30}$$

$$I' = 6A$$

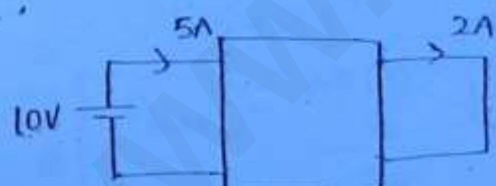
Case 2 (30V)

$$\frac{5}{10} = \frac{-i''}{30}$$

$$i'' = -15A$$

$$I = I' + I'' = 6 - 15 = -9A$$

Q. when given network satisfies reciprocity find the value of i.

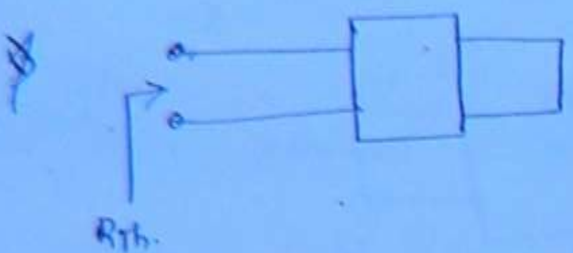


Soln Here we use norton eq coz 2A branch is short ckted. Hence to find i we go for norton eq.

Case (i) (I_{sc}).

$$\frac{2}{10} = \frac{I_{sc}}{30}$$

Case (2) (R_{Th})

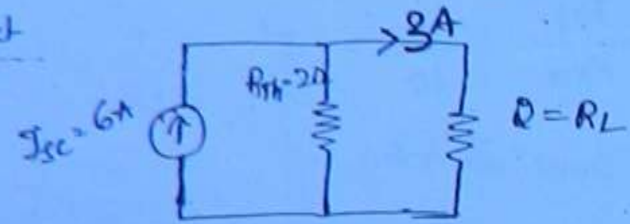


(118)

The physical connections of this circuit r similar to the first one.

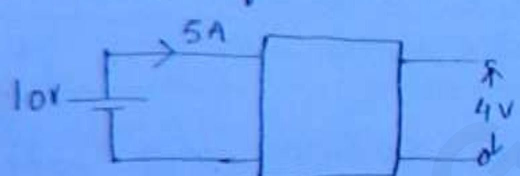
$$\therefore \quad R_{Th} = \frac{10V}{5A} = 2\Omega.$$

Eq ckt

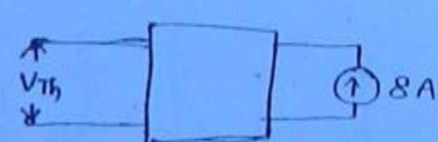


$$\therefore \quad I_{2\Omega} = 3A$$

when given network satisfies the reciprocity, find the value of i.



Case (i) (V_{Th})

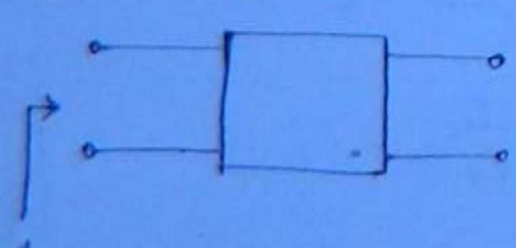


$\frac{5}{4} = \frac{8}{V}$

$$\frac{Res}{Exci} = \frac{V_{Th}}{8} = \frac{4}{5}.$$

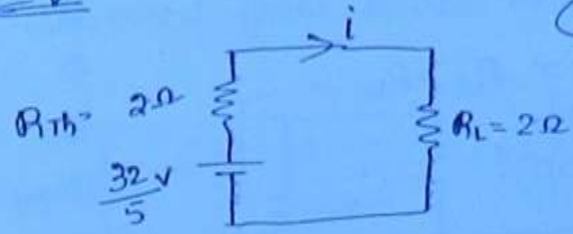
$$V_{Th} = \frac{32}{5} V.$$

Case (2) (R_{Th}).



$$R_{Th} = \frac{10}{5} = 2\Omega.$$

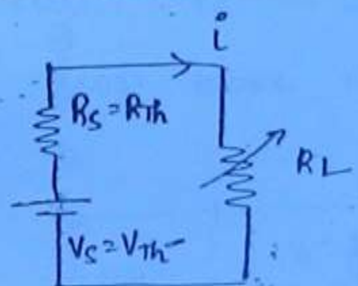
Eq.



$$i = \frac{32/5}{2+2}$$

$$i = \frac{8}{5} A = 1.6 A$$

Maximum Power Transfer theorem.



$$I = \frac{V_s}{R_s + R_L}$$

$$P_L = I^2 R_L$$

$$= \left(\frac{V_s}{R_s + R_L}\right)^2 \cdot R_L \longrightarrow (1)$$

differentiate eqn (1) wrt R_L & equate it to zero.

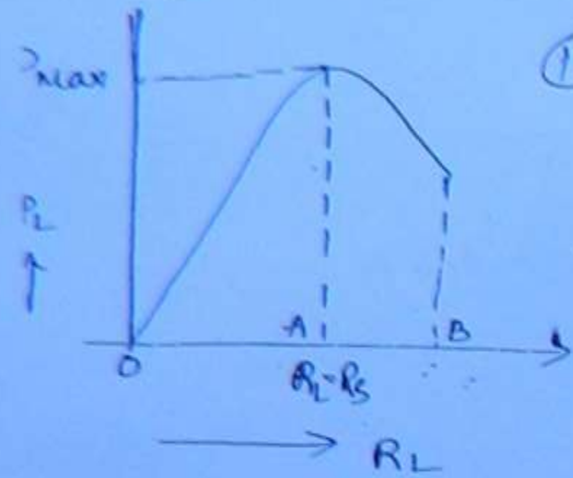
$$\Rightarrow \quad R_L = R_s$$

$$P_{max} = \frac{V_s^2}{(R_s + R_L)^2} \cdot R_L$$

** $$\boxed{P_{max} = \frac{V_s^2}{4R_L}}$$

$$\eta = \frac{o/p}{i/p} \times 100 \Rightarrow \frac{I^2 R_L}{I^2 (R_L + R_s)} \times 100$$

** $$\boxed{\eta = \frac{R_L}{R_L + R_L} \times 100 = 50\%}$$



1) OA $\longrightarrow R_S > R_L$

$\eta < 50\%$

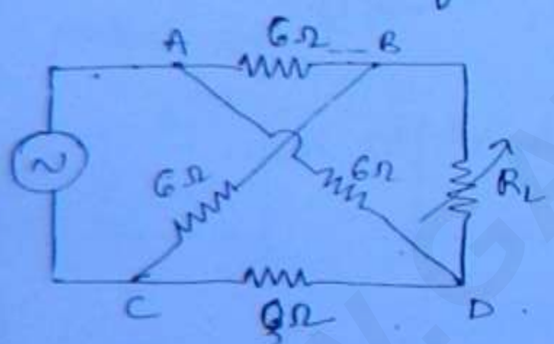
2) A $\longrightarrow R_L = R_S$

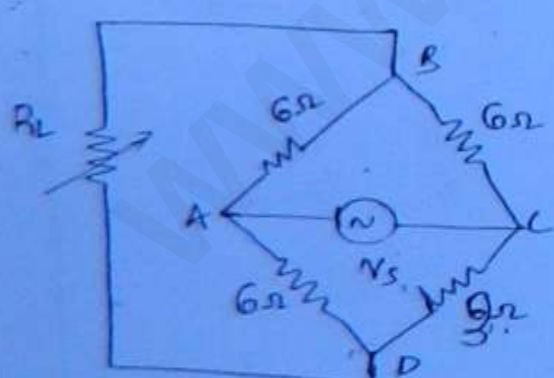
$\eta = 50\%$

3) AB $\longrightarrow R_L > R_S$

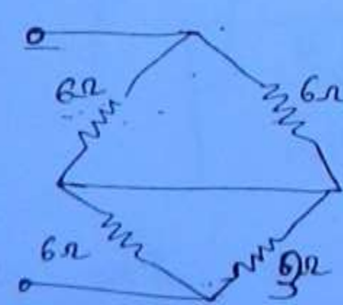
$\eta > 50\%$

In the circuit shown, at what ~~hat~~ value of R_L power delivered from source to load is maximum?





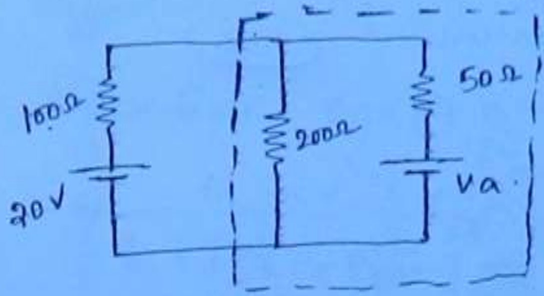
$R_{Th} \longrightarrow$



$R_{Th} = 3 + 2 = 5\Omega$.

$R_L = R_{Th} = 5\Omega$.

Q. In the circuit shown, at what value of Va; power delivered from source to load is maximum. (121)



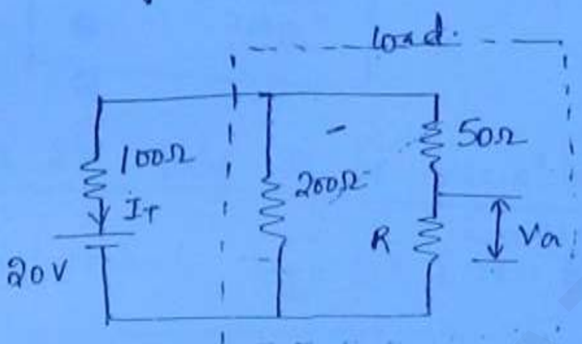
(a) 7.5 V (b) 10 V

(c) 15 V (d) 0 V

Soln.

$\frac{(R+50)\,200}{R+250} = 100$

Here Va is nothing but voltage drop across a resistor.



$(Req)_L$

$$(Req)_L = \frac{200(50+R)}{200+50+R}$$

as per MPT $Req = 100\Omega$

$\therefore \quad Req_L = R_S = 100$

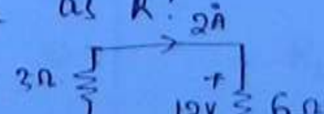
$$100 = \frac{200(50+R)}{250+R} \Rightarrow R = 150\Omega$$

$$I_T = \frac{20}{R_S + (Req)_L} = \frac{20}{100+100} = 1/10$$

$\therefore$

$$Va = \frac{I_T}{2} \cdot R = 7.5V$$

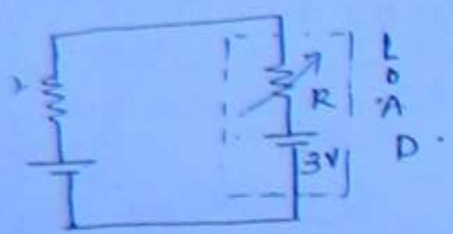
Va as R:



≃

Va can be represented as drop

In the circuit shown at what value of R, the power delivered from source to load is maximum. 122



$I = \frac{10}{R}$ $(x = 3$

$R + \frac{3}{1} = 2$

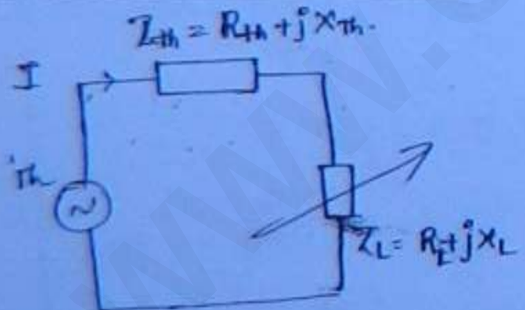
$2 - 1.2 = 0.8$

$$(R_{eq})_L = R_s = 2$$

$$I = \frac{10}{R_s + (R_{eq})_L} = \frac{10}{2+2} = 2.5\,A$$

$$I = \frac{10-3}{2+R} = 2.5 \Rightarrow R = 0.8\,\Omega$$

Maximum Power Transfer Theorem (A.C.)



$$I = \frac{V_{Th}}{(R_{th}+R_L) + j(X_{th}+X_L)}$$

$$I = \frac{V_{Th}}{\sqrt{(R_{th}+R_L)^2 + (X_{Th}+X_L)^2}}$$

$$P_L = I^2 R_L$$

$$P_L = \frac{V_{th}^2 R_L}{(R_L+R_{th})^2 + (X_L+X_{th})^2} \longrightarrow (1)$$

Case 1: Both R_L & X_L are variable.

→ Differentiate eqn ① wrt R_L and equate it to zero.

→ Differentiate eqn ① wrt X_L and equate it to zero.

$$\boxed{\begin{gathered} R_L + jX_L = R_{th} - jX_{Th} \\ Z_L = Z_{th}^* \end{gathered}}$$

$$\boxed{P_{max} = \frac{V_{th}^2}{4R_L}} \quad , \quad \boxed{\eta = 50\%}$$

Case 2: Only R_L is variable. (X_L = constant)

→ Differentiate eqn ① wrt R_L & equate it to zero

$$\boxed{R_L = \sqrt{R_{Th}^2 + (X_L + X_{Th})^2}}$$

$$\boxed{\eta > 50\%}$$

Ex $R_L > R_{Th}$

$$\eta = \frac{R_L}{R_L + R_{Th}} \times 100 > 50\%$$

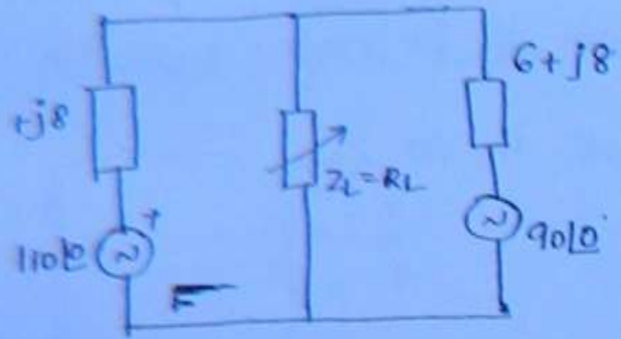
Case 3: R_L is variable ($X_L = 0$).

$$P_L = \frac{V_{th}^2 R_L}{(R_L + R_{th})^2 + X_{th}^2} \rightarrow ②$$

differentiate eqn ② wrt R_L and equate it to zero.

$$\boxed{\begin{gathered} R_L = \sqrt{R_{th}^2 + X_{th}^2} \\ R_L = |Z_{th}| \end{gathered}} \quad \& \quad \boxed{\eta > 50\%}$$

Find max. power dissipation in the load impedance.



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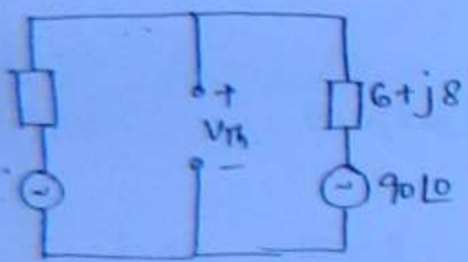
[Case.3, prob.]

= First find thevenin eq.



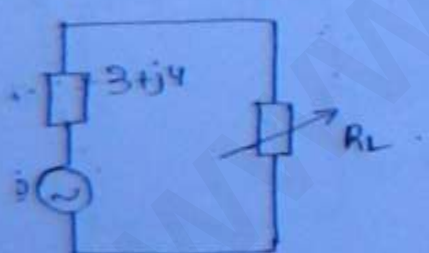
$$\frac{V_{Th} - 110\angle 0^\circ}{6+j8} + \frac{V_{Th} - 90\angle 0^\circ}{6+j8} = 0.$$

$$2V_{Th} = 200\angle 0^\circ$$

$$V_{Th} = 100\angle 0^\circ$$

$$R_{Th} = \frac{(6+j8)\,\cancel{(6+j8)}}{2\,\cancel{(12+j16)}} = \frac{\cancel{36 - 64 + j96}}{\cancel{12+j16}}$$

Eq. $Z_{Th} = 3 + j4$.



$$\Rightarrow \quad R_L = \sqrt{R_{Th}^2 + X_{Th}^2}$$

$$R_L = \sqrt{3^2 + 4^2} = 5\,\Omega.$$

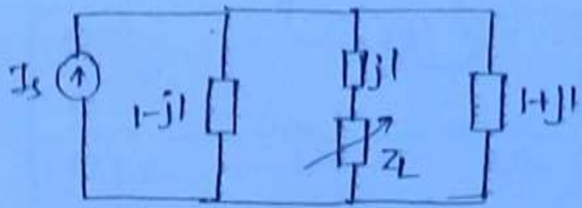
$$R_L = 5\,\Omega$$

$$I = \frac{100\angle 0^\circ}{(3+5)+j4} = \frac{100\angle 0^\circ}{8+j4} = \frac{100\angle 0^\circ}{\sqrt{8^2+4^2}}.$$

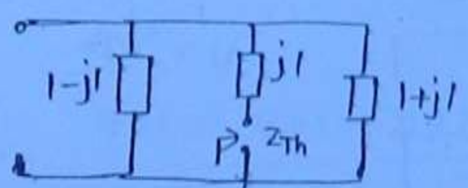
$$P = I^2 \cdot R_L = \frac{100^2}{(8^2+4^2)} \times 5.$$

P = [illegible]

Q. At what value of Z_L, power delivered from source load is max.



$\underline{Z_{Th}}$.



$$Z_{Th} = \frac{(1+j1)(1-j1)}{1+j1+1-j1} + j1$$

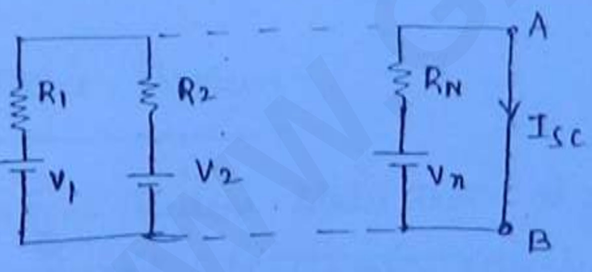
$$Z_{Th} = 1+j1.$$

Case(i) form.

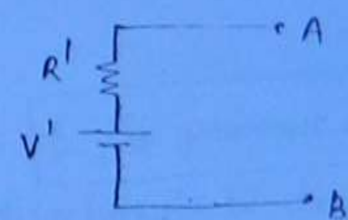
$$\therefore \quad Z_L = Z_{Th}^{*} = 1-j1$$

$$\therefore \quad Z_L = 1-j1$$

Millman's Theorem



$\simeq$



$$R' = R_{th}.$$

$$\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

**
$$\boxed{R' = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_n}\right)} = \frac{1}{G_1 + G_2 + \cdots + G_n}}$$

$$V_{oc} = I_{sc} R_{Th}$$

$$V' = I'R'$$

$$\boxed{V' = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \cdots + \frac{V_n}{R_n}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}}$$

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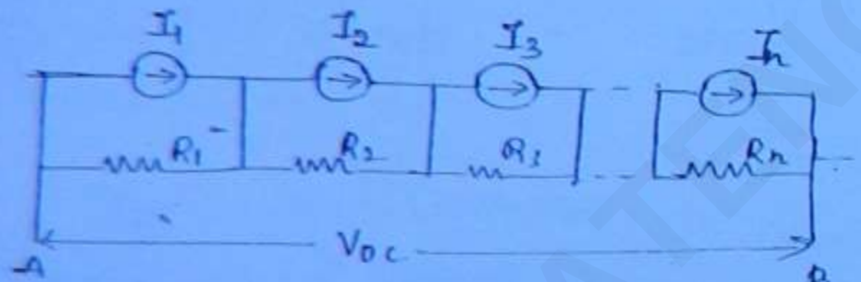
$$\boxed{V' = \frac{V_1 G_1 + V_2 G_2 + \cdots + V_n G_n}{G_1 + G_2 + \cdots + G_n}}$$

$V' = V_{Th}$



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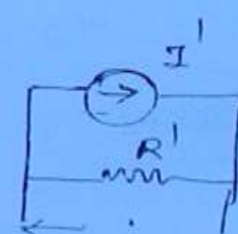

$$R' = R_{th}.$$

$$\boxed{R' = R_1 + R_2 + \cdots R_n.}$$

$$I_{sc} = \frac{V_{oc}}{R_{Th}}.$$

$$I' = \frac{V'}{R'}$$

$$\boxed{I' = \frac{I_1 R_1 + I_2 R_2 + \cdots I_n R_n}{R_1 + R_2 + \cdots + R_n}}$$

Find the value of I in the ckt shown.

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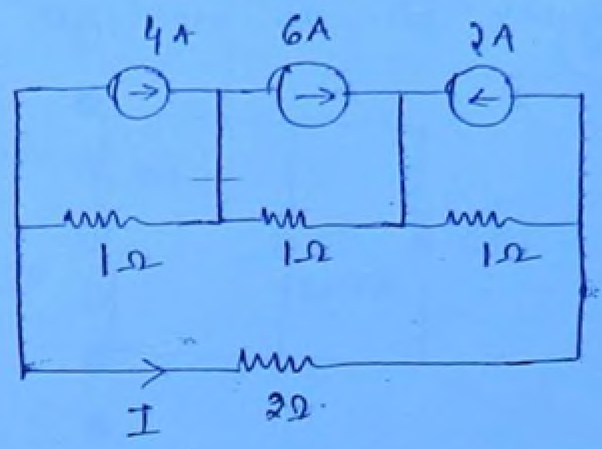
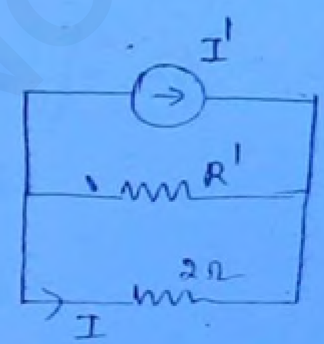


Sol.

$$I' = \frac{4\times1 + 6\times1 - 2\times1}{1+1+1} = 8/3\ A.$$

$$R' = 1+1+1 = 3\Omega.$$



$$I = \frac{-8/3 \cdot 3}{3+2}$$

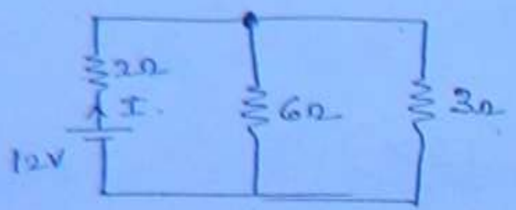
$$I = \frac{-8}{5}\ A$$

Tellegen's Theorem

→ Tellegen's theorem states that algebraic sum of the powers in any circuit (~~linear~~, non-linear, unidirectional, bi-directional, time variant and invariant elements) at any instant = 0.

And it is given by,

$$\boxed{\sum_{K=1}^{n} V_K i_K = 0}$$



(128)

$R_{eq} = 2 + \frac{6\times 3}{9} = 4\Omega$.

$I_T = \frac{12}{4} = 3A$

$I_{6\Omega} = \frac{3\times 3}{9} = 1A \Rightarrow I_{3\Omega} = 2A$.

$V_2 = 2\times 3 = 6V$, $V_3 = V_6 = 3\times 2 = 6V$

$V_2 I + V_6 I_6 + V_3 I_3 - V_s I = 6\times 3 + 6\times 1 + 6\times 2 - 12\times 3$.

$= 18 + 6 + 12 - 36$

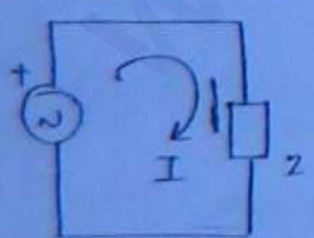
$= 0$

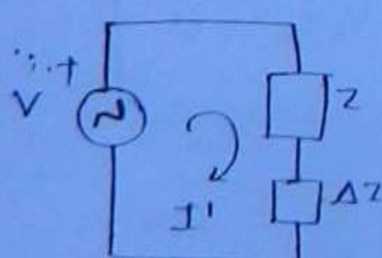
For verification of Tellegen's theorem KVL & KCL equations are used.

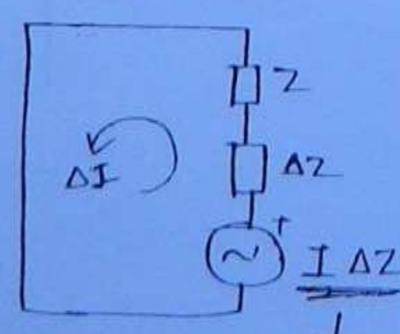
Tellegen's theorem works based on the principle of LAW of Conservation of Energy.

mpensation Theorem.

modified circuit





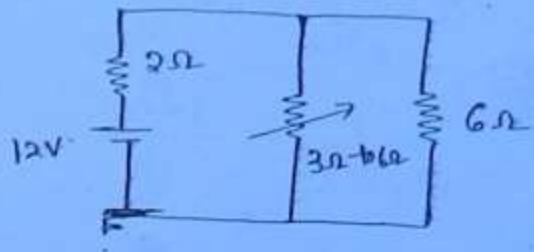


$I = V/Z$.

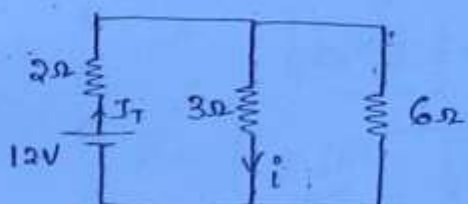
$I' = \frac{V}{Z+\Delta Z}$

Compensation emf (or)

Q. Find change in current in 2Ω & 6Ω resistor when resistance in the variable branch is changed from 3Ω to 6Ω.



Step 1: <u>Find original current</u> circulating in the variable branch.



$$R_{eq} = 2 + \frac{3\times 6}{9}$$
$$= 4\Omega$$
$$I_T = 12/4 = 3A$$
$$i_{3\Omega} = 2A$$

<u>Step 2:</u> Find compensation emf

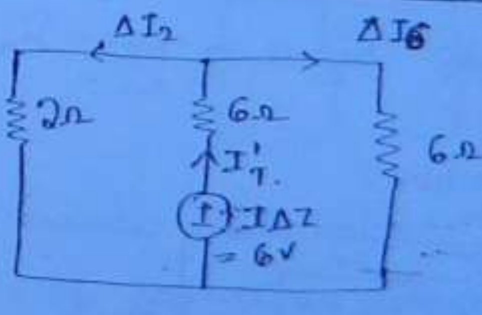
$$= I\,\Delta Z = 2(6-3) = 6V$$

<u>Step 3:</u> Develop modified circuit.

while developing modified circuit, <u>deactivate all the independant sources and connect the compensation emf in series to variable branch.</u>

$$R_{eq} = 6 + \frac{2\times 6}{2+6} \qquad 27/4$$

$$I_T' = \frac{6}{R_{eq}} \quad \therefore \frac{8}{9}\,A \cdot 0.8$$



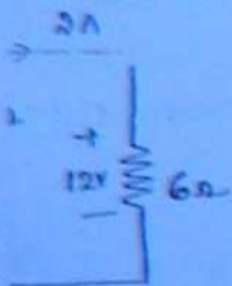
$$\Delta I_2 = I_T' \cdot \frac{6}{6+2} = 0.6\,A$$

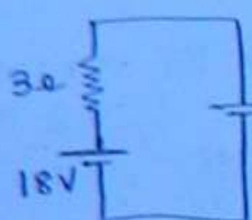
$$\Delta I_6 = I_T' - \Delta I_2 = 0.2A$$

8—

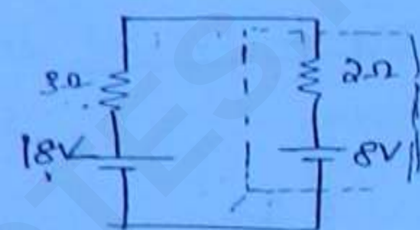
The Bridge circuits, to obtain null deflection in the ranometer, compensation Theorem is used.

STITUTION THEOREM

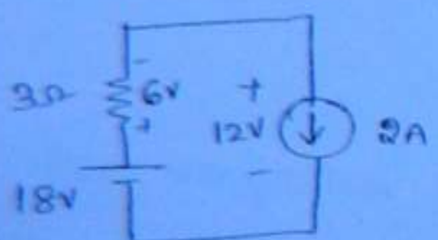




$I = \frac{18-12}{3} = 2A$ → opposing emf.



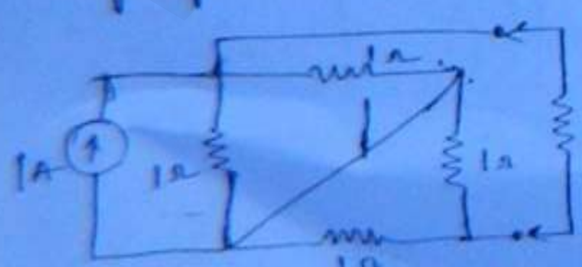
$I = \frac{18-8}{3+2} = 2A$



xtbook

Req is equal in both cases. $\therefore \frac{V}{I} = R_{eq}$.

Superposition.



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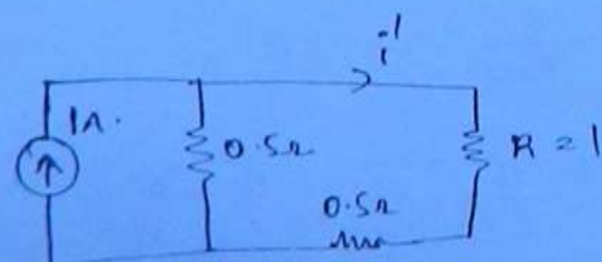

$i' = 1 \cdot \frac{0.5}{0.5+0.5+1} = 0.25A$

en 1A is O.C. ⇒ balanced bridge

3.

$(R_1 + j\omega L)(R_4 - j/\omega C_4) =$

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$$(R_1 + j\omega L_1)\left(R_4 - j/\omega C_4\right) = R_2 R_3.$$

$$\omega L_1 R_4 - \frac{R_1}{\omega C_4} = 0 \Rightarrow \omega^2 L_1 R_4 = \frac{R_1}{C_4}.$$

$$\frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4}$$

4.

$$V_{Th} = \frac{100\angle 0^\circ \cdot j4}{3+j4} \quad \frac{100\,(3-j4)\,j4}{25}$$

$$= j16\,(3-j4).$$

5.

$$P = \left(\pm\sqrt{P'} \pm \sqrt{P''} \pm \sqrt{P'''} \right)^2.$$

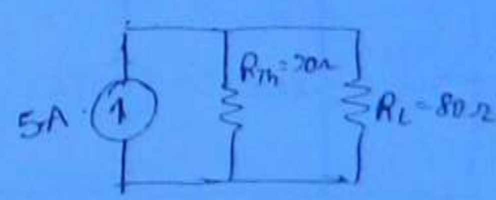
$$P_{max} = \left(\sqrt{18} + \sqrt{50} + \sqrt{98} \right)^2 = .$$

$$P_{min} = \left(\sqrt{98} - \sqrt{50} - \sqrt{18} \right)^2$$

6.

$$R_{Th} = \frac{100}{5.\cancel{00}} = \frac{100}{5} = 20\Omega.$$

$$I_L = I_{80\Omega} = \frac{5 \times 20}{80+20} = 1A$$



7. ω values is same, voltage branches can be replaced by eq. voltage source.

In the above circuit, if freq. of sources are unequal current response can be obtained only by using superposition theorem.

13. $V_{oc} = 2 \cdot \frac{2}{2+3} = 4/5\ V.$

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$100 + 100$

$\sqrt{200}$

$10\sqrt{2}$

$\frac{V-10}{6} + \frac{V-5}{4} = 0$

$\frac{2V+3V}{12} = \frac{10}{6} + \frac{5}{4}$

$\frac{5V}{12} = 5\left[\frac{4+3}{12}\right]$

$= 7$

$R_{Th} = \frac{3\times2}{5} + \frac{4}{5} = 2\Omega.$

$I_{sc} = \frac{V_{oc}}{R_{Th}} = \frac{4}{5\times2} = \frac{2}{5}\ A.$

14. $R_L = \sqrt{R_{Th}^2 + X_{Th}^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2} = 14.14\Omega$

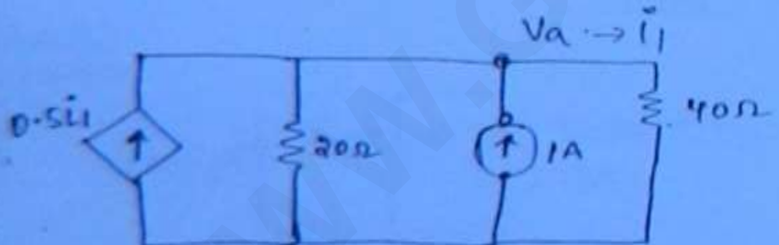
15. $P_{max} = \frac{V_s^2}{4R_L} = \frac{12^2}{4\times2} = 18W.$

17. millman's theorem.

$V' = \frac{10/6 + 5/4}{1/6 + 1/4} = 7V.$

$R' = \frac{1}{\frac{1}{6}+\frac{1}{4}} = 2.4\Omega.$

18. $R_L = R_{Th}.$



$\frac{V_a}{20} + \frac{V_a}{40} = 0.5i_1 + 1.$

$i_1 = \frac{V_a}{40}.$

$\frac{V_a}{20} + \frac{V_a}{40} = \frac{V_a}{80} + 1.$

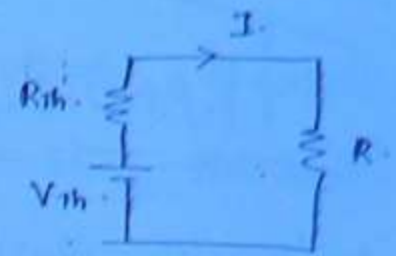
$\Rightarrow V_a = 16.$

$\therefore R_{Th} = \frac{V_a}{I_s} = \frac{16}{1} = 16\Omega$

19. $I_{4\Omega} = \frac{3}{4}A$

$I_{3\Omega} = \frac{3}{4} + 0.25 = 1A$

20. $I_{sc} = 3A$.



$$I = \frac{V_{Th}}{R + R_{Th}}$$

$$3 = \frac{V_{Th}}{R_{Th} + 0} \rightarrow (1) \qquad R = 0$$

$$1.5 = \frac{V_{Th}}{R_{Th} + 2} \rightarrow (2) \qquad R = 2.$$

$$V_{Th} = 6, \quad R_{Th} = 2.$$

$$R = 1 \Rightarrow \quad I = \frac{V_{Th}}{R_{Th} + 1} = 2A.$$

$1.5 \times 2 = 3V$

$\frac{3 \times 2}{3} = 2A$

21.

I_1	I_2	V
8A	12A	80
-8A	4A	0
10A	10A	?

$\therefore$ Same network,

network constants are same.

$$8k_1 + 12k_2 = 80$$
$$-8k_1 + 4k_2 = 0.$$
$$\Rightarrow k_1 = \qquad k_2 =$$

$$V = 10k_1 + 10k_2$$

$4k_2 = 8k_1$

$k_2 = 2k_1$

$8k_1 + 24k_1 = 80$

$32k_1 = 80$

$k_1 = \frac{80}{32}$

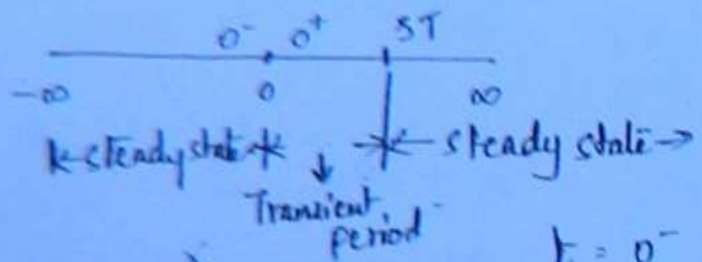
22. equal currents $\Rightarrow$ equal resistance

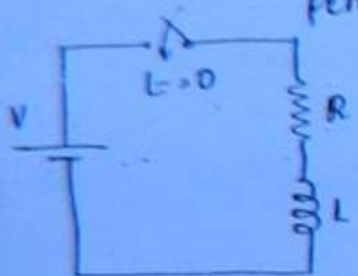
$\therefore$

23. $R_{Th} = 1 + j1$

$$Z_L = Z_{Th}^{*} = 1 - j1$$

TRANSIENTS





$t = 0^-$ $i = 0$

&

$t = 0^+$ $i = 0$ (O.C)

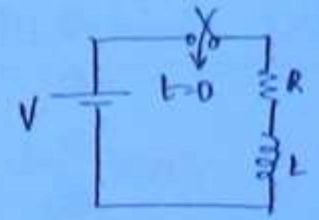
$t = \infty$ $V_L = L\dfrac{di}{dt} = 0$.: (S.C)

Transients are present in the circuit when the circuit is subjected to any changes either by changing source magnitude or while changing any circuit elements, provided circuit consists of any energy storage elements. Since,

Inductor doesn't allow sudden change of current and it stores energy in the form of magnetic field.

Capacitor doesn't allow sudden change of voltage and it stores energy in the form of electric field.

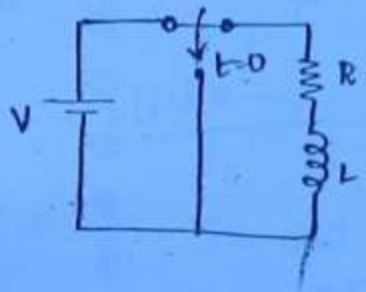
when circuit is having only resistive elements, no transients are present in the circuit. Since resistor allows sudden change of current and voltage and it doesn't store any energy.



$t=0^-$ indicates immediately before operating the switch.

$t=0^+$, indicates immediately after operating switch.

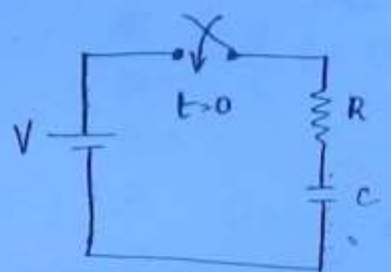
$t=\infty$ indicates steady state condition.



$t=0^-$ $i=I_0$

$t=0^+$ $i=I_0$ (current source)

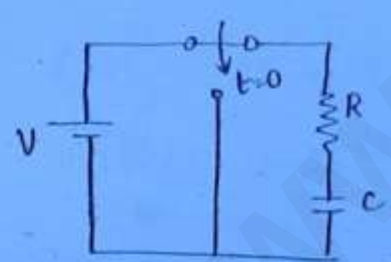
$t=\infty$ $i=0$



$t=0^-$ $V_c=0$

$t=0^+$ $V_c=0$ (S.C.)

$t=\infty$ $V_c=V$, $i=0$ (O.C)

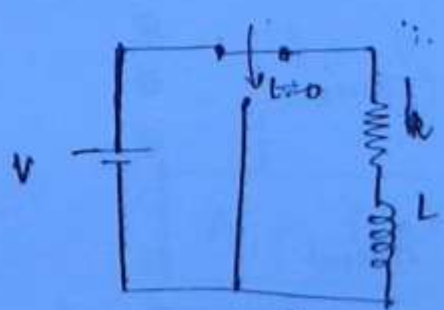


$t=0^-$ $V_c=V_0$ $(V_0=V)$.

$t=0^+$ $V_c=V_0$ (voltage source)

$t=\infty$ $V_c=0$

Source Free RL circuit



$t=0^-$ $i=I_0$

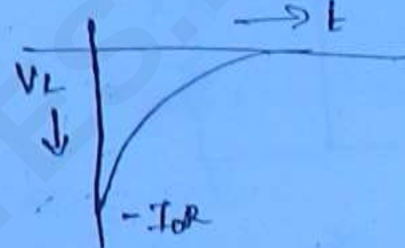
$t=0^+$ $i=I_0$

t > 0

$$iR + L\frac{di}{dt} = 0$$

$$iR = -L\frac{di}{dt}$$

$$\int_0^t -\frac{R}{L}\,dt = \int_{I_0}^{i(t)} \frac{di}{i} = \log\frac{i(t)}{i_0} \Rightarrow I_0 e^{-Rt/L} = i(t)$$



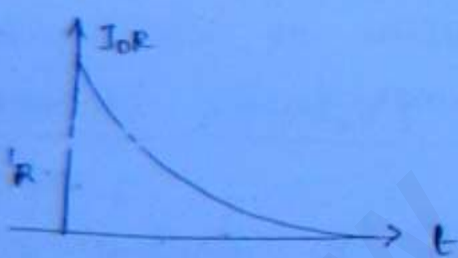
$$v_L = L\frac{di}{dt}$$

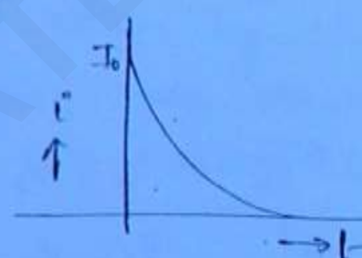
$$v_L = L\frac{d}{dt}\left(I_0 e^{-Rt/L}\right) = -I_0 R e^{-Rt/L}$$

$$v_R = iR$$

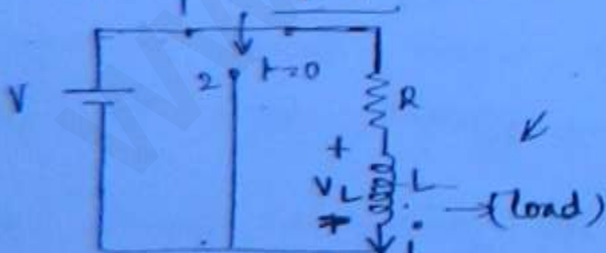
$$v_R = I_0 R e^{-Rt/L}$$

$$i(t) = I_0 e^{-\frac{R}{L}t}$$

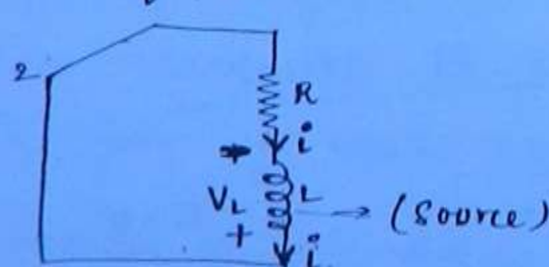




t = 0



t > 0.



In the discharging inductor current direction doesn't change but voltage polarities are reversed

- RL circuit with Source

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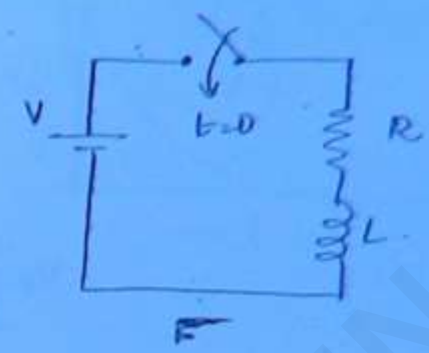
By KVL,



$$V = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

$i(t) = C.F + P.I$

CF ⇒ complementary func.

PI – particular integral.

C.F → Transient response (or) Source free response

$$\frac{di}{dt} + \frac{R}{L}i = 0 \Rightarrow i(t) = Ae^{-Rt/L}$$

P.I → Steady state response / Final value → S.C.

$i = V/R$

$i(t) = CF + PI$

$i(t) = Ae^{-Rt/L} + V/R$

$t = 0^-$ $\quad i = 0$

$t = 0^+$ $\quad i = 0$

$$0 = A + \frac{V}{R} \Rightarrow A = 0 - \frac{V}{R}$$

**

$$\boxed{A = i(0^+) - i(\infty)}$$

$$\boxed{i(t) = [i(0^+) - i(\infty)]e^{-Rt/L} + i(\infty)}$$

Note:

✓ This formula is only applical

$$i(t) = \left[i(0^+) - i(\infty) \right] e^{-\frac{R}{L}t} + i(\infty)$$

$i(0^+)$ → I.V. $\quad$ $i(\infty)$ → F.V. $\quad$ $i(\infty)$ → F.V.

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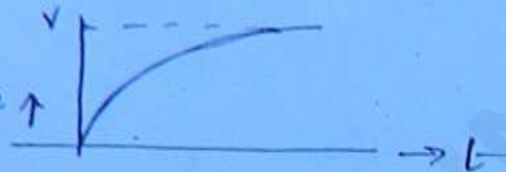
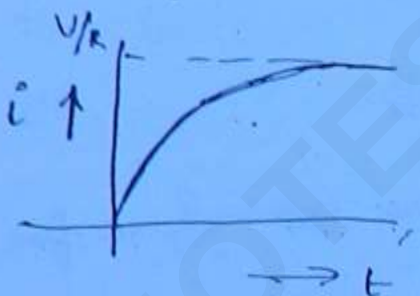
$v = iR$

$v_R = V(1 - e^{-Rt/L})$

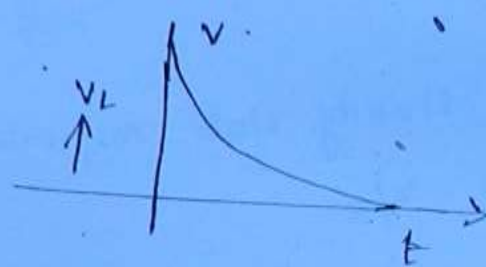


$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

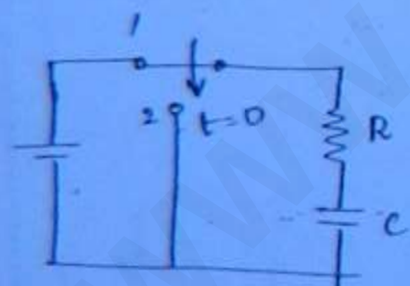


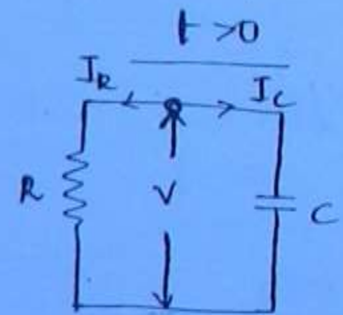
$v_L = L\frac{di}{dt} = L\frac{d}{dt}\left\{ \frac{V}{R}\left[1 - e^{-Rt/L}\right]\right\}$

$V_L = Ve^{-Rt/L}$



Source Free RC ckt.





Assume a virtual voltage source V

$t = 0^-, \quad V_c = V_0$

$t = 0^+, \quad V_c = V_0$.

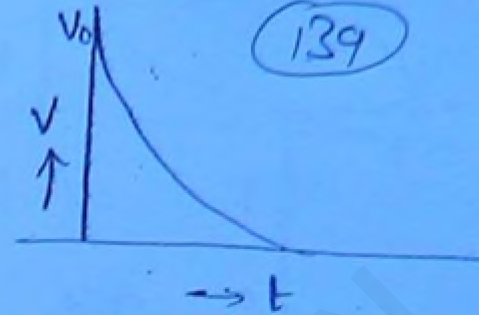
$I_R + I_C = 0$

$\frac{V}{R} + C\frac{dV}{dt} = 0 \Rightarrow \frac{V}{R} = -C\frac{dV}{dt}$

$-\frac{1}{} \; dt = dV$

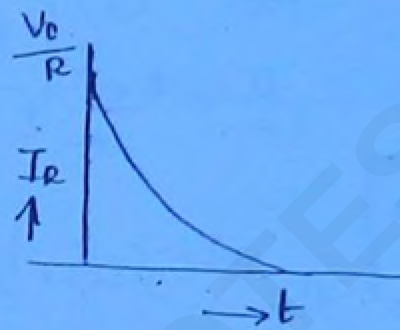
$$\int_0^t \frac{-1}{RC}\,dt = \int_{V_0}^{V(t)} \frac{dv}{V}$$

$$V(t) = V_0 e^{-t/RC}$$



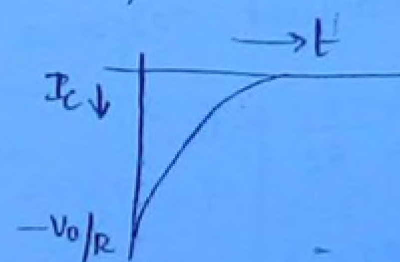
$$I_R = \frac{V}{R}$$

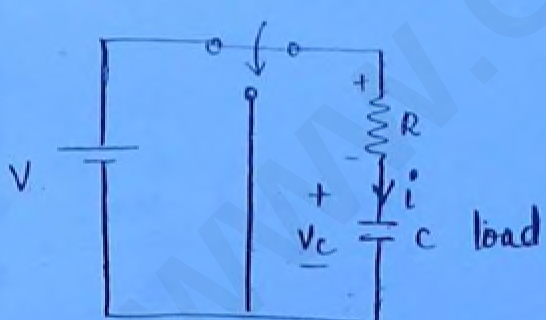
$$I_R = \frac{V_0}{R} e^{-t/RC}$$

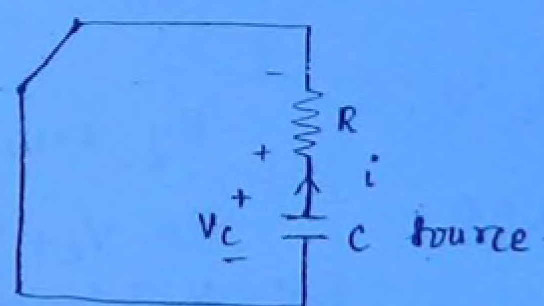


$$I_C = C\frac{dv}{dt}$$

$$I_C = C\frac{d}{dt}\left(V_0 e^{-t/RC}\right) = \frac{-V_0}{R} e^{-t/RC}$$



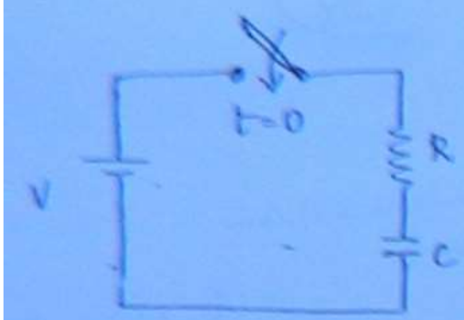




Note:

In the discharging capacitor voltage across capacitor polarities do not change. But current direction of the capacitor is reversed.

RC circuit with Source.



By KVL,

$$V = iR + \frac{1}{C}\int i\,dt$$

diff. wrt

$$0 = R\frac{di}{dt} + \frac{i}{C}.$$

$$\boxed{\frac{di}{dt} + \frac{i}{RC} = 0}$$

$$i(t) = CF + PI.$$

CF = Transient response.

$$\frac{di}{dt} + \frac{i}{RC} = 0 \Rightarrow i(t) = Ae^{-t/RC}$$

PI $\rightarrow$ Steady state :

C $\longrightarrow$ O.C $\Rightarrow i = 0 \Rightarrow i(\infty) = 0$.

$t = 0^-$, $V_c = 0$.

$t = 0^+$, $V_c = 0$.

By KVL,

$$V = V_R + V_c.$$

$t = 0^+$, $V = IR + 0 \Longrightarrow I(0^+) = V/R.$

$i(t) = CF + PI$

$i(t) = Ae^{-t/RC} + 0$

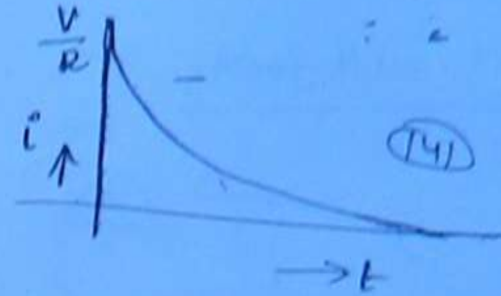
$$i(t) = \frac{V}{R}e^{-t/RC}$$

$A = i(0^+) - i(\infty)$

$A = V/R - 0$

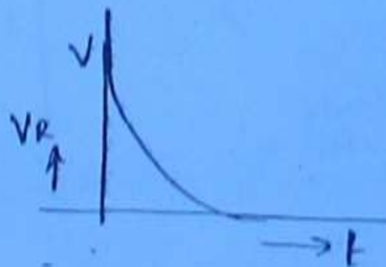
$$A = V/R$$

$$i(t) = \frac{V}{R} e^{-t/RC}$$



$$V_R = iR$$

$$V_R = V e^{-t/RC}$$

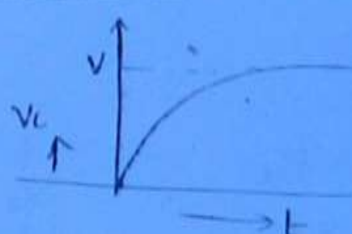


$$V_C = \frac{1}{C}\int_0^t i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC}\,dt$$

$$V_C(t) = -Ve^{-t/RC} + V$$

$$* \boxed{V_C(t) = \left[V_C(0^+) - V_C(\infty)\right] e^{-t/RC} + V_C(\infty)}$$

$$V_C(t) = V\left(1 - e^{-t/RC}\right)$$



Time Constant

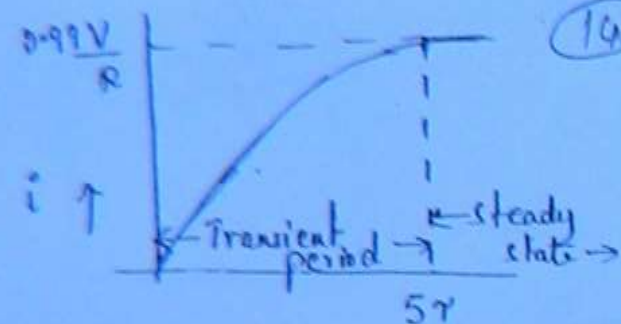
→ Time constant is the time taken for response to rise 63.2% of the max. value & is given by,

$$\left.\begin{array}{ll} \tau = L/R & - RL \\ \tau = RC & - RC \end{array}\right\} \text{unit sec.}$$

$t = \tau$

$$i(t) = \frac{V}{R}(1 - e^{-1}) = 0.63\,V/R.$$

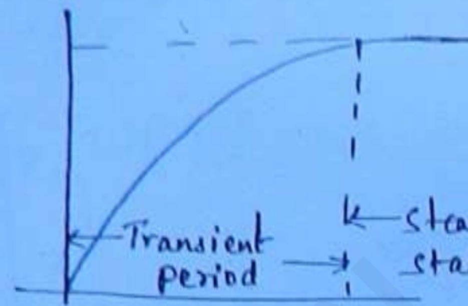
R-L with source



$\Rightarrow$ $i(t) = \frac{V}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$

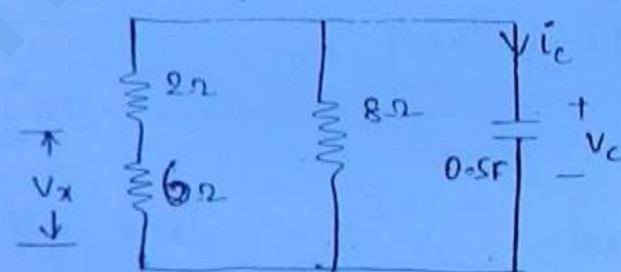
$i(t) = \frac{V}{R}\left(1 - e^{-t/\tau}\right)$

RC with source



$\Rightarrow$ $V_c = V\left(1 - e^{-t/RC}\right)$

Find response of V_c, i_c, and V_x when initial value of the capacitor is 3V. i.e. $V_0 = 3V$



First find out the equivalent resistance & capacitance.

$V_c = V_0 e^{-t/RC}$

$V_c = 3e^{-t/2}$ source free

$\because V_x = V_c \cdot \frac{6}{6+2} = V_c \cdot \frac{6}{8}$

$V_x = 3e^{-t/2} \cdot \frac{6}{8} \Rightarrow V_x = \frac{9}{4} e^{-t/2}$

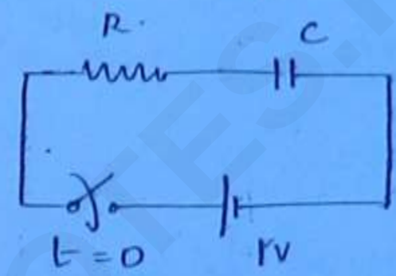
$$i_c = C \cdot \frac{dv_c}{dt} \Rightarrow i_c = C \cdot \frac{d}{dt}\left(3e^{-t/2}\right)$$

$$i_c = \frac{1}{2} \cdot \frac{-3}{2} \cdot e^{-t/2} \Rightarrow \boxed{i_c = -\frac{3}{4} e^{-t/2}}$$

Q. Find the rate of rise of voltage across capacitor at $t=0^+$.

(a) RC ✓(b) $1/RC$

c) 0 (d) V



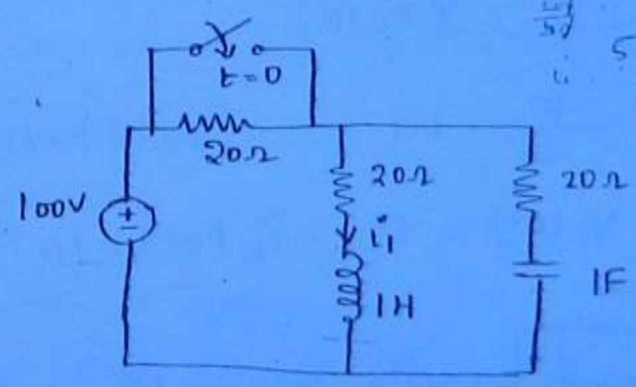
Soln:-

$$V_c = -Ve^{-t/RC} + V$$

$$\frac{dV_c}{dt} = \frac{+V}{RC} e^{-t/RC} + 0$$

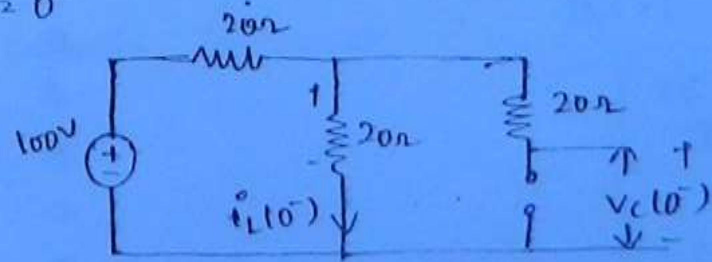
$$\left.\frac{dV_c}{dt}\right|_{t=0^+} = \frac{V}{RC} = \frac{1}{RC} \quad \because V = 1V$$

Q. Find $\frac{di_1}{dt}$ at $t=0^+$



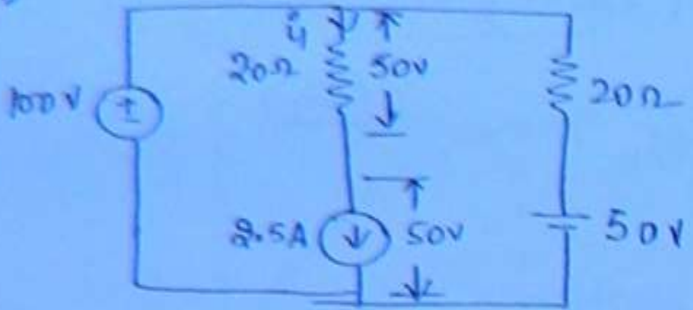
Soln. first find ckt at $t=0^-$ to calculate initial values

$t=0^-$



$$i_L(0^-) = \frac{100}{20+20} = 2.5A$$

$$V_c(0^-) = 2.5 \times 20 = 50V$$



(144) $i_L(0^+) = i_L(0^-) = 2.5A$

$V_c(0^-) = V_c(0^+) = 50V$.

$$V_L = L\frac{di_1}{dt}$$

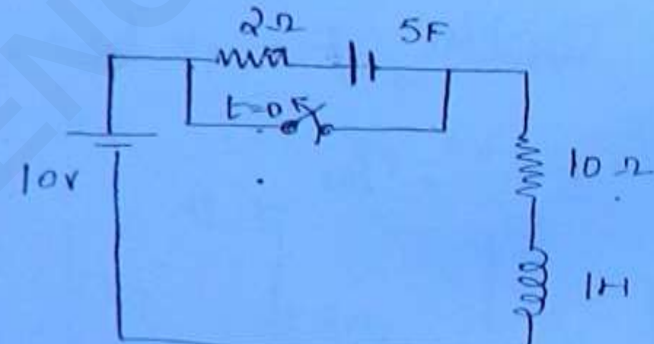
$$\frac{di_1}{dt} = \frac{V_L}{L}.$$

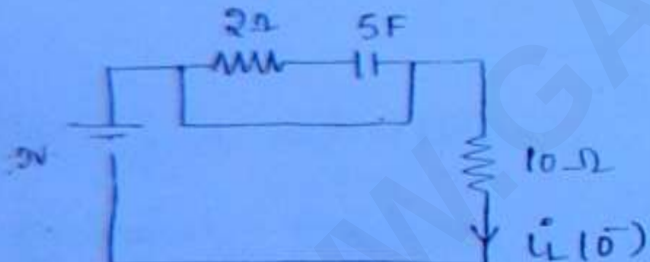
$$\left.\frac{di_1}{dt}\right|_{t=0^+} = 50\ A/s$$

Find voltage across the switch at $t = 0^+$

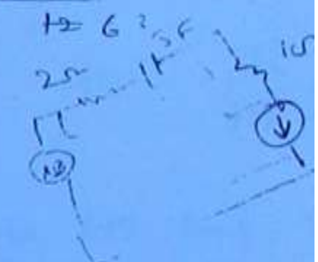
(a) 5 V (b) 10 V

(c) 0 (d) none.



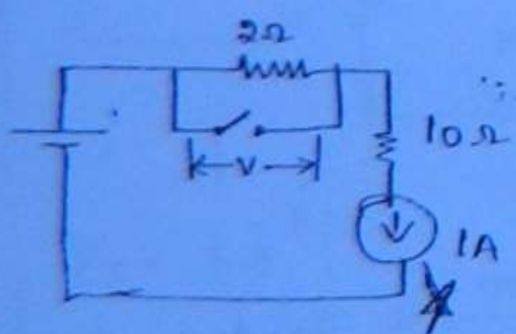


$\frac{10 \times 2}{12} = 6$ $\frac{5}{3}$



$i_L(0^-) = \frac{10}{10} = 1A$, $V_c(0^-) = 0$.

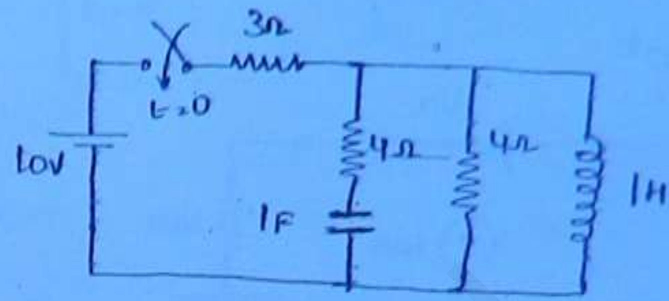
$t = 0^+$, $V_L(0^+) = 0$ $i_L(0^+) = 1A$



$V = 1 \times 2Ω$.

$\boxed{V = 2V}$

Q. Find $i_c(0^+)$, $V_L(0^+)$, $W_c(\infty)$, $W_L(\infty)$

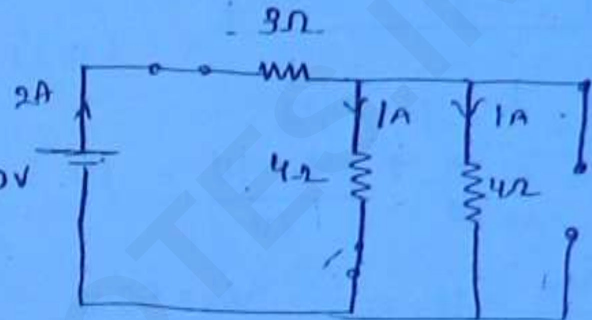


at $t=0^-$,

$V_c(0^-) = 0, \quad I_L(0^-) = 0.$

$t = 0^+$

$V_c(0^+) = 0, \quad I_L(0^+) = 0.$

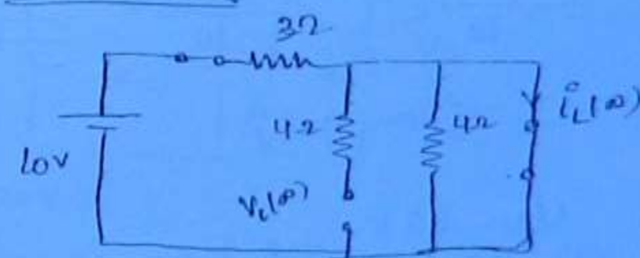


$$i_c(0^+) = \frac{10}{3+2/2} = \frac{2A}{2} = 1A$$

$i_c(0^+) = 1A.$

$$V_L(0^+) = \frac{10 \times 2}{2+3} = 4V. \Rightarrow V_L(0^+) = 4V$$

at $t = \infty$.



$i_L(\infty) = 10/3 \ A$

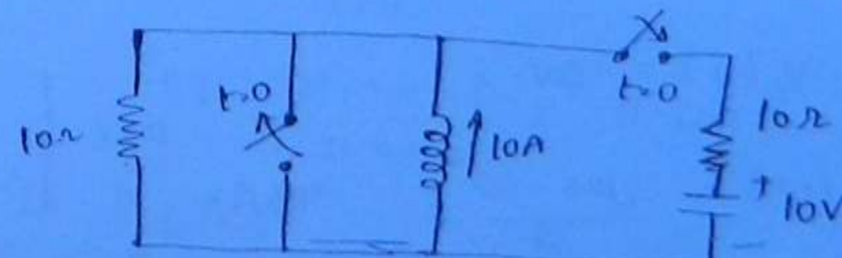
$$W_L(\infty) = \frac{1}{2} L \, i_L(\infty)^2$$

$$= \frac{1}{2} \times 1 \times \left(\frac{10}{3}\right)^2 = \frac{50}{3}$$

$V_c(\infty) = 0.$

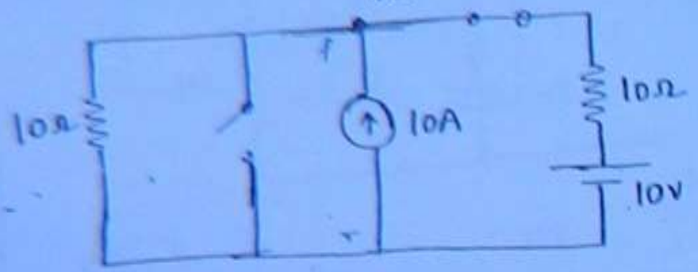
$$W_c(\infty) = \frac{1}{2} C V_c^2(\infty) = 0.$$

Q. Find voltage across inductor and current flowing through capacitor at $t=0^+$



$i_L(0^+) = i_L(0^-) = 10\,A$, $V_c(0^+) = V_c(0^-) = 10V$.

at $t = 0^+$



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By KCL,

$$\frac{V_a}{10} + \frac{V_a - 10}{10} = 10$$

$$\frac{V_a}{5} = 11 \Rightarrow V_a = 55\,V$$

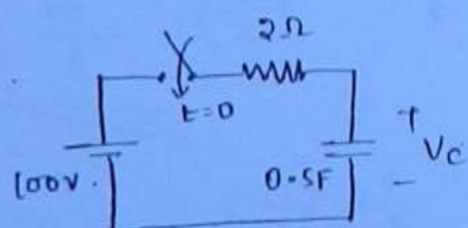
$$i_c = \frac{V_a - 10}{10} = \frac{55-10}{10} = \frac{45}{10} = 4.5A$$

$$i_c = 4.5A$$

$$V_L = V_a = 55V$$

Find $i_c(0^+)$, $i_c(1s)$.

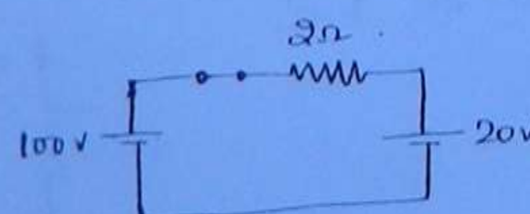
$V_c(0^+)$, $V_c(\infty)$



when $Q_0 = 10C$

$$V_0 = \frac{Q_0}{C} = \frac{10}{0.5} = 20V$$

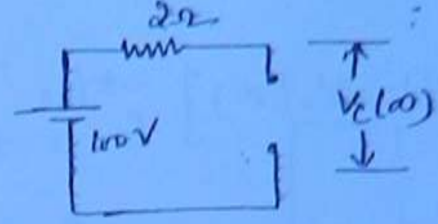
$$V_c(0^+) = V_c(0^-) = 20V.$$



$$i_c(0^+) = \frac{100-20}{2} = 40A.$$

$t = \infty$

$V_c(\infty) = 100V$ (147)



$W_c(\infty) = \frac{1}{2} C \cdot V_c^2(\infty)$

$i(\infty) = 0$

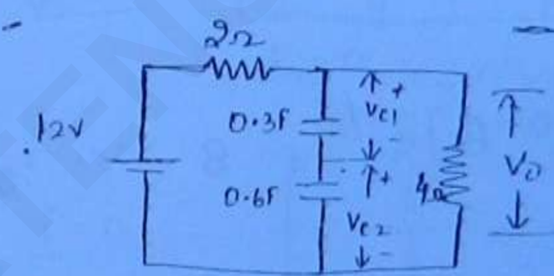
$= \frac{1}{2} \cdot 0.5 \times 100^2 = 2500J$

$i(t) = [i(0^+) - i(\infty)] e^{-t/RC} + i(\infty)$

$i(t) = 40 e^{-t}$ $RC = 1$

$i(1sec) = 40 e^{-1} = 14.7A$

Q. Find response of V_0 for the circuit shown.



(a) $22 e^{-3.75t} + 8$

(b) $22 e^{-t/3.75} + 8$

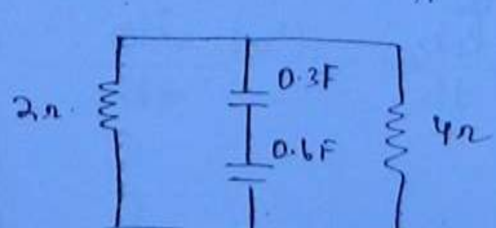
(c) $22 e^{-1.2t} + 8$

(d) $22 e^{-t/1.2} + 8$

$V_{C_1}(0) = 20V$

$V_{C_2}(0) = 10V$

Soln. Deactivate all the independant sources to find the ~~independant sources~~ time const.



$R_{eq} = \frac{2 \times 4}{2+4} = 6Ω$

$C_{eq} = \frac{0.3 \times 0.6}{0.3 + 0.6} = 0.2F$

$T = R_{eq} C_{eq}$

$-t/RC$

$$V_o(t) = \left[V_o(0^+) - V_o(\infty)\right] e^{-t/RC} + V_o(\infty) \rightarrow (1)$$

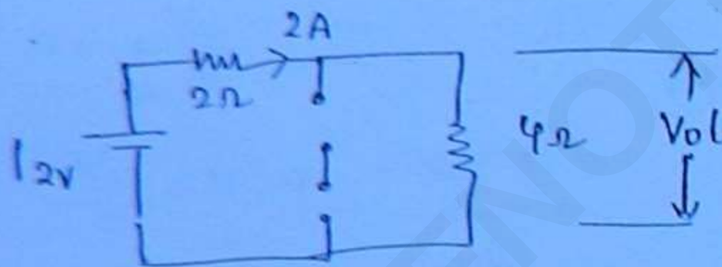
$V_o = V_{C1} + V_{C2}$

at $t = 0^+$

$V_o(0^+) = V_{C1}(0^+) + V_{C2}(0^+)$

$V_o(0^+) = 20 + 10 = 30V$

$t = \infty$



$V_o(\infty) = 8V$

in (1)

$$V_o(t) = [30-8]e^{-t/RC} + 8 = 22e^{-3.75t} + 8$$

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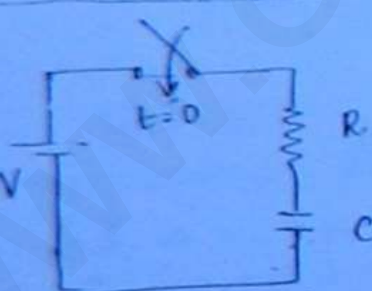
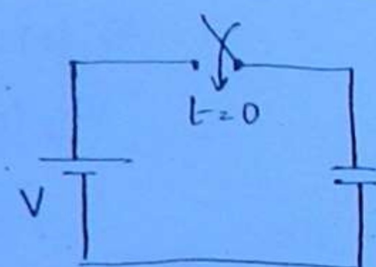
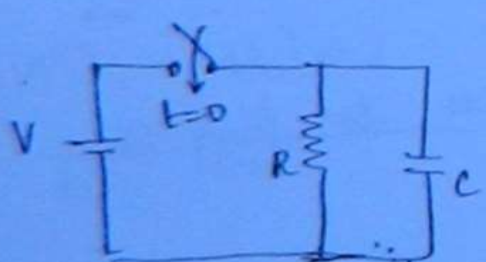
(1)



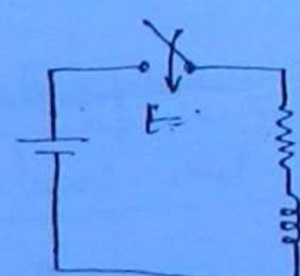
(2)

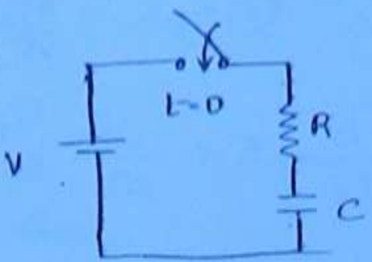


(3)



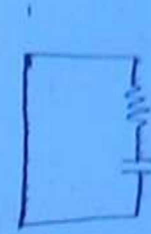
(4)



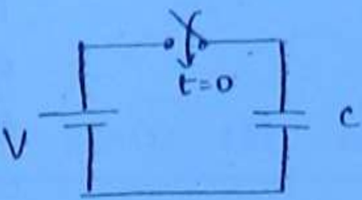
1. 

$t = 0^-$, $V_c = 0$

$t = 0^+$, $V_c = 0$

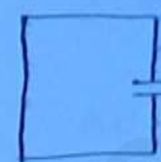


$\tau = RC$.

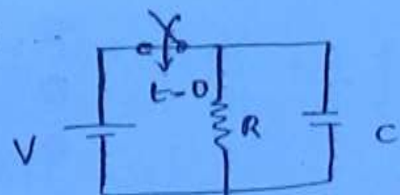
2.** 

$t = 0^-$, $V_c = 0$

$t = 0^+$, $V_c = V$

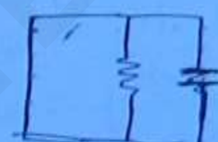


$\tau = RC = 0$.

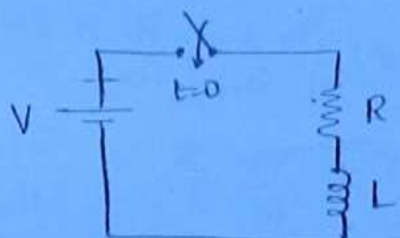
3.** 

$t = 0^-$, $V_c = 0$

$t = 0^+$, $V_c = V$



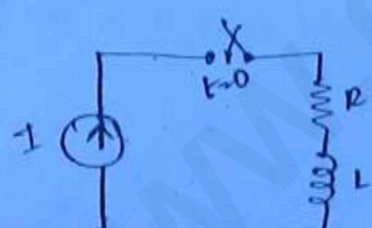
$R_{eq} = 0 \Rightarrow \tau = RC = 0$

4. 

$t = 0^-$, $i = 0$

$t = 0^+$, $i = 0$

$\tau = L/R$.

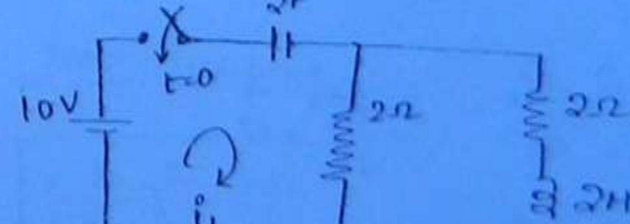
5.* 

$t = 0^-$, $i = 0$

$t = 0^+$, $i = I$

$\tau = \frac{L}{R_{eq}} = \frac{L}{\infty} = 0$

Q. Find $i_1(0^+)$, $i_2(0^+)$, $V_c(0^+)$, $\frac{di_1}{dt}(0^+)$, $\frac{di_2}{dt}(0^+)$, $\frac{d^2i_1}{dt^2}(0^+)$, $\frac{d^2i_2}{dt^2}(0^+)$.



at $t=0^-$,

$i_1(0^-) = 0$.

$i_2(0^-) = 0$.

$V_c(0^-) = 0$.

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t $t=0^+$,

$V_c(0^+) = i_2(0^+) = 0$.

$i_1(0^+) = \frac{10}{2} = 5A$.

KVL in loop 2.

$$4i_2 + 2\frac{di_2}{dt} - 2i_1 = 0. \longrightarrow (1)$$

t $t=0^+$

$$4\times 0 + 2\frac{di_2}{dt} - 2(5) = 0 \Rightarrow \frac{di_2}{dt}(0^+) = 5A/s$$

KVL in loop 1

$$10 = \frac{1}{C}\int i_1 dt + 2(i_1 - i_2)$$

diff wrt.

$$\frac{i_1}{2} + 2\frac{di_1}{dt} - 2\frac{di_2}{dt} = 0$$

$t=0^+$

$$\frac{5}{2} + 2\frac{di_1}{dt}\Big|_{0^+} - 2\times 5 = 0.$$

$$\frac{di_1(0+)}{dt} = 3.75\ A/sec.$$

diff (1) wrt

$$4\frac{di_2}{dt} + 2\frac{d^2 i_2}{dt^2} - 2\frac{di_1}{dt} = 0.$$

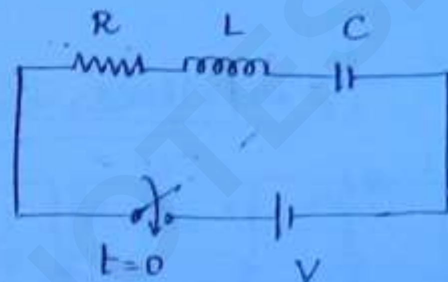
$t=0^+$ $\frac{d^2 i_2}{}$

– diff eq. ② wrt.

$$\frac{1}{2}\frac{di_1}{dt} + 2\left[\frac{d^2 i_1}{dt^2} - \frac{d^2 i_2}{dt^2}\right] = 0 .$$

$$\frac{d^2 i_1}{dt^2} =$$

RLC series ckt with DC excitation



$$V = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

diff wrt

$$0 = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C}$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0 . \qquad \frac{d}{dt} = D .$$

$$D_1, D_2 = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} .$$

$$D_1, D_2 = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

case 1:

$$\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$$ over damping

$$\alpha = \frac{-R}{2L} , \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{\ldots}}$$

$$\boxed{i(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}}$$

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Case 2:

$$\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$$ critical damping.

$$i(t) = (c_1 + c_2 t)\, e^{\alpha t}$$

Case 3:

$$\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$$ under damping

$$\alpha = -R/2L \quad , \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$\Rightarrow i(t) = (c_1 \cos\beta t + c_2 \sin\beta t)\, e^{\alpha t}$$

Damping coefficient $= \dfrac{R}{2L}$

$$\boxed{\text{Time constant} = \frac{1}{\text{damping coeff.}} = \frac{2L}{R}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$q = \frac{1/R}{\sqrt{LC}} \qquad \frac{1}{2a} = \frac{1}{2q}$$

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\therefore \omega_d = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{R^2 C}{4L}}$$

$$\boxed{\omega_d = \omega_0 \sqrt{1-\xi^2}}$$

Case 4:

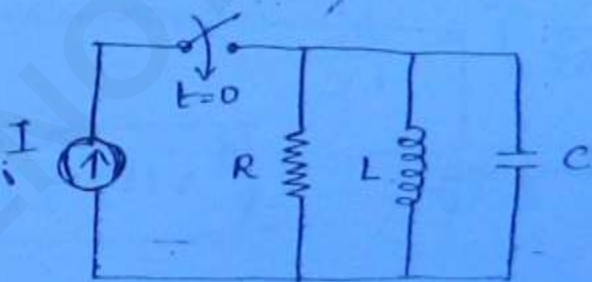
$\boxed{R = 0}$ undamping.

$\alpha = 0$ $\beta = \frac{1}{\sqrt{LC}}$

$$\boxed{i(t) = c_1 \cos\beta t + c_2 \sin\beta t}$$

RLC parallel circuit with DC excitation

$$I = \frac{V}{R} + C\frac{dv}{dt} + \frac{1}{L}\int v\,dt$$



diff w.r.t.

$$0 = \frac{1}{R}\frac{dv}{dt} + C\frac{d^2v}{dt^2} + \frac{v}{L}$$

$$\boxed{\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0}$$

$$\left(D^2 + \frac{D}{RC} + \frac{1}{LC}\right)V = 0.$$

$$D_1, D_2 = \frac{-1/RC \pm \sqrt{\left(\frac{1}{RC}\right)^2 - \frac{4}{LC}}}{2}$$

$$\boxed{D_1, D_2 = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}}$$

Case 1:

$$\left(\frac{1}{2RC}\right)^2 > \frac{1}{LC}$$ Overdamping

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$$\alpha = -1/2RC$$

$$\beta = \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\boxed{v(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}}$$

Case 2

$$\left(\frac{1}{2RC}\right)^2 = \frac{1}{LC}$$

critical damping

$$\boxed{v(t) = (c_1 + c_2 t)e^{\alpha t}}$$

Case 3:

$$\left(\frac{1}{2RC}\right)^2 < \frac{1}{LC}$$ underdamping

$$\alpha = \frac{-1}{2RC}, \quad \beta = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$$\boxed{v(t) = (c_1 \cos\beta t + c_2 \sin\beta t)e^{\alpha t}}$$

→ Damping coefficient $= \frac{1}{2RC}$

Time const. $= \frac{1}{\text{damping coeff.}} = 2RC$

$$\zeta = \frac{\frac{1}{2RC}}{\frac{1}{\sqrt{LC}}} = \frac{1}{2R}\sqrt{\frac{L}{C}}$$

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

$\omega_d = \frac{1}{\ldots}\sqrt{\ldots}$

case 4:

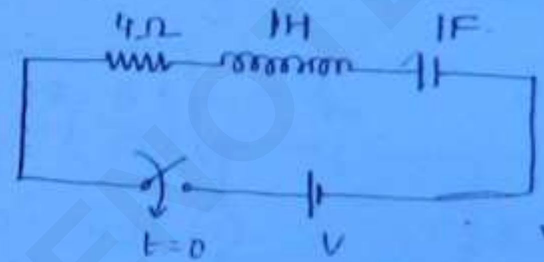
$G = 0$

$\frac{1}{R} = 0 \qquad R = \infty \qquad$ undamping

$$\beta = \frac{1}{\sqrt{LC}}$$

$$\boxed{V(t) = C_1 \cos\beta t + C_2 \sin\beta t}$$

***Q. Find current response for $t > 0$



$t \geq 0$

Note :

1. Steady state current response of RLC series circuit with DC excitation = 0.

2. $\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$

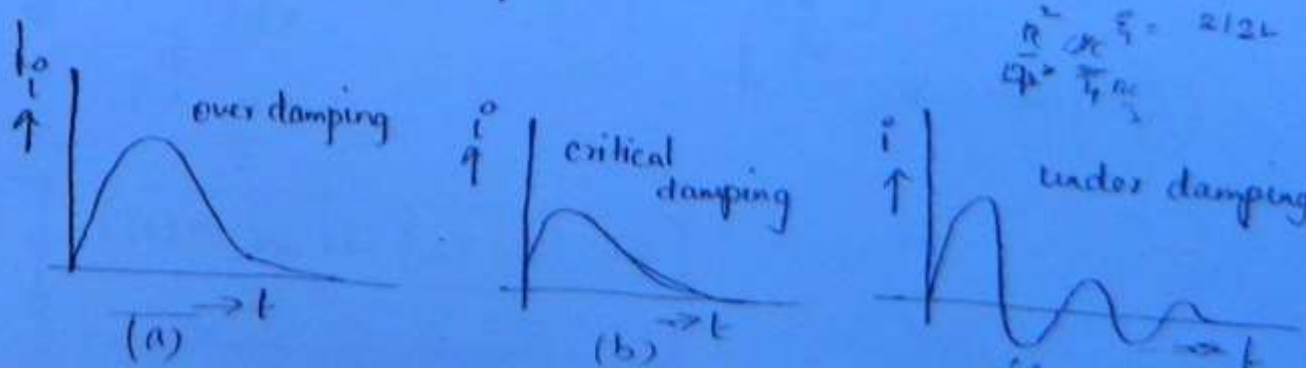
$\xi > 1 \Rightarrow$ over damped

$\xi = 1$ critically damped

$\xi < 1$ under damped

$\xi = 0$ undamped.

3.



⇒ In over & critical damping no oscillations are present.

⇒ $\xi \geq 1$ ⇒ $\frac{R}{2}\sqrt{\frac{C}{L}} \geq 1$ (for series ckt)

For under damping system more than one oscillation is present. ($\xi < 1$).

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When response is asked,

$$i(t) = c_1 e^{(\alpha-\beta)t} + c_2 e^{(\alpha+\beta)t}$$

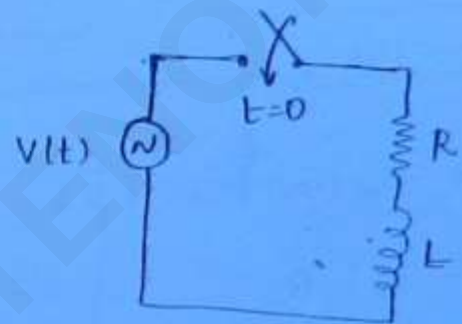
$$\alpha = \frac{-R}{2L}, \quad \beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

AC TRANSIENTS

→ Intensity of the DC transients are more than intensity of the AC transients.

→ In the AC circuit, based on selection of circuit elements, operating frequency and switching operation it is possible to obtain TRANSIENT FREE RESPONSE. But in the DC ckt, it is not possible to obtain transient free response.

$$V = iR + L\frac{di}{dt}$$



$$\boxed{\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}}$$

$$i(t) = CF + PI$$

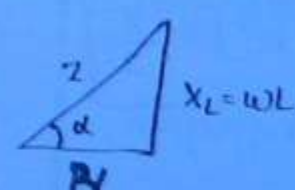
$V(t) = V_m \sin(\omega t + \theta)$

CF ⟶ Transient response

$$\frac{di}{dt} + \frac{R}{L}i = 0 \Rightarrow i(t) = Ae^{-Rt/L}$$

P.I → steady state response

$$i = \frac{V}{Z\angle\alpha}$$



$$i = \frac{V}{Z}\angle{-\alpha}$$

$$\alpha = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\boxed{i(t) = \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}$$

$i(t) = C.F + P.I$

$i(t) = Ae^{-R/L\,t} + \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)$

$t = 0^-, \quad i = 0$

$t = 0^+, \quad i = 0$

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$0 = A + \frac{V_m}{Z}\sin(\theta - \alpha)$

$A = -\frac{V_m}{Z}\sin(\theta - \alpha)$

$$\boxed{i(t) = \underbrace{-\frac{V_m}{Z}\sin(\theta - \alpha)e^{\frac{-Rt}{L}}}_{T.R} + \underbrace{\frac{V_m}{Z}\sin(\omega t + \theta - \alpha)}_{S.R}}$$

<u>Case 1:</u> $v(t) = V_m \sin(\omega t + \theta), \quad t = 0$

$\theta - \alpha = 0$

** $\theta = \alpha$

$$\boxed{\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)}$$ <u>Condition for transient free response</u>

<u>Case 2:</u> $v(t) = V_m \cos(\omega t + \theta), \quad t = 0$

** $$\boxed{\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}}$$

<u>Case 3:</u>

$i(t) = CF + PI$

$i(t) = Ae^{-\frac{R}{L}(t - t_0)} + \frac{V_m}{Z}\sin(\omega t + \theta - \alpha)$

$t = t_0^- \quad i = 0$

$t = t_0^+ \quad i = 0$

$$0 = A + \frac{V_m}{Z}\sin(\omega t_0 + \theta - \alpha)$$

$$\boxed{A = \frac{-V_m}{Z}\sin(\omega t_0 + \theta - \alpha)}$$

Case 3:

$$V(t) = V_m \sin(\omega t + \theta), \quad t = t_0$$

$$\omega t_0 + \theta - \alpha = 0$$

$$\omega t_0 = \alpha - \theta$$

**

$$\boxed{\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta}$$

Case 4:

$$V(t) = V_m \cos(\omega t + \theta), \quad t = t_0$$

$$\boxed{\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \pi/2}$$

parallel

series

	RL		RC
1. $V(t) = V_m \sin(\omega t + \theta), t = 0$ $i(t) = I_m \sin(\omega t + \theta), t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right)$	$\theta = \tan^{-1}(\omega\tau)$	$\theta = \tan^{-1}(\omega RC)$
2. $V(t) = V_m \cos(\omega t + \theta), t = 0$	$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) + \frac{\pi}{2}$	$\theta = \tan^{-1}(\omega\tau) + \pi/2$	$\theta = \tan^{-1}(\omega RC) + \frac{\pi}{2}$
3. $V(t) = V_m \sin(\omega t + \theta), t = t_0$	$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$	$\tan^{-1}(\omega\tau) - \theta$	$\tan^{-1}(\omega RC) - \theta$
4. $V(t) = V_m \cos(\omega t + \theta), t = t_0$	$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta + \frac{\pi}{2}$	$\tan^{-1}(\omega\tau) - \theta + \frac{\pi}{2}$	$\tan^{-1}(\omega RC) - \theta + \pi/2$

Note

→ In RLC ckt, it is not possible to obtain transient free response since ckt is having two energy storing element.

Ex. for underdamped RLC sys.

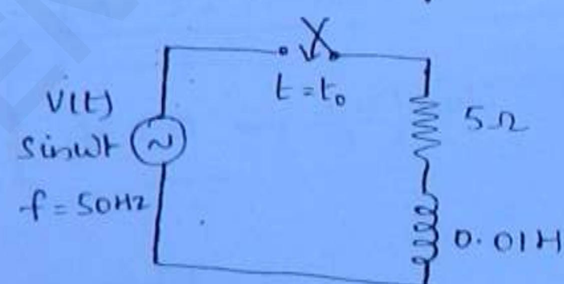
$$i(t) = (C_1 \cos\beta t + C_2 \sin\beta t)e^{\alpha t}.$$

For transient free response,

$$C_1 \cos\beta t + C_2 \sin\beta t \text{ should be } = 0$$

But cos & sineterm will never be equal to zero simultaneously. Hence, there will be no transient free response.

- At what value of t_0 transient free response is obtained?



$$\omega t_0 = \tan^{-1}\left(\frac{\omega L}{R}\right) - \theta$$

$$\theta = 0^\circ$$

$$\omega t_0 = \tan^{-1}\left(\frac{2\pi \times 50 \times 0.01}{5}\right) = \tan^{-1}(0.2\pi).$$

$$t_0 = \frac{32.14 \times \pi}{2\pi \times 50 \times 180}$$

$\tan^{-1}(0.2\pi)$ should be taken in radians

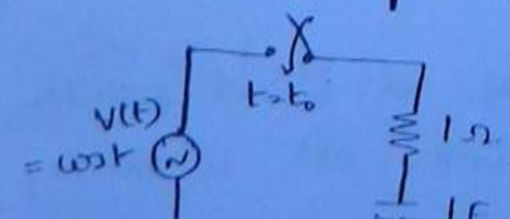
$$t_0 = 1.78 \text{ msec}.$$

$\frac{32.14}{1000 \times 18.0}$ 16.07 1.78

- At what value of t_0, transient free response is obtained?

$$\omega t_0 = \tan^{-1}(\omega RC) - \theta + \pi/2.$$

$$\theta = 0^\circ$$



$\omega = 1$.

$$t_0 = \tan^{-1}(1) + \pi/2$$

$$= \pi/4 + \pi/2 = \frac{3\pi}{4}$$

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$$t_0 = \frac{3\pi}{4} \text{ sec}.$$

#/8/11

Laplace Transforms.

$A \longrightarrow A/s$

$e^{at} \longrightarrow \frac{1}{s-a}$

$\sin\omega t \longrightarrow \frac{\omega}{s^2+\omega^2}$; $\cos\omega t \longrightarrow \frac{s}{s^2+\omega^2}$

$e^{at}\sin\omega t \longrightarrow \frac{\omega}{(s-a)^2+\omega^2}$; $u(t) \longrightarrow 1/s$

$e^{-at}\cos\omega t \longrightarrow \frac{s+a}{(s+a)^2+\omega^2}$

$\delta(t) \longrightarrow 1$

$\frac{df}{dt} \longrightarrow sF(s) - f(0^+)$

Initial value theorem.

$$f(0^+) = \lim_{s\to\infty} sF(s) = \lim_{t\to 0} f(t)$$

Find initial value of $F(s) = \frac{(2s+1)(s+3)}{s(s+2)(3s+4)}$.

$$F(s) = \frac{(2+1/s)(1+3/s)}{s(1+2/s)(3+4/s)}$$

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$$f(0^+) = \lim_{s \to \infty} sF(s) = \lim_{s \to 0} \frac{(2+1/s)(1+3/s)}{(1+2/s)(3+4/s)}$$

$$= \frac{2 \times 1}{3} = \underline{\underline{\frac{2}{3}}}$$

Find initial value of the following function.

$$f(t) = 6 + \delta(t).$$

$$F(s) = \frac{6}{s} + 1$$

Note

∴ the above function initial value theorem cannot be applied. Since, to apply the initial " " denominator power should be > numerator power.

Final value theorem.

$$f(0^+) = \lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t).$$

Find final value of the following function.

$$F(s) = \frac{(s+1)(s+3)}{s(s+2)(s+6)}.$$

$$f(\infty) = \lim_{s \to 0} sF(s) = \frac{1 \times 3}{2 \times 6} = \frac{1}{4}$$

Q. Find final value of the following function:

$f(t) = 3 + e^{2t}$.

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$F(s) = \frac{3}{s} + \frac{1}{s-2}$

Note: For the above function final value theorem cannot be applied since pole is present in right 1/2 of the s plane. [unstable system].

Q. Find final value of the following function

$F(s) = \frac{\omega}{s^2+\omega^2}$ (a) 0 (b) 1 (c) ∞ (d) none

Soln

$f(t) = \sin\omega t$.

ans: lies b/n 1 & −1.

Note: For marginally stable system also, final value theorem cannot be applied.

Q. Current flowing through 4H inductor is given by, $I(s) = \frac{10}{s(s+2)}$. Find initial voltage of the inductor.

Soln.

$V_L = L\frac{di}{dt} \Rightarrow V_L(s) = 4\left[sI(s) - i(0^+)\right]$

$V(s) = 4sI(s) - 4i(0^+)$.

$i(0^+) = \lim_{s \to \infty} sF(s) = \frac{s \cdot 10}{s(s+2)}$

$i(0^+) = 0.$

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$V(s) = 4s \cdot I(s) = \frac{4s \cdot 10}{s(s+2)}.$

$= \frac{40}{(s+2)}$

$V(0^+) = \lim_{s \to \infty} sV(s) = \lim_{s \to \infty} \frac{s \cdot 40}{s[1+2/s]} = 40.$

$V(0^+) = 40V.$

Exceptional case in inductor.

$i_L = \frac{1}{L} \int_{-\infty}^{t} V dt$

$i_L(t) = \frac{1}{L} \int_{-\infty}^{0^-} V dt + \frac{1}{L} \int_{0^-}^{t} V dt$

$i_L(t) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{t} V dt$

0^+

$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} V dt$ ↳ inductor doesn't allow sudden current change = 0

if

$i_L(0^+) = i_L(0^-) + 0.$

$i_L(0^+) = i_L(0^-) + \frac{1}{L} \int_{0^-}^{0^+} V dt.$

$\boxed{\text{then} \quad V \to \delta(t)}$

$$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} \delta(t)\,dt \rightarrow \text{area} = 1.$$

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$$\boxed{i_L(0^+) = \frac{1}{L}} \rightarrow (1) \qquad i_L(0^-) = 0.$$

$$W(0^+) = \frac{1}{2} L\, i_L^2(0^+) = \frac{1}{2} \cdot L \left(\frac{1}{L}\right)^2$$

$$\boxed{W(0^+) = \frac{1}{2L}}$$

Note :-

From above relation it is concluded that inductor doesn't allow instantaneous change for given i/p.

→ from above relation it is concluded that inductor allows instantaneous changes for voltage impulse function. (Imp)

In case of capacitor

$$V_c = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

$$V_c(t) = \frac{1}{C}\int_{-\infty}^{0^-} i\,dt + \frac{1}{C}\int_{0^-}^{t} i\,dt$$

$$V_c(t) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i\,dt$$

$t = 0^+$,

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i\,dt$$

note :- $V_c(0^+) = V_c(0^-) + 0$

From the above relation, it is concluded that capacitor

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i\,dt$$

when $i \rightarrow \delta(t)$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} \delta(t)\,dt$$

area = 1.

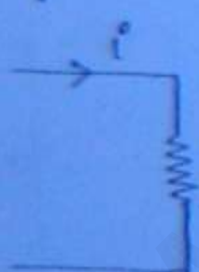
$$\boxed{V_c(0^+) = 1/C}$$

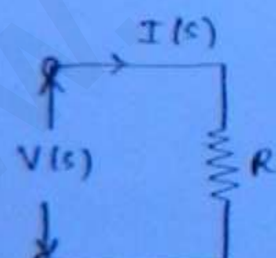
$$W_c(0^+) = \frac{1}{2} C V_c^2(0^+) = \frac{1}{2} C \left[\frac{1}{C}\right]^2$$

$$\boxed{W_c(0^+) = \frac{1}{2C}}$$

Note: from the above relation it is concluded that capacitor ~~doesn't~~ allow instantaneous voltage change for a ~~given tipped~~ current impulse function.

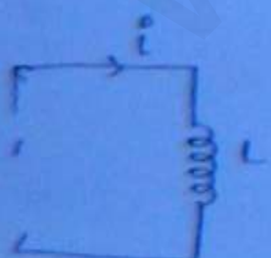
Laplace domain

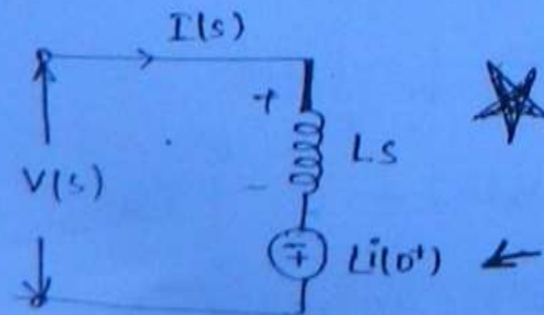




$V(t) = R\,i(t)$

$V(s) = R\,I(s)$





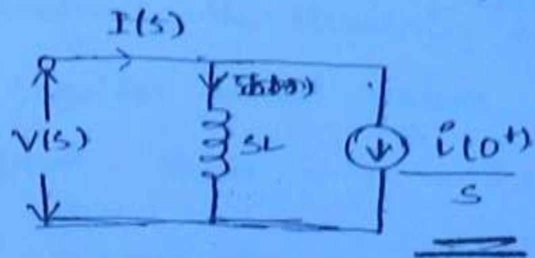
$V = L\frac{di}{dt}$

$V(s) = L[sI(s) - i(0^+)]$

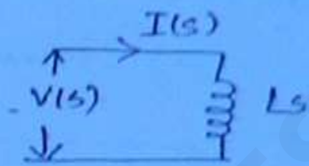
$Z(s) = \frac{V(s)}{I(s)} = L(s) - L\,i(0^+)$

$$LsI(s) = V(s) + Li(0^+)$$

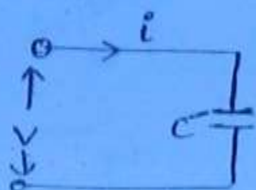
$$I(s) = \frac{V(s)}{Ls} + \frac{\cancel{L}i(0^+)}{\cancel{L}s} \rightarrow (\text{kcl}).$$



$i(0^+) = 0.$



Capacitor

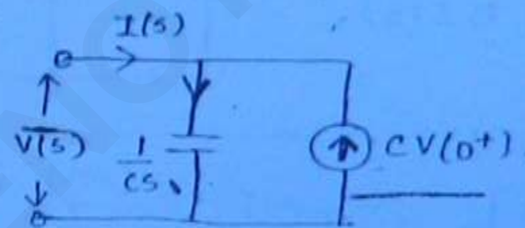


$$i = C\frac{dv}{dt}$$

$$I(s) = C[sV(s) - V(0^+)]$$

$$I(s) = \frac{V(s)}{1/Cs} - CV(0^+)$$

↳ kcl.



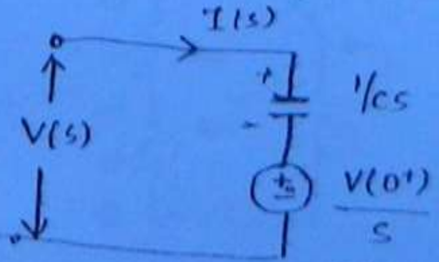
$$Y(s) = \frac{I(s)}{V(s)}$$

$$Y(s) = Cs - \frac{CV(0^+)}{V(s)}$$

$V = L\frac{di}{dt} = sLI(s)$

$$\frac{V(s)}{1/Cs} = I(s) + CV(0^+)$$

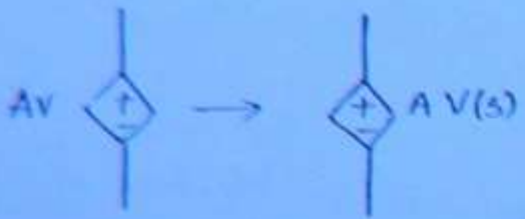
$$V(s) = \frac{1}{Cs}I(s) + \frac{CV(0^+)}{Cs} \rightarrow \text{KVL}.$$

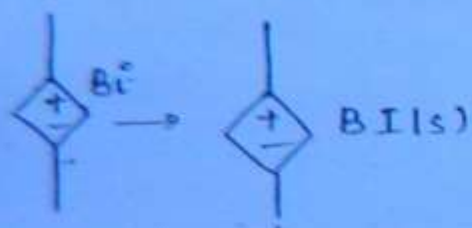


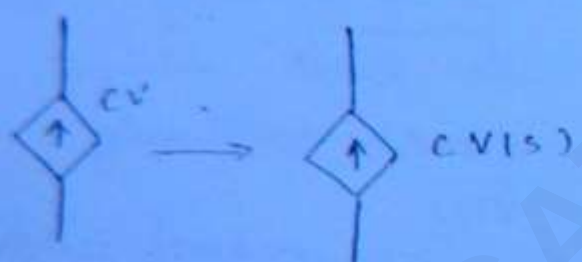
$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{Cs} + \frac{V(0^+)}{sI(s)}$$

$V(0^+) = 0.$

ansformation of dependant sources from time domain to s-domain







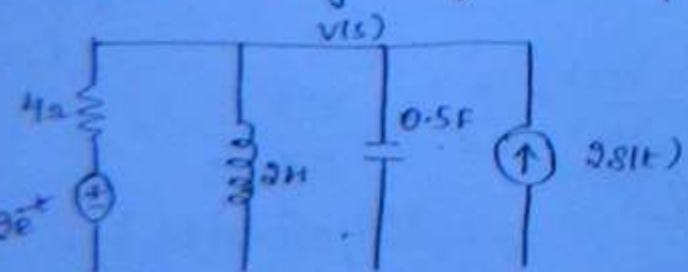
$V(s) = R I(s) + L\frac{di}{dt}$

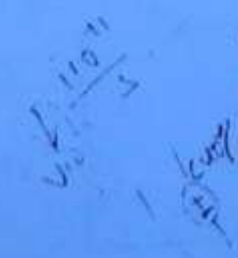
$V(s) = \cancel{R I(s)} + sL I(s) - L i(0^+)$

$V(s) = s L I(s) - i(0^+)$

$\frac{V(s)}{sLI(s) - Li(0)} \; \frac{1}{sL} = \frac{Li(0^+)}{sL}$

Find V(s) when initial current of the inductor is 2A and initial voltage of the capacitor is 3V





Soln

$i_L(0^+) = 2A$.

$V_0 = 3V$.

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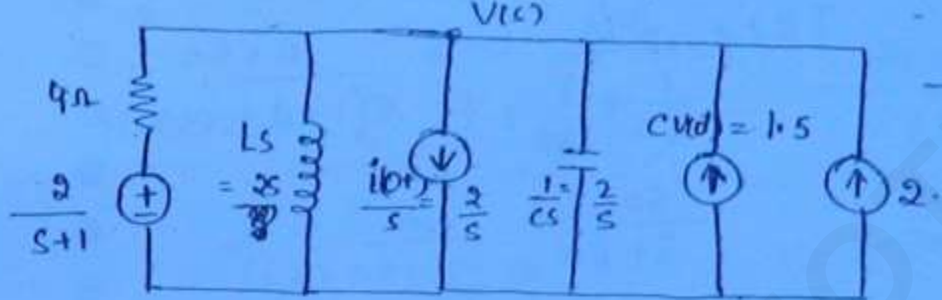
* Whenever, the elements in circuit are in parallel, then consider initial values as current sources & vice versa i.e series ⟶ voltage sources.

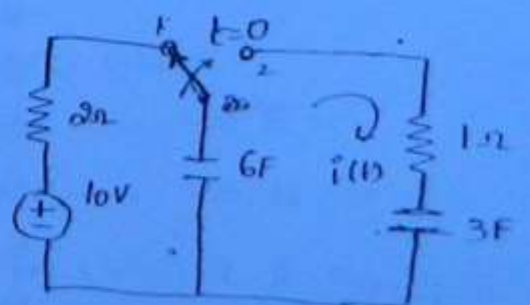


⟶ in s-domain

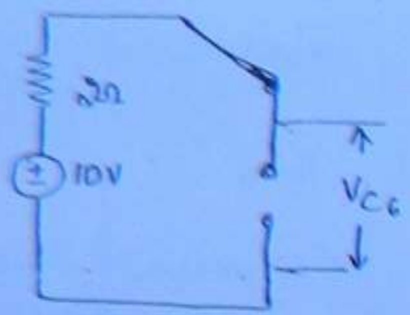
$$\frac{V(s) - 2/s+1}{4} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{V(s)}{2/s} = 1.5 + 2$$

$$\frac{(s+1)V(s) - 2}{4(s+1)} + \frac{V(s)}{2s} + \frac{2}{s} + \frac{s \cdot Vs}{2} = 3.5$$

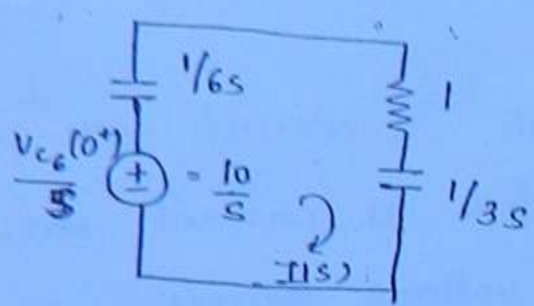
Q. Find $i(t)$ for $t > 0$



$t = 0^-$



$t > 0$



$V_{C6}(0^-) = 10V$

$V_{C3}(0^-) = 0$

$$I(s) = \frac{10/s}{\frac{1}{6s} + \frac{1}{3s} + 1} = \frac{10 \times 6s}{s(1+2+6s)}$$

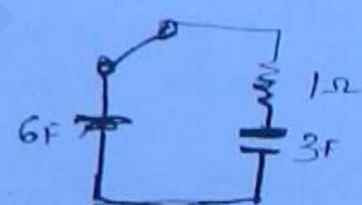
$$= \frac{60}{3+6s} = \frac{20}{1+2s}$$

$$= \frac{10}{s+1/2} \Rightarrow \quad i(t) = 10e^{-t/2}$$

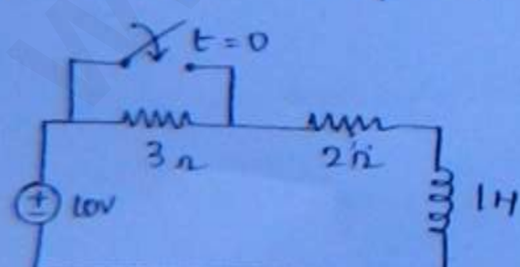
-2 using classical method.

$$i(t) = \frac{V_0}{R} e^{-t/RC}$$

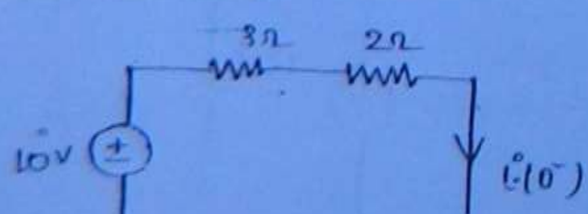
$$i(t) = \frac{10}{1} e^{-t/2}$$



Find current response for $t > 0$.



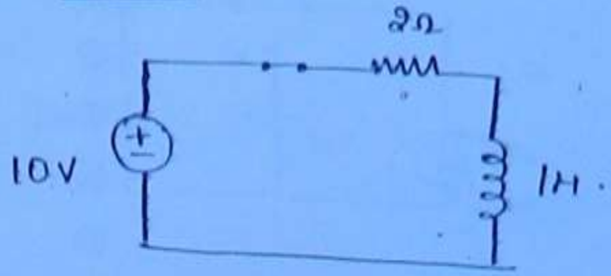
at $t = 0^-$,



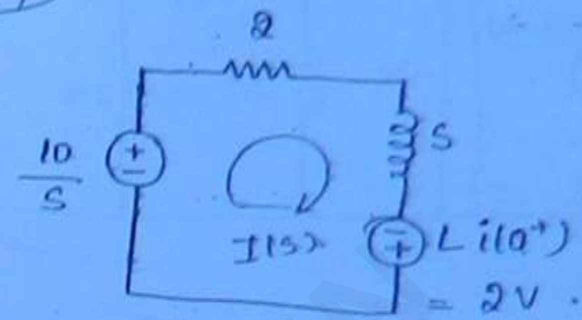
$i(0^-) = 2A$

$5 - [5-2]e^{-2t/1}$
$= 5 - 3e$

$t > 0$



S domain $\longrightarrow$



$$2 + \frac{10}{s} = (2+s)\,I(s)$$

$$\Rightarrow \quad I(s) = \frac{2s+10}{s(2+s)}$$

$$I(s) = \frac{5}{s} - \frac{3}{s+2}$$

$$i(t) = 5 - 3e^{-2t}$$

$\longrightarrow$ obj – classical

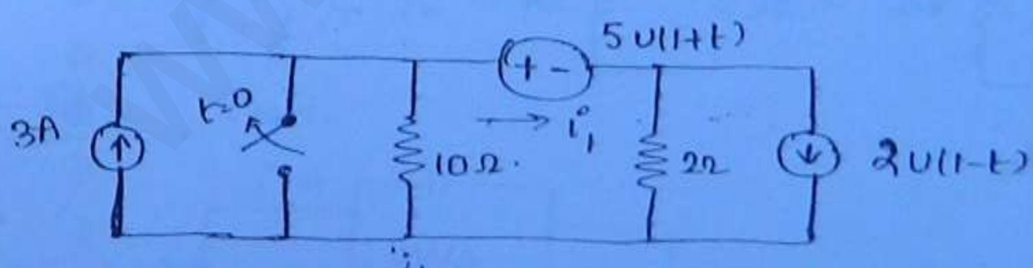
Subj – L.T

For obj

$$i(t) = [\,i(0^+) - i(\infty)\,]e^{-Rt/L} + i(\infty)$$

$$= (2-5)\,e^{-2t} + 5 = 5 - 3e^{-2t}$$

Q. Find the value of i_1 at $t = -2$ sec.



$U(t) \rightarrow 0$ to ∞

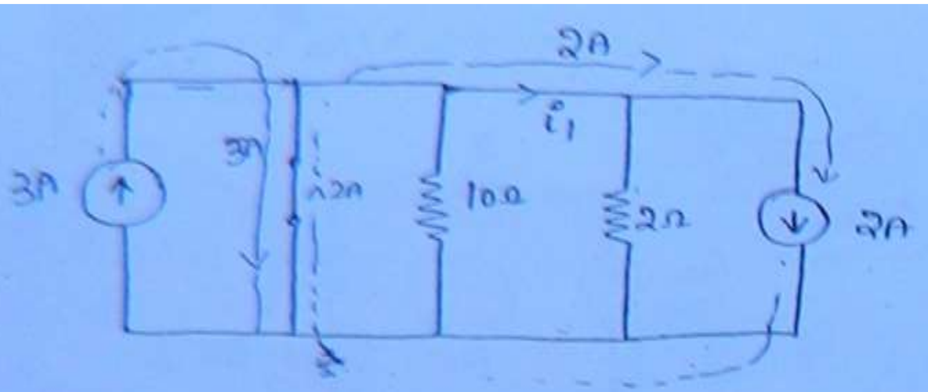
$U(-t) \rightarrow -\infty$ to 0

D.C $\rightarrow -\infty$ to ∞

$U(1+t) \rightarrow -1$ to ∞

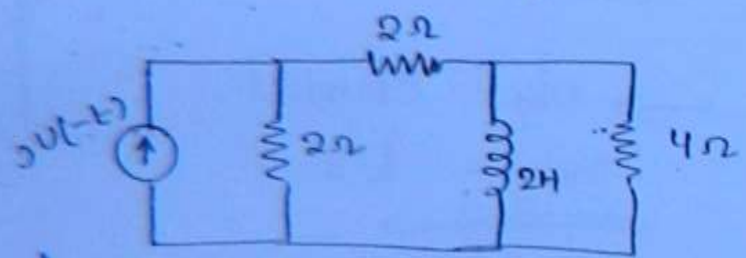
$U(1+t) \rightarrow -1$ to ∞

$U(1-t) \rightarrow -\infty$ to 1

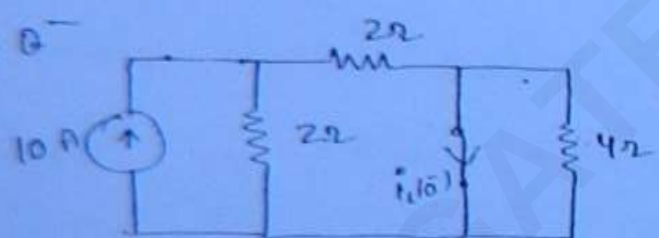


$$i_1\Big|_{t\to 2} = 2A$$

Find current response in the inductor for $t > 0$

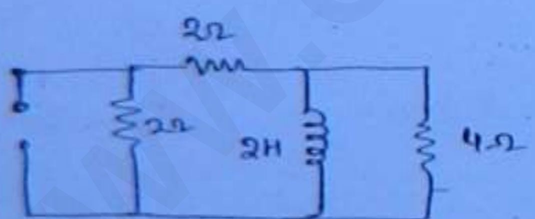


$t = 0^-$

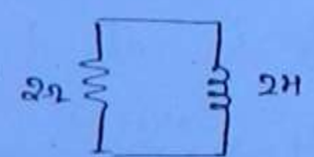


$\Rightarrow \quad i_L(0^-) = 5A$

$t > 0$



$\Rightarrow$

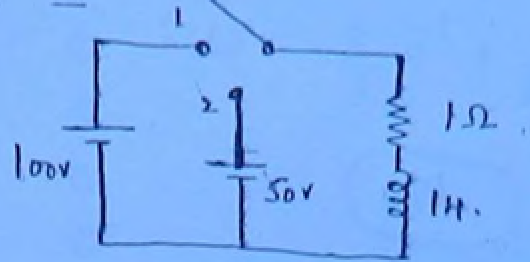


$\Rightarrow \quad \tau = \frac{L}{R} = 1$

$$i(t) = I_0 e^{-Rt/L}$$

$$= 5e^{-t}$$

In the circuit shown, at $t = 0$ sec, the switch is connected to 1. After 1 time constant, the switch is transferred to position 2. Find the current response when the switch is at position 2.



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Soln. position - 1

$$i(t) = 100\left[1 - e^{-t}\right] \qquad \left[\frac{V}{R}\left[1-e^{-Rt/L}\right]\right]$$

position 2.

$$i(t) = \left[i(t_0) - i(\infty)\right] e^{\frac{-R(t-t_0)}{L}} + i(\infty)$$

50

$50 - [50 - 63.2]\, e^{(t-1)}$

$+ 13 e^{-(t-1)}$

at $t = t_0 = 1s \Rightarrow 1\tau$ (one time const.)

$i(t_0) = 100(1 - e^{-1}) = 63.2A$ ($\because$ from position 1)

at position 2,

$$i(t) = \left[63.2 - 50\right] e^{-(t-1)} + 50$$

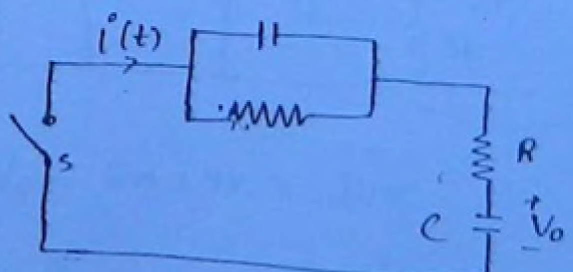
$$i(t) = 50 + 13.2 e^{-(t-1)}$$

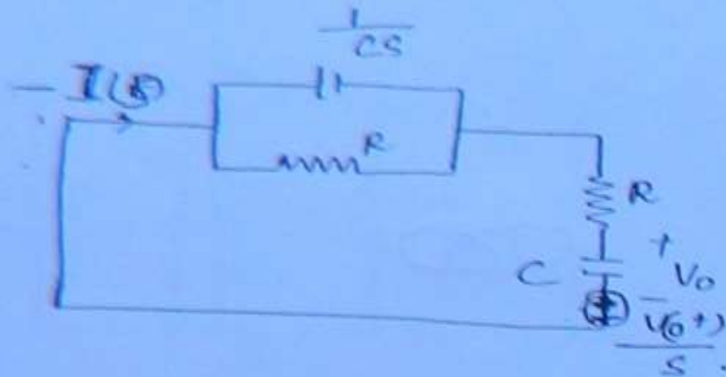
$f(t) - F(s)$

$f(t+1) - F(s)e^{s}$

Pg - 34 Conv.

1.





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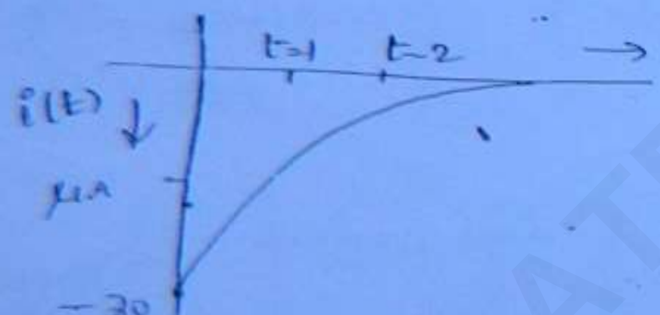
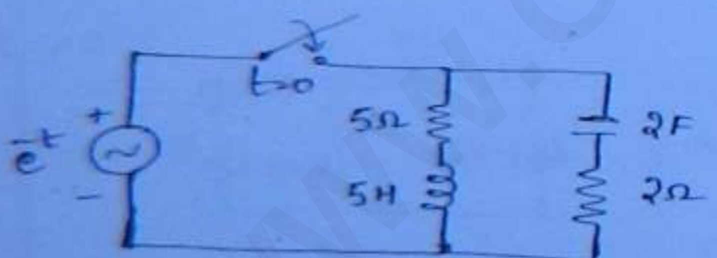
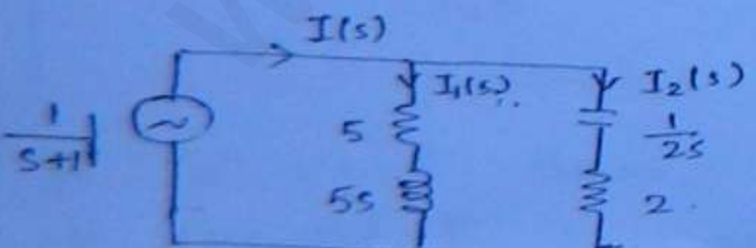
$$I(s) = \frac{-V(0^+)/s}{R + \frac{1}{Cs} + \frac{R \cdot 1/Cs}{R + 1/Cs}}$$

$$i(t) = -\left[14.46e^{-2.62t} + 5.54e^{-0.38t}\right] \mu A$$





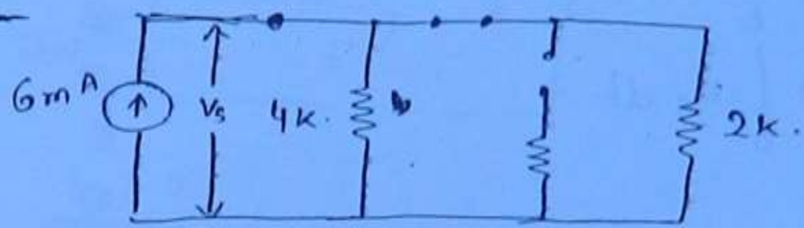


$$I(s) = I_1(s) + I_2(s) = \frac{1/s+1}{5+5s} + \frac{1/s+1}{2+1/2s}$$

$$i(t) = \frac{2}{3}e^{-t} + \frac{1}{5}te^{-t} - \frac{1}{6}e^{-t/4}$$

4.

$t = 0^-$



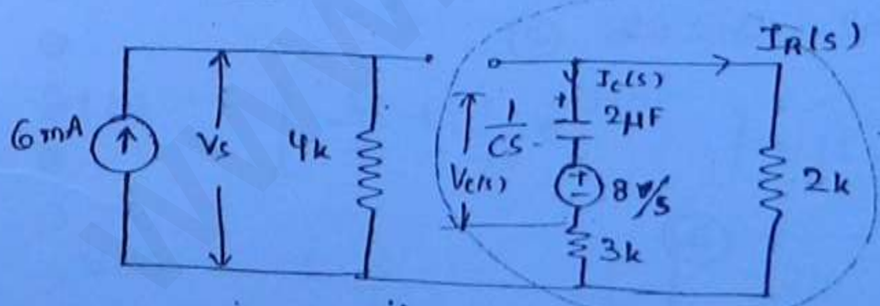
$$I_{4k} = \frac{6m \times 2k}{4k + 2k} = \frac{12}{6k}$$

$$I_{4k} = 2mA$$

$$V_{4k} = 2mA \times 4k = 8V$$

$$\Rightarrow V_c(0^-) = V_{4k} = \underline{8V} \Rightarrow V_c(0^-) = 8V$$

at $t = 0^+$



$\rightarrow$ S domain

$$V_s = 6m \times 4k = 24V$$

$$I_R(s) \quad \frac{V(0^+)/s}{3k + 2k + 1/Cs} = \frac{8/s}{5k + \frac{500k}{S}}$$

$$= \frac{8m}{-} = 8m$$

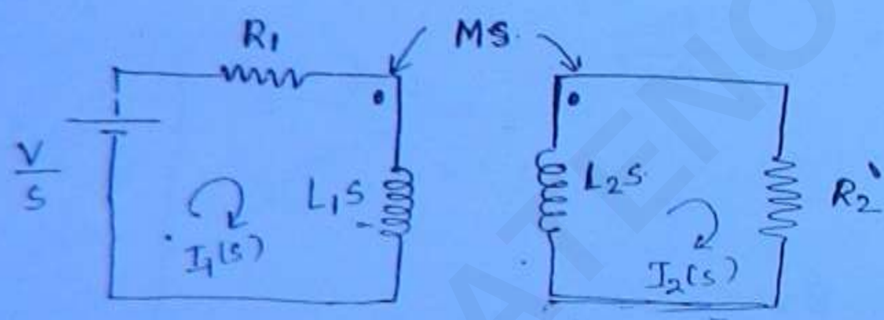
$$i_R(t) = 1.6\, e^{-100t}\ mA.$$

$$i_c(t) = -i_R(t) = -1.6\, e^{-100t}\ mA.$$ 176

$$V_c = \frac{1}{C}\int_{-\infty}^{t} i_c\, dt$$

$$= \frac{1}{C}\int_{-\infty}^{0} i_c\, dt + \frac{1}{C}\int_{0}^{t} i_c\, dt$$

$$V_c = 8 + \frac{1}{C}\int_{0}^{t} i_c\, dt$$



$$-\frac{V}{S} + (R_1 + L_1 S) I_1(S) - MS\, I_2(S) = 0 \longrightarrow (1)$$

$$(R_2 + L_2 S) I_2(S) - MS\, I_1(S) = 0 \longrightarrow (2)$$

From eq. (2)

$$I_2(S) = \frac{MS\, I_1(S)}{R_2 + L_2 S} \longrightarrow (3)$$

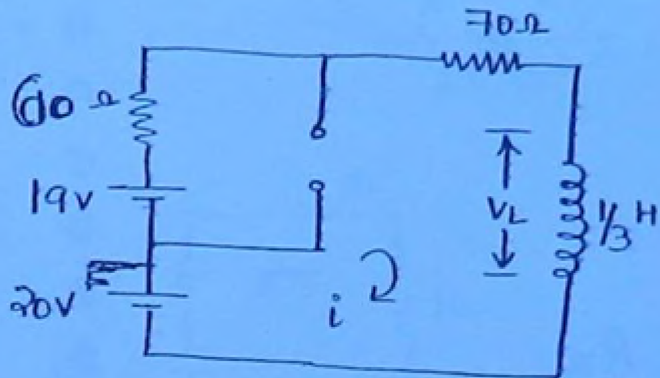
Sub eq (3) in (1)

$$i_1(t) = \left[5 - e^{-t/s}\right] u(t)$$

6.

$t = 0^-$

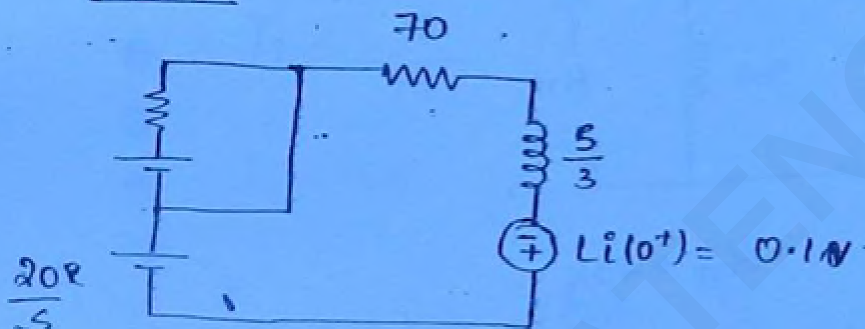
177



$$i(0^-) = \frac{20+19}{60+70}$$

$$i(0^-) = 0.3A$$

$t > 0$



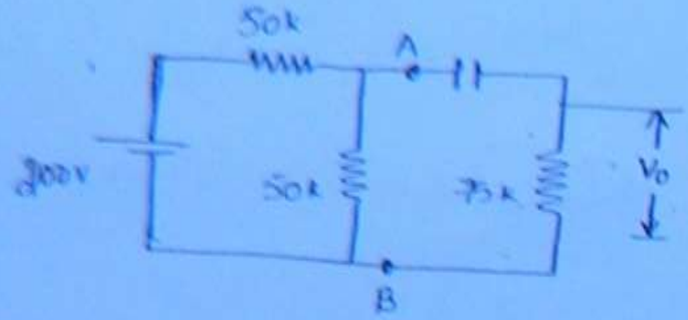
$$I(s) = \frac{\frac{20}{s} + 0.1}{70 + s/3} = \frac{3(20 + 0.1s)}{s[s+210]}.$$

$$= \frac{2/7}{s} + \frac{1/70}{s+210}$$

$$i(t) = \left[\frac{2}{7} + \frac{1}{70}e^{-210t}\right]u(t)$$

$$V(t) = L\frac{di}{dt} = \frac{1}{3}\left[\frac{-210}{70}e^{-210t}\right]$$

$$V(t) = -e^{-210t}$$



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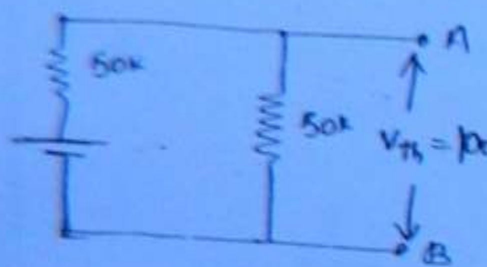
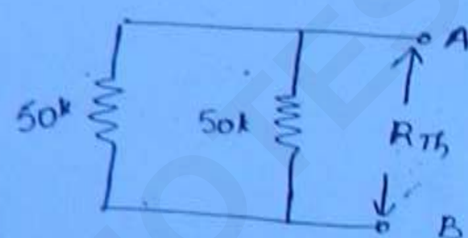
Thevenin eq. across AB.

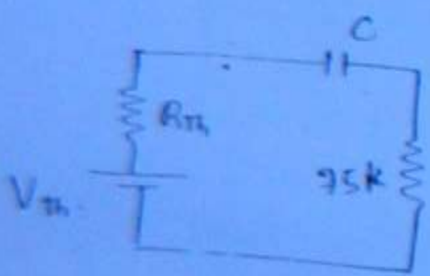
Case i) (V_{Th}).



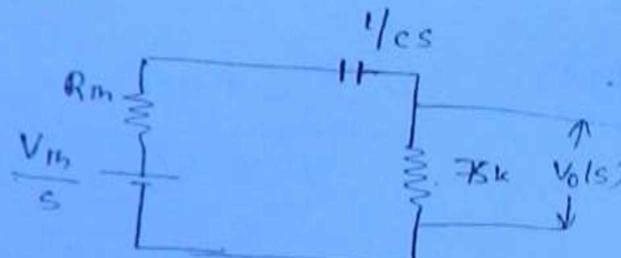
Case ii) R_{Th}.



$R_{Th} = 25k$.



S domain



$$V_0(s) = \frac{V_{Th}}{s} \cdot \frac{75k}{75k + \frac{1}{Cs} + R_{Th}}$$

$$V_0(t) = \mathcal{L}^{-1} V_0(s) \Rightarrow V_0(t) = 75e^{-10t}.$$

at $t = t_0$

$$V_0(t) = 75e^{-(t-t_0)}$$

$t = t_0$

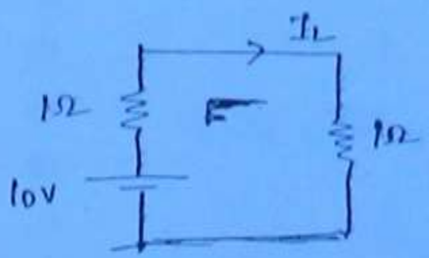
$$V_0 = 75V$$

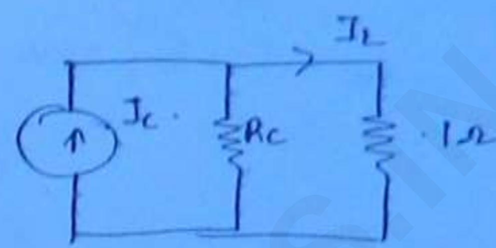
$$t = 25\times10^{-3} \Rightarrow V_0(t) = e^{-10(25\times10^{-3} - t_0)}$$

Theorems obj.

25. $i = \dfrac{V_1 - V_2}{1 \times 10^6} \Rightarrow R_{CRO} = \dfrac{V_2}{i}$

28.





$I_L = \dfrac{10}{1+1}$

$I_L = 5A$

$I_L = I_C \cdot \dfrac{R_C}{R_C + 1}$

$5 = \dfrac{I_C R_C}{R_C + 1}$

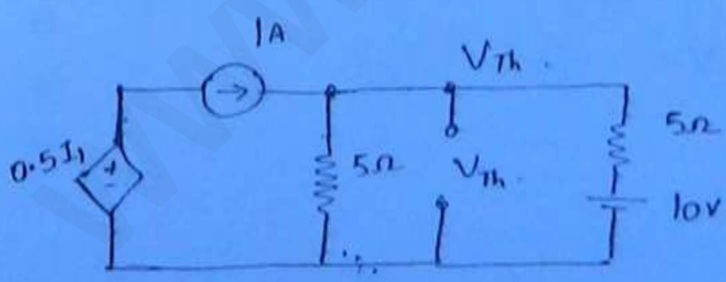
ans (b) Check the values

30.

$P_{max} = \dfrac{V_s^2}{4 R_L} \Rightarrow R_L = R_s = 100\Omega$

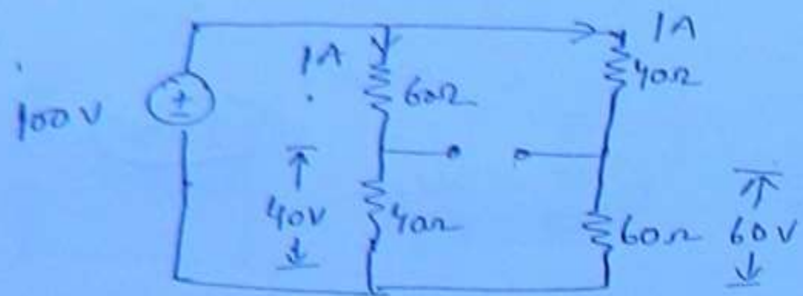
$P_{max} = 0.25W = $ (c)

31.



$1 = \dfrac{V_{Th}}{5} + \dfrac{V_{Th} - 10}{5} \Rightarrow V_{Th} = 7.5V$

$R_{Th} = 5/2 = 2.5\Omega$.



$$V_{Th} = 60 - 40 = 20V.$$

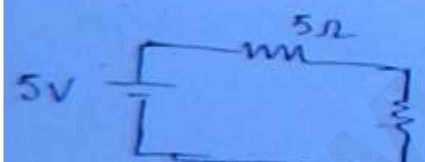
ote :

In balanced bridge voltage across adjacent branches is same.

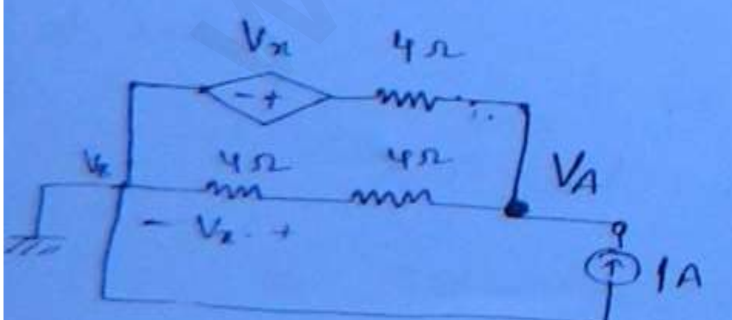
$E = E_1$, $I = 0$ (O.C) , $V = 5$. $\Rightarrow$ $V_{oc} = 5V$.

$E = 0$, $I = 1$, $V = 5$ $\Rightarrow$ $R_{Th} = 5\Omega$

$E = \cancel{E_2}$, $E = E_1$ $I \to 5\Omega$.

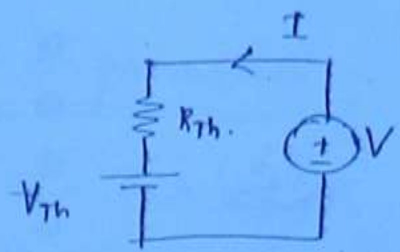


$5\Omega = R_L$ $\Rightarrow$ $V = 2.5V$



$$1 = \frac{V_A}{8} + \frac{V_A - V_x}{4} \qquad V_x = V_A/2$$

39.



$V - V_{Th} = I R_{Th}$

$V = V_{Th} + R_{Th} \cdot I \rightarrow (1)$

$I = \dfrac{V - V_{Th}}{R_{Th}}$

$I = 0.2V - 2$

$0.2V = I + 2 \rightarrow$

$V = 5I + 10 \rightarrow (2)$

Compare (1) & (2)

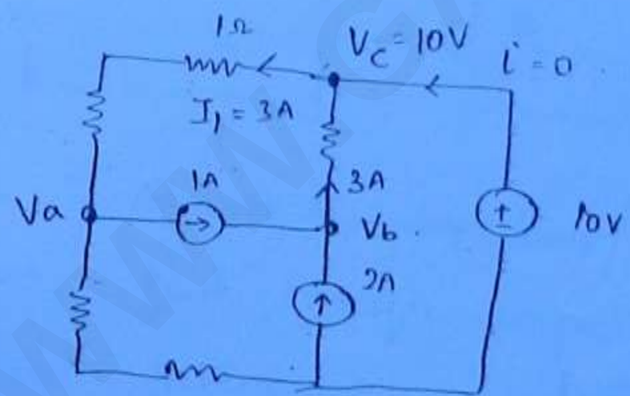
$V_{Th} = 10V, \quad R_{Th} = 5\Omega$

40.

$Z_L = Z_s^* \Rightarrow Z_L = 3 + j4$

$P_{max} = \dfrac{V_s^2}{4R_L} = \dfrac{240 \times 240}{4 \times 3} = 4.8kW.$

41.



$\dfrac{V_a}{2} + \dfrac{V_a - 10}{2} + 1 = 0$

$V_a = 4V.$

$I_1 = \dfrac{10 - 4}{1 + 1} = 3A$

44.

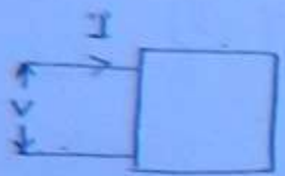
$I_{sc} = 16\angle 0 \dfrac{25}{25 + 15 + j30}$

$I_{sc} = 6.4 - j4.8$

TWO PORT N/Ws

A pair of terminals at which signal may enter or leave from the network is called as port.

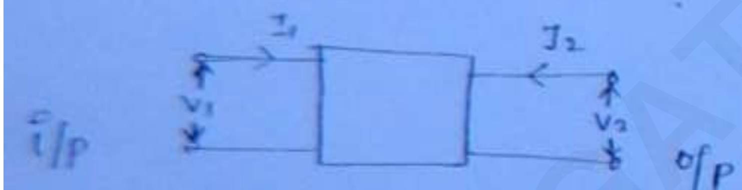
When n/w is having 1 pair of terminals then it is called as single port n/w.



Ex:- motor

Ex. Generator.

When n/w is having two pair of terminals, then the n/w is called as 2 port n/w.



Ex. Transformer.

<u>Classification of parameters</u>.

- Z [Open circuit parameters].
- Y [Short circuit "].
- h [Hybrid parameters].
- g [Inverse Hybrid parameters].
- ABCD [Transmission line parameters].

  abcd [Inverse " " "].

Z parameters

$V_1 = Z_{11}I_1 + Z_{12}I_2 \longrightarrow (1)$ KVL

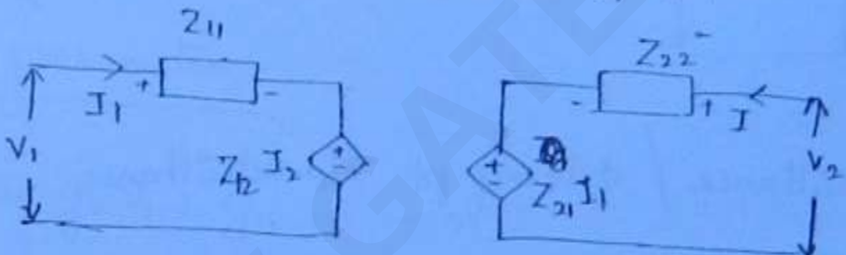
$V_2 = Z_{21}I_1 + Z_{22}I_2 \longrightarrow (2)$ KVL

$V_1, V_2 \} \longrightarrow$ dependant variables.

$I_1, I_2 \} \longrightarrow$ independant variables [source]

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$. $Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ $Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$.



$Z_{11} \Rightarrow$ Open ckt i/p impedance / driving pt. i/p impedance

$Z_{21} \Rightarrow$ fwd transfer impedance.

$Z_{12} \Rightarrow$ reverse " "

$Z_{22} \Rightarrow$ O.C. o/p impedance (or) driving point o/p impedance.

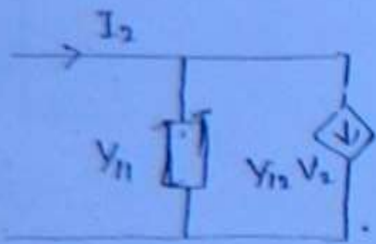
Parameters..

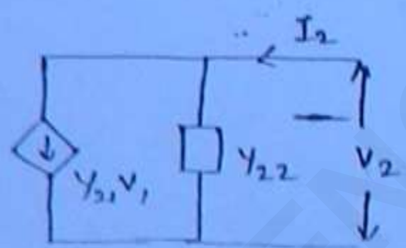
$I_1 = Y_{11} V_1 + Y_{12} V_2 \longrightarrow (1)$ kcl

$I_2 = Y_{21} V_1 + Y_{22} V_2 \longrightarrow (2)$ kcl

$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$. $\quad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$.

$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ $\quad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$.





$Y_{11} \rightarrow$ Short ckt i/p admittance / driving pt. i/p admittance.

$Y_{21} \rightarrow$ fwd transfer admittance.

$Y_{12} \rightarrow$ reverse " " .

$Y_{22} \rightarrow$ Short ckt o/p admittance / driving pt. o/p admittance.

parameters

$V_1 = h_{11} I_1 + h_{21} V_2 \longrightarrow (1)$

$I_2 = h_{21} I_1 + h_{22} V_2 \longrightarrow (2)$

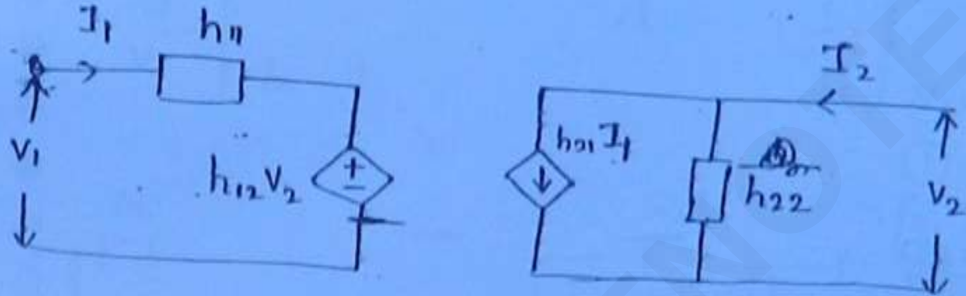
$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ $\left(h_{11} \neq z_{11} \, , \; h_{11} = \frac{1}{Y_{11}} \right)$.

(Ω)

$$h_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \quad \left[h_{21} = \frac{Y_{21}}{Y_{11}} = \frac{I_2/V_1}{I_1/V_1} \right]$$
[no unit]

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} \quad \left(h_{12} = \frac{Z_{12}}{Z_{22}} \right)$$

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0} \quad \left(h_{22} \neq Y_{22} \;,\; h_{22} = \frac{1}{Z_{22}} \right)$$



g parameters

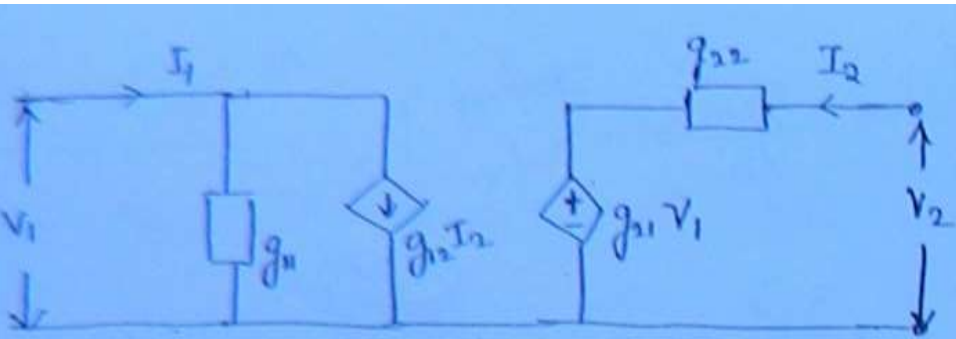
$$I_1 = g_{11} V_1 + g_{12} I_2 \longrightarrow (1)$$
$$V_2 = g_{21} V_1 + g_{22} I_2 \longrightarrow (2)$$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0} \quad \left(g_{11} \neq Y_{11} \,,\; g_{11} = \frac{1}{Z_{11}} \right)$$
(mho)

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0} \quad \left(g_{21} = \frac{Z_{21}}{Z_{11}} \right)$$

$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0} \quad \left(g_{12} = \frac{Y_{12}}{Y_{22}} \right)$$

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0} \quad \left(g_{22} \neq Z_{22} \,,\; g_{22} = \frac{1}{\ldots} \right)$$



$V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$

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ABCD parameters

$V_1 = AV_2 - BI_2 \longrightarrow (1)$

$I_1 = CV_2 - DI_2 \longrightarrow (2)$

$A = \left.\dfrac{V_1}{V_2}\right|_{I_2=0}$ $\left[A = \dfrac{Z_{11}}{Z_{22}}\right]$

$C = \left.\dfrac{I_1}{V_2}\right|_{I_2=0}$ $\left[C = \dfrac{1}{Z_{21}}\right]$

(mho) S]

$B = \left.\dfrac{-V_1}{I_2}\right|_{V_2=0}$ $\left[B = \dfrac{-1}{Y_{21}}\right]$

(Ω)

$D = \left.\dfrac{-I_1}{I_2}\right|_{V_2=0}$ $\left(D = \dfrac{-Y_{11}}{Y_{21}}\right)$

Note: For ABCD parameters, it is not possible to develop equivalent circuit $\because$ both eqn 1 & 2 are developed wrt I/P.

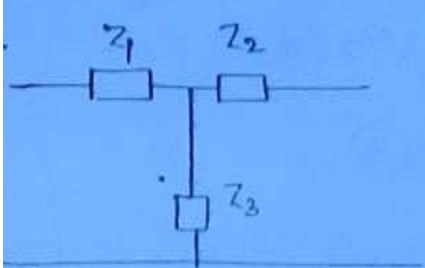
abcd parameters

$$V_2 = aV_1 - bI_1$$

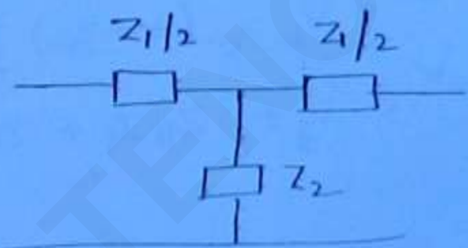
$$I_2 = cV_1 - dI_1$$

$$a = \left.\frac{V_2}{I_1}\right|_{I_1=0} \qquad b = \left.\frac{-V_2}{I_1}\right|_{V_1=0}$$

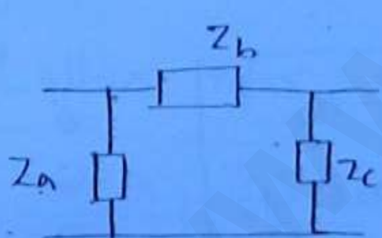
$$c = \left.\frac{I_2}{V_1}\right|_{I_1=0} \qquad d = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$$



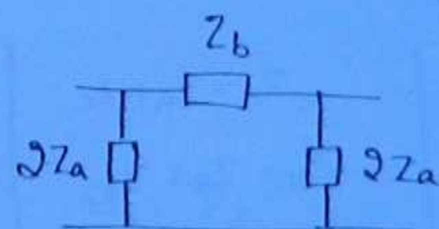
Unsymmetrical T



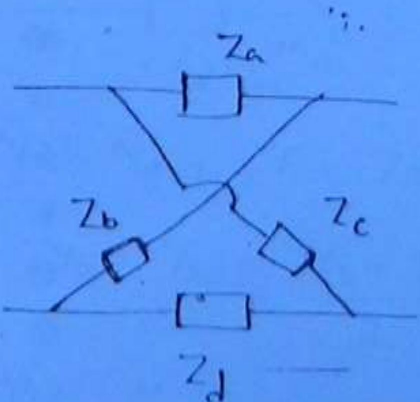
Symmetrical T



Unsymmetrical π



Symmetrical π



$\left.\begin{array}{l} Z_a = Z_d \\ Z_b = Z_c \end{array}\right\} \rightarrow$ Symmetrical lattice

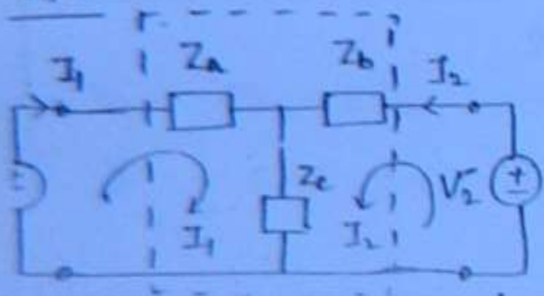
$\left.\begin{array}{l} Z_a \neq Z_d \\ Z_b \neq Z_c \end{array}\right\}$ unsymmetrical lattice

*

Symmetrical	Reciprocal
$Z_{11} = Z_{22}$	$Z_{12} = Z_{21}$
$Y_{11} = Y_{22}$	$Y_{12} = Y_{21}$
$h_{11}h_{22} - h_{21}h_{12} = 1$	$h_{12} = -h_{21}$
$A = D$	$AD - BC = 1$

(188)

n/w



$V_1 = (Z_a + Z_c) I_1 + Z_c I_2 \longrightarrow (1)$

$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow (2)$

$V_2 = Z_c I_1 + (Z_b + Z_c) I_2 \longrightarrow (3)$

$V_2 = Z_{21} I_1 + Z_{22} I_2 \longrightarrow (4)$

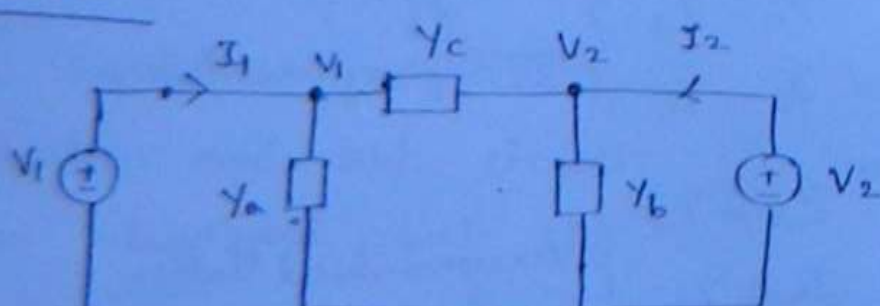
$$\boxed{\begin{aligned} Z_{11} &= Z_a + Z_c \\ Z_{21} &= Z_{12} = Z_c \\ Z_{22} &= Z_b + Z_c \end{aligned}}$$

$Z_{11} = Z_a + Z_c$

$Z_{11} = Z_a + Z_{12}$

$$\boxed{\begin{aligned} Z_a &= Z_{11} - Z_{12} \\ Z_b &= Z_{22} - Z_{12} \\ Z_c &= Z_{12} = Z_{21} \end{aligned}}$$

Π n/w



$I_1 = V_1 Y_a + (V_1 - V_2) Y_c$

$I_1 = (Y_a + Y_c) V_1 - Y_c V_2 \rightarrow (1)$

$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (2)$ 189

$I_2 = V_2 Y_b + (V_2 - V_1) Y_c$

$I_2 = -Y_c V_1 + (Y_b + Y_c) V_2 \rightarrow (3)$

$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (4)$

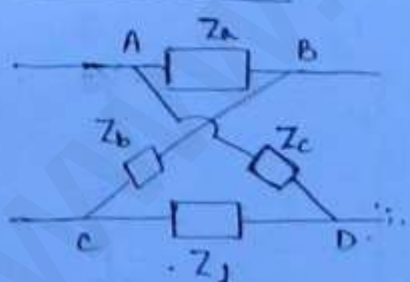
$$\boxed{\begin{aligned} Y_{11} &= Y_a + Y_c \\ Y_{21} &= Y_{12} = -Y_c \\ Y_{22} &= Y_b + Y_c \end{aligned}}$$

$Y_{11} = Y_a + Y_c$

$\rightarrow Y_{11} = Y_a - Y_{12}$

$$\boxed{\begin{aligned} Y_a &= Y_{11} + Y_{12} \\ Y_a &= Y_{22} + Y_{12} \\ Y_c &= -Y_{12} = -Y_{21} \end{aligned}}$$

Lattice n/w



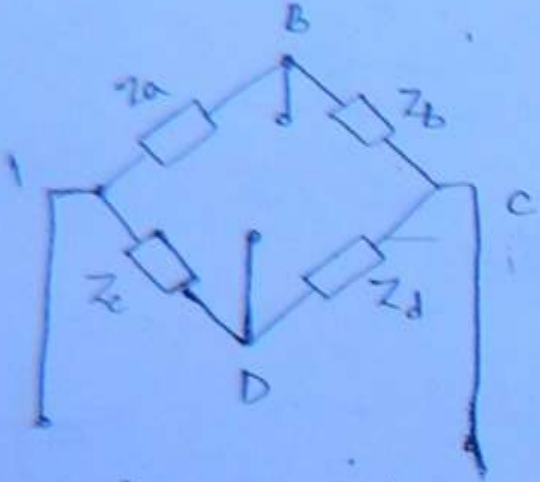
$Z_a = Z_d$, $Z_b = Z_c$ } sy-lattice

$Z_{11} = Z_{22} = \frac{Z_b + Z_a}{2}$ ✓

$Z_{12} = Z_{21} = \frac{Z_b - Z_a}{2}$

$Y_a = Y_d$, $Y_b = Y_c$

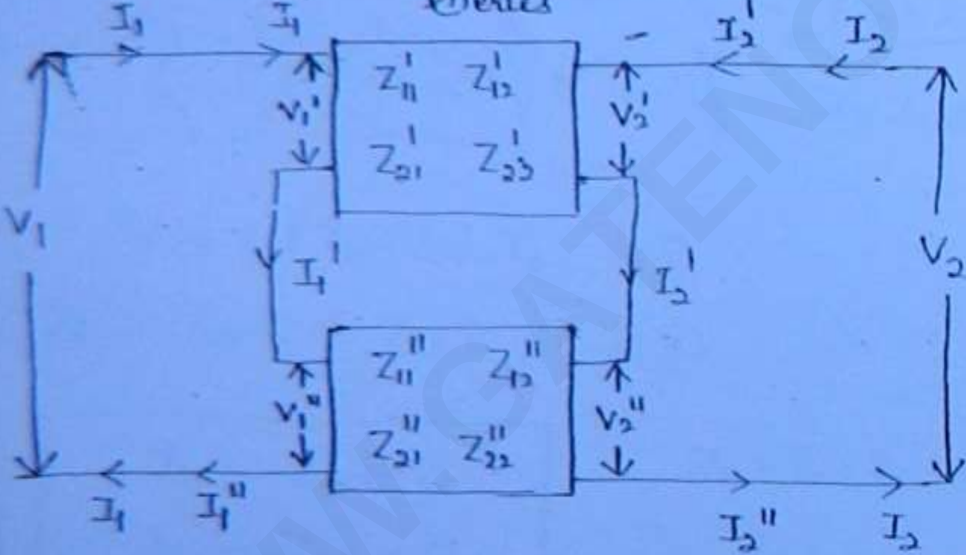
$Y_{11} = Y_{22} = \frac{Y_b + Y_a}{2}$



Bridge n/w

Equivalent impedance parameters

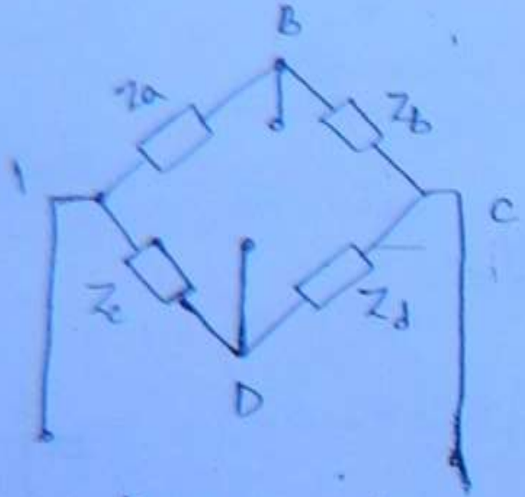
Series



$V_1 = V_1' + V_1''$ $\qquad$ $V_2 = V_2' + V_2''$

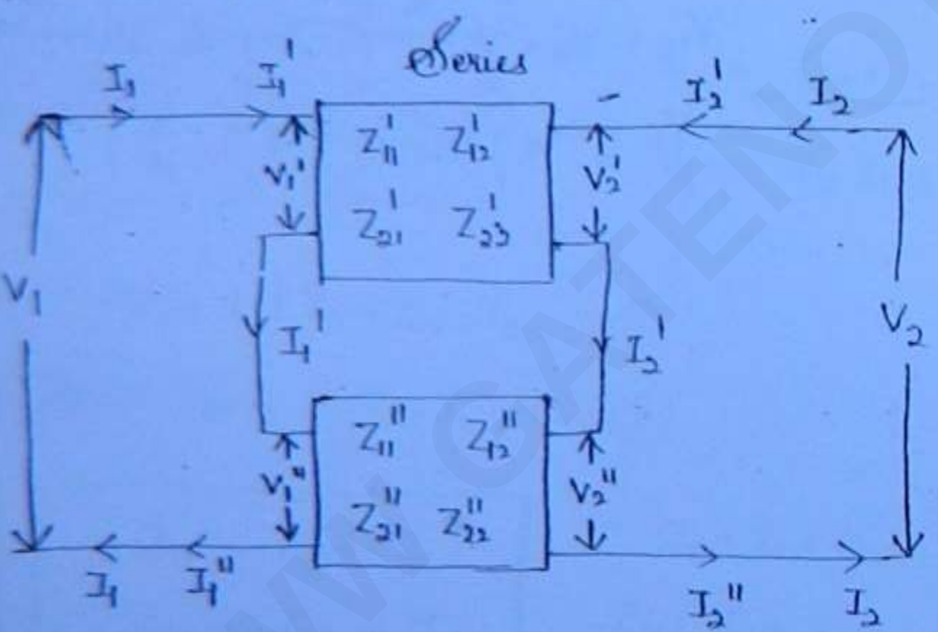
$I_1 = I_1' = I_1''$ $\qquad$ $I_2 = I_2' = I_2''$

$$\boxed{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} + \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix}}$$



Bridge n/w

Equivalent impedance parameters

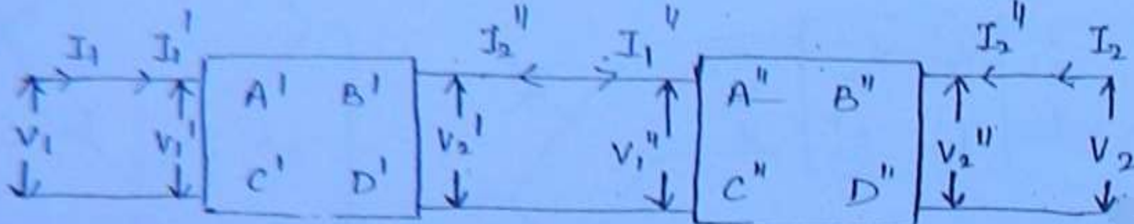


$V_1 = V_1' + V_1''$ $\qquad$ $V_2 = V_2' + V_2''$

$I_1 = I_1' = I_1''$ $\qquad$ $I_2 = I_2' = I_2''$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Z_{11}' & Z_{12}' \\ Z_{21}' & Z_{22}' \end{bmatrix} + \begin{bmatrix} Z_{11}'' & Z_{12}'' \\ Z_{21}'' & Z_{22}'' \end{bmatrix}$$

Cascade / Tandem n/w



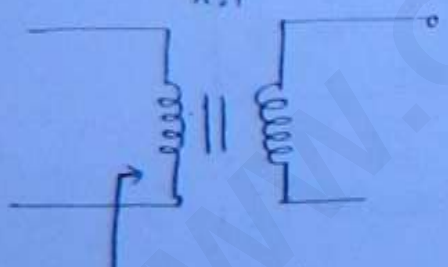
$V_1 = V_1'$ $\qquad$ $V_2' = V_1''$ $\qquad$ $V_2 = V_2''$

$I_1 = I_1'$ $\qquad$ $I_1'' = -I_2'$ $\qquad$ $I_2 = I_2''$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}$$

3/11

Find Z, Y & ABCD parameters of the n/w shown.



E:

In the ideal T/F it is not possible to find impedance and admittance values. Since, self and mutual inductance of ideal T/F are ∞.

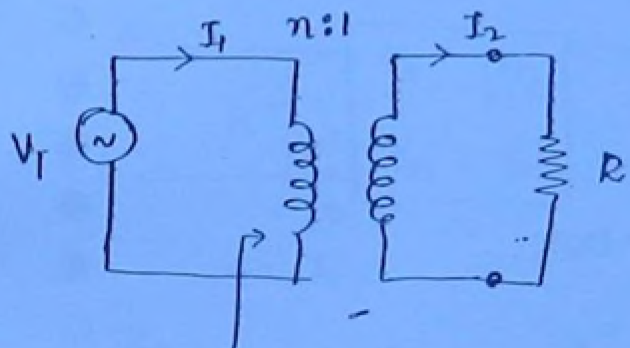
$$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{n}{1}$$

$A = \frac{V_1}{V_2} = n.$

$D = \frac{-I_1}{I_2} = \frac{+1}{n}.$

(∵ (−) sign is only for the current direction but not the parameter).

Equivalent resistance.



Ideal T/F.

find the eq. i/p impedance resistance.

impedance

$I_1^2 R_1 = I_2^2 R_2 \Rightarrow R_1 = \left(\frac{I_2}{I_1}\right)^2 R_2$

$\boxed{R_1 = n^2 R.}$

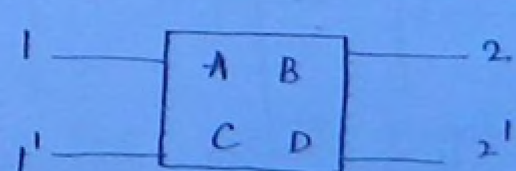
→ if inductor is given $L_1 = n^2 L$

→ " capacitor " " $C_1 = \frac{C}{n^2}$

Q. Find eq. impedance wrt

(i) 1 − 1'

(ii) 2 − 2'



) 1—1'

$$Z_{eq} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} \Bigg|_{I_2 = 0}$$

$$\boxed{Z_{eq} = \frac{A}{C}\ \Omega}$$

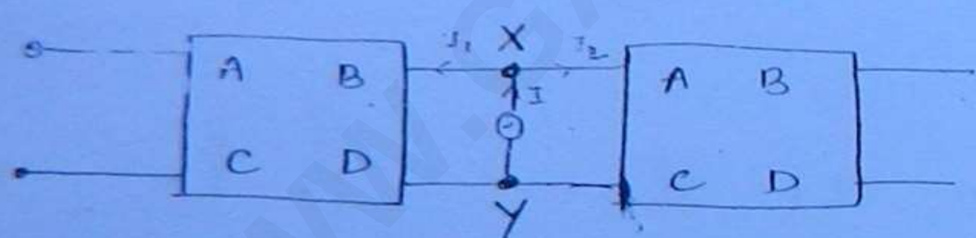
2—2'

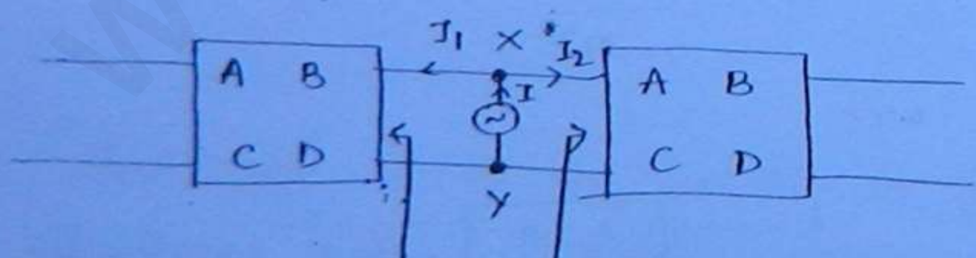
$$Z_{eq} = \frac{V_2}{I_2} \Bigg|_{I_1 = 0}$$

$$I_1 = CV_2 - DI_2 \Rightarrow 0 = CV_2 - DI_2$$

$$\boxed{Z_{eq} = \frac{D}{C}\ \Omega}$$

Find eq. impedance wrt X & Y.

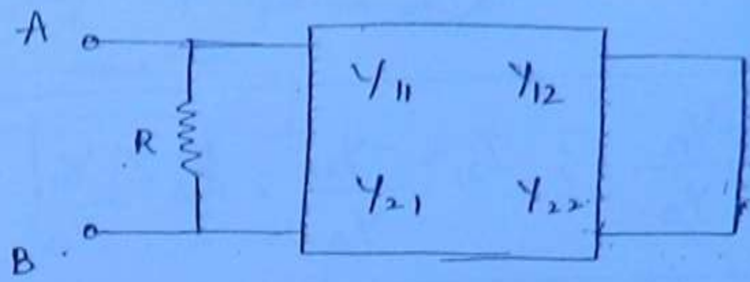




(from prev prob) $Z_{eq_1} = \frac{D}{C}$ $Z_{eq_2} = A/C$ $Z_1 \parallel Z_2$.

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{AD/C^2}{(A+D)/C}$$

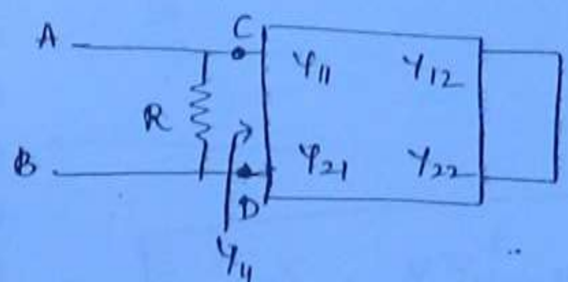
Q. Find eq admittance wrt A & B.



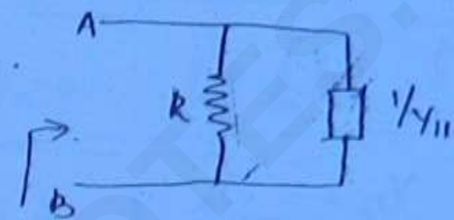
$I_1 = Y_{11} V_1 + Y_{12} V_2$ (1)
$I_2 = Y_{21} V_1 + Y_{22} V_2$

$1/Y_{11}$

Soln.



$\Rightarrow$



$$Y_{eq} = \frac{1}{R} + Y_{11}$$

$$Y \rightleftharpoons Z$$

Z

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \longrightarrow (1)$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{Z_{11} Z_{22} - Z_{12} Z_{21}} \begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \longrightarrow (2)$$

Y

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \longrightarrow (3)$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{Y_{11} Y_{22} - Y_{12} Y_{21}} \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \longrightarrow (4)$$

From 1 & 4

$$Z_{11} = \frac{Y_{22}}{Y_\Delta} \; ; \; Z_{12} = \frac{-Y_{12}}{Y_\Delta} \; ; \; Z_{21} = \frac{-Y_{21}}{Y_\Delta} \; ; \; Z_{22} = \frac{Y_{11}}{Y_\Delta}$$

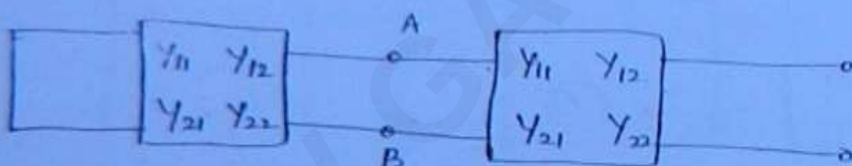
$$Y_\Delta = Y_{11}Y_{22} - Y_{12}Y_{21}$$

From 2 & 3

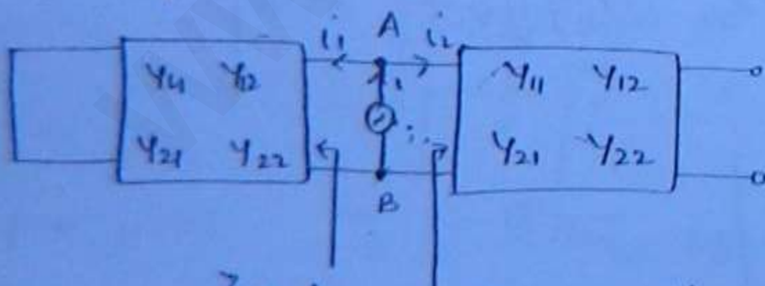
$$Y_{11} = \frac{Z_{22}}{Z_\Delta} \; ; \; Y_{12} = \frac{-Z_{12}}{Z_\Delta} \; ; \; Y_{21} = \frac{-Z_{21}}{Z_\Delta} \; ; \; Y_{22} = \frac{Z_{11}}{Z_\Delta}$$

$$Z_\Delta = Z_{11}Z_{22} - Z_{12}Z_{21}$$

Find eq. impedance wrt A & B.



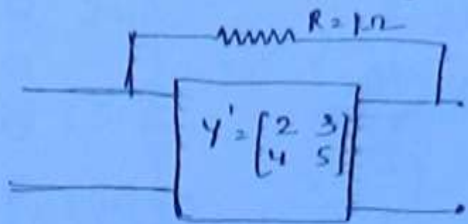
~~Zeq~~



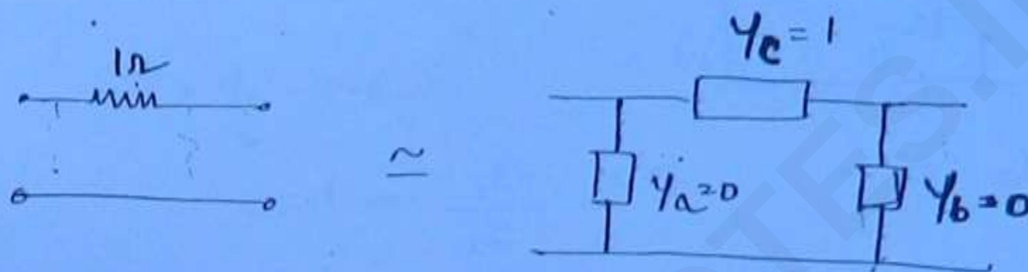
$Z_1 = \frac{1}{Y_{22}}$ $\qquad Z_{11} = \frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}Y_{21}}$

$Z_{eq} = \frac{Z_1 \cdot Z_{11}}{\quad} = Y_{22} /$ [illegible]

Q. When two 2-port n/ws are connected in parallel, find eq. Y parameters.



<u>Soln.</u>



$$Y_{11}'' = Y_a + Y_c = 1.$$

$$Y_{22}'' = Y_b + Y_c = 1$$

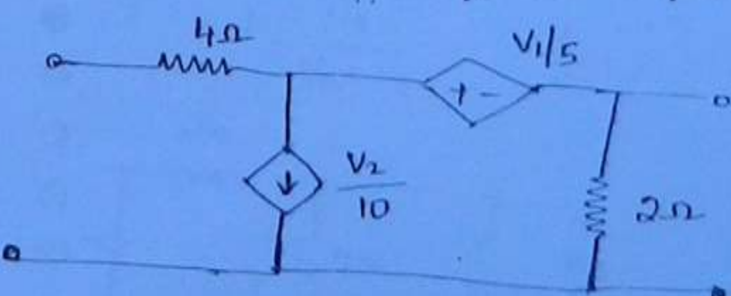
$$Y_{12}'' = Y_{21}'' = -Y_c = -1.$$

$$Y'' = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}.$$

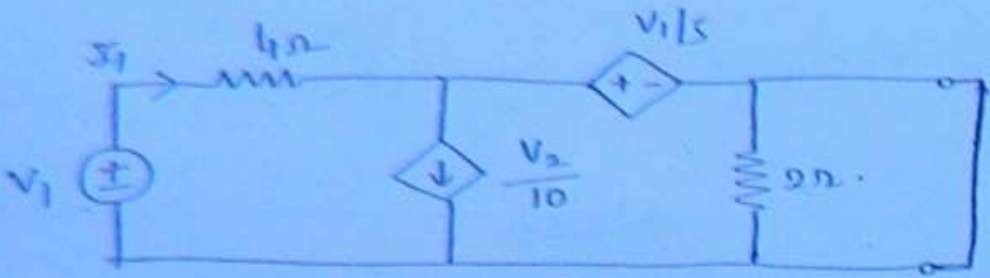
$$[Y] = [Y'] + [Y''] = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

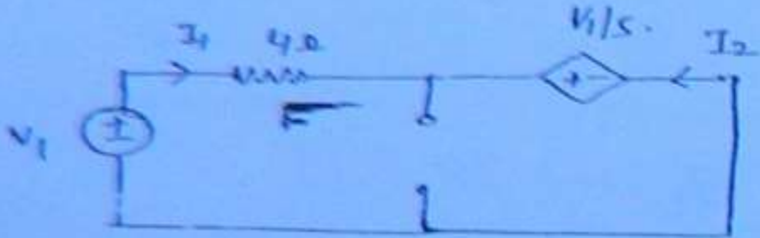
$$[Y] = \begin{bmatrix} 3 & 2 \\ 3 & 6 \end{bmatrix}.$$

<u>Q.</u> Find B & D of the n/w shown.



$V_1 = AV_2 \ldots$





$I_1 = -I_2 \Rightarrow$ $D = \frac{-I_1}{I_2} = 1$

$\underline{D = 1}$

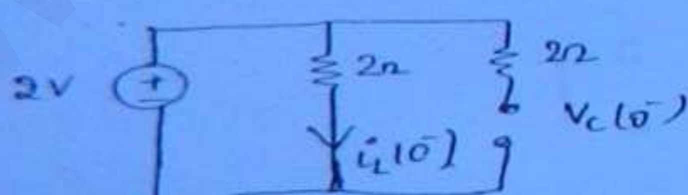
$V_1 - 4I_1 - V_1/5 = 0 \Rightarrow V_1 + 4I_2 - V_1/5 = 0$

$\frac{4V_1}{5} = -4I_2 \Rightarrow \frac{V_1}{I_2} = -5$

$\underline{B = -5}$

<u>nsients W.B.</u>

$t = 0^-$



$i_L(0^-) = \frac{2}{2} = 1A$

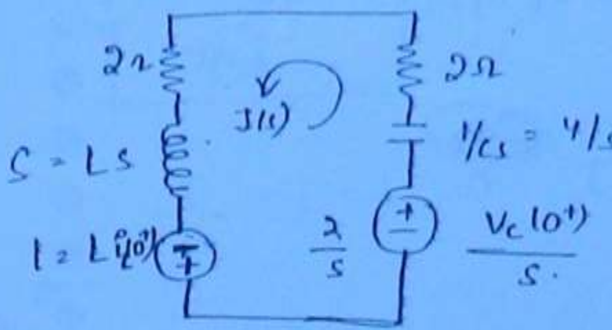
$V_c(0^-) = 2V$

$t > 0,$ $V_c(0^+) = 2V$

$i_L(0^+) = 1A$

S - domain

m-1



$$I(s) = \frac{2/s + 1}{2 + 2 + \frac{4}{s} + s}$$

$$i(t) = e^{-t/1/2}$$

$$i(t) = e^{-2t}.$$

$\tau = 1/2 = 0.5$

(b).

m-2. Time const. $= \frac{2L}{R} = \frac{2 \times 1}{4} = \underline{1/2}$ sec

current in inductor is decaying ∴ not oscillatory.

8. i, In the above circuit energy transformation is continuously done b/n inductor & capacitors. Thereby output response is oscillatory.

ii, In the above circuit no energy loss is present.

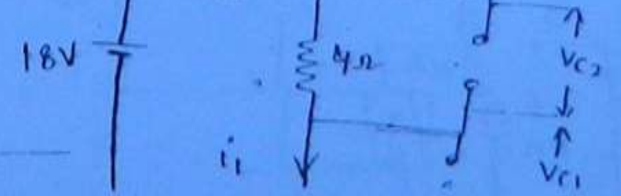
∴ resistance = 0.

Ans (d)

2.

$I = 1 \cdot \frac{1/8}{1/2 + 1/8} = \frac{1}{5}$; $V = I \cdot 1/2 = \underline{0.1 V}$

4.

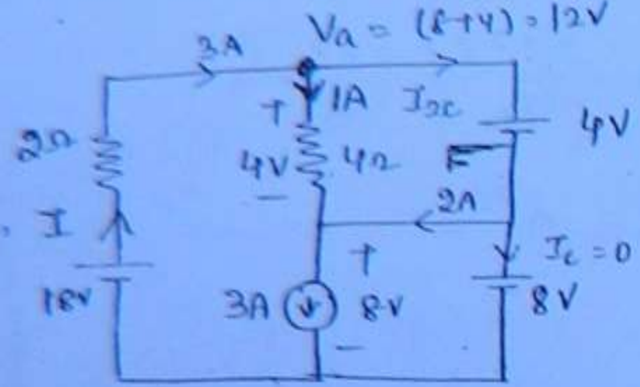


$$i_L = \frac{18}{2+4} = 3A.$$

$-V_{C_2} = 12 \cdot \frac{C}{C+2C} = 4V.$

$V_{C_1} = 12 - 4 = 8V.$

(200)



$3 = 1 + I_{2C}$

$I_2 = 2A$

$I = \frac{18-12}{2} = 3A.$

$V = iR + L\frac{di}{dt} \Rightarrow 18 = 6 \times 1 + L(6)$

$\Rightarrow L = 2H.$

$H(s) = \frac{V_0(s)}{V_i(s)} \Rightarrow V_0(s) = \frac{1}{(s-2)(s-3)}.$

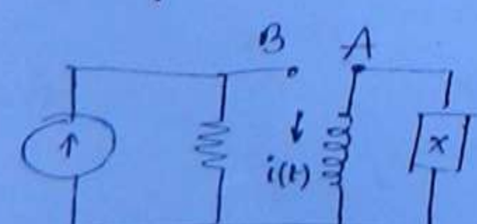
Initial & final value theorems.

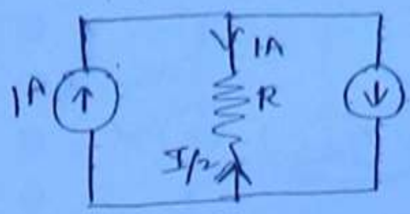
at ~~t=∞~~, energy in the inductor is totally dissipated. ∴ current in R = 0. $|_{t=\infty}$

$i(t) = I\sin(\omega t + 30°)$ → lead angle

current leads → $X =$ Capacitor

$i(0^+) = \frac{I}{2}$

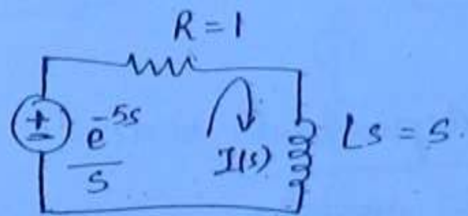




$i(0^+) = I/2$

$$\frac{I}{2} - 1 = 1 \Rightarrow I = 4A.$$

(11)



(12).

$$H(s) = \frac{1}{s+1}, \quad V_i = \cos t.$$

$$H(j\omega) = \frac{1}{j\omega+1} = \frac{V_o}{V_i} \Rightarrow \frac{1}{j+1.} = \frac{1}{\sqrt{2}}\angle -45^\circ = \frac{V_o}{V_i}$$

$$V_o = \frac{V_i}{\sqrt{2}}\angle -45 = \frac{1}{\sqrt{2}}\cos(t-45^\circ)$$

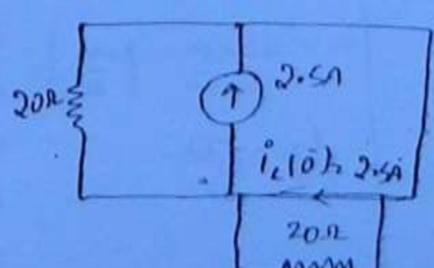
(13).

$$V_R = V_L = V/2, \qquad V_L = V/2.$$

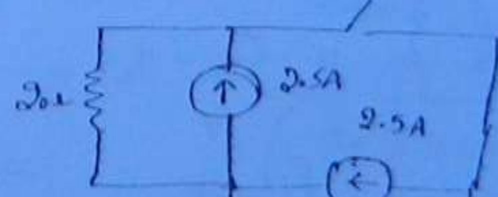
$$Ve^{-Rt/L} = V/2 \Rightarrow e^{-t/\tau} = \frac{1}{2}.$$

$$t = -\tau \ln(1/2).$$

(15) $t = 0^-$

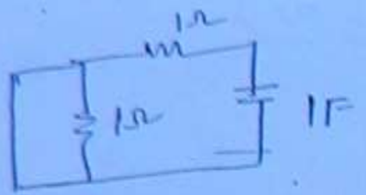


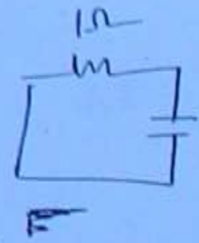
$t = 0^+$



$V_x = -(2.5)\times(20) = -50V$.

16.





$\Rightarrow \tau = RC = 1\,sec$

Note.

while finding τ deactivate all independant sources.
τ is calculated after operating the switch.

$t = 0^+$

$iR + L\frac{di}{dt} + E_2 = 0$.

$-8R + 2(3) - 4R = 0$

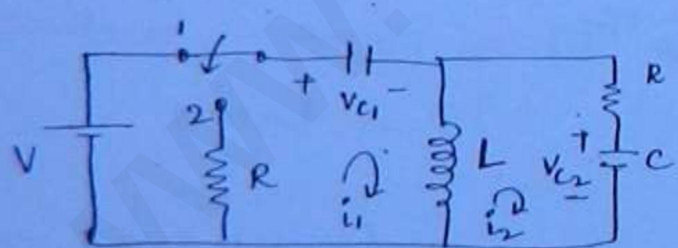
$R = 0.5\Omega$.

$t = \infty$, $L \to SC$

$E_2 = -i(\infty)R$

$E = -4R$.

$v(t) = (C_1 + C_2 t)e^{\alpha t} \Rightarrow$ critically damped.

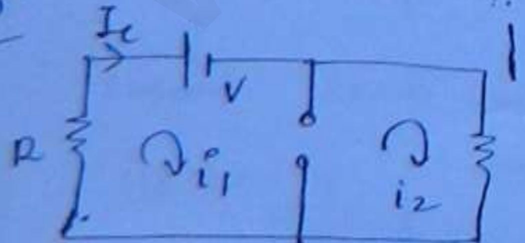


$V_{C_1} = V$

$V_{C_2} = 0$

$i_L = 0$.

0^+



$I_c = \frac{V}{R+R} = \frac{V}{2R}$

$i_1 = -I_c = \frac{-V}{2R}$

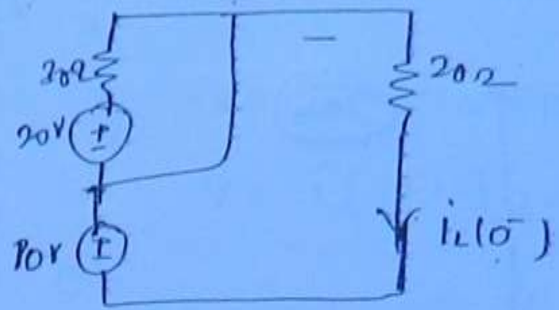
$i_2 = -V$

$i_L = i_1 - i_2$

$= \frac{-V}{2R} - \left(\frac{-V}{2R}\right)$

$i_L = 0$

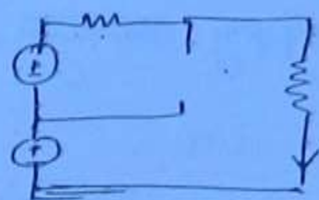
Q23.



$i_L(0^-) = 10/20$

$i_L(0^-) = 0.5A = i_L(0^+)$

$t = \infty$

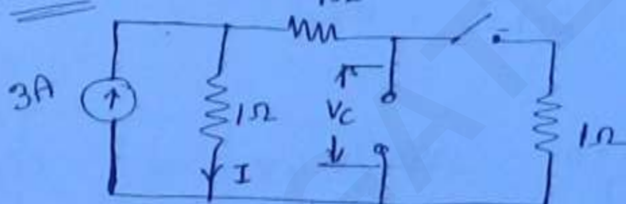


$$i(\infty) = \frac{10+20}{30+20} = 0.6$$

$$\tau = \frac{1}{50}$$

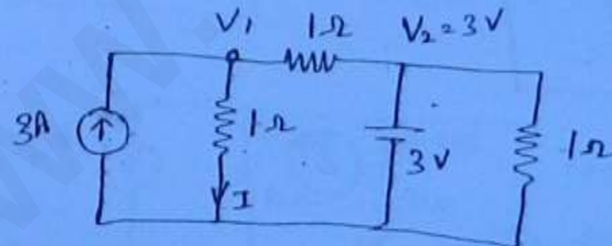
$$i(t) = 0.6 - 0.1e^{-50t}$$

Q25 $t = 0^-$



$V_c = 3 \times 1 = 3V$

$t = 0^+$



$$3 = \frac{V_1}{1} + \frac{V_1 - 3}{1}$$

$V_1 = 3$

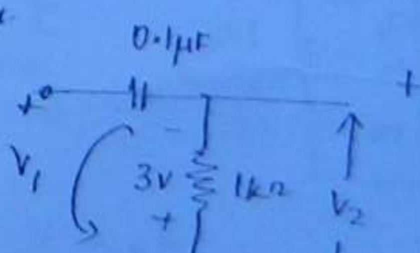
$$I = \frac{V_1}{1} = \frac{3}{1} = 3A$$

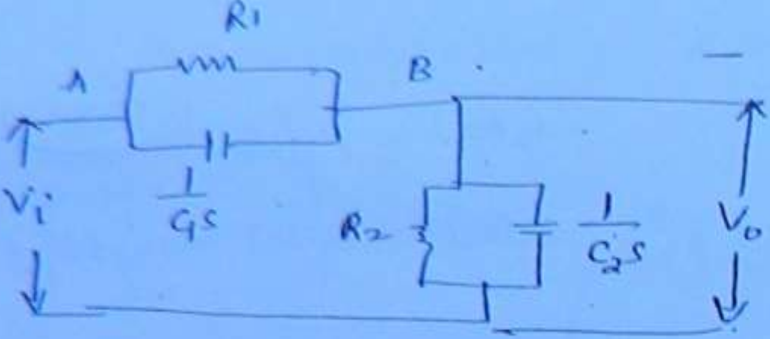
Q26.

$\tau = RC$

$= 10^3 \times 10^{-7} \rightarrow \tau = 10^{-4}$ sec

$V_2 = -3V$

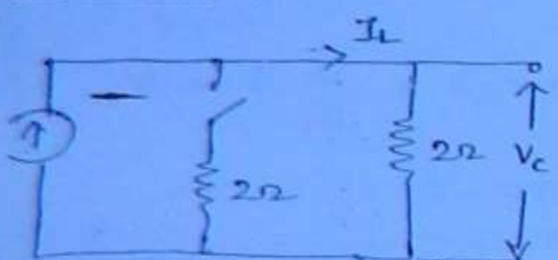




Tramform in s domain & apply voltage division

ans. (c)

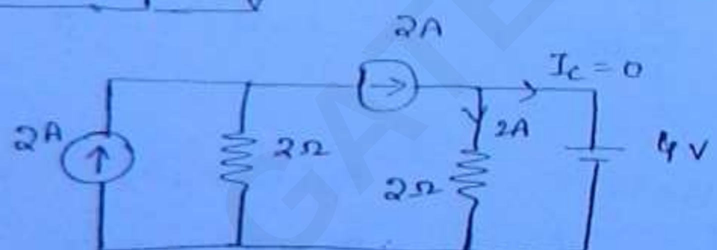
$t = 0^-$



$V_c = 2 \times 2 = 4\,V$

$I_L = 2A$

$t = 0^+$



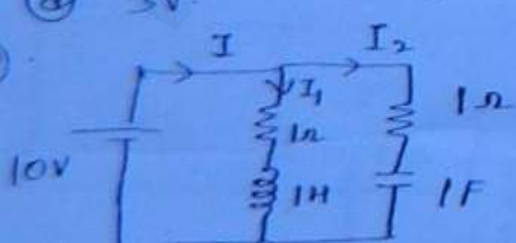
$V(0^+) = 4V$

$I_c = C \frac{dv}{dt}$ (b)

$i(t) = [i(0^+) - i(\infty)]\, e^{-t/RC} + i(\infty)$ (b)

(d) 3V.

(b)



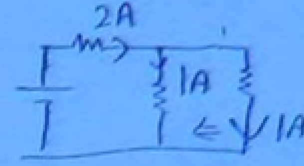
$I = I_1 + I_2$

$I_1 = \frac{V}{R}\left(1 - e^{-Rt/L}\right)$

$I_2 = \frac{V}{-} e^{-t/RC} = 10\, e^{-t}$

40. ⓓ

41. ~~$\frac{V}{10}$ = ... ⇒ ...~~ $i_L = 1$



$i_L = 2A$

$V = i[10+5] = 2[15] = 30V$

ⓐ

42. (a) $Z_{12} = 20$

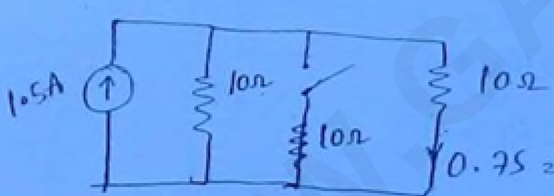
(b) $Y_{12} = \frac{-Z_{12}}{Z_{11}Z_{22} - Z_{12}Z_{21}} = 0.2$

(c) $h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0} = \frac{Z_{12}}{Z_{22}} = 20/10 = 2.0$

✓(d) $A = \frac{V_1}{V_2}\Big|_{I_2=0}$ $\left(A = \frac{Z_{11}}{Z_{21}}\right)$

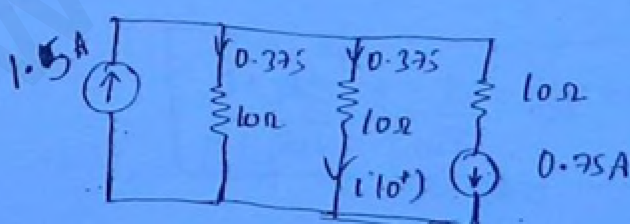
$\frac{20}{10} = 2$

43. $t = 0^-$



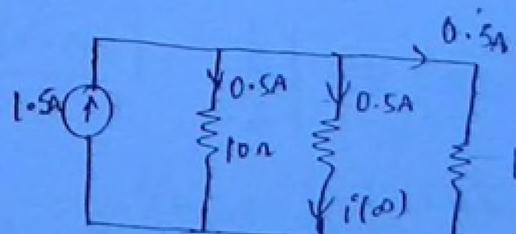
$0.75 = i_L(0^-)$.

$t = 0^+$



$i_L(0^+) = 0.75A$.

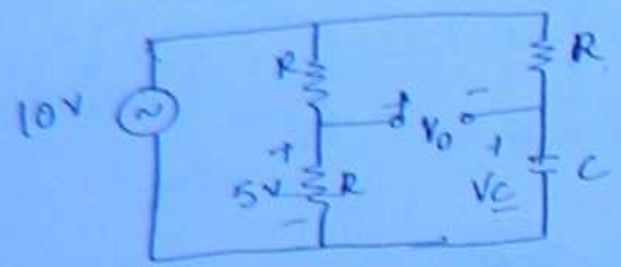
$t = \infty$



ⓐ

Two port W.B

1.



$$V_c = \frac{-jx_c}{R-jx_c}$$

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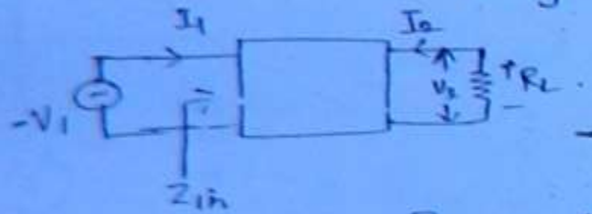
$$-5 + V_0 + V_c = 0$$

$$V_0 = 5 - V_c \Rightarrow V_0 = 5 + \frac{10jx_c}{R-jx_c}$$

$$|V_0| = 5\,\frac{|R+jx_c|}{|R-jx_c|} \Rightarrow |V_0| = 5V$$



$$V_2 = -I_2 R_L$$

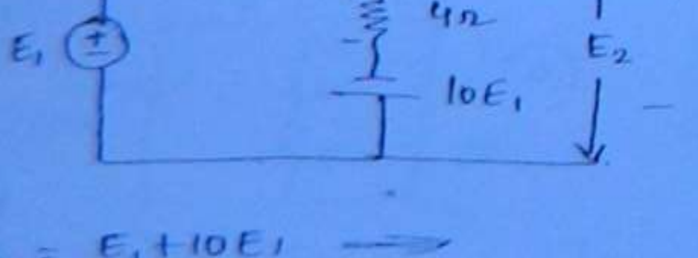
$$Z_{in} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2}$$

$$Z_{in} = \frac{A(-I_2R_L) - B(I_2)}{C(-I_2R_L) - DI_2} \Rightarrow \frac{AR_L + B}{CR_L + D}$$

$$= \frac{1\times 10 + 2}{1\times 10 + 3} = 12/13$$

$$\boxed{Z_{in} = \frac{AR_L + B}{CR_L + D}, \quad Y_{in} = \frac{1}{Z_{in}} = \frac{CR_L + D}{AR_L + B}}$$

addition of matrices (b)



$$Z_{21} = \frac{E_2}{I_1}\Big|_{I_2=0}$$

$$Z_{11} = \frac{E_1}{I_1}\Big|_{I_2=0} = \frac{6}{11}$$

$$I_1 = \frac{E_1 + 10E_1}{} \longrightarrow \qquad E_1 = \frac{6}{}\, I_1$$

5. $\pi - T$ transform.

6. $I(s) = \dfrac{V(s)}{Z(s)} = \dfrac{1/s}{(s+2)/(s+3)} =$

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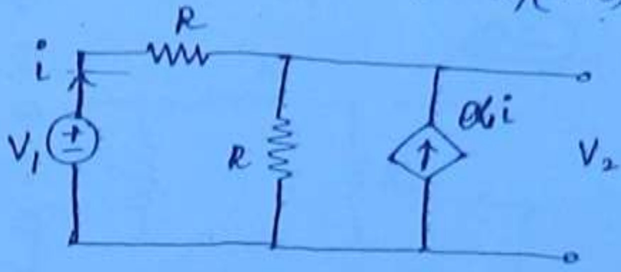
7.

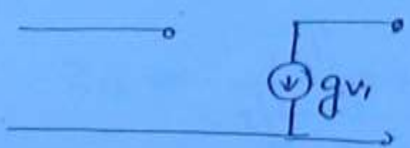


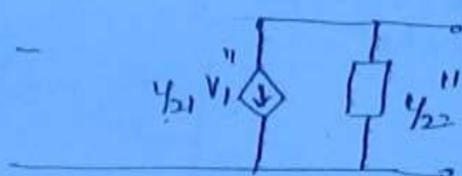
$i + \alpha i = V_2/R \rightarrow (1)$

$i = \dfrac{V_1 - V_2}{R} \rightarrow (2)$

(c).

8.





$y_{11} = y_{22} = -y_{12} = -y_{21} = Y$.

$Y_{21}'' = g$.

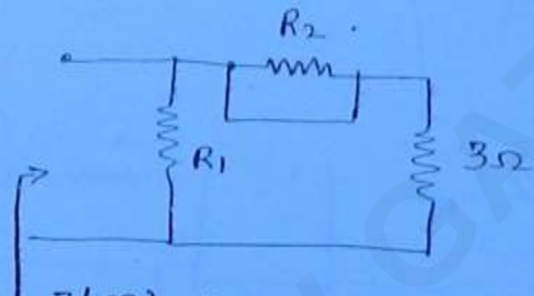
$y_{21} = -Y$.

$Y_{21} = y_{21} + Y_{21}''$

(c)

$Y_{21} = -Y + g$.

12.



$Z(\infty) = 2$.

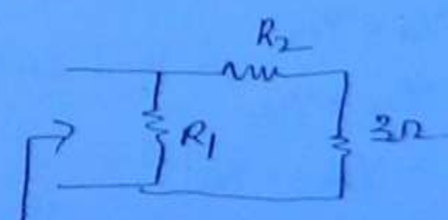
$s \rightarrow \infty$

$X_c = 1/sc = 0$

$C \rightarrow S.C.$

$2 = \dfrac{R_1 3}{3 + R_1} \Rightarrow R_1 = 6\Omega$.

$s \rightarrow 0 \Rightarrow X_c = \infty \Rightarrow C \rightarrow O.C$



$Z(0) = 3$ ∴

$3 = \dfrac{R_1(3 + R_2)}{R_1 + R_2 + 3}$

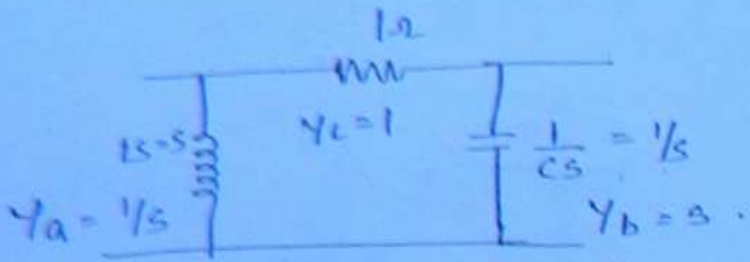
(b)

$R_2 = 3\Omega$

13.

$\dfrac{V_c}{V_i} = \dfrac{1/100\mu s}{10k + 10m\,s + 1/100\mu s}$

(d).

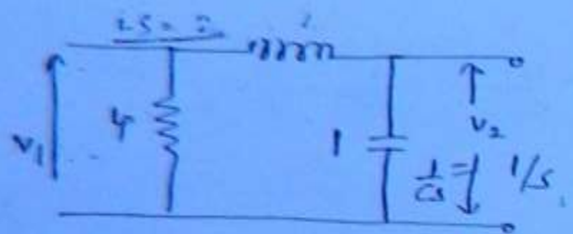


(208)

$$Y_{11} = Y_a + Y_c =$$

$$Y_{22} = Y_s + Y_c.$$

$$Y_{12} = Y_{21} = -Y_c.$$

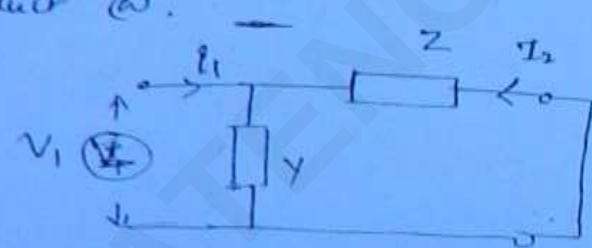


$$V_2(s) = V_1(s) \cdot \frac{1/s}{2s + 1/s}.$$

$$= 1/1+2s^2$$

(d) 2 is product (a).

$$D = \frac{-I_1}{I_2}\Big|_{V_2=0}$$



$$I_2 = \frac{-I_1 \, 1/Y}{Z + 1/Y} \Rightarrow D = 1 + YZ.$$

π — T.

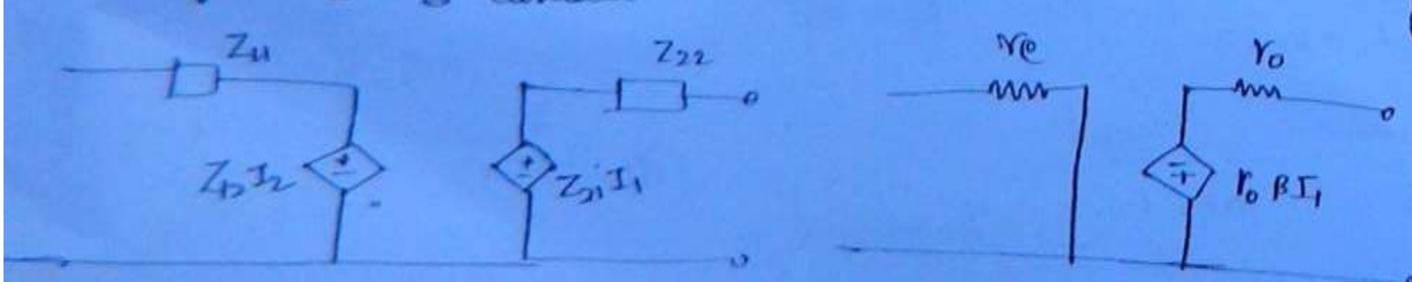
$$Y_{11} = 5, \quad Y_{12} = Y_{22} = -1, \quad Y_{22} = 1.$$

$$Y_1 = Y_{11} + Y_{12}$$

$$Y_2 = Y_{22} + Y_{12}$$

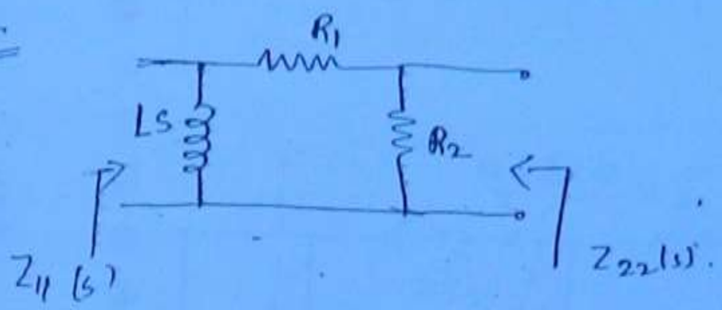
$$Y_3 = -Y_{12} = -Y_{21}.$$

transform to S domain



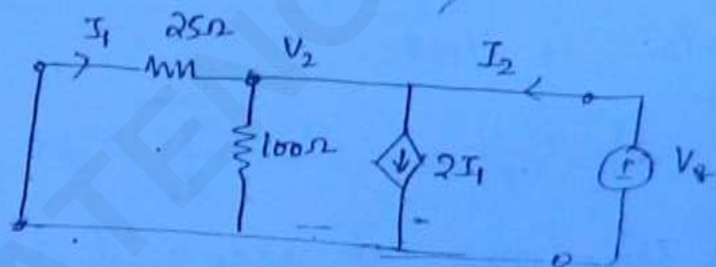
27.
$$Z_1 = Z_{11} - Z_{12}$$
$$Z_2 = Z_{22} - Z_{12}$$
$$Z_3 = Z_{12} = Z_{21}$$

28.



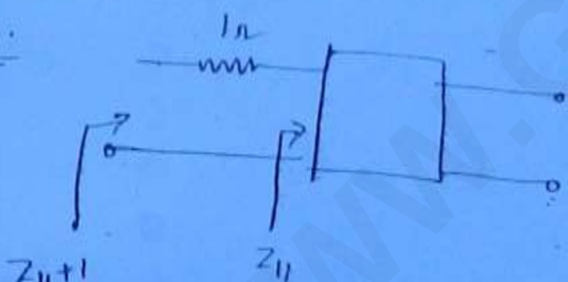
$$Z_{11} = \frac{Ls(R_1+R_2)}{R_1+R_2+Ls}, \qquad Z_{22}(s) = \frac{R_2(Ls+R_1)}{R_1+R_2+Ls}$$

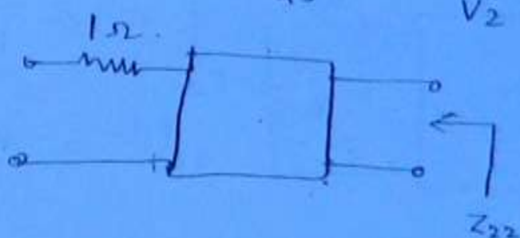
29.
$$Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}.$$



$$I_1 = \frac{-V_2}{25} \Rightarrow \frac{I_1}{V_2} = -0.04.$$

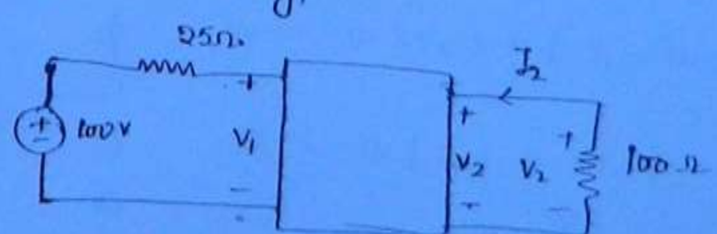
30.





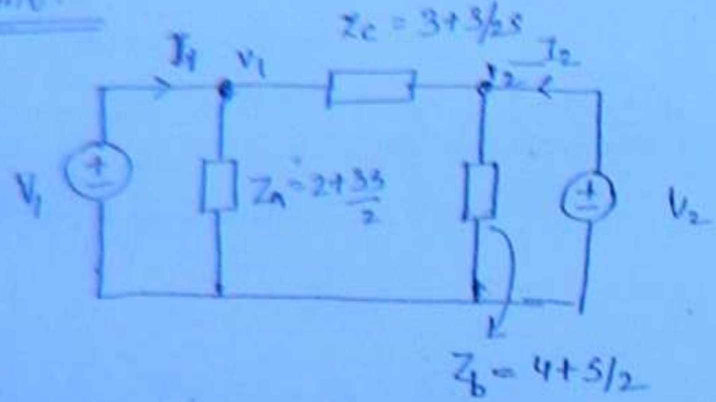
- whenever a linear element is added, there will be no variation in $Z_{12} = \frac{V_1}{I_2}$ & $Z_{21} = \frac{V_2}{I_1}$ but Z_{11} will vary.

31.



$$V_2 = -100 I_2, \quad I_2 = -V_2/100.$$

Conv.



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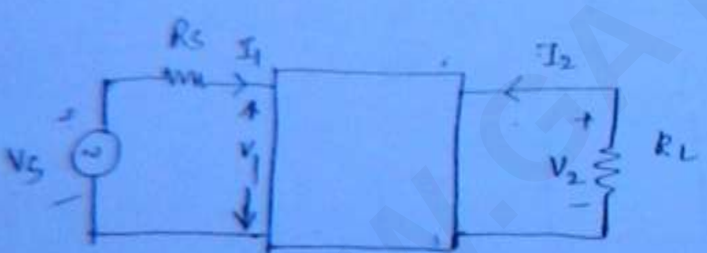
$$I_1 = \frac{V_1}{Z_A} + \frac{V_1 - V_2}{Z_C}$$

$$I_1 = V_1\left[\frac{1}{Z_A} + \frac{1}{Z_C}\right] - \frac{V_2}{Z_C}$$

$$I_1 = V_1 Y_{11} + Y_{12} V_2 .$$

$$Y_{11} = \frac{1}{Z_A} + \frac{1}{Z_C}, \quad Y_{12} = \frac{-1}{Z_C}$$

Ily apply the same proc. at node 2 V_2 & find Y_{22}, Y_{21}



$V_s = V_1 + I_1 R_s$ $\quad$ $V_1 = V_s - I_1 R_s$

$V_2 = -I_2 R_L$

$V_1 = V_s - I_1 R_s$

$V_2(s) = -I_2(s) \cdot 1$

$V_1(s) = \frac{1}{s} - 2 I_1(s)$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} z \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

$$\begin{bmatrix} 1/s - 2 I_1(s) \\ \ldots \end{bmatrix} = \begin{bmatrix} z \end{bmatrix} \begin{bmatrix} I_1(s) \\ \ldots \end{bmatrix}$$

Solve the above matrix & find $I_2(s)$.

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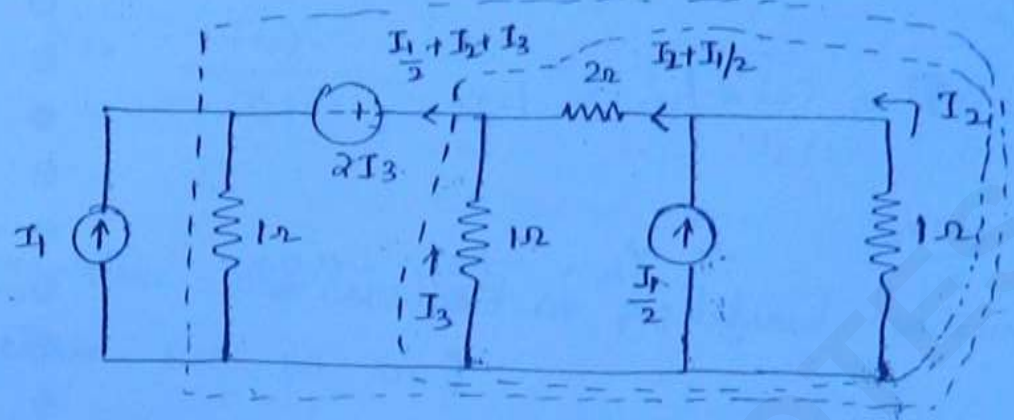
$V_2(s) = -I_2(s)$

$$V_2(t) = L^{-1}[V_2(s)] = [0.037 + 0.0456\,e^{-1.9t} - 0.083\,e^{-7.08t}].$$

3.



$$1\times I_2 + 2\left(I_2+\frac{I_1}{2}\right) + 2I_3 + 1\left(\frac{3I_1}{2}+I_2+I_3\right)=0 \rightarrow (1)$$

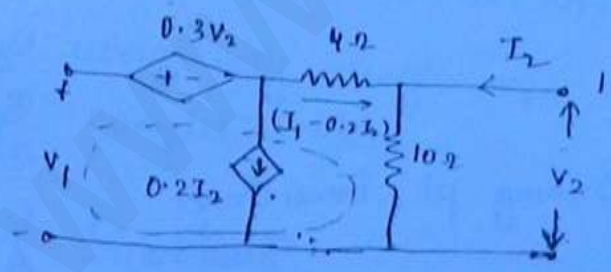
$$(1\times I_2) + 2\left(I_2+\frac{I_1}{2}\right) - (I_3\times 1) = 0 \rightarrow (2)$$

from eq. (2)

$$I_3 = I_2\times 1 + 2\left(I_2+\frac{I_1}{2}\right). \rightarrow (3)$$

Sub eq. (3) in (1) Ans $= -11/26$

4.



$$V_1 = 0.3V_2 + 4(I_1 - 0.2I_2) + V_2 \rightarrow (1)$$

$$V_2 = 10(I_1 + 0.8I_2) \rightarrow (2)$$

$V_1 = z_{11}I_1 + z_{12}I_2$ sub (2) in (1)

$V_1 = 17I_1 + 9.6\,I_2$

$V_2 = 10I_1 + 8I_2$.

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$8I_2 = V_2 - 10I_1 \Rightarrow I_2 = \frac{V_2}{8} - \frac{10}{8}I_1 \rightarrow (5)$

Substitute eq. (5) in eqn (1).

$V_1 = h_{11}I_1 + h_{12}V_2$.

$V_1 = 5I_1 + 1.2V_2 \rightarrow (4) \Rightarrow h_{12} = 1.2$.

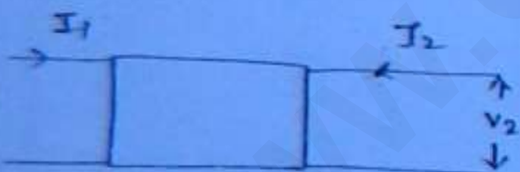
$I_1 = \frac{V_1}{5} - \frac{1.2V_2}{5} \Rightarrow Y_{12} = -\frac{1.2}{5} = -0.24$.

NETWORK FUNCTIONS

$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$

$Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$

Immittance functions.



$Z_{11}(s) = \frac{V_1(s)}{I_1(s)}$, $Y_{11}(s) = \frac{I_1(s)}{V_1(s)}$

Driving pt. immittance function

$Z_{22}(s) = \frac{V_2(s)}{I_2(s)}$, $Y_{22}(s) = \frac{I_2(s)}{V_2(s)}$

$Z_{12}(s) = \frac{V_1(s)}{I_2(s)}$

Impedance Transfer ratio

$Z_{21}(s) = V_2(s)$

3. $Y_{12}(s) = \frac{I_1(s)}{V_2(s)}$ $Y_{21}(s) = \frac{I_2(s)}{V_1(s)}$. admittance transfer ratio

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4. $G_{12} = \frac{V_1(s)}{V_2(s)}$. $G_{21} = \frac{V_2(s)}{V_1(s)}$. Voltage transfer ratio

5. $\alpha_{12} = \frac{I_1(s)}{I_2(s)}$ $\alpha_{21} = \frac{I_2(s)}{I_1(s)}$. Current transfer ratio

→ Parameters are calculated at predefined conditions (either O.C or S.C).

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

→ To calculate network function, no predefined conditions are required.

→ To design the n/w simultaneously 4 parameters are required. i.e. $Z_{11}, Z_{12}, Z_{21}, Z_{22}$.

→ By using only single n/w function, it is possible to design complete n/w.

Network Synthesis:

→ In the n/w synthesis for a given function, n/w is designed.

Single port → $Z(s)$, $Y(s)$ } immittance

Two port } 1. $Z_{11}(s)$, $Z_{22}(s)$, $Y_{11}(s)$, $Y_{22}(s)$ - driving pt immittance func.

In the network synthesis, for a given function, it is possible to design the following networks.

(214)

- Series — Foster I form
- Parallel — foster II form

} Partial fraction of Expansion

- Ladder
  - Cauer-I form
  - Cauer-II form

} Continued fraction of Expansion.

$F(s)$ should be PRF (+ve real func.).

$R \geq 0, \quad L \geq 0, \quad C \geq 0$.

If $F(s)$ is PRF, $\frac{1}{F(s)}$ is also PRF.

If $F_1(s)$ & $F_2(s)$ are PRF

i, $F(s) = F_1(s) + F_2(s) \longrightarrow$ PRF

ii, $F(s) = F_1(s) - F_2(s)$

(a) $F_1(s) > F_2(s) \longrightarrow$ PRF

(b) $F_1(s) < F_2(s) \longrightarrow$ -ve.

All the poles of the function should be present in the left half of the plane.

Imaginary poles and zeroes should be conjugate pair.

In the partial fraction of Expansion, residue should be the real.

→ Numerator and denominator polynomial should satisfy Hurwitz criteria.

→ The highest power of numerator and denominator polynomial should differ by atmost UNITY. This condition prohibits multiple poles and zeros at ∞.

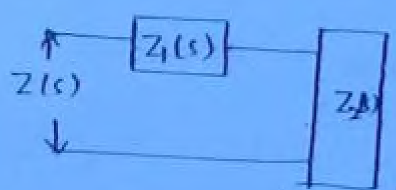
→ The lowest power of numerator and denominator polynomial should differ by atmost UNITY. This condition prohibits multiple poles and zeros at origin.

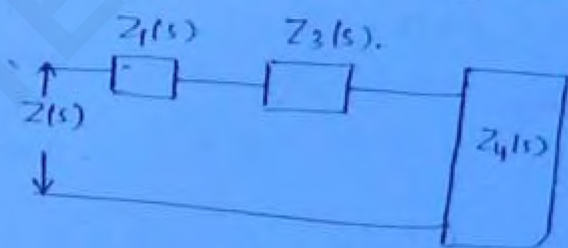
$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$$Z_2(s) = Z_3(s) + Z_4(s)$$

$$Z_4(s) = Z_2(s) - Z_3(s)$$





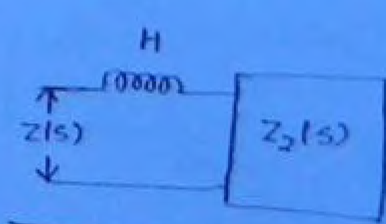
1. Removal of pole at ∞

$$Z(s) = \frac{b_{n+1} s^{n+1} + b_n s^n + \cdots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0}$$

$$Z(s) = \frac{b_{n+1} s^{n+1}}{a_n s^n} + Z_2(s) \qquad \left(H = \frac{b_{n+1}}{a_n} \right)$$

$$Z(s) = Hs + Z_2(s)$$

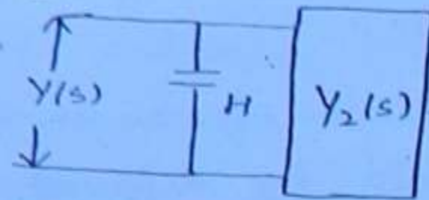
$$Z_2(s) = Z(s) - H(s)$$



$B_c = SC$

$Y(s) = Hs + Y_2(s)$ 216

$Y_2(s) = Y(s) - Hs$.



Removal of pole at origin

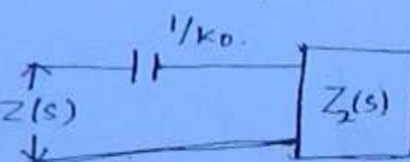
$$Z(s) = \frac{b_0 + \cdots + b_{n-1}s^{n-1} + b_n s^n}{s(a_0 + \cdots + a_{n-1}s^{n-1} + a_n s^n)}.$$

$$Z(s) = \frac{b_0}{s a_0} + Z_2(s) \qquad \left(\frac{b_0}{a_0} = k_0 \right)$$

$$Z(s) = \frac{k_0}{s} + Z_2(s)$$

$X_c = 1/sC$

$$Z_2(s) = Z(s) - \frac{k_0}{s}$$

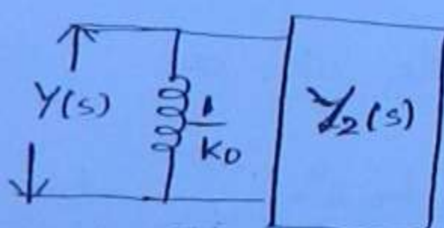


Y. for admittance func.

$B_L = 1/Ls$

$$Y(s) = \frac{k_0}{s} + Y_2(s)$$

$$Y_2(s) = Y(s) - \frac{k_0}{s}$$



Removal of Conjugate pair of poles

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

$Z_1(s) \rightarrow$ poles $\rightarrow \pm j\omega$.

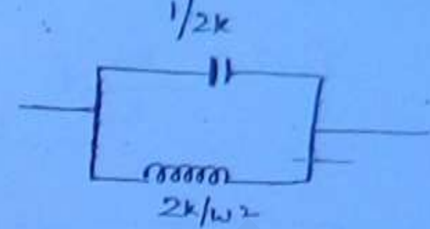
$$Z_1(s) = \frac{k_1}{s + j\omega} + \frac{k_2}{s - j\omega} \qquad (k_1 = k_2 = k).$$

$$Z_1(s) = \frac{k}{\quad} + k$$

$$Z_1(s) = \frac{2ks}{s^2+\omega^2}$$

(217)

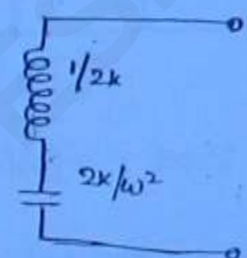
& $$Z_1(s) = \frac{1}{\frac{s^2}{2ks} + \frac{\omega^2}{2ks}} = \frac{1}{Y_a + Y_b}$$



$B_c = sC$
$B_L = 1/Ls$

IIly with admittance function,

$$Y(s) = \frac{1}{\frac{s}{2k} + \frac{\omega^2}{2ks}} = \frac{1}{Z_a + Z_b}$$



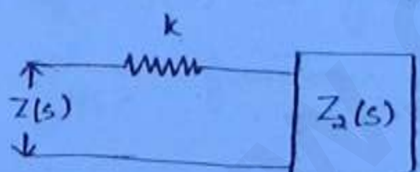
4. Removal of constant

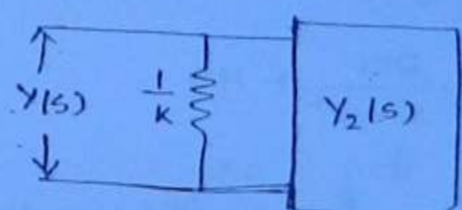
$Z(s) = k + Z_2(s)$

$Z_2(s) = Z(s) - k$

$Y(s) = k + Y_2(s)$

$Y_2(s) = Y(s) - k$



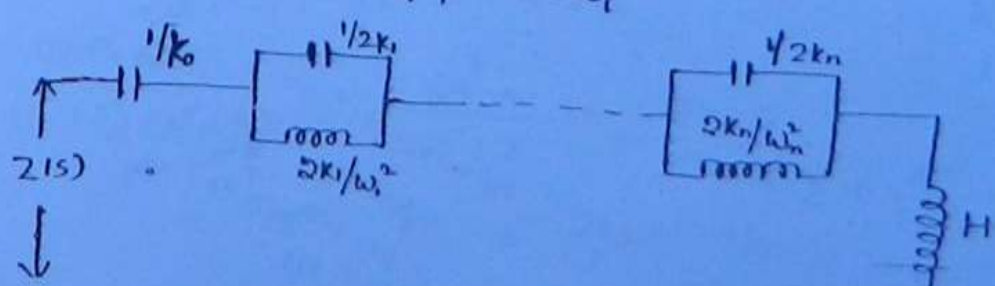


LC n/w. – Foster I form (Series)

$X_c = 1/sC$

$X_L = Ls$

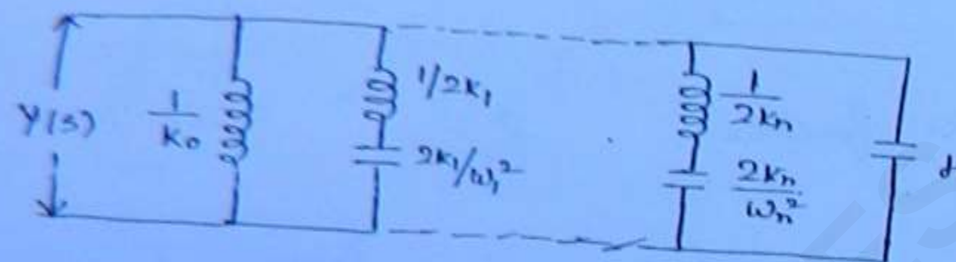
$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$



LC n/w Foster-II form [Parallel]

$$Y(s) = \frac{K_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$ (218)

$\frac{1}{Ls}$



Obtain foster I & II form of Z(s)

$$Z(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)}$$

P → ∞ ✓

P → origin

P → conjugate pair ✓

$$Z(s) = \frac{K_0}{s} + \frac{2ks}{s^2+\omega^2} + Hs$$

$$\frac{(s^2+2)(s^2+4)}{s(s^2+3)} = \frac{K_0}{s} + \frac{2ks}{s^2+\omega^2} + Hs.$$

$K_0 = 8/3$

$2k = 1/3$

$H = 1$

$$\frac{3s^2+8}{s(s^2+3)} + s$$

$$\frac{s^4+6s^2+8}{s^3+3s}$$

$$s^3+3s \,\big)\, s^4+6s^2+8 \,\big(\, s$$
$$s^4+3s^2$$
$$3s^2+8$$

$K_0 s^2 + \omega^2 K_0 + 2ks^2$

$K_0 = 8$

$K_0 = 8/3$

$K_0 + 2k = 3$

$2k = 3 - 8/3 = 1/3$

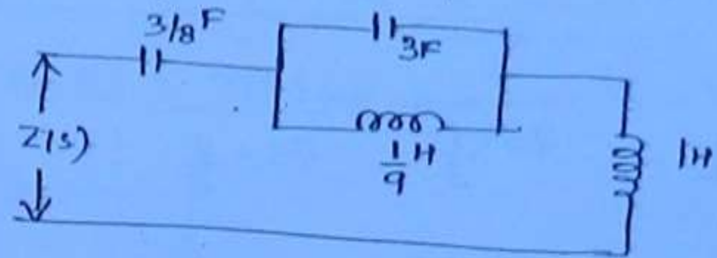
$$Z(s) = \frac{8/3}{s} + \frac{s/3}{s^2+3} + s.$$

$$Z(s) = \frac{1}{\quad} + \frac{1}{\quad}$$

$B_c = SC$ —

$B_L = 1/LS$.

$\left(\frac{1}{Y_a + Y_b} \right)$



Foster-II form.

$$Z(s) = \frac{(s^2+2)(s^2+4)}{s(s^2+3)} \Rightarrow Y(s) = \frac{S(s^2+3)}{(s^2+2)(s^2+4)}$$

Note: (1) if F(s) is given. same function is utilized to obtain foster I & IInd form.

(2). If Z(s) is given, to obtain foster IInd form, $\frac{1}{Z(s)}$ function is used.

(3) If Y(s) is given, to obtain foster 1st form, $\frac{1}{Y(s)}$ function is used.

$$\therefore Y(s) = \frac{S(s^2+3)}{(s^2+2)(s^2+4)}$$

no pole at ∞
" " " origin
conjugate pair of poles = 2.

$$\therefore Y(s) = \frac{2k_1 s}{s^2+\omega_1^2} + \frac{2k_2 s}{s^2+\omega_2^2}$$

$$\frac{s(s^2+3)}{(s^2+2)(s^2+4)} = \frac{2k_1 s}{s^2+2} + \frac{2k_2 s}{s^2+4}$$

$2k_1 \rightarrow 1/2$, $2k_2 \rightarrow 1/2$.

$$Y(s) = \frac{s/2}{s^2+2} + \frac{s/2}{s^2+4}$$

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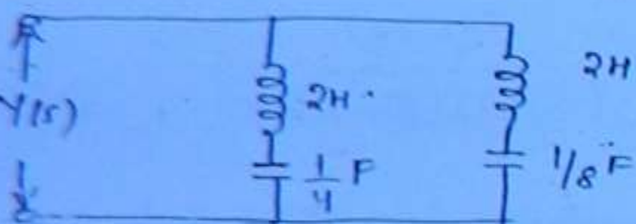
$$Y(s) = \frac{1}{\frac{s^2}{s/2}+\frac{2}{s/2}} + \frac{1}{\frac{s^2}{s/2}+\frac{4}{s/2}} \Rightarrow \frac{1}{2s+\frac{4}{s}} + \frac{1}{2s+\frac{8}{s}}$$

$$Y(s) = \frac{1}{Z_a+Z_b} + \frac{1}{Z_c+Z_d}$$

$X_L = Ls$

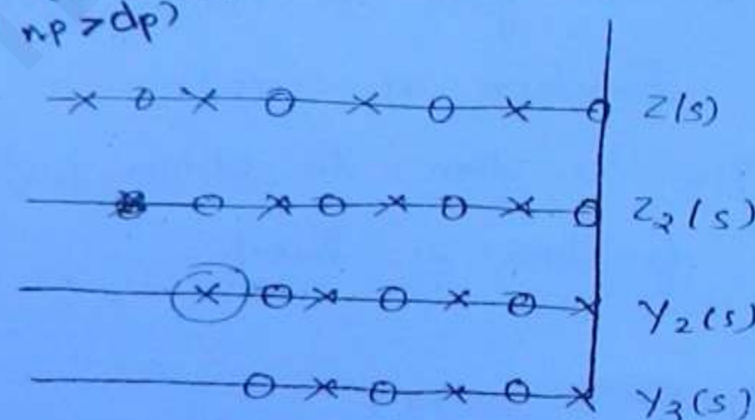
$X_C = 1/Cs$



LC n/w Cauer-I form. (ladder)

(np > dp)

Removal of pole at ∞.



$Z_2(s) = Z(s) - H_1 s$

$Y_2(s) = \frac{1}{Z_2(s)}$

$Y_3(s) = Y_2(s) - H_2 s$

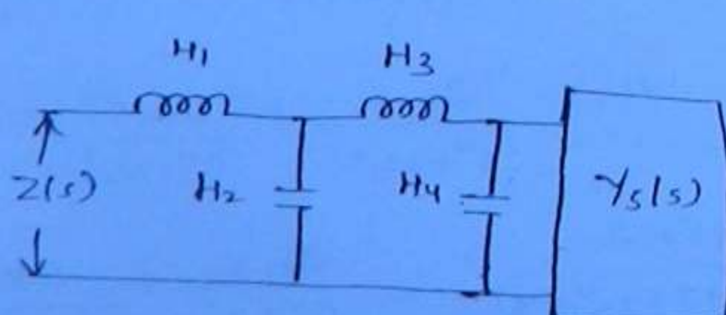
$Z_3(s) = \frac{1}{Y_3(s)}$

$Z_4(s) = Z_3(s) - H_3 s$

$Y_4 = \frac{1}{Z_4(s)}$

$Y_5(s) = Y_4(s) - H_4 s$

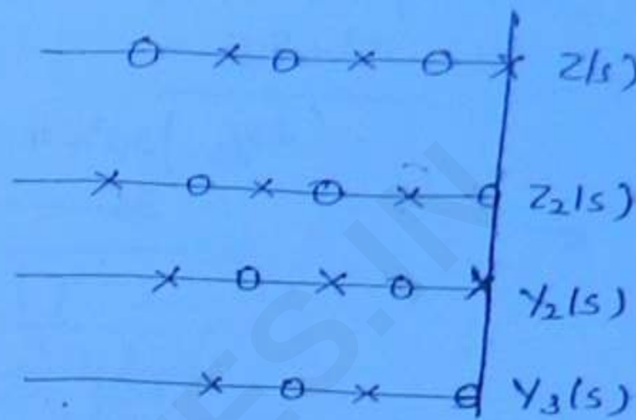
$Y_4(s) = H_4(s) + Y_5(s) \ldots$



LC n/w, Cauer – II form.

Removal of pole at origin.

$Z_2(s) = Z(s) - \frac{K_{01}}{s}$

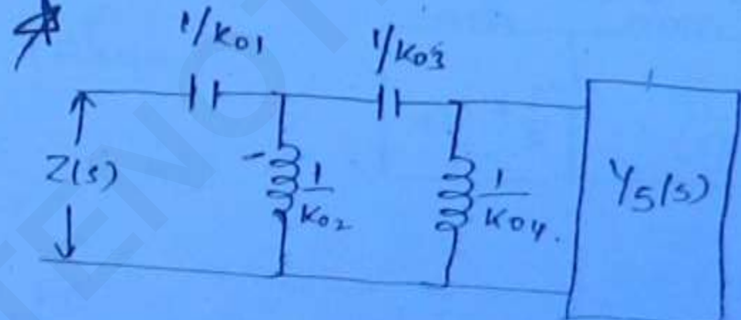


$Y_2(s) = \frac{1}{Z_2(s)}$

$Y_3(s) = Y_2(s) - \frac{K_{02}}{s}$

$Z_3(s) = \frac{1}{Y_3(s)}$

$Z_4(s) = Z_3(s) - \frac{K_{03}}{s}$



$Y_4(s) = \frac{1}{Z_4(s)}$

$Y_5(s) = Y_4(s) - \frac{K_{04}}{s}$

$Y_4(s) = Y_5(s) + K_{04}/s$.

Q. obtain Cauer – I & II form.

$Z(s) = \frac{s^3+2s}{s^4+5s^2+4}$

sol.

Cauer – I

No pole at ∞, $\therefore$ consider $Y(s)$

$Y(s) = \frac{s^4+5s^2+4}{s^3+2s}$

$$s^3+2s \,)\, s^4+5s^2+4 \,(\, s \quad \xrightarrow{C_1} Y \qquad B_C = sC$$

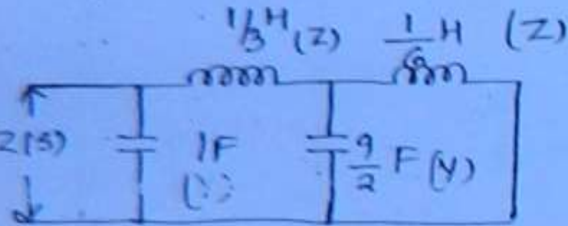
(222)

$$\frac{s^4+2s^2}{3s^2+4 \,)\, s^3+2s \,(\, s/3} \quad \xrightarrow{L_1} Z$$

$$\frac{s^3+4s/3}{2s/3 \,)\, 3s^2+4 \,(\, 9s/2} \quad \xrightarrow{C_2} Y$$

$$\frac{3s^2}{4 \,)\, 2s/3 \,(\, s/6} \quad \xrightarrow{L_2} Z$$

$$\frac{2s/3}{\times}$$



$$Z(s) = \cfrac{1}{s + \cfrac{1}{\cfrac{s}{3} + \cfrac{1}{\cfrac{9s}{2} + \cfrac{1}{s/6}}}} \Rightarrow \frac{s^3+2s}{s^4+5s^2+4}$$

Cauer-II form. $P \rightarrow$ origin

$$Z(s) = \frac{s^3+2s}{s^4+5s^2+4}$$

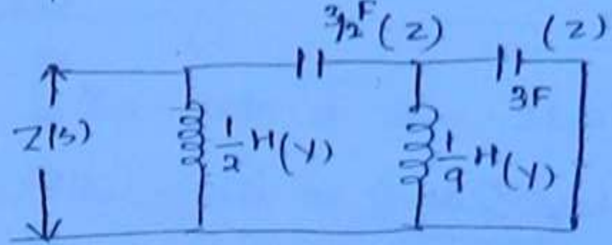
$$Y(s) = \frac{s^4+5s^2+4}{s^3+2s} \; ; \quad Y(s) = \frac{s^4+5s^2+4}{s(s^2+2)}$$

$$2s+s^3 \,)\, 4+5s^2+s^4 \,(\, \frac{2}{s} \quad \xrightarrow{L_1} Y$$

$$\frac{4+2s^2}{3s^2+s^4 \,)\, 2s+s^3 \,(\, \frac{2}{3s}} \quad \xrightarrow{C_2} Z$$

$$\frac{2s+s^3/3}{2s^3/3 \,)\, 3s^2+s^4 \,(\, 9} \quad \xrightarrow{L_2} Y$$

$B_L = 1/Ls$. $X_C = 1/sC$



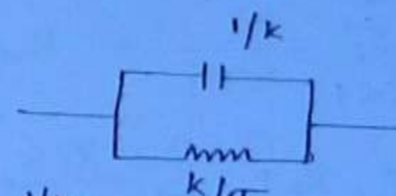
$$Z(s) = \cfrac{1}{\cfrac{2}{s} + \cfrac{1}{\cfrac{2}{3s} + \cfrac{1}{\cfrac{9}{s} + \cfrac{1}{1/3s}}}}$$

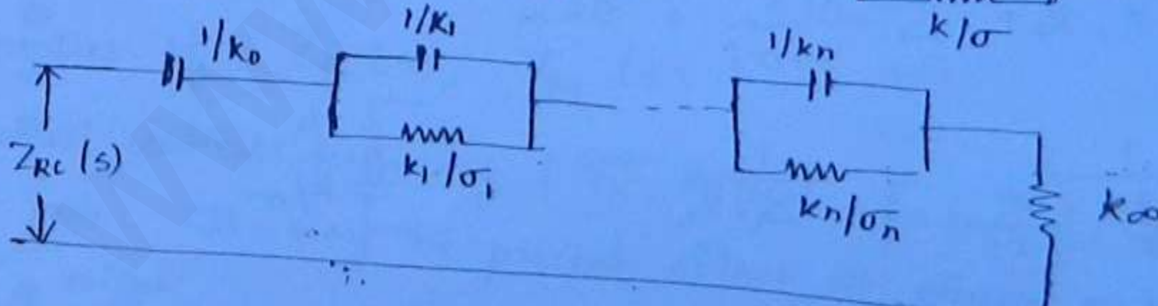
RC n/w Foster-I form (Series)

$$Z_{LC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2+\omega_i^2} + Hs$$

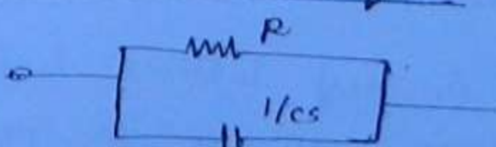
$$Z_{RC}(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i} + k_\infty$$

$$\frac{k}{s+\sigma} = \frac{1}{s/k + \sigma/k} = \frac{1}{Y_a + Y_b}$$



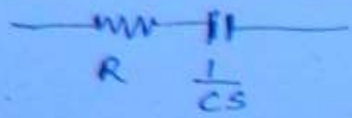


RC n/w Foster II form.



$$Z_{RC}(s) = \frac{R \cdot 1/cs}{\ } = R$$

$$Z_{RC} = \frac{R}{RC\left[s+1/RC\right]} = \frac{1}{s+\frac{1}{RC}}$$



$$Y_{RC}(s) = \frac{1}{R+\frac{1}{Cs}}$$

$$Y_{RC}(s) = \frac{Cs}{RC\left(s+1/RC\right)}$$

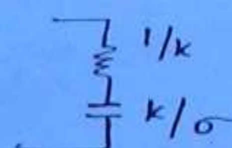
$$\Rightarrow \frac{Y_{RC}(s)}{s} = \frac{1}{R\left(s+1/RC\right)}$$

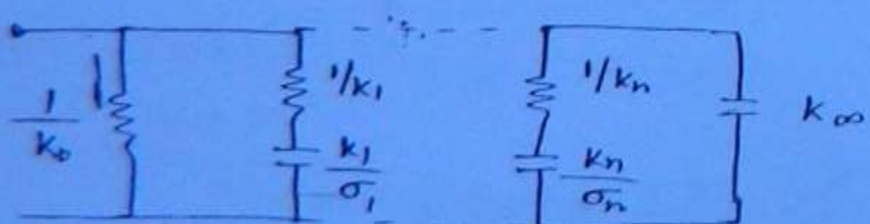
$\therefore$ Pole zero pattern of $Z_{RC}(s)$ & $\frac{Y_{RC}(s)}{s}$ are identical.

$$\frac{(s)}{s} = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{k_i}{s+\sigma_i} + k_\infty$$

$$Y_{RC}(s) = k_0 + \sum_{i=1}^{n} \frac{k_i s}{s+\sigma_i} + s k_\infty$$

$$\frac{ks}{s+\sigma} = \frac{1}{\frac{1}{k}+\frac{\sigma}{ks}} = \frac{1}{Z_a+Z_b} \Rightarrow$$





Properties of LC network

1. Either pole or zero should be present at origin.
2. Either pole or zero should be present at ∞.
3. Poles and zeros are arranged alternately on $j\omega$ axis.
4. In the partial fraction of expansion, residue should be +ve real.
5. $\frac{dx}{d\omega} > 0 \Rightarrow$ slope is +ve

$$Z(s) = \frac{k_0}{s} + \sum_{i=1}^{n} \frac{2k_i s}{s^2 + \omega_i^2} + Hs$$

$$jx = \frac{k_0}{j\omega} + \sum_{i=1}^{n} \frac{2k_i j\omega}{-\omega^2 + \omega_i^2} + j\omega H$$

$Z = j(x_L - x_c)$

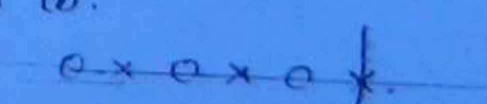
$Z = jx$.

$$x = \frac{-k_0}{\omega} + \sum_{i=1}^{n} \frac{2k_i \omega}{-\omega^2 + \omega_i^2} + H\omega.$$

$$\frac{dx}{d\omega} = \frac{k_0}{\omega^2} + \cdots \Rightarrow \frac{dx}{d\omega} > 0.$$

Properties of $Z_{RC}(s)$, $Y_{RL}(s)$

1. Lowest critical frequency (1st critical frequency) is due to pole. It may be present either at origin or nearer to origin.
2. Poles and zeros are arranged alternately on -ve real axis.
3. Highest critical freq. is due to zero, it may present either at ∞ (or) nearer to ∞.



1. $\frac{dZ_{RC}}{ds} < 0$ slope -ve & $\frac{dY_{RL}}{ds} < 0$ (226)

$\dot{X}_C = 1/sC$

$Z_{RC}(0) > Z_{RC}(\infty)$

$S = 0$ $S = \infty$

$Y_{RL}(0) > Y_{RL}(\infty)$

$X_C(0) = \infty$ $X_C(\infty) = 0$

Properties of $Y_{RC}(s)$ & $Z_{RL}(s)$.

$$Y_{RC} = K_0 + \sum_{i=1}^{n} \frac{k_i s}{s+\sigma_i} + K_\infty s.$$

Highest critical frequency is due to pole. It may be present at ∞ or nearer to ∞.

poles and zeros are arranged alternately on -ve real axis. $\times\ 0\ \times\ 0\ \times$

Lowest critical frequency is due to zero, it may be present either at origin or nearer to origin.

$\times\ 0\ \times\ 0\ \times\ 0$ Y_{RC} $0\ \times\ 0\ \times\ 0\ \times$ Z_{RC}.

$\frac{dY_{RC}}{ds} > 0$ Slope is +ve & $\frac{dZ_{RL}}{ds} > 0$

$Z_{RL}(0) < Z_{RL}(\infty)$

$Y_{RC}(0) < Y_{RC}(\infty)$

$S = 0$	$S = \infty$
$X_L = Ls$ $X_L = 0$	$X_L = \infty$

origin

– RC n/w Cauer-I form. (ladder)

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LC $\rightarrow Z_2(s) = Z(s) - H_1(s)$.

$Y_2(s) = \frac{1}{Z_2(s)}$

$Y_3(s) = Y_2(s) - H_2 s$. $(B_c = sc)$.

$Z_3(s) = 1/Y_3(s)$

$Z_4(s) = Z_3(s) - k_2$.

$Y_4(s) = 1/Z_4(s) \Rightarrow Y_5(s) = Y_4(s) - H_2 S$

$Y_4(s) = Y_5(s) + H_2(s)$

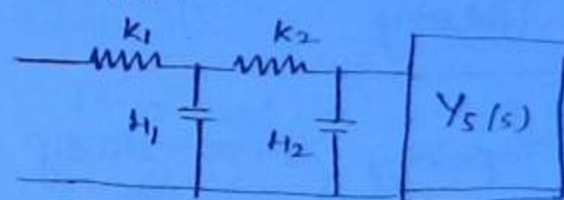
RC $\rightarrow$ step 1: Removal of constant from $Z(s)$

$Z_2(s) = Z(s) - k_1$

$Y_2(s) = \frac{1}{Z_2(s)}$

Step 2: Removal of pole at ∞ from $Y(s)$.

$Y_3(s) = Y_2(s) - H_1 s$.



Step 1 & step 2 are alternately repeated until the total function is realized.

RC n/w Cauer-II form.

LC $\Rightarrow Z_2(s) = Z(s) - \frac{K_{01}}{S}$

$Y_2(s) = \frac{1}{Z_2(s)}$

$Y_3(S) = Y_2(s) - \frac{K_{02}}{s}$ $(B_L = \frac{1}{\omega L})$

$$Z_3(s) = \frac{1}{Y_3(s)}$$

$$Z_4(s) = Z_3(s) - \frac{K_{02}}{s}$$

$$Y_4(s) = 1/Z_4(s).$$

$$Y_5(s) = Y_4(s) - K_2.$$

$$Y_4(s) = Y_5(s) + K_2.$$

Note :- Step 1 & step 2 are alternately repeated until the total function is realized.

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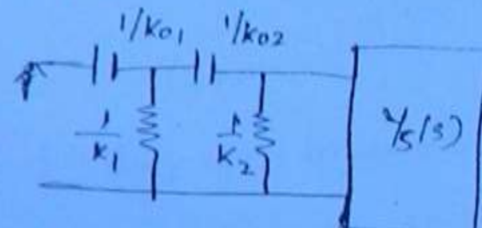
RC $\longrightarrow$ Step 1 : Removal of pole at origin from $Z(s)$

$$Z_2(s) = Z(s) - \frac{K_{01}}{s}$$

$$Y_2(s) = \frac{1}{Z_2(s)}.$$

Step 2 : Removal of constant from $Y(s)$

$$Y_3(s) = Y_2(s) - K_1$$

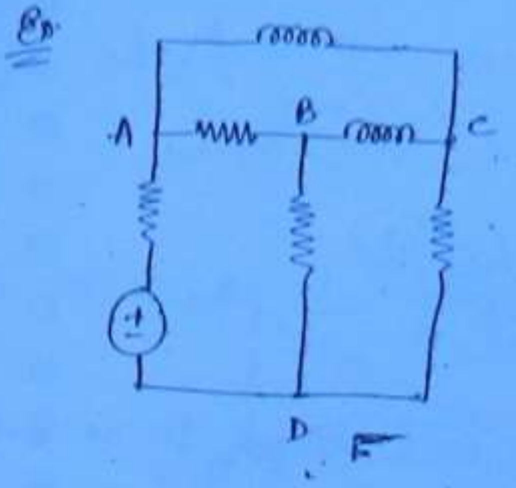


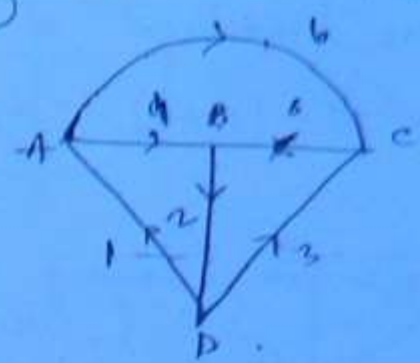
Graph Theory

Network Topology is a study of the n/w properties by investigating interconnection b/n branches and nodes, it mainly concentrates on the geometry of the network.

In the network topology, any network is replaced by graph, To develop the graph each element is replaced by either st. line or arc of the semicircle, voltage source is replaced by short circuit & current source is replaced by O.C. and graph retains all the nodes of the original n/w.

Ex.





$C = N-1$

$C = M$

$$\boxed{\text{No. of branches of n/w} \geq \text{no. of branches of graph}}$$

in +ve Augmented Incident matrix

	1	2	3	4	5	6
A	+1			-1		-1
B		-1		+1	+1	
C			+1		-1	+1
D	-1	+1	-1			

Reduced incident matrix

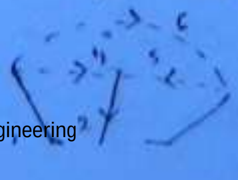
	1	2	3	4	5	6
A	+1			-1		-1
B		-1		+1	+1	
C			+1		-1	+1

D → ref

→ All the information regarding the graph can be represented mathematically in concised form is called as, incidence matrix.

→ For a given graph augmented incidence matrix is unique

→ Tree is a connected sub graph, it connects all the nodes of the n/w but it does not consist of any closed path.



Tree {1, 2, 3}

– Total tree branches = $N-1 = 4-1 = 3$ N = no. of nodes.

(tod.) links = $b-(N-1) = 6-(4-1) = 3$. b = no. of branches

Ex.

& Tree

The tree is invalid, because there is no interconnection between the nodes.

The set of branches which are disconnected, to form a tree is called as co-tree. (complementary tree).

The branch which form a tree is called as tree branch. (twig). Generally it is indicated by solid line (or) thick line.

Total no. of tree branches = $N-1$

The branch which is disconnected to form a tree is called as link. also known as chord. Generally it is indicated by dotted lines.

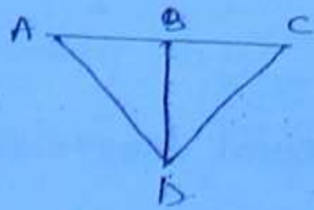
Total no. of links = $l = b-(N-1)$.

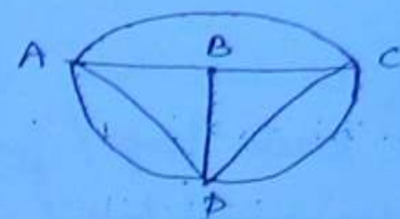
For a given graph tree is not unique.

Total no. of possible trees = N^{N-2}

Note :- The above formula can be applied when Connections should be present b/n all the nodes.

E.g.





For the above two graphs the formula is not applicable.

⇒ Total no. of possible trees for ~~any~~ graph ⇒

$= \det |AA^T|$, where A = reduced incidence matrix.

Node Pair voltages.

Total no. of node pair voltages $= \frac{N(N-1)}{2} = {}^{N}C_2$

$= \frac{4(4-1)}{2} = 6$

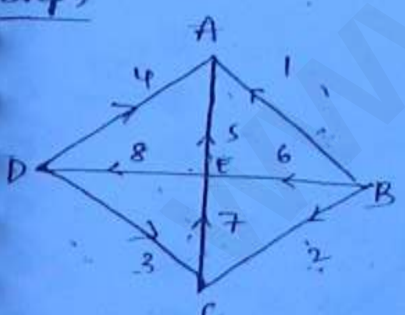
→ Total no. of edges $= \frac{N(N-1)}{2}$

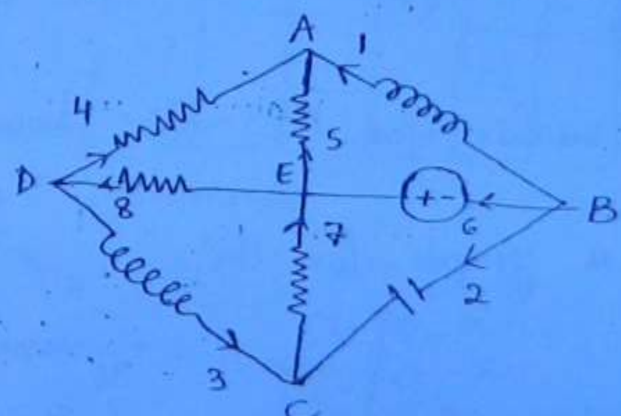
⇒ edge = branch

8/11

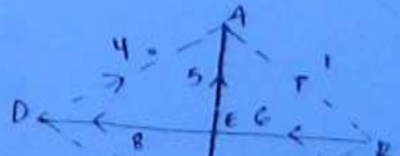
Develop a graph ~~branch matrix~~ for the network shown

Step 1





Step 2 Develop a tree for the graph.



Step 3:

Identify total no. of basic loops / fundamental loops. (or) independant loops / f - loops.

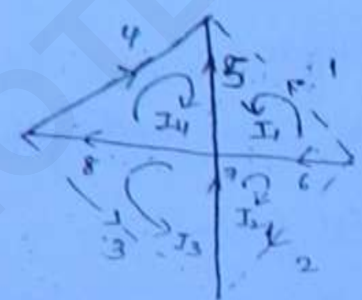
Basic loop should consist of only one link.

Total no. of basic loops = total no. of links.

$$l = b - (N-1).$$

Basic loop direction is same as the link current direction.

Tie set matrix [C]   U C_b



[C] =

	v_1 1	v_2 2	v_3 3	v_4 4	v_5 5	v_6 6	v_7 7	v_8 8
J_1	+1				-1	-1		
J_2		+1				-1	+1	
J_3			+1				+1	+1
J_4				+1	-1			+1

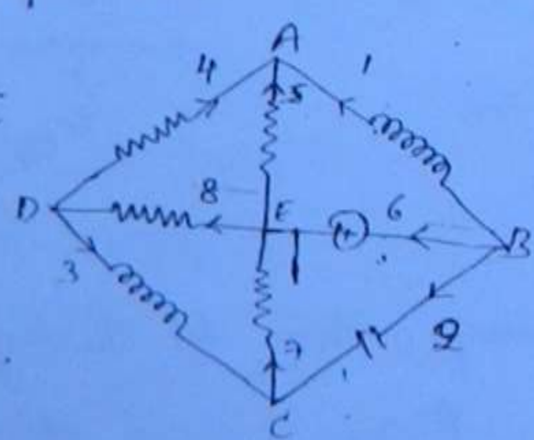
KVL.

$$v_1 - v_5 - V_6 = 0$$
$$V_2 - V_6 + V_7 = 0$$
$$V_3 + V_7 + V_8 = 0$$
$$V_4 - V_5 + V_8 = 0$$

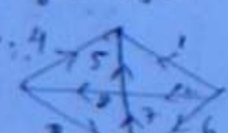
$$[C] = [U : C_b] \quad \text{voltages}$$

Cut-set matrix.

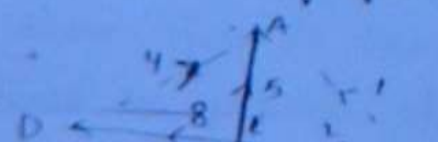
Develop cut-set matrix for the n/w shown.



Step 1: Develop a graph for the given n/w.

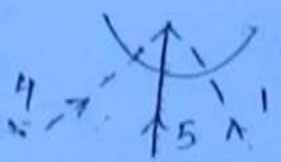


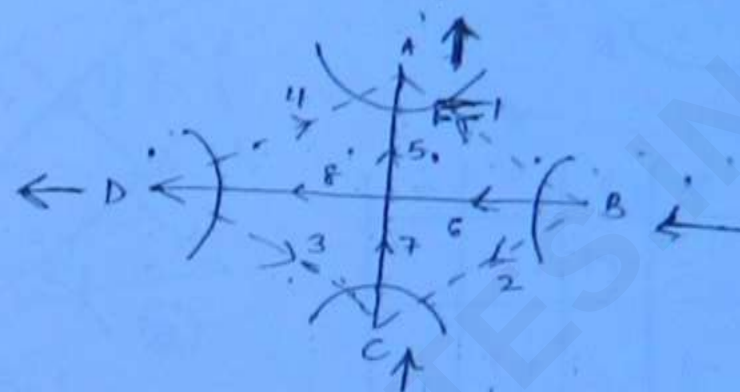
Step 2: Develop a tree for a graph.



3. i, Identify total no. of basic cut sets / fundamental cut sets / f-cut sets

ii, Basic cut set should consist of only one tree branch.





iii, Total no. of basic cut sets = total no. of tree branches
= N-1

iv, Basic cut set direction is same as the tree branch current direction.

cut-set matrix = [B] =

	i_1 1	i_2 2	i_3 3	i_4 4	i_5 5	i_6 6	i_7 7	i_8 8
	Bt					U		
A	+1			+1	+1			
B	+1	+1				+1		
C		-1	-1				+1	
D			-1	-1				+1

kcl

$i_1 + i_4 + i_5 = 0$

$i_1 + i_2 + i_6 = 0$

$-i_2 - i_3 + i_7 = 0$

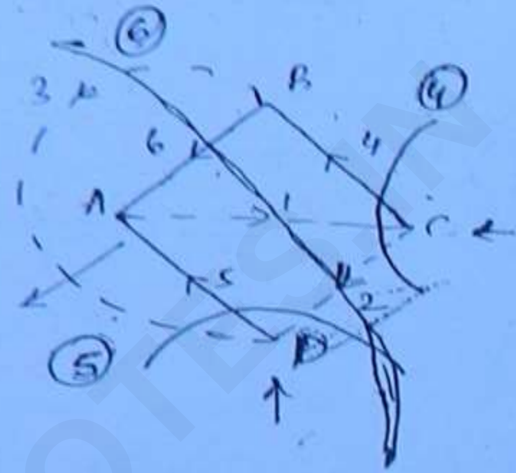
$-i_3 - i_4 + i_8 = 0$

currents

$[B] = [B_t : U]$

?. Develop cut set matrix for the graph shown.

Assume 4, 5, 6 as tree branches.

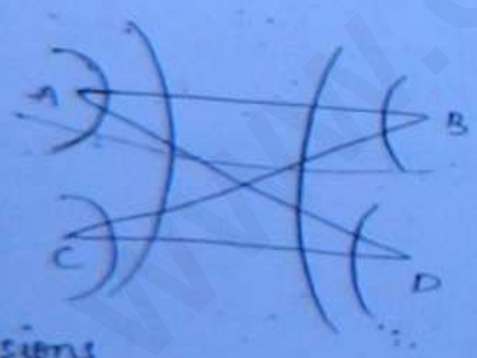


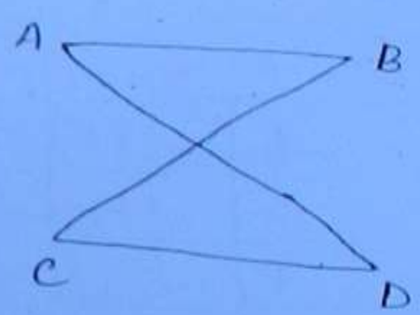
	1	2	3	4	5	6
4	-1	+1		+1		
5		-1	-1		+1	
6	-1	+1	+1			+1

Identify total no. of cut sets of the graph shown.

(a) 3 (b) 4

(c) 5 ✓(d) 6





clusions

Total no. of possible trees = N^{N-2}

Tie set matrix is not unique, total no. of possible Tie set matrix = N^{N-2}.

3. Cut set matrix is not unique; Total no. of possible cut set matrices = N^{N-2}

4. $[C]$ = $[U : C_b]$ (Tie set)

$[B]$ = $[B_l : U]$ (Cut set)

$B_l = -(C_b^T)$

$[B_l] = -[C_b^T]$

$[C_b] = -[B_l^T]$

5. The rank of the tie set matrix = total no. of links.

Rank = $l = b - (N-1)$

6. Rank of the cut set matrix = total no. of tree branches.

= $(N-1)$

7. Rank of the incidence matrix = $N-1$

Duality

R ⟷ G ; Series ⟷ parallel

L ⟷ C ; O.C ⟷ S.C

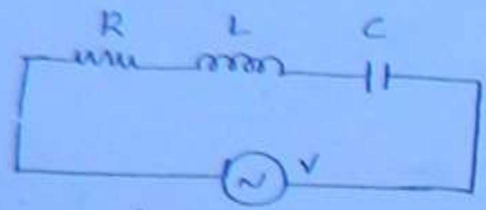
V ⟷ I ; Tie set ⟷ Cut set

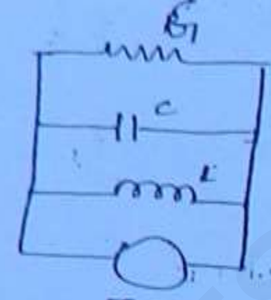
KVL ⟷ KCL ; Thevenin's ⟷ Norton's

loop (mesh) ⟷ node ; Foster I form ⟷ foster II form

$\frac{dv}{dt} \longleftrightarrow \frac{di}{dt}$

Duality doesnt mean equivalence. But it means, mathematical representation of both the networks are identical.

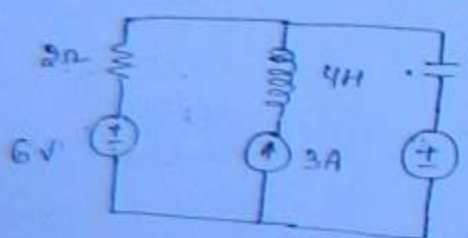


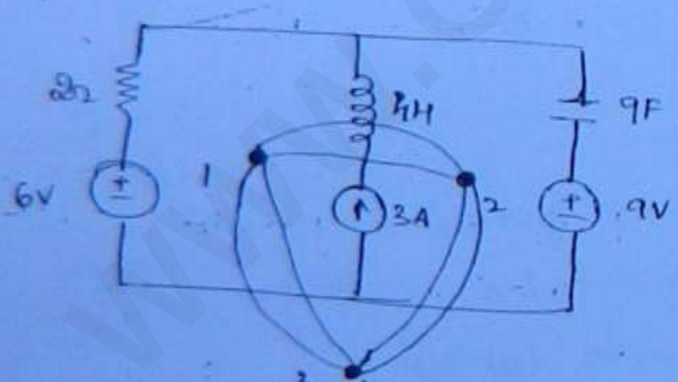


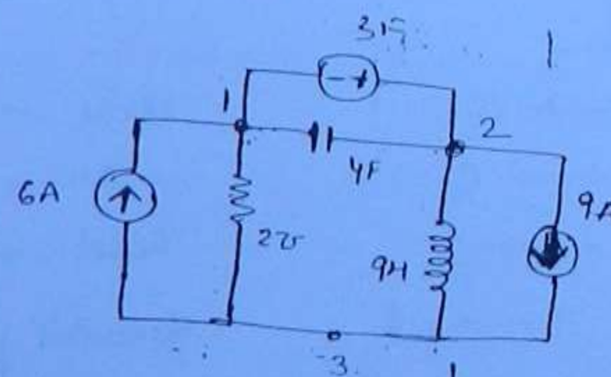
$$V = IR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$
$\hookrightarrow$ KVL

$$I = VG + C\frac{dv}{dt} + \frac{1}{L}\int V\,dt \rightarrow \text{KCL}$$

Draw the dual of the n/w shown.







Ie:

when voltage source drives a current in CW direction arrow mark of the current source is indicated towards respective node.

when current source drives a current in CW direction +ve sign is assigned to respective node.

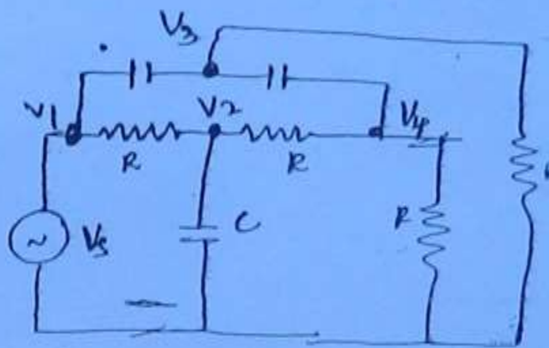
W.B

(1) f - loops = $l = b-(N-1) = 6-(4-1) = \underline{3}$

(3) $\frac{N(N-1)}{2} = \frac{10(10-1)}{2} = 45$

(4)

(6)



$e = M = 4$

$e = N-1 = 5-1 = 4$

min = 3 ($V_s = V_1$ not considered)

(7) f-cutset = $N-1 = 8-1 = 7$

(8) $N^{N-2} = 4^{4-2} = \underline{16}$

(9) f loops = $l = b-(N-1) \Rightarrow 3 = b-(4-1)$

$b = 6$

Conv.

1. Develop graph & then develop Tie set matrix.

Network Synthesis . W.B.

$$TF = \frac{1}{RCs+1} = \frac{1}{RC\left[s+\frac{1}{RC}\right]}$$ (d) 238

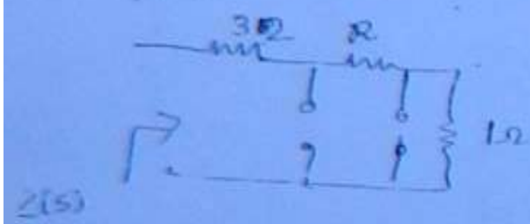
$$Y(s) = \frac{s^2+2.5s+1}{s^2+4s+3} = \frac{(s+0.5)(s+2)}{(s+1)(s+3)}$$

1. lowest critical freq → pole = ✗ → 0.5 ⇒ zero
2. alternate pole zero ✓

✗ 3.

4. $Y(0) = \frac{0.5\times 2}{3\times 1} = 1/3$.

$s=0$, $X_c = \frac{1}{sC} = \infty$, $C \to 0.C$



$$Z(0) = \frac{3\times 8}{3} = 4+R$$

$\Rightarrow 8 = 4+R \Rightarrow R = 4\Omega$.

for C value, find $Z(s)\big|_{s=1}$.

$$Y(s) = \frac{I(s)}{V(s)} = \frac{1/s}{1/s+1/s+1/\tau} = \frac{\tau s+1}{2\tau s+1}$$

$$= \frac{\tau s+1/2+1/2}{2\tau s+1} = \frac{\tau s+1/2}{2\tau s+1} + \frac{1/2}{2\tau s+1}$$

↓ 2 ⇓ $\left(\frac{1}{Z_a+Z_b}\right)$ → R_2

R_1 L

$$Z(s) = s + 2 + \frac{3}{s}$$

$L=1$, $R=2$, $C=1/3$ $Z(s)$ is PRF

10. $Z(j\omega) = \dfrac{j\omega+\alpha}{j\omega+\beta} = \dfrac{\angle \tan^{-1}(\omega/\alpha)}{\angle \tan^{-1}(\omega/\beta)} = \tan^{-1}\left(\dfrac{\omega}{\alpha}\right) - \tan^{-1}\left(\dfrac{\omega}{\beta}\right)$

θ (+ive) = $\alpha < \beta$

239

11. $s^2+s+1 \,)\, s^2+2s+2 \,(\, 1 \rightarrow Z,\ R$

s^2+s+1

$s+1 \,)\, s^2+s+1 \,(\, s \rightarrow C,\ Y$

s^2+s

$1 \,)\, s+1 \,(\, s \rightarrow L,\ Z$

s

$1 \,)\, 1 \,(\, 1 \rightarrow R$

1

0

(d)

12. $Z(s) = \dfrac{k\,(s+3)}{(s+j+1)(s+1-j)} = \dfrac{k(s+3)}{(s^2+2s+2)}$

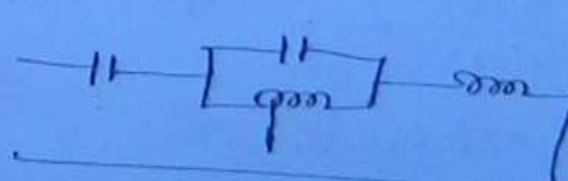
13. $Z(s) = \dfrac{k(s^2+20^2)(s^2+60^2)}{s(s^2+40^2)}$

Note – Imaginary poles & zeros should be conjugate pair. Only for LC n/w imaginary poles and zeros are present.

$P \to \infty \longrightarrow Hs$

$P \to$ origin $\to \dfrac{k_0}{s}$

$P \to$ conjugate pair.



15. $Z(s) = \dfrac{(s+1)}{(s+\sqrt{2})(s+\frac{1}{\sqrt{2}})} \Rightarrow Z(j\omega) = \dfrac{j\omega+1}{(j\omega+\sqrt{2})(j\omega+\frac{1}{\sqrt{2}})}$

$i(t) = \sin t \Rightarrow \omega = 1$.

240

$$Z(j1) = \frac{1+j1}{(\sqrt{2}+j)\left(\frac{1}{\sqrt{2}}+j\right)}$$

$V = I Z$.

$$|Z| = \frac{\sqrt{2}}{(\sqrt{3})(\sqrt{3/2})} = 2/3$$

$|V| = |Z| = 2/3$.

3.

ZRC.

1. lowest $\longrightarrow$ pole.
2. alternate

(3) high $\rightarrow$ zero

5.

In the continued fraction expansion, if all the quotients have the sign then it satisfies Hurwitz.

$q(s) = s^5 + 3s^3 + s$

$Q'(s) = 5s^4 + 9s^2 + 1$

s^5	1	3	1
s^4	5	9	1
s^3	6/5	4/5	

$$\psi(s) = \frac{Q(s)}{Q'(s)} = \quad 5s^4+9s^2+1\,)\; s^5+3s^3+s\,(\ldots$$

q satisfies Hurwitz

7.

$$F(s) = \frac{P(s)}{q(s)} = \frac{m_1(s)+n_1(s)}{m_2(s)+n_2(s)}$$

$$F(s) = \frac{m_1+n_1}{} \times \frac{m_2-n_2}{}$$

$$F(s) = \frac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} + \frac{n_1 m_2 - m_1 n_2}{m_2^2 - n_2^2}$$ 241

$= \text{even } F(s) + \text{odd } F(s)$

$= \text{Real } F(s) + \text{Im } F(s)$

$s^{even} \cdot s^{even} = s^{even}$

$s^{odd} \cdot s^{odd} = s^{odd}$

$s^{even} \cdot s^{odd} = s^{odd}$

Ex.

$s^{even} = (j\omega)^4 = \omega^4 = Re$

$s^{odd} = (j\omega)^2 = -j\omega^3 = \underline{Im}$

also $F(s) = PRF$

$\therefore \dfrac{m_1 m_2 - n_1 n_2}{m_2^2 - n_2^2} \geq 0.$

$m_1 m_2 - n_1 n_2 = 0$

$$\boxed{\frac{m_1 \,(\text{even})}{n_2 \,(\text{odd})} = \frac{n_1 \,(\text{odd})}{m_2 \,(\text{even})}}$$ **

23. $s^3 + 4s \,)\, 2s^4 + \ldots \,(2s + \ldots$

$2s \to L$, Z

$\Rightarrow L = 2H$

24. $Z_{11} = \dfrac{(R_2 + Ls) R_1}{R_1 + R_2 + Ls}$, $Z_{22} = \dfrac{R_2 (R_2 + Ls)}{R_1 + R_2 + Ls}$ R

25. equal denominators.

$Z_2(s) = PRF \therefore$

$\{Re\, Z_1(\omega) > R_{neg}$

30. $Z(s) = \dfrac{2s+1}{s(s+1)} = \dfrac{A}{s} + \dfrac{B}{s+1} = \dfrac{1}{s} + \dfrac{1}{s+1}$

$=$

31. $A = \left.\dfrac{V_1}{V_2}\right|_{I_2 = 0} \rightarrow A = \dfrac{Z_{11}}{Z_{21}}$

$B = -V_1 \,|$

$$\therefore \quad B = -\left(\frac{z_{11}z_{22} - z_{12}z_{21}}{z_{21}}\right) =$$ 242

$$D = \frac{-I_1}{I_2}\bigg|_{V_2=0}, \quad D = \frac{-y_{11}}{y_{21}} = \left(\frac{z_{22}/z_\Delta}{z_{21}/z_\Delta}\right) = \frac{z_{22}}{z_{21}}$$

Conv.

1.
$$Z(s) = \frac{s^2+1}{s(s^2+2^2)}.$$

$P \rightarrow \infty$ ✗

$P \rightarrow$ origin ✓, $P \rightarrow$ conjugate pair ✓

$$Z(s) = \frac{k_0}{s} + \frac{2ks}{s^2+w^2}$$

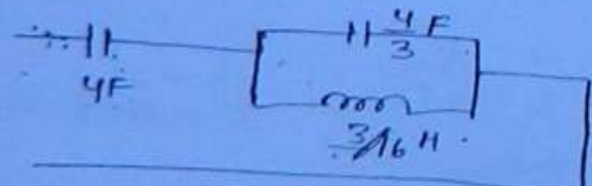
$$\frac{s^2+1}{s^2(s^2+4)} = \frac{k_0}{s} + \frac{2ks}{s^2+4}$$

$K_0 \rightarrow 1/4$

$2k \rightarrow 3/4$.

$$Z(s) = \frac{1/4}{s} + \frac{3s/4}{s^2+4}$$

$$Z(s) = \frac{1}{4s} + \frac{1}{4s/3 + 16/3s}$$



1. (b)

$$Z(s) = \frac{(s+1)(s+3)}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2} + C$$

2.

$$Z_D(s) = \frac{K(s+2)(s+6)}{s(s+4)(s+8)}$$

$$Z_D(s)\Big|_{s=-3} = +1 \Rightarrow K = .5$$

$$Z_D(s) = \frac{1.87}{s} + \frac{1.25}{s+4} + \frac{1.875}{s+8}$$

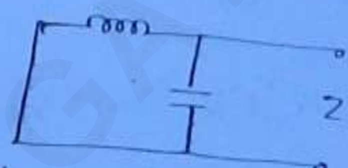
Resonance

1.

$$Q = \frac{\omega L}{R} = \frac{X_L}{R} = \frac{1000}{0.1} = 10^4$$

$$BW = \frac{f_0}{Q} = \frac{10 \times 10^6}{10^4} = 1\,kHz$$

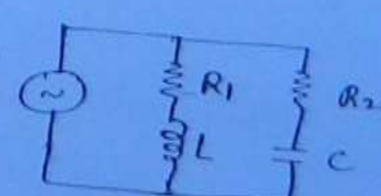
2. Find Z_{th}



$Z = \infty$ $Z = \infty$ for ideal current source

$$\omega_0 = \frac{1}{\sqrt{LC}} = 4\ rad/sec \Rightarrow \text{Resonant condition}$$

* Q. In the ckt shown, at what value of R_1, R_2 circuit resonant for all frequencies.



$$B_L = B_C$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega L}{R_1^2 + (\omega L)^2} = \frac{1/\omega C}{\dots}$$

$$\frac{1}{\frac{R_1^2}{\omega L} + \omega L} = \frac{1}{R_2^2 \omega C + \frac{1}{\omega C}}$$

$\omega \rightarrow L = R_2^2 C \Rightarrow R_2 = \sqrt{\frac{L}{C}}$.

$\frac{1}{\omega} \rightarrow \frac{R_1^2}{L} = \frac{1}{C} \Rightarrow R_1 = \sqrt{\frac{L}{C}}$.

$R_1 = R_2 = \sqrt{\frac{L}{C}}$.

$$Y(s) = \frac{s^2 + 0.5s + 100}{5s} = \frac{s}{5} + 0.1 + \frac{20}{s}.$$

$L = \frac{1}{20} H$, $G = 0.1$, $C = 1/5$.

$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$15 = \sqrt{V_R^2 + 9^2} \Rightarrow V_R = 12V$

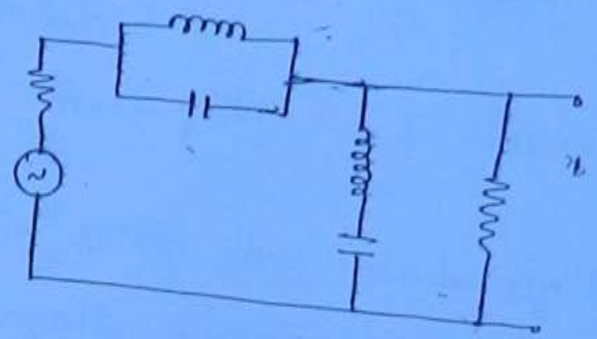
$20 = \sqrt{V_R^2 + V_L^2} \Rightarrow 20^2 = 12^2 + V_L^2 \Rightarrow V_L = 16V$

$\therefore V_L - V_C = 9$

$V_C = 7$

∴ Low pass filter.

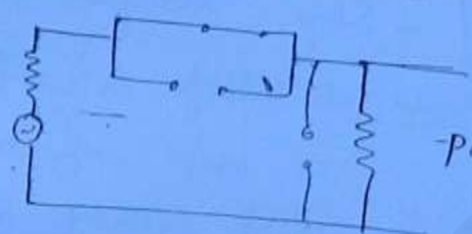
Q. Identify type of n/w for the n/w shown



at $f = 0$,

$X_C = \infty \rightarrow C \rightarrow O.C$

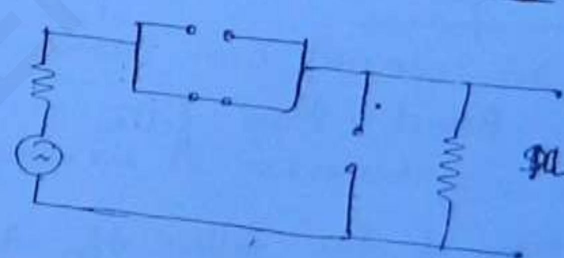
$X_L = 0 \rightarrow L \rightarrow S.C$



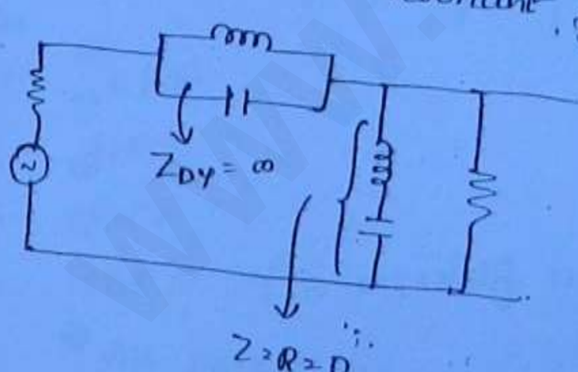
at $f = \infty$,

$X_C = 0 \rightarrow C = S.C$

$X_L = \infty \rightarrow L = O.C$

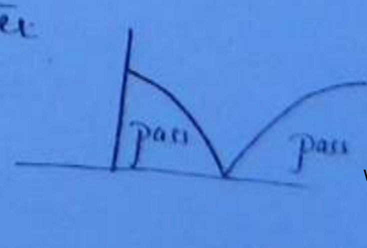


to verify whether BEF or all pass filter we have to check resonant freq.

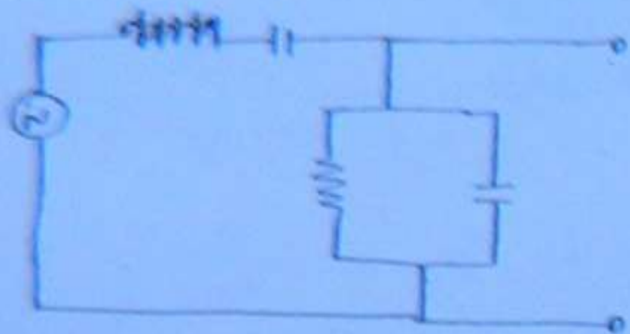


at $f = f_0$

when BEF eliminates only few frequencies, then it is also called as notch filter

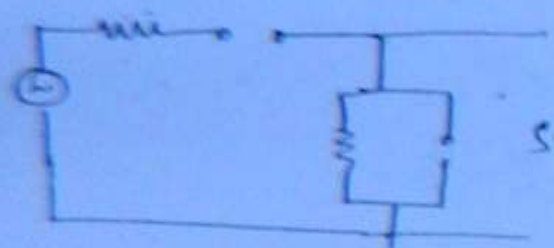


Identify type of the n/w for the n/w shown.



at $f=0$,

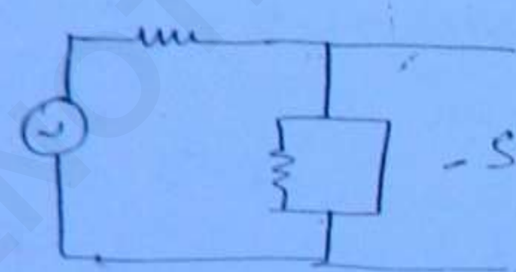
$X_C \to \infty \Rightarrow C = O.C.$



Stop

at $f=\infty$,

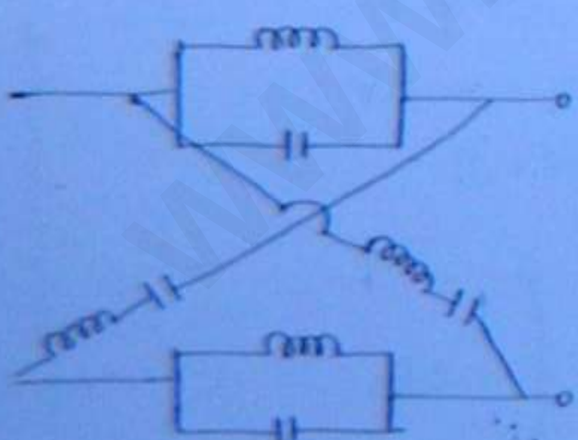
$X_C \to 0$, $S.C \to C$.



- Stop

$\Rightarrow$ Band Pass filter.

Identify type of the filter of the n/w shown



(a) LPF

(b) BPF

(c) BEF

(d) All pass filter ✓

FILTERS

Based on components present in the filter, filters are classified as

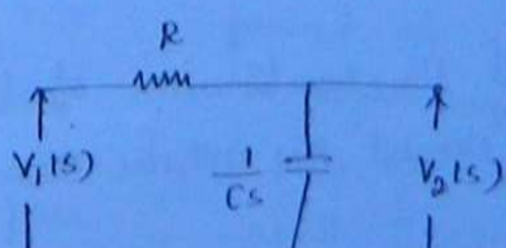
1. Active filter 2. Passive filter.

→ Active filters are made up of op-amp & capacitor.
→ Generally inductor is not used in the active filter, since size of the inductor is bulky & cost is high.
→ In the active filter, it is possible to increase gain of the system.
→ Passive filter is made up of series and parallel LC sections. (reactive n/w)
→ In passive filter it is not possible to increase gain of the system.
→ Based on frequency of operation, filters are classified as

1. LPF
2. HPF
3. BPF
4. BEF (BSF)
5. All pass filter.

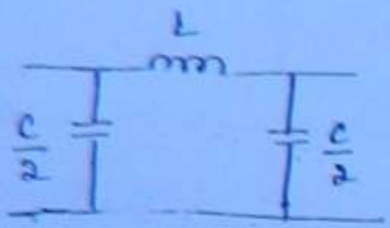
LPF

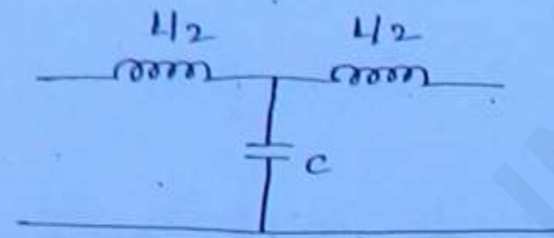
First order



$$V_2(s) = V_1(s)\,\frac{1/Cs}{R + 1/Cs} = \frac{V_1}{1 + RCs}$$

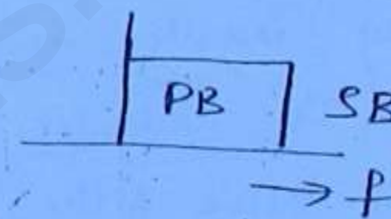
Second order





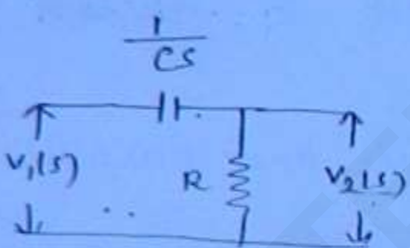
T

$$X_C = \frac{1}{2\pi f C}, \quad X_L = 2\pi f L$$



HPF

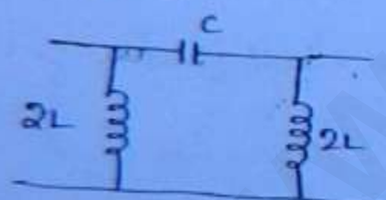
First order



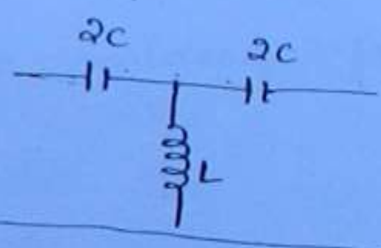
$$V_2(s) = V_1(s)\,\frac{R}{R + 1/Cs}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{RCs}{1 + RCs}$$

Second order



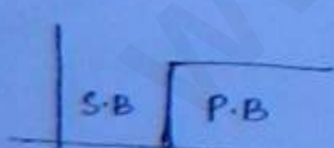
π



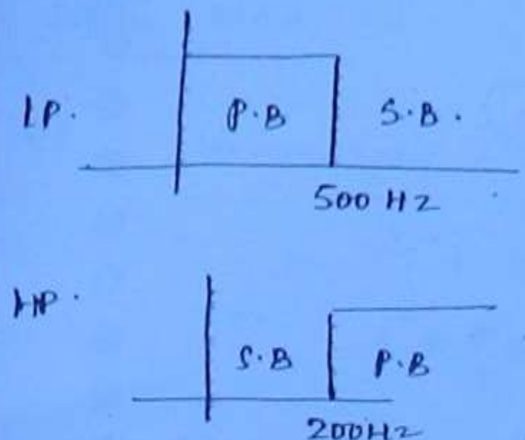
T

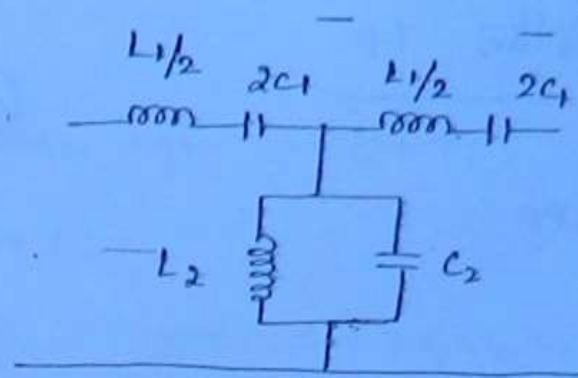
$$X_C = \frac{1}{2\pi f C}$$

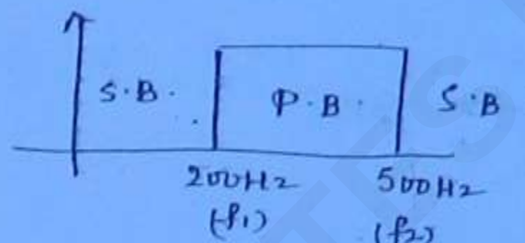
$$X_L = 2\pi f L$$



BPF can be obtained with the combination of LPF & HPF. cut off freq. of LPF should be greater than, cut off freq of HPF

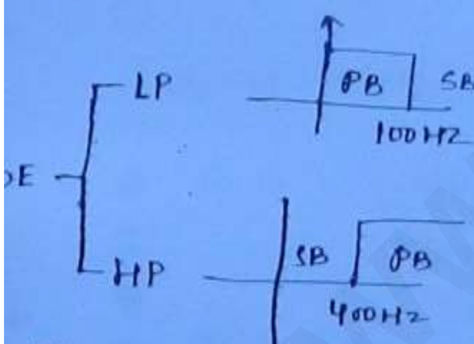


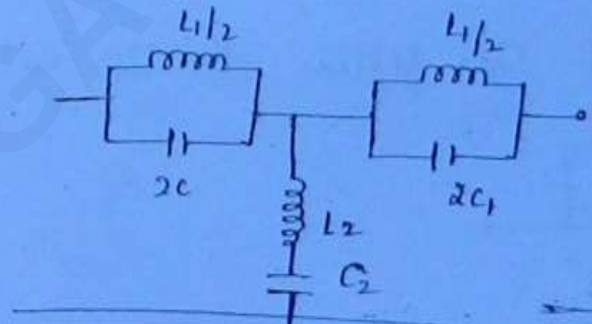


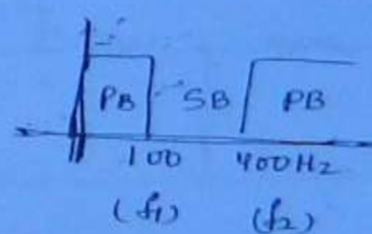


BEF / BSF

BEF can be obtained by connecting LPF & HPF in parallel and cut off frequency of the HPF should be greater than cut off freq. of LPF







First Order filter T.F

$\frac{1}{1+\tau s} \longrightarrow$ LPF

$\frac{\tau s}{1+\tau s} \longrightarrow$ HPF $\tau = RC$

$\frac{1-\tau s}{1+\tau s} \longrightarrow$ All pass

Note :-

1. In All pass filter poles are present in LHP & zeros are present in right half plane.
2. Poles and zeros are in symmetric about $j\omega$ axis.

Second Order filters T.F

$$\frac{P}{s^2+as+b} \longrightarrow LPF$$

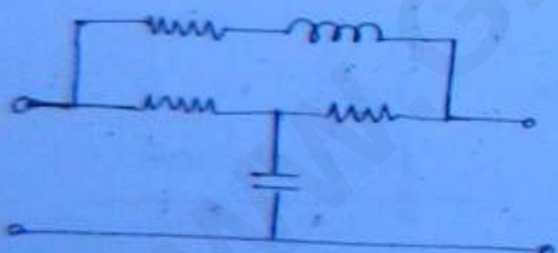
$$\frac{Ps^2}{s^2+as+b} \longrightarrow HPF$$

$$\frac{Ps}{s^2+as+b} \longrightarrow BPF$$

$$\frac{Ps^2+q}{s^2+as+b} \longrightarrow BEF$$

$$\frac{s^2-ps+q}{s^2+as+b} \longrightarrow \text{All pass filter.}$$

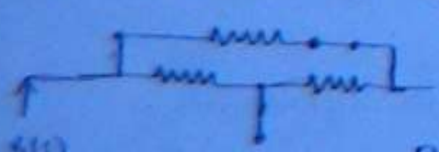
Identify type of the filter for the network shown



$f=0$,

$X_C = \frac{1}{2\pi f C} = \infty \Rightarrow C \to O.C$

$X_L = 2\pi f L = 0 \Rightarrow L \to S.C.$

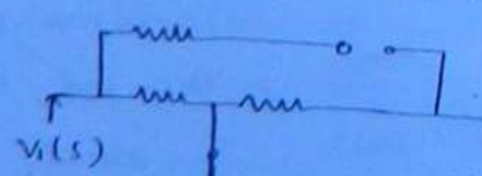


Pass

at $f=\infty$,

$X_C = 0 \Rightarrow C \to S.C.$

$X_C = \infty \quad L \to O.C.$



Stop

Filters WB

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① denominator - charac. eqn. - $(1+\tau s)^2$

$= \tau^2 s^2 + 2\tau s + 1$.

$\Rightarrow s^2 + \frac{2}{\tau}s + \frac{1}{\tau^2} \simeq s^2 + 2\xi\omega_n s + \omega_n^2$.

$\omega_n = 1/\tau$.

$2\xi \cdot \frac{1}{\tau} = \frac{2}{\tau} \Rightarrow \xi = 1 \rightarrow$ critically damped.

9. $B_1 = R/L_1$

$B_2 = R/L_2 = \frac{R}{L_1/4} = \frac{4R}{L_1} \Rightarrow \frac{B_1}{B_2} = 1/4$.

Resonance WB cont-

10. $\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) = 0 \quad s^2 + 20s + 10^6 = 0$.

$\omega_2 - \omega_1 = BW = \frac{R}{L} = 20, \quad \omega_0^2 = 10^6$.

$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{10^3}{20} = 50$

11. $V_L = 2V_C$

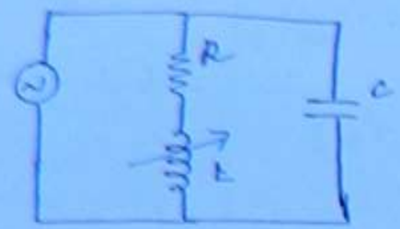
$IX_L = 2IX_C \Rightarrow X_C = \frac{X_L}{2}$

$\cos\theta = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} \Rightarrow \cos 45^\circ = \frac{20}{\sqrt{20^2 + \left(X_L - \frac{X_L}{2}\right)^2}}$

$\Rightarrow X_L = 40\Omega$.

(b):

(252)



$$Z_{eq} = \frac{(R+jX_L)(-jX_c)}{R+jX_L-jX_c}$$

$$Z_{eq} = \frac{X_LX_c - jRX_c}{R+j(X_L-X_c)} \Rightarrow \frac{(X_LX_c - jRX_c)[R-j(X_L-X_c)]}{R^2+(X_L-X_c)^2}$$

$$= \frac{RX_LX_c - RX_c(X_L-X_c) - j[R^2X_c + X_LX_c(X_L-X_c)]}{(\quad)}$$

$Im(Z_{eq}) = 0$ $\quad$ $R^2 - X_LX_c + X_L^2 = 0$

$X_L^2 - X_LX_c + R^2 = 0$

$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$

$\omega_n = R$, $\quad 2\xi\omega_n = -X_c \Rightarrow \xi = \frac{-X_c}{2R}$

$\xi < -1$

$\frac{-X_c}{2R} < -1 \Rightarrow \frac{X_c}{2R} > 1 \Rightarrow X_c > 2R$

$X_c > 10$ $\quad$ (b)

13. $\xi \geq 1$

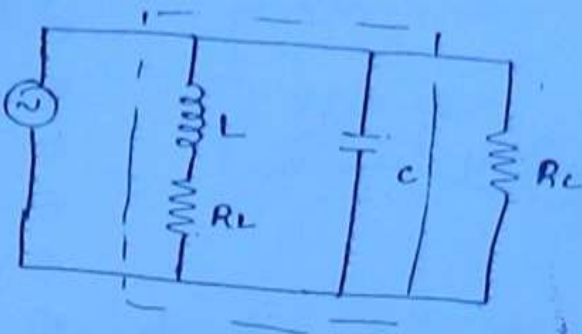
$$\frac{-R}{2}\sqrt{\frac{C}{L}} \geq 1 \quad \text{(c)}$$

Note: (1) In over & critical damping, no oscillations are present.

In the underdamping system more than one oscillation are present.

Conv.

(1)



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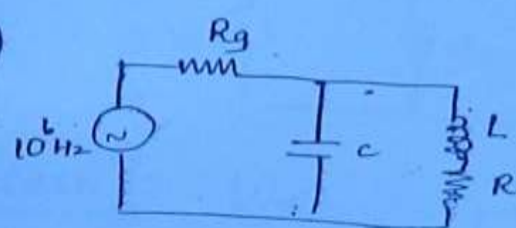
R_c doesnt influence the imaginary part of Z_{eq}. Hence can be neglected.

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R_L^2}{L^2}}$$

(2)



$$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Rg can be neglected ∵ it doesnt influence the Im(Zeq).

$$10^6 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \rightarrow (1)$$

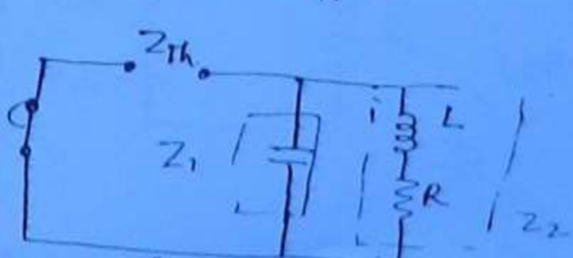
$$Q = \frac{\omega L}{R} \Rightarrow \frac{R}{L} = \frac{\omega}{Q}$$

$$\frac{R}{L} = \frac{2\pi f}{7} \Rightarrow \frac{R}{L} = \frac{2\pi \times 10^6}{7} \rightarrow (2)$$

sub. (2) in (1).

$L = 0.2\text{mH}$

$R = 180\,\Omega$

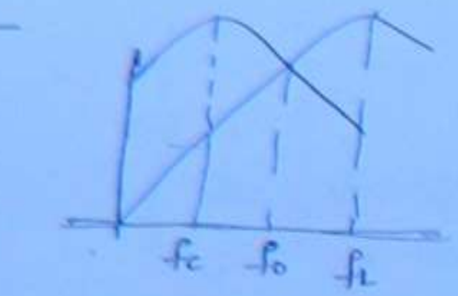


$Z_1 = -jX_c$

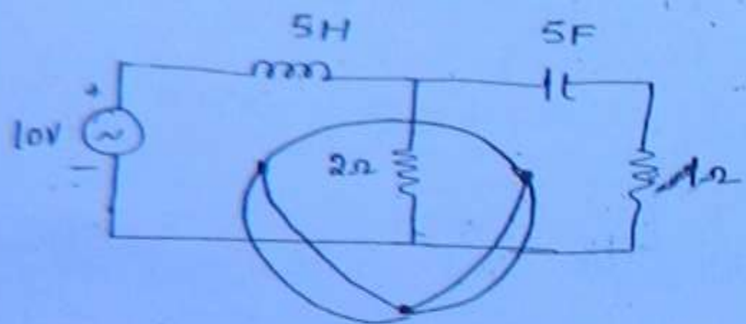
$Z_2 = R + jX_L$

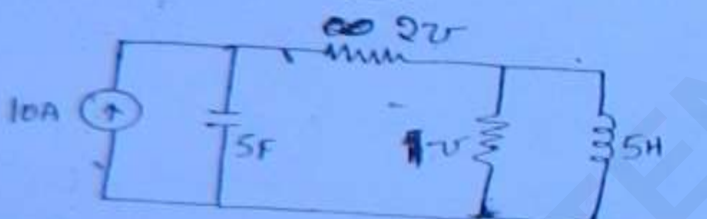
$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

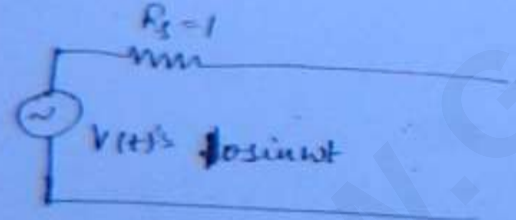
$|Z_{Th}| = 5.3\text{k}\Omega$



heorems conv.



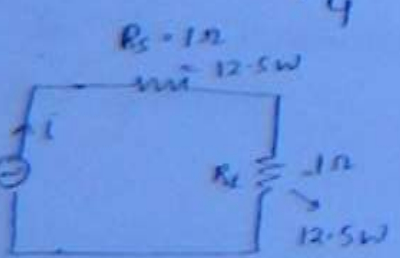




$R_L = R_s = 1$

$$P_{max} = \frac{V_s^2}{4R_L}$$

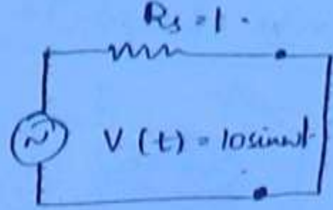
$$= \frac{(10/\sqrt{2})^2}{4} = \frac{100}{8} = 12.5W$$



$P_T = 12.5 + 12.5$

$P_T = 25W$

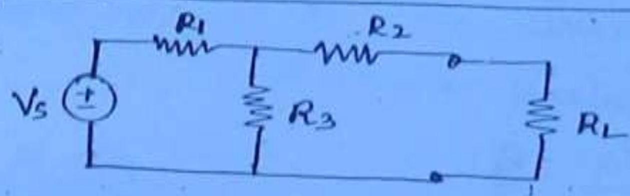
i,



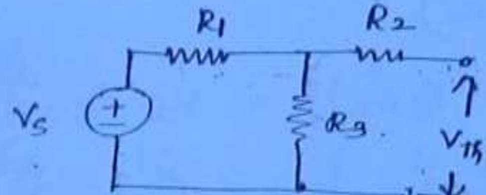
$$P = \frac{V_s^2}{R_s}$$

$$= \frac{(10/\sqrt{2})^2}{1} = 50W$$

Proof of thevenin's theorem.

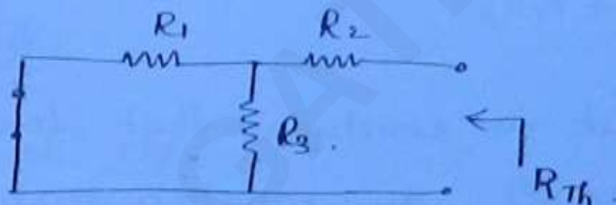


Case 1: (V_{Th})



$$V_{Th} = \frac{V_s R_3}{R_1 + R_3} \rightarrow (1)$$

Case 2:



$$R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3} \rightarrow (2)$$

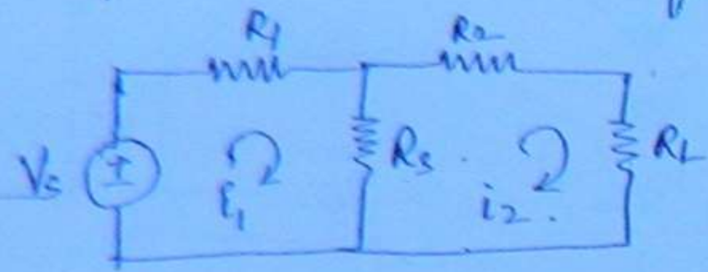
Case 3:

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$I_L = \frac{V_s \cdot R_3 / R_1 + R_3}{(R_L + R_2)(R_1 + R_3) + R_1 R_3 / (R_1 + R_3)}$$

$$I_L = \frac{V_s R_2}{(R_1 + R_2)(R_1 + \ldots) + \ldots} \rightarrow (4)$$

using conventional method find out I_L.



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$$V_S = (R_1+R_3)\, i_1 - i_2 R_3 .$$

$$i_1 R_3 = i_2 (R_2+R_3+R_L) .$$

$$i_2 (R_2+R_3+R_L) = \frac{V_S + i_2 R_3}{(R_1+R_3)} \cdot R_3$$

$$i_2 \left[(R_2+R_3+R_L)(R_1+R_3) - (R_3^2) \right] = V_S \cdot R_3 .$$

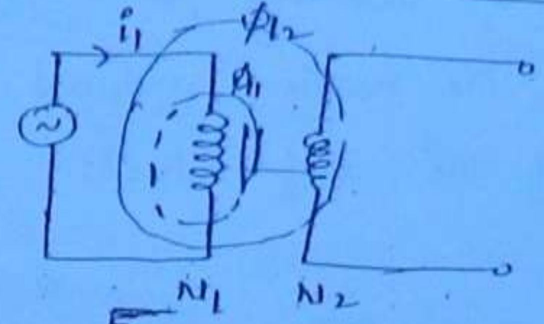
$$i_2 \left[R_2 R_1 + R_2 R_3 + R_1 R_3 + \cancel{R_3^2} + R_L R_1 + R_L R_3 - \cancel{R_3^2} \right] = V_S R_3 .$$

$$i_2 = \frac{V_S R_3}{(R_1+R_3)(R_L+R_2) + R_1 R_3} = i_L' \rightarrow (8)$$

from the above calculation it is concluded that load current of eq (4) & eq (8) are equal. Hence thevenins theorem is proved.

Magnetic Coupled circuits

Case i)



(257)

$I \Rightarrow \phi_1$ < $\phi_{11} \rightarrow$ leakage flux ; $\phi_{12} \rightarrow$ useful flux (or) mutual flux.

$$e_1 \propto \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{dt}$$

$$e_1 = -N_1 \frac{d\phi_1}{di_1} \cdot \frac{di_1}{dt} \qquad \left(L = \frac{N\phi}{I} \right)$$

$$\boxed{e_1 = -L_1 \frac{di_1}{dt}}$$ → self induced emf.

$$e_2 \propto \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{dt}$$

$$e_2 = -N_2 \frac{d\phi_{12}}{di_1} \cdot \frac{di_1}{dt}$$

$$\boxed{M_{21} = \frac{N_2 \phi_{12}}{I_1}}$$

$$\boxed{e_2 = -M_{21} \frac{di_1}{dt}}$$ → mutual induced emf.

en current is either entering or leaving at oth dotted terminals, sign of the mutual induced ltage is same as the sign of the self induced ltage.

hen one current is entering and other current is leaving at dotted terminal, sign of the mutual nduced voltage is opposite to the sign of self induced voltage.

The amount of magnetic coupling between the inductors is expressed by coefficient of coupling (K)

$$K = \frac{\text{useful flux}}{\text{Total flux}} = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$$

For ideal system $K = 1$. ($\because$ leakage $\phi_L = 0$ $\therefore$ $\phi_{12} = \phi_1$, $\phi_{21} = \phi_2$)

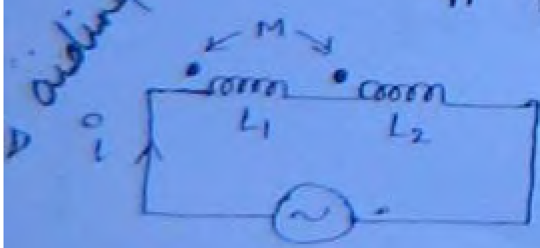
For practical system, the range of K is $0 < K < 1$

$$M = K\sqrt{L_1 L_2}.$$

mutual inductance ↓ → self inductance

coeff. of coupling

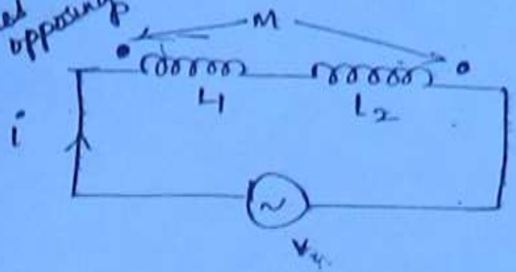
aiding



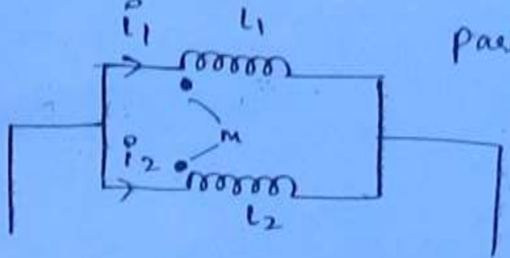
$$V = L_1\frac{di}{dt} + \frac{Mdi}{dt} + L_2\frac{di}{dt} + \frac{Mdi}{dt}$$

$$L_{eq}\cdot\frac{di}{dt} = (L_1 + L_2 + 2M)\frac{di}{dt}$$

Series opposing

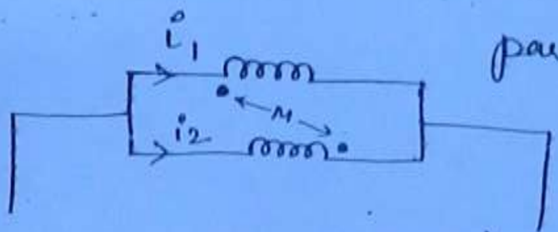


$$L_{eq} = L_1 + L_2 - 2M.$$



Parallel aiding

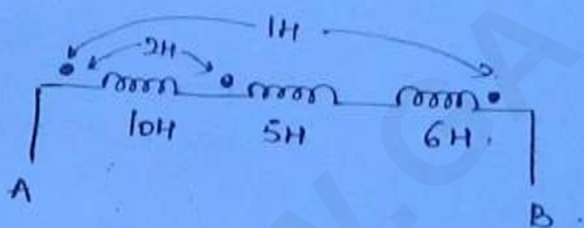
$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}.$$



parallel opposing.

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Q. Find eq. inductance w.r.t A & B.



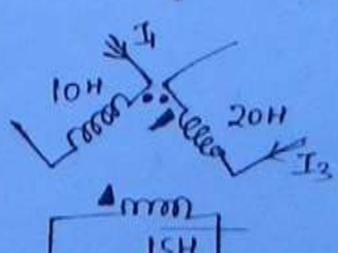
$15 + 4 = 19$

$19 + 6 - 2 = \underline{23H}$

$$L_{eq} = L_1 + L_2 + L_2 \pm 2M_1 \pm 2M_2 \pm 2M_3$$

$$L_{eq} = 10 + 5 + 6 + 4 - 2 = \underline{23H}$$

Q. Develop inductance matrix for the network shown.



• 1H

▲ 2H

260