6

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192

-: HAND WRITTEN NOTES:-

OF

Electronic & communication
~~Mechanical~~ ENGG.

ECE

I

Electronic & Communication
ENGG.

-: SUBJECT:-

CONTROL SYSTEM

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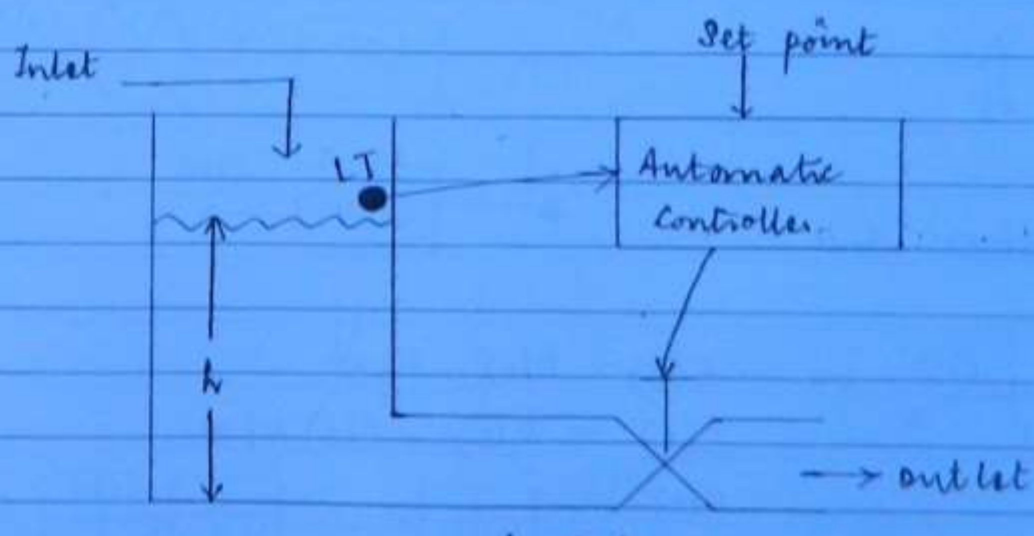
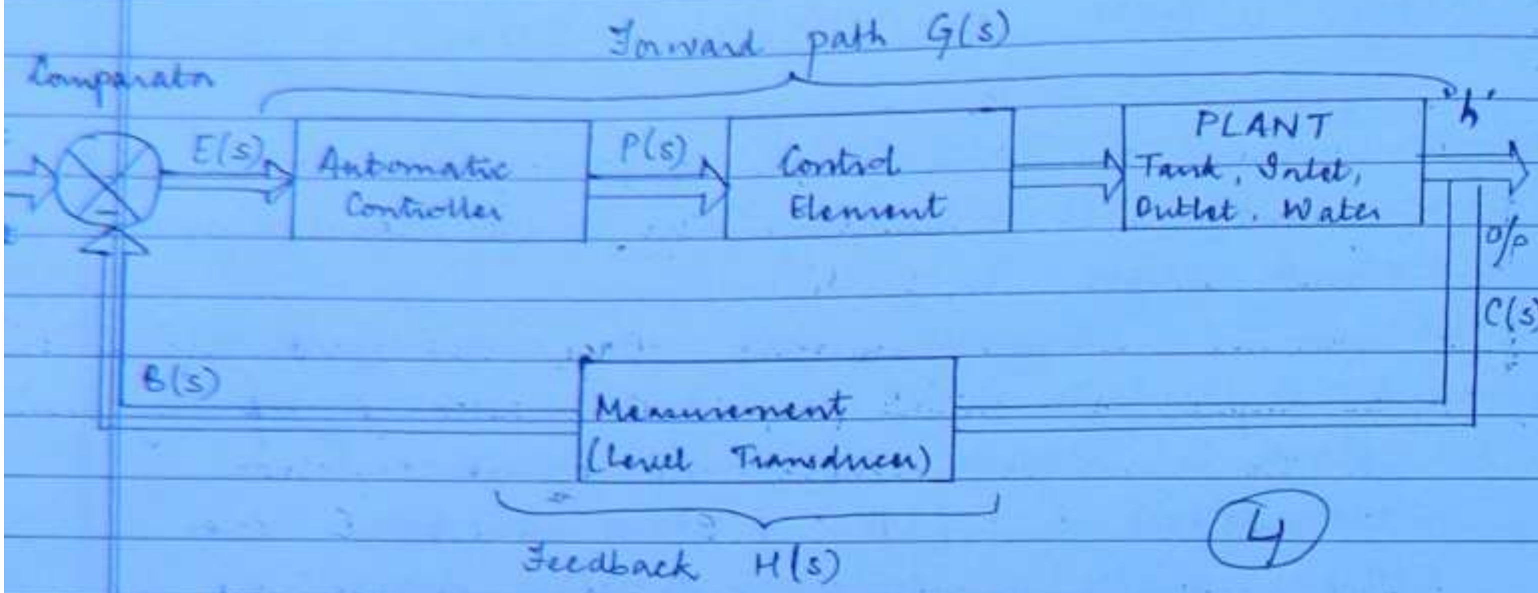
2

INTRODUCTION TO CONTROL SYSTEMS

1. Consider a liquid level control system whose control objective is to maintain the water level in the tank at a height 'h'
2. Controller is an automatic device with error signal $E(s)$ as input & controller output $P(s)$ affecting the dynamics on the plant to achieve the control objective.
   ∴ Controller output $P = f(e)$ where e = error
3. The different modes of controller operation are proportional, proportional + integral & proportional + int + derivative
4. There are 2 basic control loop configurations
   i) Closed loop (or) Feedback control system
   → In this configuration the changes in the output are measured through feedback & compared with the input or "set point" to achieve control objective
   → Feedback implies measurement (sensors or transducers)



(control element)



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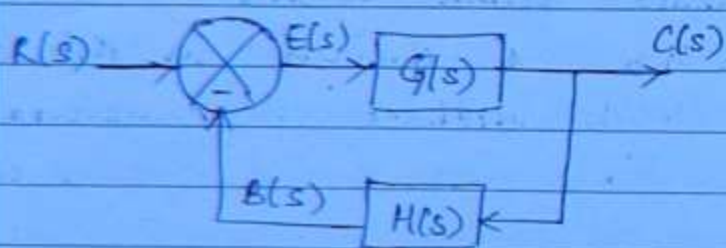
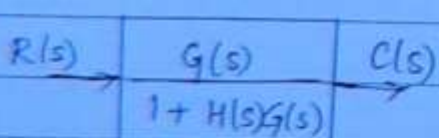
→ Control Canonical Form



→ Eq Mathematical Form



$$E(s) = R(s) - B(s)$$

$$\frac{C(s)}{G(s)} = R(s) - C(s)H(s)$$

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

$$C(s) - [H(s)G(s) + 1] = G(s)R(s)$$

stable subject any disturbance (always to this

ii) Open Loop Control Systems -

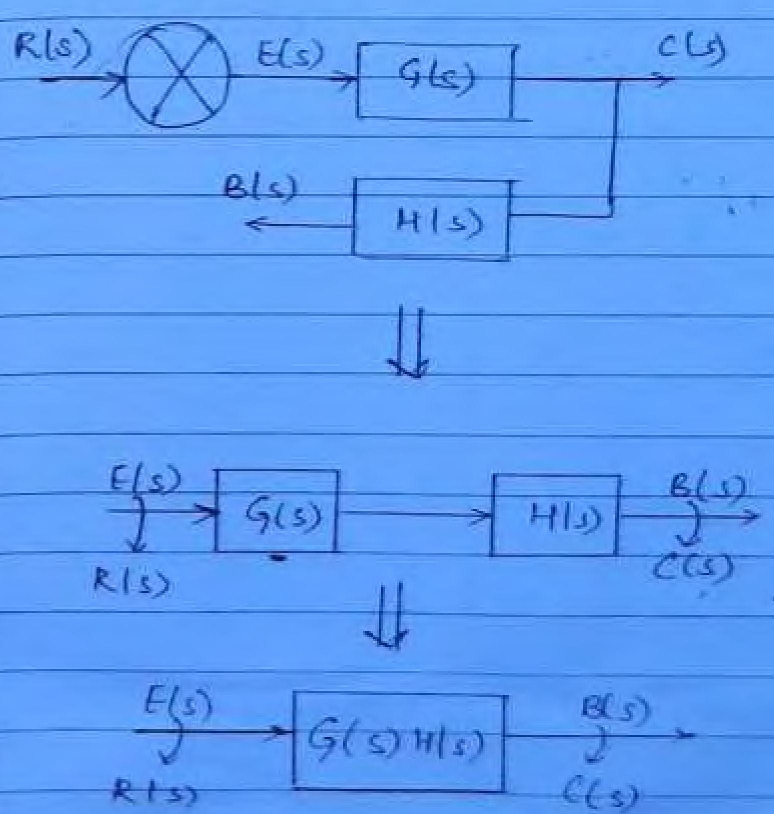
→ In this configuration, the feedback or measurement is not connected to the forward path or controller (open loop)

→ These systems are conditional control systems formulated under the basic condition that the system is not subject to any type of disturbance

→ Feedback in an open loop system except for displaying the information about the output has no major significance. This insignificance of feedback is termed as elimination / removal of feedback

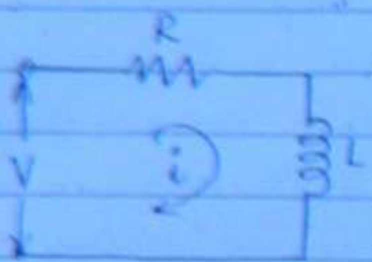
→ Open Loop systems are more stable than closed loop systems & hence do not require any performance analysis

→ The application of Open Loop control is limited because they are conditional control systems



CONCEPT OF TRANSFER FUNCTIONS -

It is a mathematical model representing a control system relating input & output in the form of ~~the~~ a ratio ie $\frac{\text{Output}}{\text{Input}}$



$$V = Ri + L\frac{di}{dt}$$

Applying L.T

$$V(s) = I(s)R + Ls\,I(s)$$

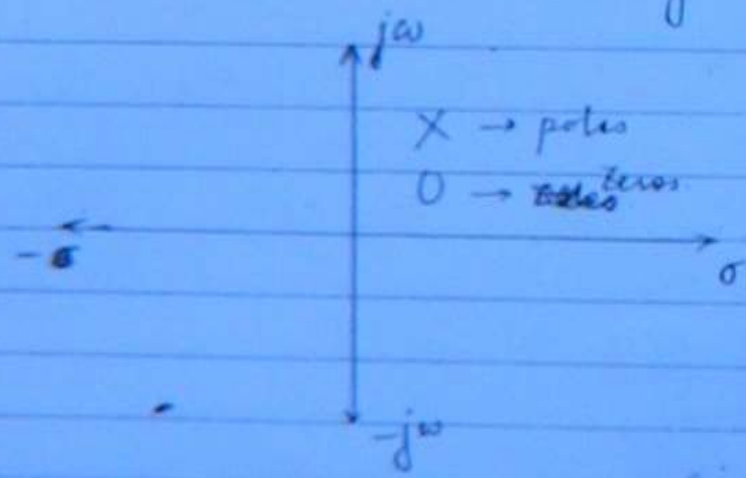
$$= I(s)[R + Ls]$$

$$\frac{I(s)}{V(s)} = \frac{1}{R + Ls}$$

$$\frac{I(s)}{V(s)} = \frac{1/L}{(s + R/L)}$$

$$TF = F(s) = \frac{N(s)}{D(s)} = \frac{C(s)}{R(s)} = \frac{k(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)} \quad \text{--- (1)}$$

k = system gain



$s = \sigma + j\omega$
(s - plane)

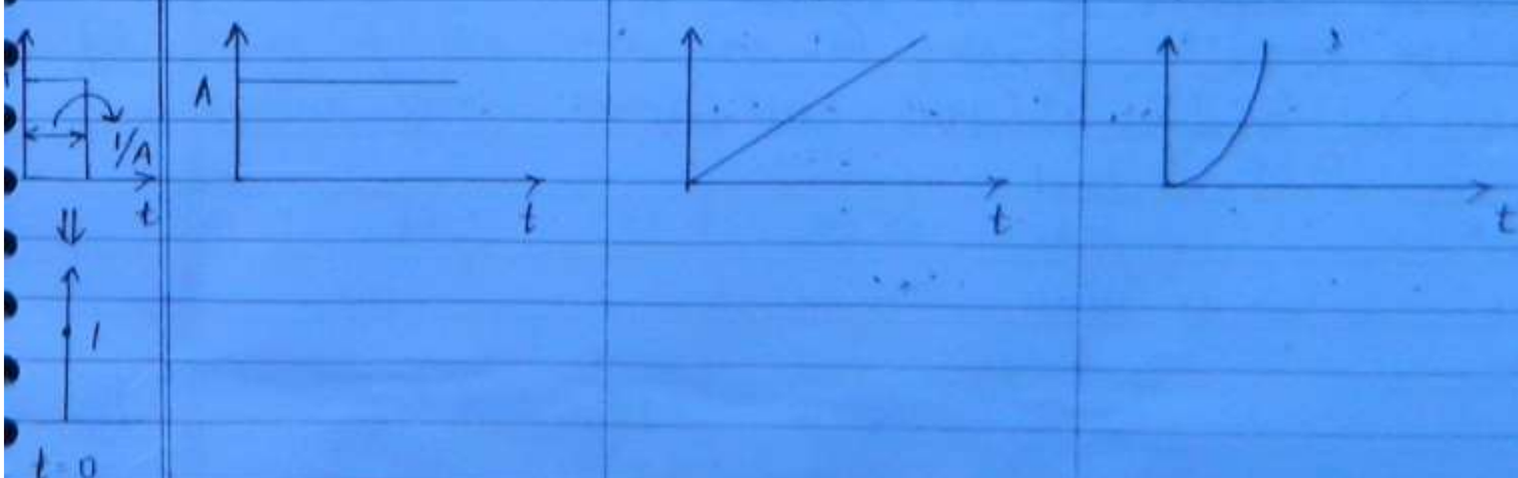
* From eq ①, transfer function of LTI system may be defined as ratio of laplace transform of output to laplace transform of input under the assumption that systems initial conditions are zero.

* Poles & zeros are those critical frequencies which make the transfer function infinity or zero. Poles & zeros represent system parameters only.

Singularity Functions of Transfer Functions –

The standard time domain test signals are also known as singularity functions which define the transfer function of LTI system.

Impulse signal	Step Signal	Ramp Signal	Parabolic Signal
$\delta(t) = 1 \quad t=0$ $= 0 \quad t \neq 0$	$r(t) = A\,u(t)$ $u(t) = 1 \quad t>0$ $= 0 \quad t<0$	$r(t) = At \quad t>0$ $= 0 \quad t<0$	$r(t) = At^2/2 \quad t>0$ $= 0 \quad t<0$
L.T $\Delta(s) = 1$	LT $R(s) = \frac{A}{s}$	LT $R(s) = \frac{A}{s^2}$	LT $R(s) = \frac{A}{s^3}$



$$T.F = F(s) = \frac{C(s)}{R(s)}$$

$$C(s) = F(s) \times R(s)$$

Let $R(s)$ = Impulse signal $= 1$

$C(s)$ = Impulse Response

$= F(s) \times 1$

$= T.F = F(s) \longrightarrow (2)$

$\mathcal{L}$ (Impulse Response) = T.F.

[WEIGHTING FUNCTION]

$\frac{d}{dt}$ (Parabolic Response) = Ramp response

$\frac{d}{dt}$ (Ramp Response) = Step response

$\frac{d}{dt}$ (Step Response) = Impulse Response

$\mathcal{L}$ (Impulse Response) = T.F

The impulse response of a system is

$c(t) = -te^{-t} + 2e^{-t} \quad (t > 0)$

Its open loop transfer function will be

a) $\frac{2s+1}{(s+1)^2}$ b) $\frac{2s+1}{s+1}$

c) $\frac{2s+1}{s^2}$ d) $\frac{2s+1}{s}$

$c(t) = -te^{-t} + 2e^{-t}$

$T.F = C(s) = \frac{-1}{(s+1)^2} + \frac{2}{s+1} = \frac{2s+1}{(s+1)^2}$

$$TF = \frac{2s+1}{(s+1)^2}$$

$$\frac{G(s)}{1+H(s)G(s)} = \frac{2s+1}{(s+1)^2}$$

$H(s)G(s) = ?$

Ideally $H(s) = 1$

$$\frac{G(s)}{1+G(s)} = \frac{2s+1}{(s+1)^2}$$

$\Rightarrow G(s)\,[(s+1)^2] = [1+G(s)]\,(2s+1)$

$\Rightarrow G(s)\,[(s+1)^2 - (2s+1)] = 2s+1$

$\rightarrow \boxed{G(s) = \frac{2s+1}{s^2}}$ — (c)

Shortcut

$$TF = \frac{2s+1}{(s+1)^2}$$

$$G(s) = \frac{2s+1}{(s+1)^2 - (2s+1)}$$ only for $H(s)=1$ $\quad \frac{N}{D-N}$

$$G(s) = \frac{2s+1}{s^2}$$

CWB

chapter 2

Q8. What is the open loop DC gain of a unity negative feedback system having closed loop T.F $\frac{s+4}{s^2+7s+13}$

a) $\frac{4}{13}$ b) $\frac{4}{9}$ c) 4 d) 13

Sol

$$TF = \frac{s+4}{s^2+7s+13}$$

$$G(s)H(s) = \frac{s+4}{s^2+7s+13-(s+4)} = \frac{s+4}{s^2+6s+9}$$

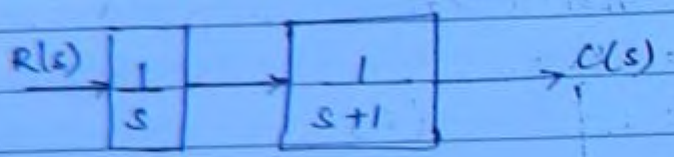
$$G(s) = \frac{s+4}{s^2+6s+9} \qquad H(s) = 1$$

$s = 0$

$$G(s) = \frac{0+4}{0+0+9}$$

$$G(s) = \frac{4}{9} \quad . \text{(b)}$$

Q3 Find the impulse response of the system



$$\text{Impulse response} = L^{-1}(TF)$$

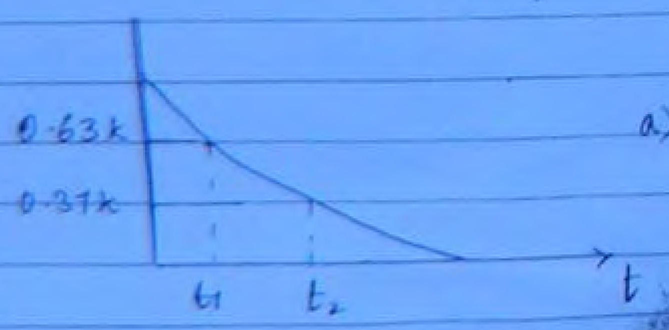
$$= L^{-1}\left[\frac{1}{s(s+1)}\right] \longrightarrow \text{partial fractions}$$

$$= L^{-1}\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

$$= 1 - e^{-t} \quad \text{Ans}$$

———x———

Q4 The Impulse Response of the system having T.F $\frac{C(s)}{R(s)} = \frac{k}{(s+\alpha)}$ is shown in figure



The value of α shall be

a) t_1 b) $\frac{1}{t_1}$ c) t_2 d) $\frac{1}{t_2}$

Sol Impulse Response $= L^{-1}(T.F) = L^{-1}\frac{k}{s+\alpha} = ke^{-\alpha t}$ (11)

At $t = t_2$

$$Ke^{-\alpha t_2} = 0.37\,K$$

$$e^{-1} = 0.37$$

$$ke^{-\alpha t_2} = ke^{-1}$$

$$\alpha t_2 = 1$$

$$\boxed{\alpha = \frac{1}{t_2}}$$ Ans

Q5. A certain CS has input R(t) & C(t) output. If the input is first passed through a block having TF e^{-s} and then applied to the system, the modified output will be

a) $c(t-1)\,u(t)$

b) $c(t)\,u(t-1)$

c) $c(t-1)\,u(t-1)$

d) None

Sol (c) By Laplace transformation theorem

$$R(s) \rightarrow \boxed{F(s)} \rightarrow C(s)$$

$$C(s) = R(s)F(s)$$

$$R(s) \rightarrow \boxed{e^{-s}} \rightarrow \boxed{F(s)} \rightarrow C_m(s)$$

$$C_m(s) = R(s)e^{-s}F(s)$$

$$C_m(s) = C(s)e^{-s}$$

$$\boxed{L^{-1}F(s)e^{-as} = f(t-a)\,u(t-a)}$$

$$\boxed{L^{-1}C(s)e^{-s} = C(t-1)\,u(t-1)} \longrightarrow \text{Ans}$$

Q The Unit Step Response of the system is
$Y(t) = te^{-t}$ $(t > 0)$
Its T.F will be

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a) $\frac{1}{(s+1)^2}$ b) $\frac{1}{(s+1)}$

c) $\frac{s}{(s+1)^2}$ d) $\frac{s}{(s+1)}$

$$T.F = \cancel{L(te^{-t})} \frac{C(s)}{R(s)}$$

$$= \frac{\frac{1}{(s+1)^2}}{\frac{1}{s}} = \frac{s}{(s+1)^2} \quad -(c)$$

(or)

$$\frac{d}{dt}(\text{Step Response}) = \text{Impulse Response}$$

$$\frac{d}{dt} Y(t) = Y'(t) = -te^{-t} + e^{-t}$$

$$L(Y'(t)) = T.F = \frac{-1}{(s+1)^2} + \frac{1}{(s+1)}$$

$$T.F = \frac{s}{(s+1)^2}$$

Q. The step response of the system is

$$c(t) = 5 - \frac{4}{8}e^{-2t} + 8e^{-t} \quad (t > 0)$$ (13)

The gain of the T.F. in time constant form will be

a) -7.5 b) -7 c) 7.5 d) 7.

Sol

$$T.F = \frac{C(s)}{R(s)}$$

$\frac{d}{dt}$ (Step response) = Impulse Response

$$\frac{d}{dt}(c(t)) = c'(t) = \frac{-4}{8}(-2)e^{-2t} + 8(-1)e^{-t}$$

$$= e^{-2t} - 8e^{-t}$$

$$L(c'(t)) = T.F = \frac{1}{(s+2)} - \frac{8}{(s+1)}$$

$$= \frac{(s+1) - 8(s+2)}{(s+2)(s+1)}$$

$$= \frac{-7s - 15}{(s+2)(s+1)}$$

$$= \frac{-7(s + 15/7)}{(s+2)(s+1)}$$

Time Constant form $(1 \pm Ts)$

T → depend only on poles

→ known as natural time constant (GATE)

$$T.F = \frac{-7 \times \frac{15}{7}\left[1 + \frac{7s}{15}\right]}{2[1+0.5s]\,1\,[1+s]}$$

$$= \frac{-7.5\,(1 + 7s/15)}{(1+0.5s)(1+s)}$$

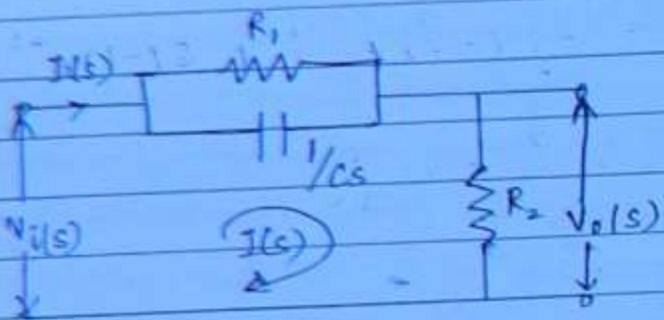
$\boxed{K = -7.5}$ (a)

Introduction to COMPENSATORS

Compensators in Control Systems are used for improving the performance specifications ie transient & steady state response characteristics. They are of 3 types -

I. Lead Compensators

It is used for improving the transient state or speed of response of the system.



$$V_i(s) = I(s)\left[\frac{R_1}{R_1Cs+1} + R_2\right]$$

$$= I(s)\left[\frac{R_1 + R_2 + R_1R_2Cs}{R_1Cs+1}\right]$$

$$V_0(s) = I(s)R_2$$

$$\frac{V_0(s)}{V_i(s)} = \frac{R_2(R_1Cs+1)}{R_1 + R_2 + R_1R_2Cs}$$

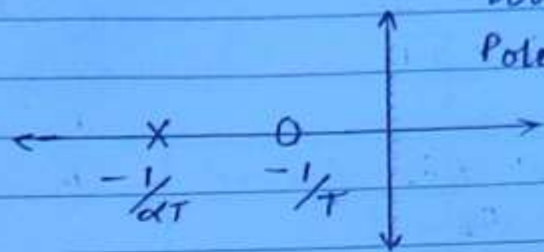
$$\boxed{T = R_1C \qquad \alpha = \text{Attenuation Gain} = \frac{R_2}{R_2+R_1} \quad (\alpha < 1)}$$

$$= \frac{R_2(R_1Cs+1)}{R_1+R_2\left[1 + \frac{R_1R_2Cs}{R_1+R_2}\right]}$$

$$\boxed{\frac{V_0(S)}{V_i(s)} = \frac{\alpha(1+Ts)}{(1+\alpha Ts)}}$$

Zero at $s = -1/T$

Pole at $s = -1/\alpha T$

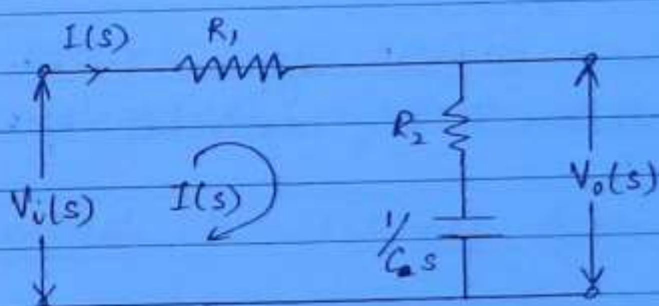


NOTE → The frequency nearest to origin is a zero in a LEAD compensator. Therefore its effect is not dominating.

If zero is added to a system TF it will represent ~~behave as~~ a LEAD compensator.

II. <u>Lag compensator</u>

It is used for improving the steady state response characteristics of the system ie elimination of steady state error b/w output & input



$$V_i(s) = I(s) \times \left[R_1 + R_2 + \frac{1}{Cs}\right]$$

$$= I(s)\left[\frac{R_1 Cs + R_2 Cs + 1}{Cs}\right]$$

$$V_o(s) = I(s)\left[R_2 + \frac{1}{Cs}\right]$$

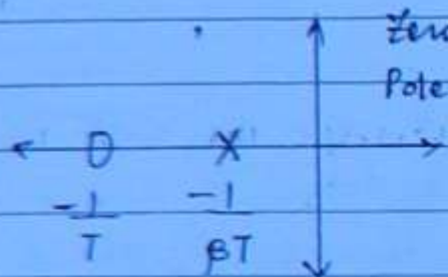
$$= I(s)\left[\frac{R_2 Cs + 1}{Cs}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2Cs+1}{R_1Cs+R_2Cs+1}$$

$$\boxed{T = R_2C \qquad \beta = \frac{R_1+R_2}{R_2} = \frac{1}{\alpha} \quad (\beta > 1)}$$

$$= \frac{R_2Cs+1}{R_2Cs\frac{(R_1+R_2)}{R_2}+1}$$

$$\boxed{\frac{V_o(s)}{V_i(s)} = \frac{1+Ts}{1+\beta Ts}}$$



Zero at $s = -1/T$

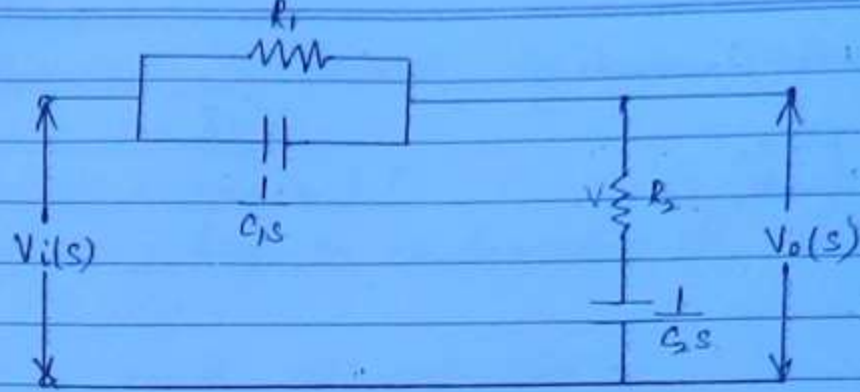
Pole at $s = -1/\beta T$

NOTE → If a pole is added to a system T.F in terms of compensation, it represents a LAG Compensator.

III LEAD LAG (or) LAG-LEAD compensators

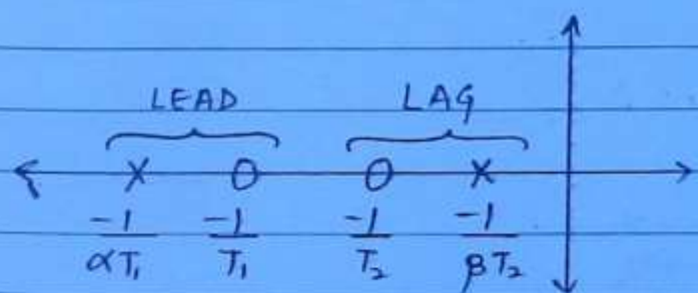
→ It is used for improving both transient & steady state response characteristics

→ It exhibits both LEAD & LAG characteristics in its frequency response



$$\frac{V_o(s)}{V_i(s)} = \frac{\alpha(1+T_1 s)}{(1+\alpha T_1 s)} \frac{(1+T_2 s)}{(1+\beta T_2 s)}$$

$$\boxed{T_1 = R_1 C_1 \quad / \quad T_2 = R_2 C_2}$$



MECHANICAL SYSTEMS -

All mechanical systems are classified into two types -

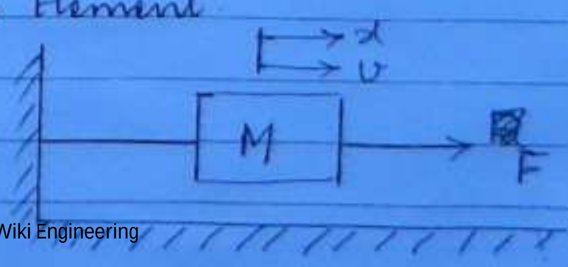
I. Mechanical Translational Systems -

Input = Force (F)

Output = Linear displacement (x) or Linear Velocity (v)

The 3 ideal elements are -

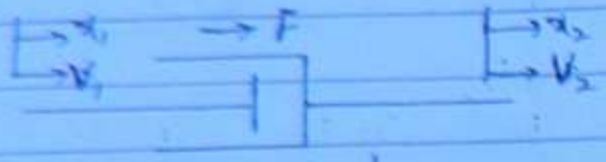
1. Mass Element



$$F = M\frac{dv}{dt}$$

$$F = M\frac{d^2x}{dt^2}$$

4. Damper Element (Friction)



f (or) B

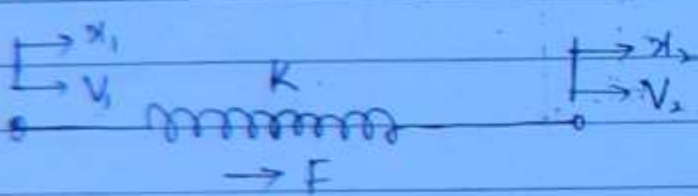
$$F = f(V_1 - V_2) = f\,V$$

$(V = V_1 - V_2)$

$$F = f\,\frac{d}{dt}(x_1 - x_2) = f\,\frac{dx}{dt}$$

$(x = x_1 - x_2)$

5. Spring Element (Stiffness)



$$F = K\int (V_1 - V_2)\,dt = K\int V\,dt$$

$(V = V_1 - V_2)$

$$F = K(x_1 - x_2) = Kx$$

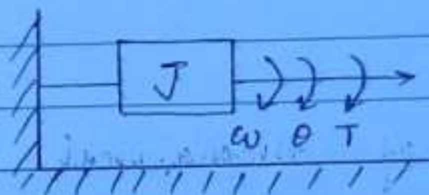
$(x = x_1 - x_2)$

II Mechanical Rotational System

Input = Torque (T)
Output = Angular displacement (θ) (or) Angular Velocity (ω)

The 3 ideal elements are -

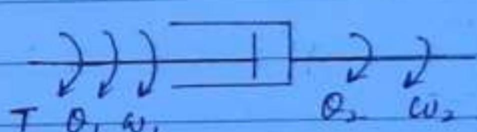
1. Inertia Element



$$T = J\frac{d\omega}{dt}$$

$$T = J\frac{d^2\theta}{dt^2}$$

2. Torsional Damper Element (Friction)



f (or) B

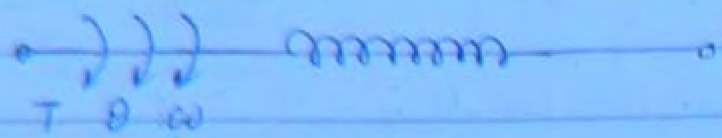
$$T = f(\omega_1 - \omega_2) = f\omega$$

$$(\omega = \omega_1 - \omega_2)$$

$$T = f\frac{d}{dt}(\theta_1 - \theta_2) = f\frac{d\theta}{dt}$$

$$(\theta = \theta_1 - \theta_2)$$

3. Torsional Spring Element (Stiffness)



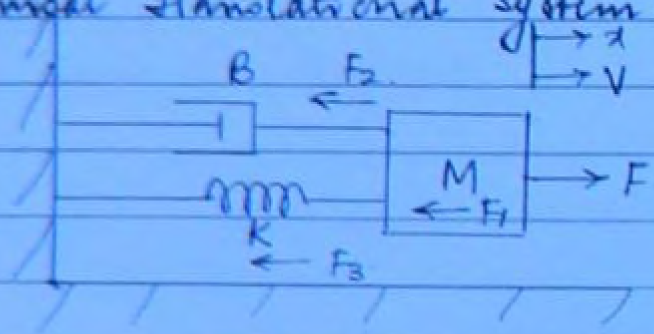
$$T = K \int \omega \, dt$$

$$T = K\theta$$

ANALOGOUS SYSTEMS –

The electrical equivalents of mechanical elements are known as analogous systems.

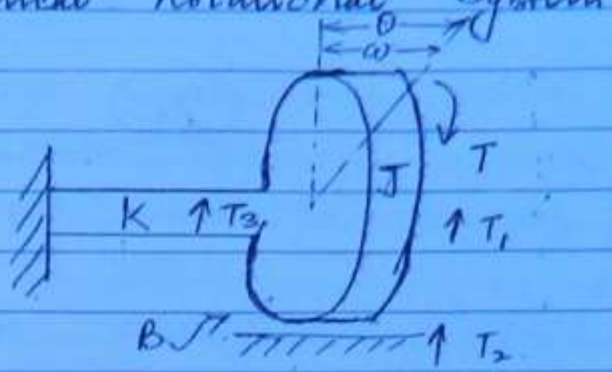
I. Mechanical Translational System



$$F = F_1 + F_2 + F_3$$

$$\left.\begin{aligned} F &= M\frac{dV}{dt} + BV + K\int V \, dt \\ F &= M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx \end{aligned}\right] \text{—— } (1)$$

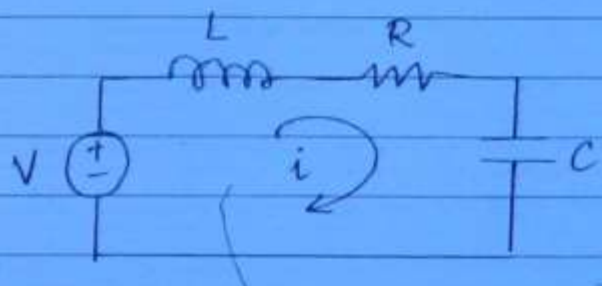
II. Mechanical Rotational System



$$T = T_1 + T_2 + T_3$$

$$\left.\begin{aligned} T &= J\frac{d\omega}{dt} + B\omega + K\int \omega\, dt \\ T &= J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta \end{aligned}\right] \leftarrow (2)$$

III Electrical System



$$V = V_1 + V_2 + V_3$$

$$\left.\begin{aligned} V &= L\frac{di}{dt} + Ri + \frac{1}{C}\int i\, dt \\ V &= L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + Cq \end{aligned}\right] - (3)$$

$\left[i = \frac{dq}{dt} \quad (q = \text{change})\right]$

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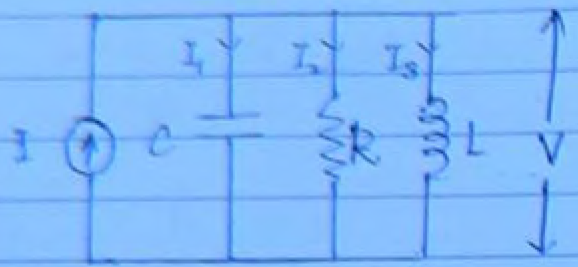
IV Electrical Systems -



$$I = I_1 + I_2 + I_3$$

$$I = C\frac{dV}{dt} + \frac{V}{R} + \frac{1}{L}\int V\,dt$$

$$V = \frac{d\phi}{dt} \qquad \phi = \text{flux}$$

$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{\phi}{L} \quad \text{— (4)}$$

Comparing (1) → (4)

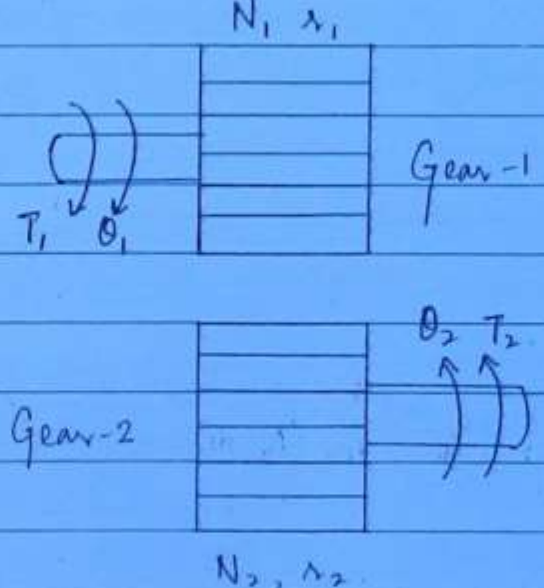
1. F - T - V - Analogy
2. F - T - I - Analogy

Mass/Inertia element & spring element are known as "CONSERVATIVE ELEMENTS"

F - T - V - I
M - J - L - C
B - B - R - 1/R
K - K - 1/C - 1/L
V - ω - i - V
x - θ - q - φ

GEARS -

* Gears are mechanical devices which are used as intermediate elements between electrical motors & load.
* They are used for stepping up / stepping down either torque or speed.
* They are analogous to electrical transformers.



N = No of teeth on the circumference of gear wheel

T = Torque on gear wheel (N-m)

r = radius of gear wheel (m)

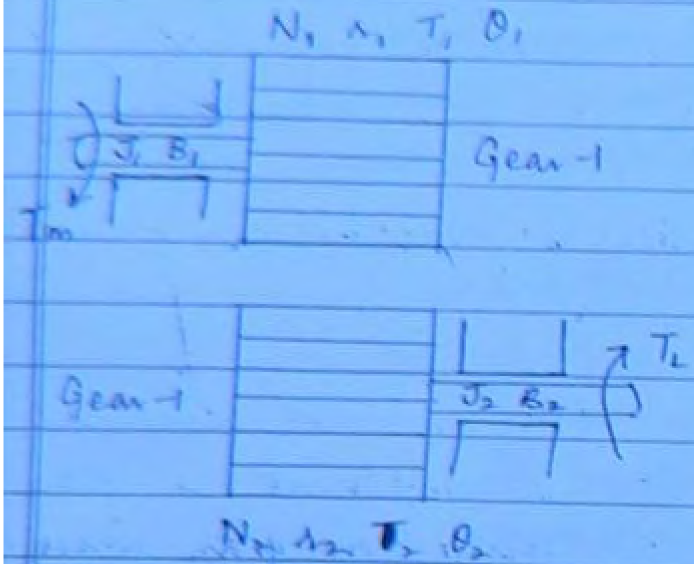
θ = angular disp (rad)

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} = \frac{\omega_2}{\omega_1} = \frac{\alpha_2}{\alpha_1}$$

For 'n' gear wheels -

$$\boxed{\frac{N_x}{N_y} = \frac{T_x}{T_y} = \frac{r_x}{r_y} = \frac{\theta_y}{\theta_x} = \frac{\omega_y}{\omega_x} = \frac{\alpha_y}{\alpha_x}}$$

Dynamics of Gear connected b/w Motor & Load -



T_M = Motor torque (N-m)
T_1 = Torque on gear-1 due to T_m (N-m)
T_2 = Torque on gear-2 due to T_1 (N-m)
T_L = Torque on Load due to T_2 (N-m)

$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + T_1$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_1}{N_2} = \frac{T_1}{T_2} \Rightarrow T_1 = \left(\frac{N_1}{N_2}\right) T_2$$

$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \cancel{T_2} \left(\frac{N_1}{N_2}\right) T_L$$

$$\frac{N_1}{N_2} = \frac{\theta_2}{\theta_1} = \frac{\dot{\theta}_2}{\dot{\theta}_1} = \frac{\ddot{\theta}_2}{\ddot{\theta}_1} \Rightarrow \ddot{\theta}_2 = \left(\frac{N_1}{N_2}\right)\ddot{\theta}_1 \text{ or } \ddot{\theta}_1 = \left(\frac{N_2}{N_1}\right)\ddot{\theta}_2$$

$$\Rightarrow \dot{\theta}_2 = \left(\frac{N_1}{N_2}\right)\dot{\theta}_1 \text{ or } \dot{\theta}_1 = \left(\frac{N_2}{N_1}\right)\dot{\theta}_2$$

25

I Equivalent Inertia & Friction for Motor Side Gear (Gear-1)

$$T_M = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)^2 J_2 \frac{d^2\theta_1}{dt^2} + \left(\frac{N_1}{N_2}\right)^2 B_2 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$T_m = \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right] \frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right] \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

$$\boxed{J_{eq_1} = J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2} \qquad \boxed{B_{eq_1} = B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2}$$

II Equivalent Inertia & Friction for Load Side Gear (Gear-2)

$$T_M = \left(\frac{N_2}{N_1}\right) J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right) B_1 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) J_2 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_1}{N_2}\right) B_2 \frac{d\theta_2}{dt} + \left(\frac{N_1}{N_2}\right) T_L$$

multiplying both sides by $\frac{N_2}{N_1}$

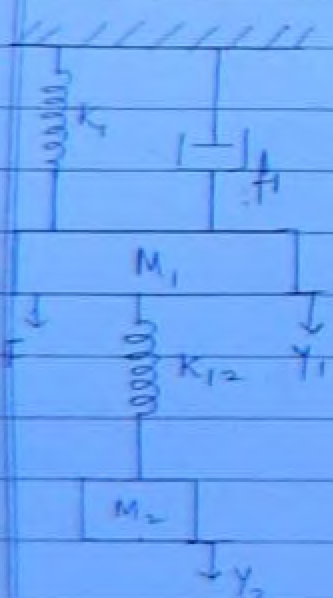
$$\frac{N_2}{N_1} T_M = \left(\frac{N_2}{N_1}\right)^2 J_1 \frac{d^2\theta_2}{dt^2} + \left(\frac{N_2}{N_1}\right)^2 B_1 \frac{d\theta_2}{dt} + \cancel{\frac{N_1}{N_2}} J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$\frac{N_2}{N_1} T_M = \left[\left(\frac{N_2}{N_1}\right)^2 J_1 + J_2\right] \frac{d^2\theta_2}{dt^2} + \left[\left(\frac{N_2}{N_1}\right)^2 B_1 + B_2\right] \frac{d\theta_2}{dt} + T_L$$

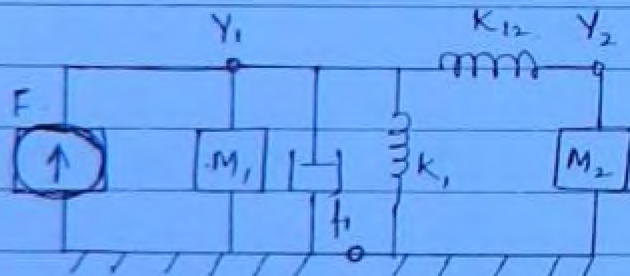
$$\boxed{J_{eq_2} = J_2 + \left(\frac{N_2}{N_1}\right)^2 J_1} \qquad \boxed{B_{eq_2} = B_2 + B_1 \left(\frac{N_2}{N_1}\right)^2}$$

NODAL METHOD -

* No. of nodes = No. of displacements
* Take an additional node which is a reference node
* Connect mass or inertial mass elements between the principal node and reference only.
* Connect the spring and damping elements either between the principal nodes or between principal node & reference depending on their position
* Obtain the nodal diagram & write the describing differential equations at each node.



Nodal Diagram
(Mech Equivalent diagram)



At node Y_1

$$F = M_1 \frac{d^2 Y_1}{dt^2} + f_1 \frac{dY_1}{dt} + K_1 Y_1 + K_{12}(Y_1 - Y_2)$$

At node Y_2

$$0 = M_2 \frac{d^2 Y_2}{dt^2} + K_{12}(Y_2 - Y_1)$$

Transfer function $\frac{Y_1(s)}{F(s)}$

$$F(s) = (M_1 s^2 + f_1 s + k_1 + k_{12}) Y_1(s) - k_{12} Y_2(s)$$

$$0 = (M_2 s^2 + k_{12}) Y_2(s) - k_{12} Y_1(s)$$

$$Y_2(s) = \left[\frac{k_{12}}{M_2 s^2 + k_{12}} \right] Y_1(s)$$

$$F(s) = \left\{ (M_1 s^2 + f_1 s + k_1 + k_{12}) - \frac{k_{12}^2}{M_2 s^2 + k_{12}} \right\} Y_1(s)$$

$$F(s) = \left\{ \frac{(M_1 s^2 + f_1 s + k_1 + k_{12})(M_2 s^2 + k_{12}) - k_{12}^2}{M_2 s^2 + k_{12}} \right\} Y_1(s)$$

$$\frac{Y_1(s)}{F(s)} = \frac{M_2 s^2 + k_{12}}{(M_1 s^2 + f_1 s + k_1 + k_{12})(M_2 s^2 + k_{12}) - k_{12}^2}$$

1 Mass element $\rightarrow$ order - 2

2 Mass element $\rightarrow$ order - 4

3 Mass element $\rightarrow$ order - 6

$\vdots$

$\boxed{n \text{ Mass elements} \rightarrow \text{order} - 2n}$

F-V ANALOGY

$$V = L_1 \frac{d^2 q_1}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} + \frac{q_1 - q_2}{C_{12}}$$

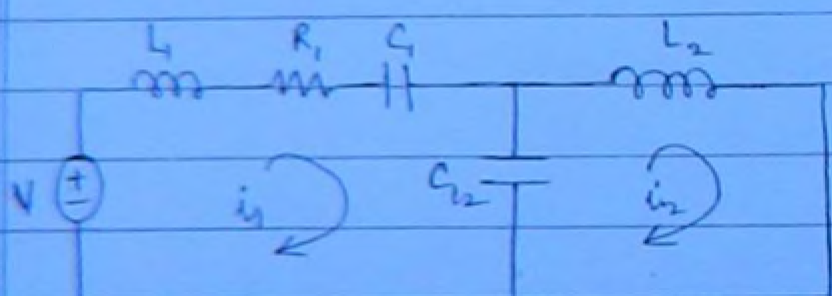
$$0 = L_2 \frac{d^2 q_2}{dt^2} + \frac{q_2 - q_1}{C_{12}}$$

$$i_1 = \frac{dq_1}{dt} \Rightarrow q_1 = \int i_1 \, dt$$

$$i_2 = \frac{dq_2}{dt} \Rightarrow q_2 = \int i_2 \, dt$$

$$V = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 \, dt + \frac{1}{C_{12}} \int (i_1 - i_2) \, dt$$

$$0 = L_2 \frac{di_2}{dt} + \frac{1}{C_{12}} \int (i_2 - i_1) \, dt$$



F - I ANALOGY

$$I = C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{R_1}\frac{d\phi_1}{dt} + \frac{\phi_1}{L_1} + \frac{\phi_1 - \phi_2}{L_{12}}$$

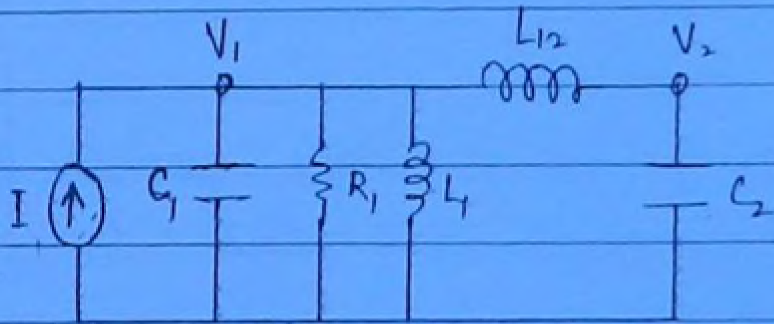
$$0 = C_2 \frac{d^2\phi_2}{dt^2} + \frac{\phi_2 - \phi_1}{L_{12}}$$

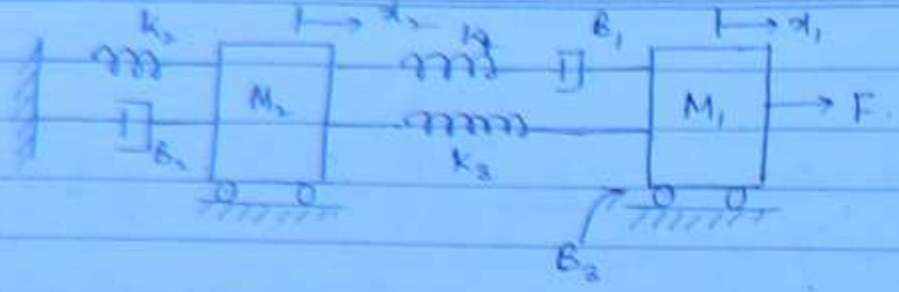
$$V_1 = \frac{d\phi_1}{dt} \Rightarrow \phi_1 = \int V_1 \, dt$$

$$V_2 = \frac{d\phi_2}{dt} \Rightarrow \phi_2 = \int V_2 \, dt$$

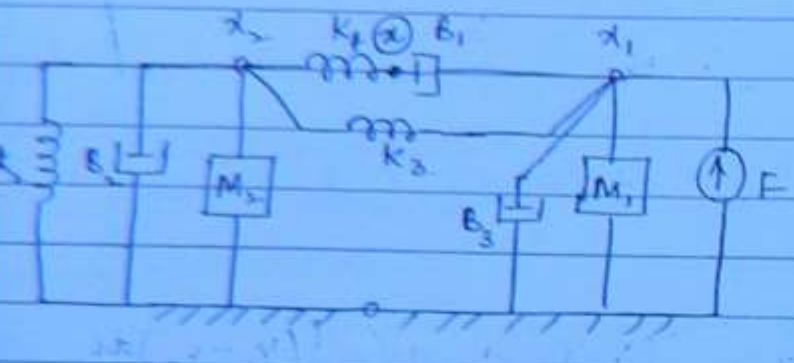
$$I = C_1 \frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int V_1 \, dt + \frac{1}{L_{12}}\int (V_1 - V_2)\, dt$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{L_{12}}\int (V_2 - V_1)\, dt$$





Nodal Diagram -



At node x_1

$$F = M_1 \frac{d^2x_1}{dt^2} + B_3 \frac{dx_1}{dt} + k_3(x_1 - x_2) + B_1 \frac{d}{dt}(x_1 - x)$$

At dummy node x

$$0 = B_1 \frac{d}{dt}(x - x_1) + K_1(x - x_2)$$

At node x_2

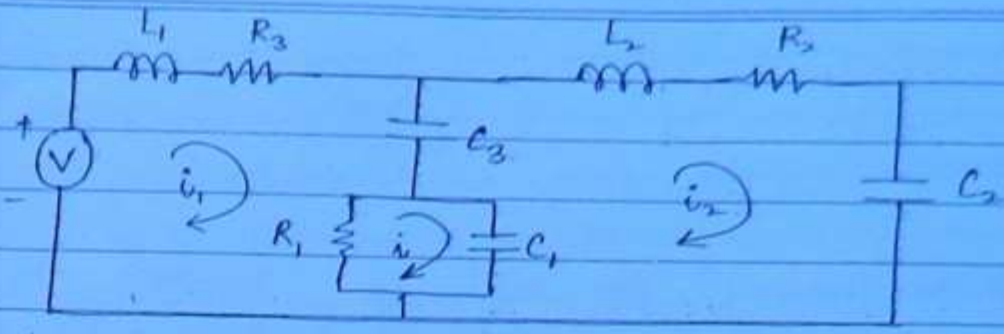
$$0 = M_2 \frac{d^2x_2}{dt^2} + B_2 \frac{dx_2}{dt} + k_2 x_2 + k_3(x_2 - x_1) + k_1(x_2 - x)$$

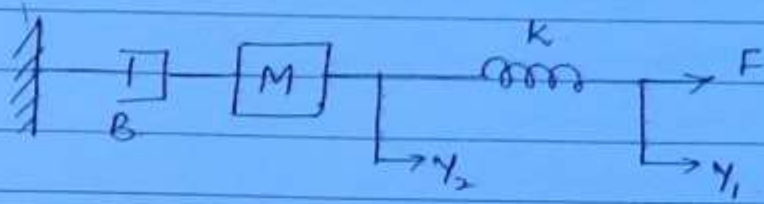
F-V ANALOGY

$$V = L_1 \frac{di_1}{dt} + R_3 i_1 + \frac{1}{C_3}\int (i_1 - i_2) + R_1(i_1 - i)$$

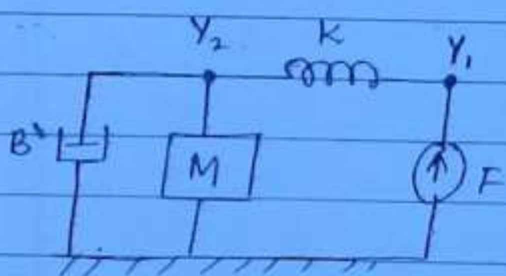
$$0 = R_1(i - i_1) + \frac{1}{C_1}\int (i - i_2)\,dt$$

$$0 = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + \frac{1}{C_3}\int (i_2 - i_1)\,dt + \frac{1}{C_1}\int (i_2 - i)\,dt$$





Nodal diagram -



Only one node as only one mass is present so only one displacement

∴ Y_1 is dummy node

At dummy node Y_1

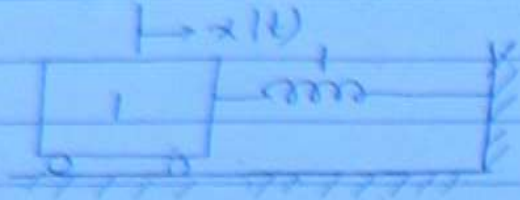
$$F = K(Y_1 - Y_2)$$

At node Y_2

$$0 = M\frac{d^2Y_2}{dt^2} + B\frac{dY_2}{dt} + K(Y_2 - Y_1)$$

$$F = K(Y_1 - Y_2) = M\frac{d^2Y_2}{dt^2} + B\frac{dY_2}{dt}$$

Q



For unit impulse force the eqⁿ for resulting oscillation will be -

a) sint b) sin 2t

c) $\sqrt{2}$ sint d) sin $\sqrt{t}$

Ans.

$$F = M\frac{d^2x}{dt^2} + Kx$$

$M \to 1$, $K \to 1$

$$F(s) = (s^2+1)X(s)$$

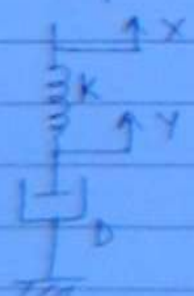
$$X(s) = \frac{F(s)}{s^2+1}$$

F(s) = Impulse force = 1

$$X(s) = \frac{1}{s^2+1}$$

$x(t) = \sin t \to$ (a)

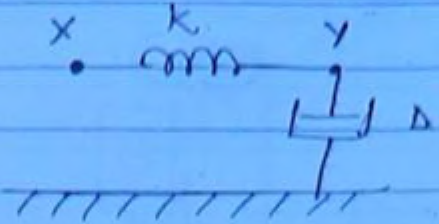
Q Find the pole of mechanical system -



a) $\frac{-K}{D}$ b) 0, $\frac{-K}{D}$

c) $-KD$ d) $\frac{-D}{K}$

Sol



At mode Y

$$0 = D\frac{dY}{dt} + k(Y - X)$$

$$D\frac{dY}{dt} + kY = kX$$

$$[Ds + k]\,Y(s) = k\,X(s)$$

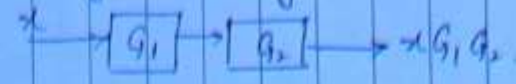
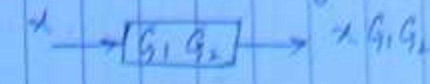
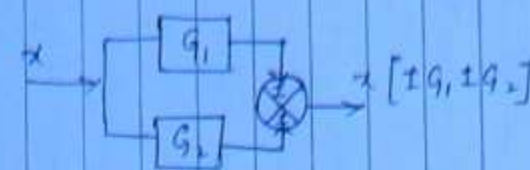
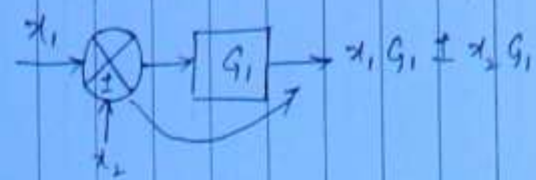
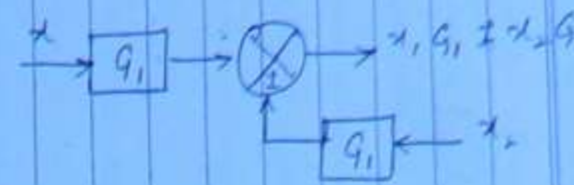
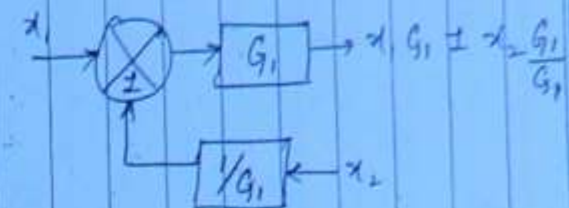
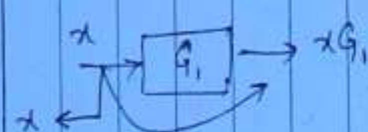
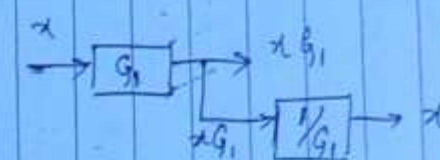
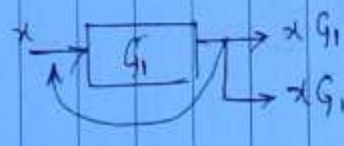
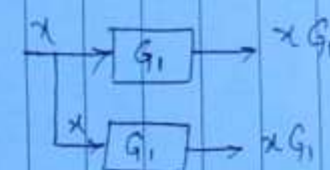
Dummy T.F

$$\frac{Y(s)}{X(s)} = \frac{k}{Ds + k}$$

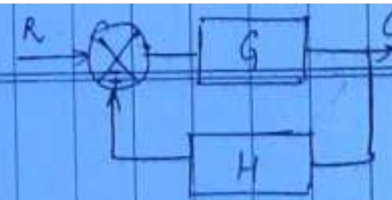
pole $\Rightarrow$ $Ds + k = 0$

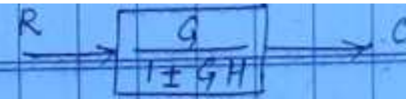
$$s = \frac{-k}{D} \quad \text{—(a)}$$

SIGNAL FLOW GRAPH

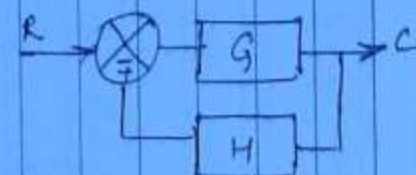
Rules	Original Diagram	Equivalent Diagram
1. Combining blocks in series		
2. Combining blocks in parallel		
3. Shifting the summing element after the block		
4. Shifting the summing element before the block		
5. Shifting the take off point after the block		
6. Shifting the take - off point before the block		

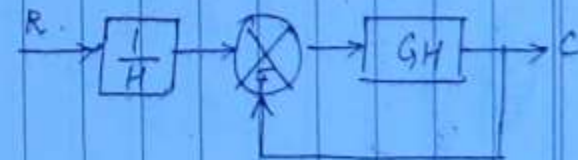
7. Transfer function of closed loop control system





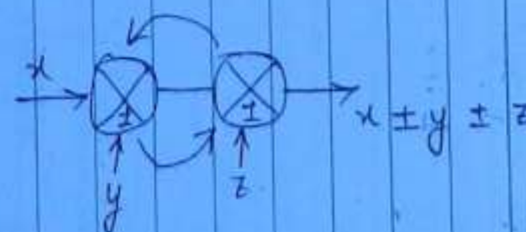
8. Block diagram transformation

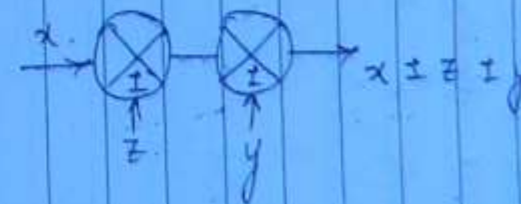




$$\frac{C}{R} = \frac{1}{H} \times \frac{GH}{1 \pm GH} = \frac{G}{1 \pm GH}$$

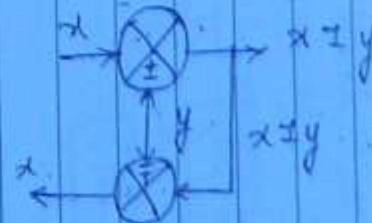
9. Interchanging summing elements



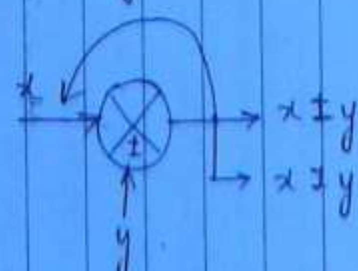


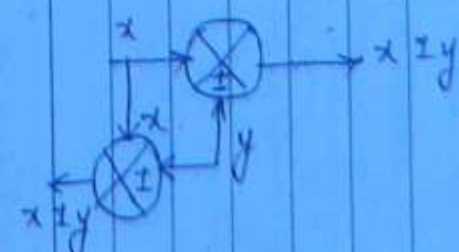
CRITICAL RULES -

10. Shifting the take off point after summing element



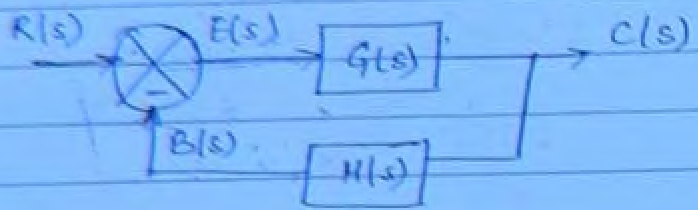
11. Shifting the take-off point before summing element

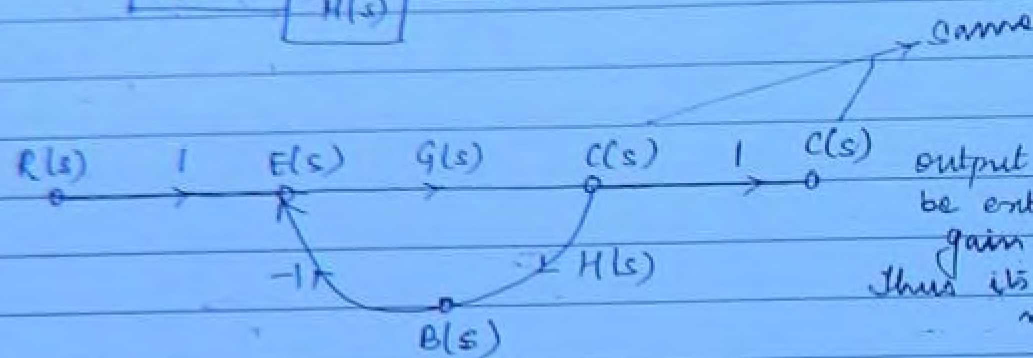




SIGNAL FLOW GRAPH for CLCS -

Its a graphical representation of control system in which nodes representing each of the system variables are connected by direct branches.





output node can be extented with gain 1.
Thus its an output node by definition.

TERMINOLOGY OF SFG'S.

1. NODE -

It represents a system variable & is equal to the sum of all incoming signals at it.

2. INPUT NODE or SOURCE NODE -

Its a node having only outgoing branches

3. OUTPUT NODE or SINK NODE -

Its a node having only incoming branches

4. MIXED or CHAIN NODE -

Its a node having both incoming & outgoing branches.

5. PATH -

It is the traversal of the connected branches in the direction of branch arrow such that no node is traversed more than once.

6. FORWARD PATH -

Its a path from input ~~node~~ node to output node

7. LOOP -

Its a path which originates & terminates at the same nod

Self loop at defined input is not ~~connected~~ valid

Self loop at ~~not~~ defined output node is valid

cuz output node can be extended

NOTE -

Self loops on defined input nodes are not valid loops.

Self loops / loops on output nodes are valid loops.

8. NON TOUCHING LOOPS -

Two or more loops are said to be non-touching if they do not have a common node

MASON'S GAIN FORMULA -

The overall gain or T.F $= \dfrac{\Sigma P_k \Delta_k}{\Delta}$

$$= \frac{P_1 \Delta_1 + P_2 \Delta_2 + \cdots + P_k \Delta_k}{\Delta}$$

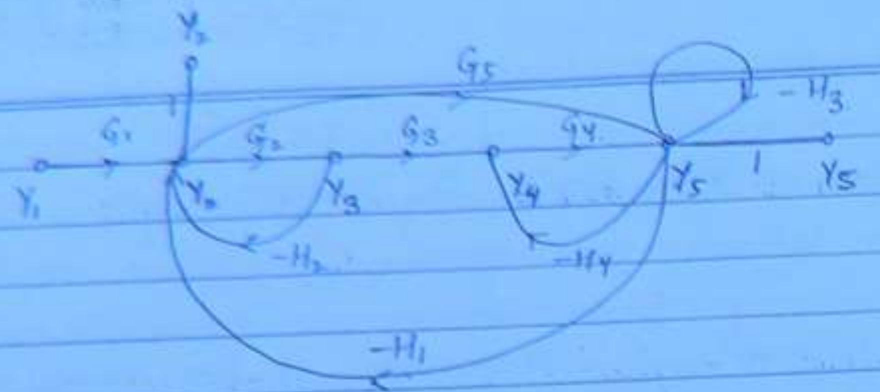
where

P_K = Path gain of K^{th} forward path

$\Delta = 1 - \left[\begin{array}{c}\text{Sum of loop gains} \\ \text{of all individual} \\ \text{loops}\end{array}\right] + \left[\begin{array}{c}\text{Sum of gain} \\ \text{products of two} \\ \text{non touching loops}\end{array}\right] - \left[\begin{array}{c}\text{Sum of gain products} \\ \text{of three non touching} \\ \text{loops}\end{array}\right] \cdots$

Δ_K = It is that value of "Δ" obtained by removing all the loops touching K^{th} forward path.

8.



case 1) $\boxed{\dfrac{Y_5}{Y_1}}$

1) Forward paths -

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_2 G_5$

2) To find Δ

i) Individual loops

$I_1 = -G_2 H_2$ $\qquad$ $I_4 = -G_2 G_3 G_4 H_1$

$I_2 = -G_4 H_4$ $\qquad$ $I_5 = -G_5 H_1$

$I_3 = -H_3$

ii) Two NTL's

$L_1 = I_1 I_2 = G_2 H_2 G_4 H_4$

$L_2 = I_1 I_3 = G_2 H_2 H_3$

~~3) To find Δ_1 & Δ_2~~

~~$\Delta_1 = \Delta_2 = 1$~~

$$\Delta = 1 - \begin{bmatrix} -G_2 H_2 - G_4 H_4 - H_3 \\ -G_2 G_3 G_4 H_1 - G_5 H_1 \end{bmatrix} + \begin{bmatrix} G_2 H_2 G_4 H_4 \\ + G_2 H_2 H_3 \end{bmatrix}$$

3) To find Δ_1 & Δ_2

$\Delta_1 = \Delta_2 = 1$

$$\frac{Y_5}{Y_1} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\frac{Y_5}{Y_1} = \frac{G_1G_2G_3G_4 + G_1G_5}{\Delta} \quad \text{--- (1)}$$

case 2) $\boxed{\frac{Y_5}{Y_2}} = \frac{[Y_5/Y_1]}{[Y_2/Y_1]}$

To find $\boxed{\frac{Y_2}{Y_1}}$

1) Forward path

$P_1 = G_1$

2) To find Δ

Δ is same as above

[Δ is independent of F.P]

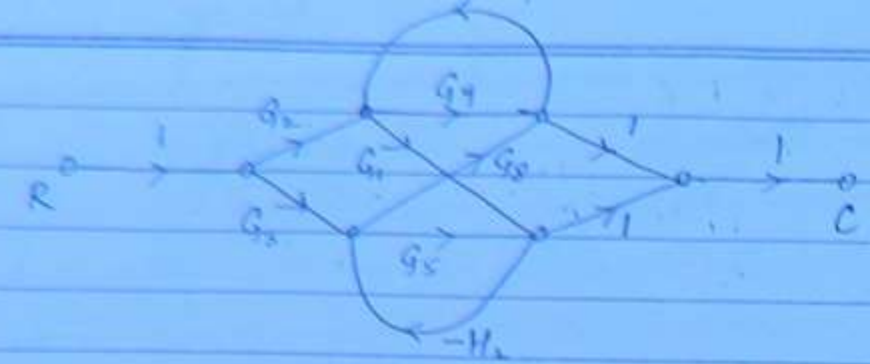
3) To find Δ_1

$\Delta_1 = 1 - [-G_4H_4 - H_3]$

$$\frac{Y_2}{Y_1} = \frac{P_1\Delta_1}{\Delta} = \frac{G_1[1 + G_4H_4 + H_3]}{\Delta} \quad \text{--- (2)}$$

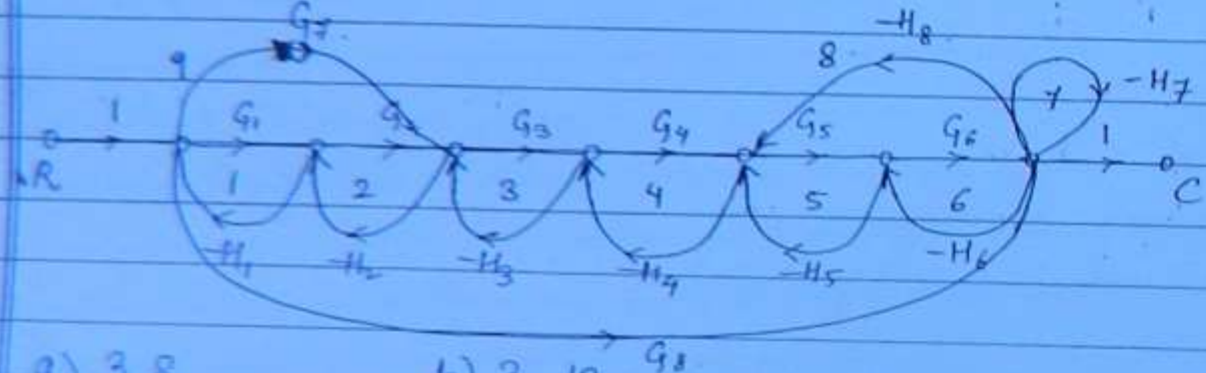
$$\frac{Y_5}{Y_2} = \frac{Y_5/Y_1}{Y_2/Y_1} = \frac{\cancel{G_1}G_2G_3G_4 + \cancel{G_1}G_5}{\cancel{G_1}[1 + G_4H_4 + H_3]}$$

$$\frac{Y_5}{Y_2} = \frac{G_2G_3G_4 + G_5}{1 + G_4H_4 + H_3} \quad \text{--- (3)}$$



$$\frac{C}{R} = \frac{G_2G_4[1+G_5H_2] + G_3G_5[1+G_4H_1] + G_2G_1[1] + G_3G_8[1] + G_2G_1(-H_2)G_8[1] + G_3G_8(-H_1)G_1[1]}{1-\left[-G_4H_1 - G_5H_2 + G_1G_8H_1H_2\right] + \left[G_4H_1\,G_5H_2\right]}$$

Q For S.F.G find No. of forward paths & feedback paths -



a) 3, 8 b) 3, 10

c) 3, 9 d) 3, 11

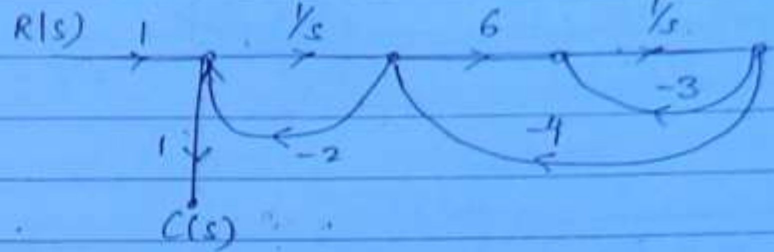
$L_{10} = G_8(-H_8)(-H_4)(-H_3)(-H_2)(-H_1)$

$L_{11} = G_8(-H_6)(-H_5)(-H_4)(-H_3)(-H_2)(-H_1)$

Ans (d)

CWB chapter-1 (41)

(5)



$\frac{C(s)}{R(s)} = ?$

Sol

$$\frac{C}{R} = \frac{1\cdot\left\{1-\left(\frac{-3}{s}-\frac{24}{s}\right)\right\}}{1-\left[\frac{-2}{s}\ \frac{-3}{s}\ \frac{-24}{s}\right]+\frac{6}{s^2}}$$

$$= \frac{\frac{s+27}{\cancel{s}}}{\frac{s^2+29s+6}{s^{\cancel{2}}}} = \frac{s(s+27)}{s^2+29s+6} \quad (d)$$

(4)

$$V_i(s) = Z_1(s)\,I_1(s) + Z_3(s)\left[I_1(s) - I_2(s)\right]$$

$$= Z_1 I_1 + Z_3 I_1 - Z_3 I_2$$

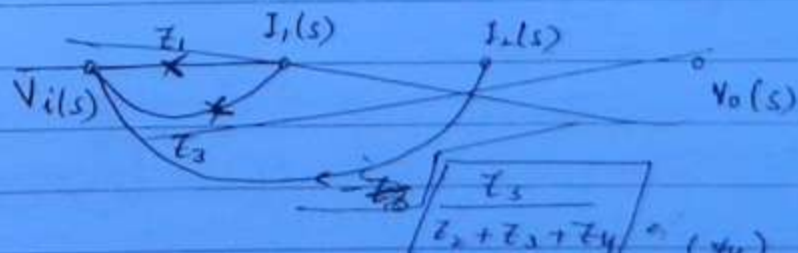
$$I_1 = \frac{V_i}{Z_1+Z_3} + I_2\cdot\frac{Z_3}{Z_1+Z_3}$$

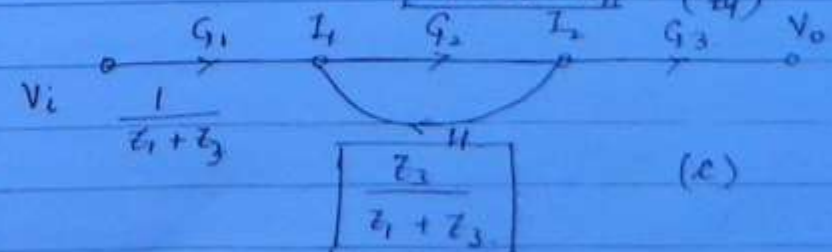
$$V_o(s) = Z_4 I_2$$

$$Z_2 I_2 + Z_4 I_2 + Z_3(I_2 - I_1) = 0$$

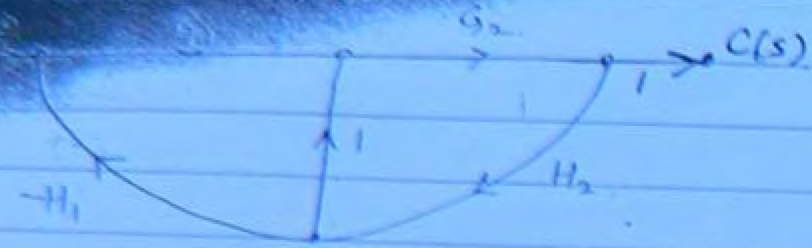
$$Z_2 I_2 + Z_4 I_2 + Z_3 I_2 - Z_3 I_1 = 0$$

$$I_2 = \frac{Z_3}{Z_2+Z_3+Z_4}\, I_1$$





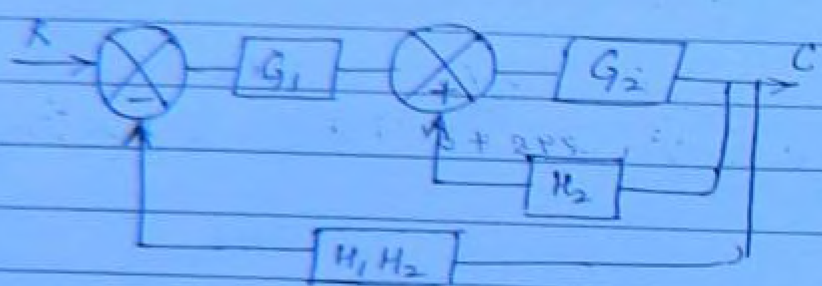
(c)

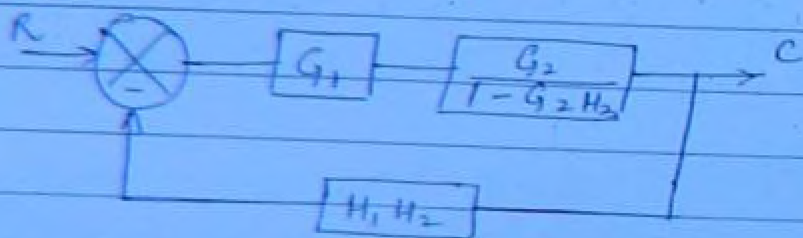


$$\frac{C}{R} = \frac{G_1 G_2 [1]}{1 - [\cancel{G_1 H_1} + G_2 H_2 - G_1 G_2 H_1 H_2]}$$

(c)

OR



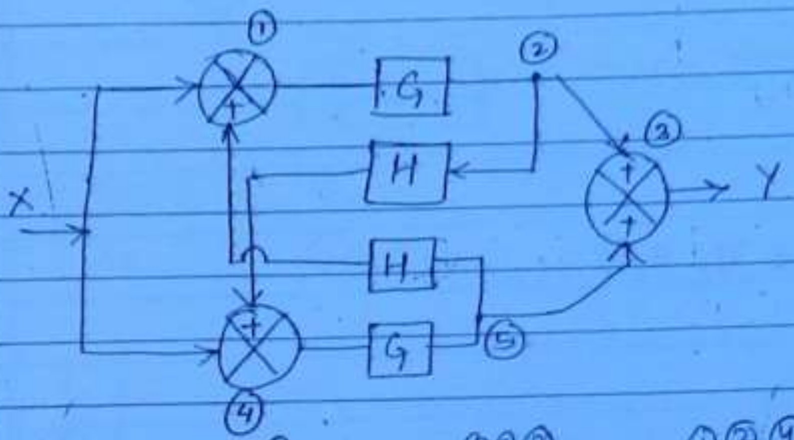


$$\frac{C}{R} = \frac{\dfrac{G_1 G_2}{1 - G_2 H_2}}{1 + \dfrac{G_1 G_2 (H_1 H_2)}{1 - G_2 H_2}}$$

$$\frac{C}{R} = \frac{G_1 G_2}{1 - G_2 H_2 + G_1 G_2 H_1 H_2}$$

Q. $\frac{Y}{X}$ equals (B)

a) $\frac{G}{1-GH}$ b) $\frac{2G}{1-GH}$ c) $\frac{GH}{1-GH}$ d) $\frac{2GH}{1-GH}$

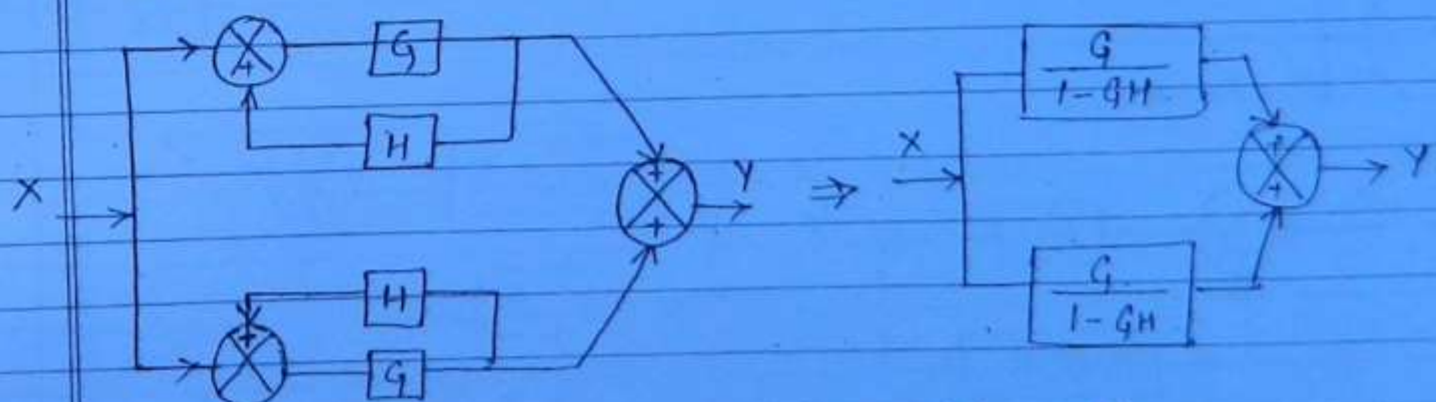


Sol.

$$\frac{Y}{X} = \frac{G[1] + G[1] + G^2H[1] + G^2H[1]}{1-[G^2H^2]}$$

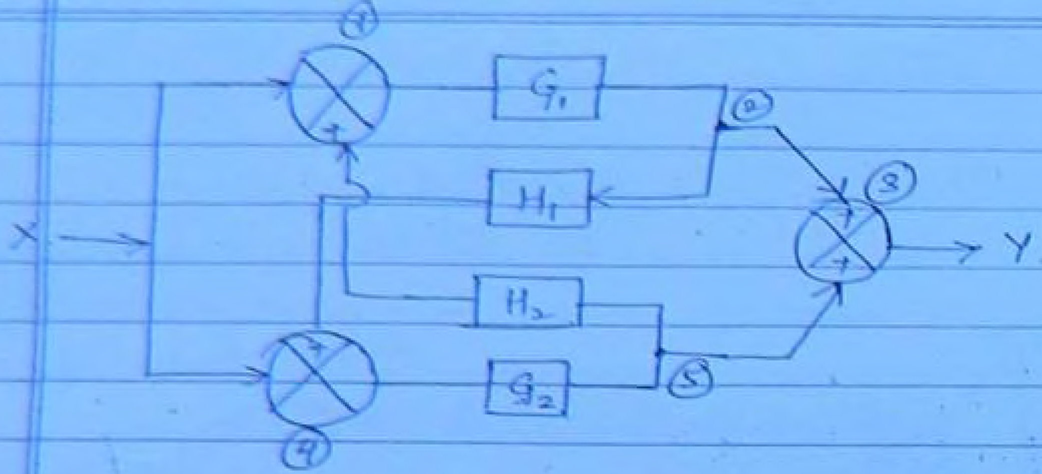
$$\frac{Y}{X} = \frac{2G + 2G^2H}{1-G^2H^2} = \frac{2G\,\cancel{[1+GH]}}{(1-GH)\cancel{(1+GH)}} = \frac{2G}{1-GH} \quad -(b)$$

OR



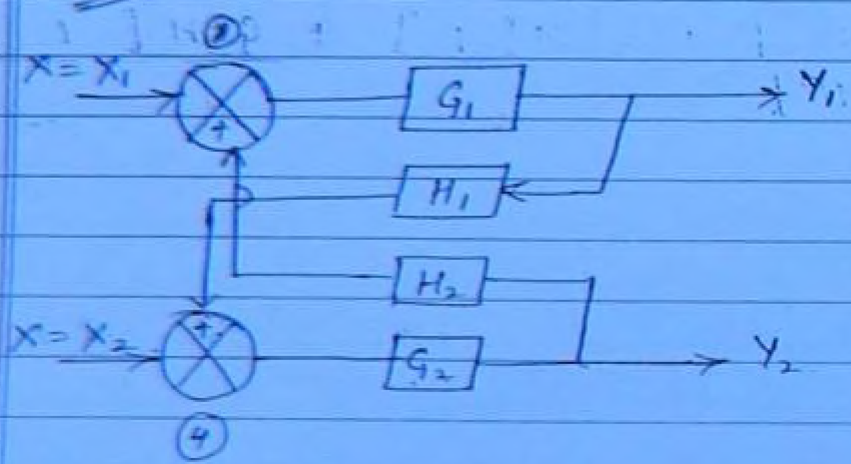
$$\frac{Y}{X} = \frac{2G}{1-GH}$$

Q

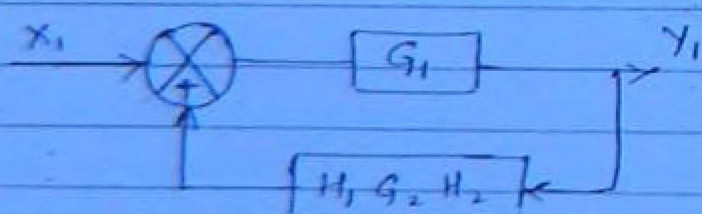


$$\frac{Y}{X} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

OR

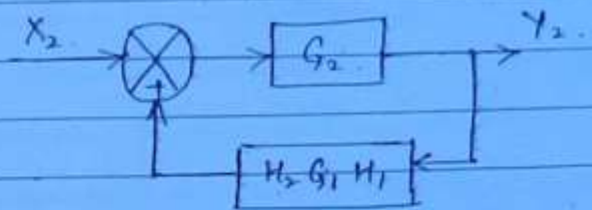


$\rightarrow \left.\dfrac{Y_1}{X_1}\right|_{Y_2 = X_2 = 0}$



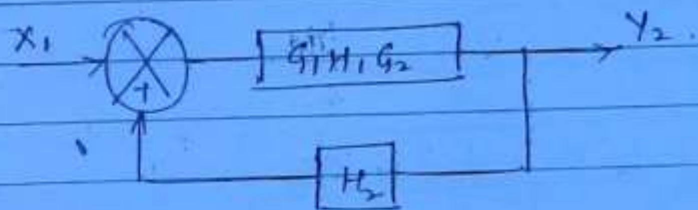
$$\frac{Y_1}{X_1} = \frac{G_1}{1 - G_1 H_1 G_2 H_2}$$

se 2 → $\left.\dfrac{Y_2}{X_2}\right|_{Y_1 = X_1 = 0}$



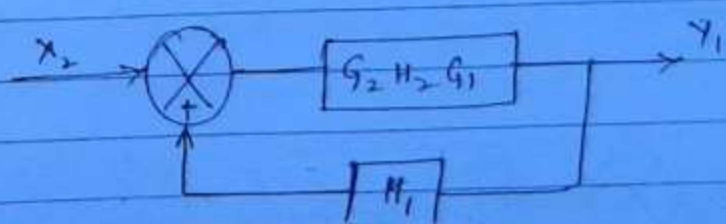
$$\frac{Y_2}{X_2} = \frac{G_2}{1 - G_1 H_1 G_2 H_2}$$

se 3 $\left.\dfrac{Y_2}{X_1}\right|_{Y_1 = X_2 = 0}$



$$\frac{Y_2}{X_1} = \frac{G_1 H_1 G_2}{1 - G_1 H_1 G_2 H_2}$$

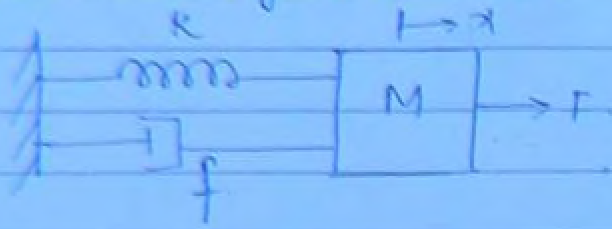
case 4 → $\left.\dfrac{Y_1}{X_2}\right|_{Y_2 = X_1 = 0}$



$$\frac{Y_1}{X_2} = \frac{G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

$$\frac{Y}{X} = \frac{Y_1}{X_1} + \frac{Y_2}{X_2} + \frac{Y_1}{X_2} + \frac{Y_2}{X_1} = \frac{G_1 + G_2 + G_1 H_1 G_2 + G_2 H_2 G_1}{1 - G_1 H_1 G_2 H_2}$$

NV Q5. Draw an integrator based electronic ckt?



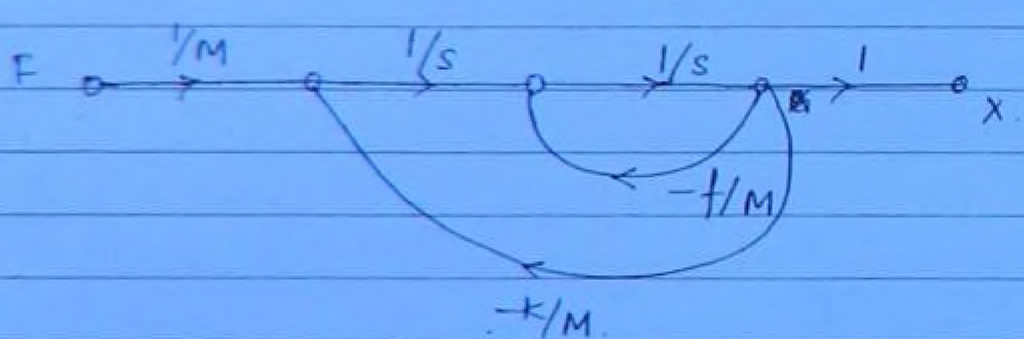
Sol

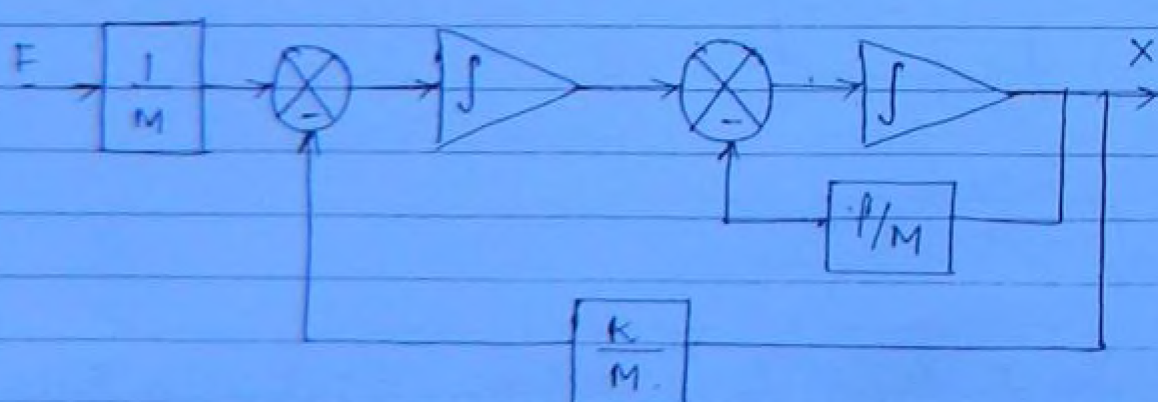
$$F = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + Kx$$

$$F(s) = (Ms^2 + fs + K)X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + fs + K} = \frac{1/M}{s^2 + \frac{f}{M}s + \frac{K}{M}}$$

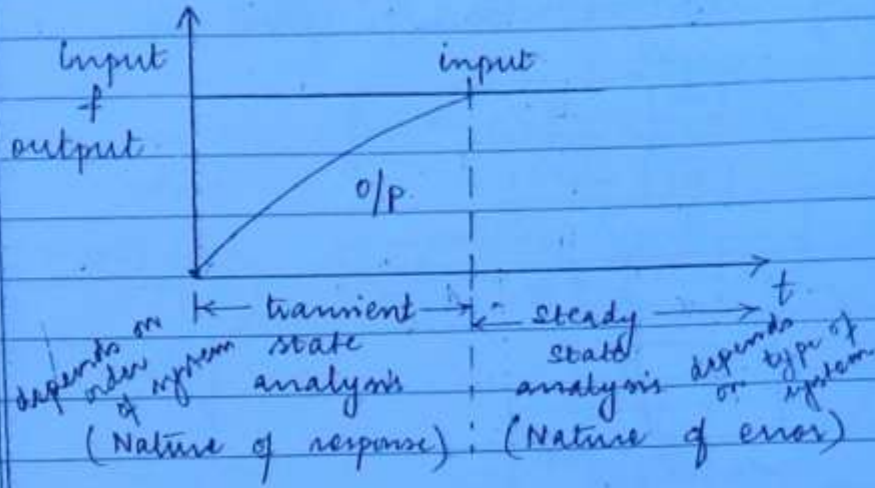
$$= \frac{\frac{1/M}{s^2}}{1 + \frac{f/M}{s} + \frac{K/M}{s^2}} = \frac{\frac{1/M}{s^2}}{1 - \left[\frac{-f/M}{s} - \frac{K/M}{s^2}\right]}$$





TIME DOMAIN ANALYSIS

(47)



STANDARD TEST SIGNALS -

1. Sudden input → step signal
2. Velocity type input → Ramp signal
3. Acceleration type input → Parabolic signal

} (time domain) T.D Analysis

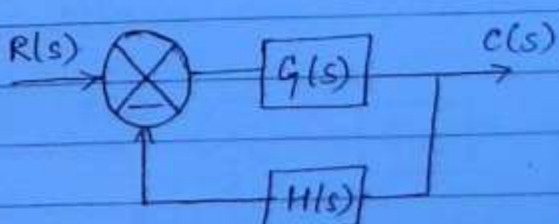
4. Sudden shocks → Impulse signals } stability Analysis

Signals ① & ④ ⇒ Bounded inputs — ∴ for transient analysis we use only step signal

Signals ② & ③ ⇒ Un-bounded inputs

TYPE & ORDER OF A SYSTEM

Every T.F representing a control system has certain type & order



Type

$$G(s)\,H(s) = \frac{K(1+T_a s)}{s^P(1+T_1 s)}$$

$P=0 \Rightarrow$ Type - 0 system

$P=1 \Rightarrow$ Type - 1 system

⋮

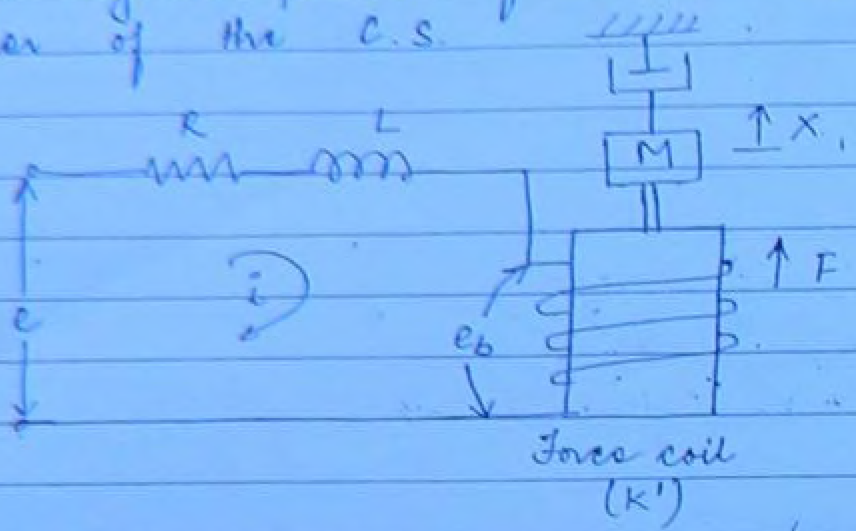
$P=n \Rightarrow$ Type - n system

The no. of open loop poles at origin gives type of C.S

Order - $1 + G(s)H(s) = 0$

The highest power of characteristic eq^n gives the order of the C.S.

eg



→ e = input voltage

→ e_b = transducted voltage ($\propto$ speed)

Electrical system:

$$e = iR + L\frac{di}{dt} + e_b$$

$$e - e_b = iR + L\frac{di}{dt}$$

$$E(s) - E_b(s) = I_s\left[R + Ls\right] \rightarrow (1)$$

→ $e_b \propto$ speed

$$e_b \propto \frac{dx}{dt}$$

$$e_b = k_b\frac{dx}{dt}$$

$$E_b(s) = k_b\, s\, X(s) \rightarrow (2)$$

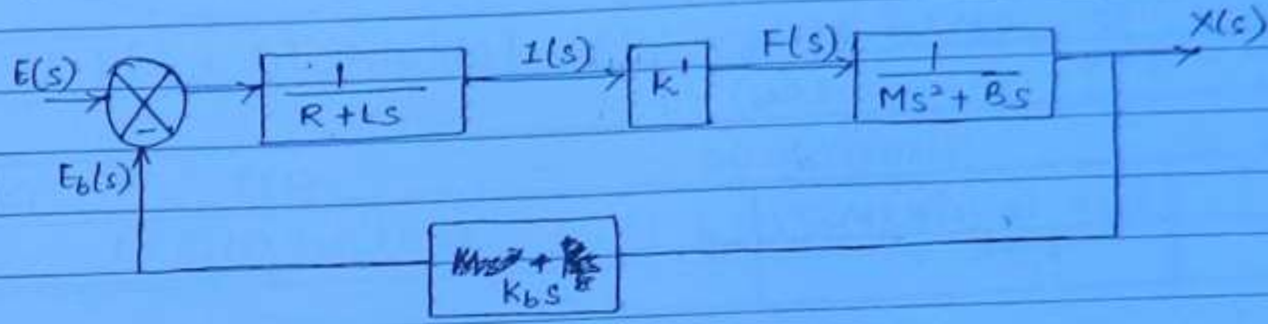
→ Force coil $F \propto i \Rightarrow F = k' i$

$$\Rightarrow F(s) = k' I(s) \rightarrow (3)$$

→ Mechanical system

$$F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt}$$

$$\Rightarrow F(s) = [Ms^2 + Bs]\,X(s) \quad \text{--- (4)}$$



$$G(s)H(s) = \frac{K'K_b\cancel{s}}{(R+Ls)(Ms^2+B\cancel{s})} = \underline{\text{Type 0}} \text{ system}$$

$$1 + G(s)H(s) = 0 \Rightarrow 1 + \frac{K'K_b}{(R+Ls)(Ms+B)} = 0$$

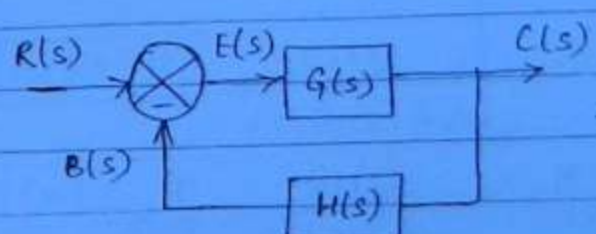
$$\Rightarrow (R+Ls)(Ms+B) + K'K_b = 0$$

<u>Order 2</u> system

STEADY STATE RESPONSE ANALYSIS -

It deals with the estimation of magnitude of steady state error between input and output & depends on type of the control system.

To obtain an expression for error

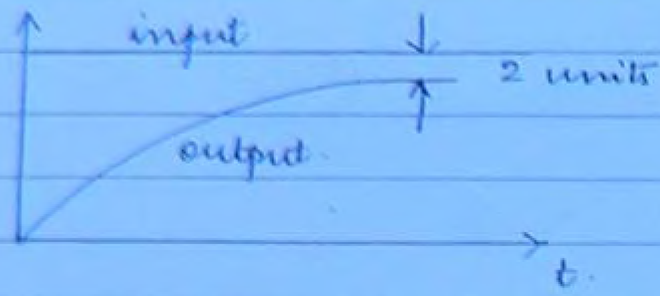


$$E(s) = R(s) - B(s)$$
$$= R(s) - C(s)H(s)$$
$$= R(s) - E(s)G(s)H(s)$$

$$E(s)[1 + G(s)H(s)] = R(s)$$

$$\boxed{E(s) = \frac{R(s)}{1 + H(s)G(s)}}$$

Error



$$\lim_{t \to \infty} e(t) = 2 \text{ units} \quad (e_{ss})$$

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t)}$$

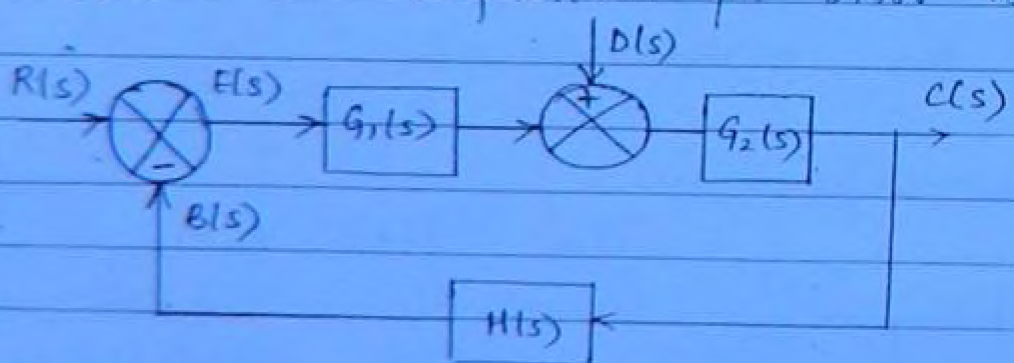
Applying final value theorem

$$\boxed{e_{ss} = \lim_{s \to 0} s.E(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s.\, R(s)}{1+G(s)H(s)}$$

$$e_{ss} = \frac{\lim_{s \to 0} \{s.R(s)\}}{1+\lim_{s \to 0} \{H(s)G(s)\}}$$

→ To obtain an expression for error with disturbances -



$$E(s) = R(s) - B(s)$$

$$E(s) = R(s) - C(s)H(s)$$

$$C(s) = [E(s)\,G_1(s) + D(s)]\,G_2(s)$$

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$$C(s) = E(s)G_1(s)G_2(s) + D(s)G_2(s)$$

$$E(s) = R(s) - E(s)G_1(s)G_2(s)H(s) - D(s)G_2(s)H(s)$$

$$E(s)\,[1 + G_1(s)G_2(s)H(s)] = R(s) - D(s)G_2(s)H(s)$$

$$\boxed{E(s) = \frac{R(s)}{1+G_1(s)G_2(s)H(s)} - \frac{D(s)G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}}$$

$$e_{ss} = \lim_{s\to 0} s.\,E(s)$$

$$\boxed{e_{ss} = \lim_{s\to 0} \frac{s.\,R(s)}{1+G_1(s)G_2(s)H(s)} - \lim_{s\to 0} \frac{s.\,D(s)G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}}$$

Error due to R → put $D(s)=0$
Error due to D → put $R(s)=0$
Error due to comparison → neither $=0$

CWB chapter 2

5 $R=0$ (take given)

$$e_{ss} = -\lim_{s\to 0} s\,\frac{D(s)G_2(s)H(s)}{1+G_1(s)G_2(s)H(s)}$$

$$e_{ss} = -\lim_{s\to 0} \frac{s.\frac{1}{s}\,G_2(s)}{1+G_1(s)G_2(s)}$$

$$|e_{ss}| = \lim_{s\to 0} \frac{G_2(s)}{1+G_1(s)G_2(s)}$$

$$|e_{ss}| = \lim_{s\to 0} \frac{G_2(s)}{G_1(s)G_2(s)} = \lim_{s\to 0}\left[\frac{1}{G_1(s)}\right] \rightarrow (c)$$

Steady state error for all types of inputs -

a) Step input -

$$R(s) = \frac{A}{s}$$

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$$e_{ss} = \frac{\lim_{s\to 0} s \cdot \frac{A}{s}}{1 + G(s)H(s)}$$

$$= \frac{A}{1 + \lim_{s\to 0} G(s)H(s)}$$

K_p = position error constant

$$= \lim_{s\to 0} G(s)H(s)$$

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$

b) Ramp input -

$$R(s) = \frac{A}{s^2}$$

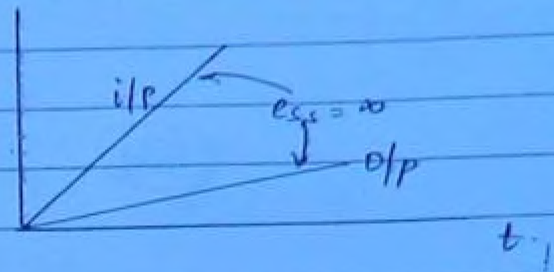
$$e_{ss} = \frac{\lim_{s\to 0} \cancel{s} \cdot \frac{A}{s^{\cancel{2}}}}{1 + G(s)H(s)}$$

$$= \frac{A}{\lim_{s\to 0} s + \lim_{s\to 0} s \cdot G(s)H(s)} = \frac{A}{K_v}$$

K_v = Velocity error constant

$$= \lim_{s\to 0} s \cdot G(s)H(s)$$

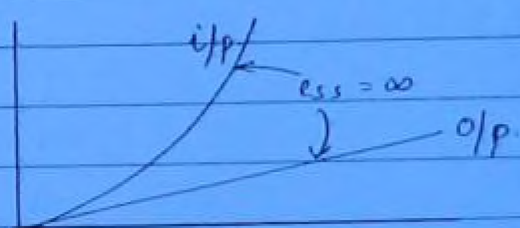
$$\boxed{e_{ss} = \frac{A}{K_v}}$$



c) Parabolic Input -

$$R(s) = \frac{A}{s^3}$$

$$e_{ss} = \frac{\lim\limits_{s \to 0} s \cdot \frac{A}{s^3}}{1 + \frac{k(1+Tas)}{1+T_1 s}} = \frac{A}{\lim\limits_{s \to 0} s^2 + \lim\limits_{s \to 0} s^2 \frac{k(1+Tas)}{1+T_1 s}} = \frac{A}{0} = \infty$$



	Step Input	Ramp Input	Parabolic Input
TYPE-0	$\frac{A}{1+k}$ $K_p = K$	∞ $K_v = 0$	∞ $K_A = 0$
TYPE-1	0 $K_p = \infty$	$\frac{A}{K}$ $K_v = k$	∞ $K_A = 0$
TYPE-2	0	0	$\frac{A}{K}$

Observations -

1. $e_{ss} \propto \frac{1}{K}$ (54)

as $K\uparrow$ $\therefore e_{ss}\downarrow$

2. The maximum type number of linear control system is 2. Beyond type-2 the system exhibits non-linear characteristics more dominantly.

CWB chapter 3

(21)

$$r(t) = 2 + 3t + 4t^2$$

$$R(s) = \frac{2}{s} + \frac{3}{s^2} + \frac{4.2}{s^3}$$

$$= \frac{2}{s} + \frac{3}{s^2} + \frac{8}{s^3}$$

$$G(s) = \frac{10}{s^2(4+s)}$$

$$e_{ss} = 0 + 0 + \frac{A}{K}$$

$A = 8$

$$K = K_A = \lim_{s \to 0} s^2 \times \frac{10}{s^2(4+s)} = \frac{10}{4}$$

$$e_{ss} = \frac{8 \times 4}{10} = 3.2 \text{ units} \quad \text{-(d)}$$

(18) Type-1 $e_{ss} = \frac{1}{K} \Rightarrow \frac{5}{100} = \frac{1}{K}$

(25) Type - 0 $\xrightarrow[\text{Integrator}]{1/s}$ TYPE - 1

$e_{ss} = \frac{1}{1+k}$ $\qquad e_{ss} = \frac{1}{K} = \frac{1}{4} = 0.25$ units (d)

$0.2 = \frac{1}{1+k} \Rightarrow k = 4$

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(23) $r(t) = [1 - t^2]\, 3u(t)$

$R(s) = \frac{3}{s} - \frac{6}{s^3}$

$e_{ss} = \frac{A}{1+K} - \frac{A}{K}$

$e_{ss} = \frac{3}{1+K_p} - \frac{6}{K_a}$ (d)

(4) $e_{ss} = \frac{1}{1+k}$

$k = k_p = \lim_{s \to 0} \frac{10}{s^2 + 14s + 50}$

$k = \frac{10}{50}$

$e_{ss} = \frac{1}{1+k} = \frac{1}{1+0.2} = 0.83$

$e_{ss} = 0.83$ units (b)

ERROR SERIES-

$$E(s) = R(s) \cdot \frac{1}{1+G(s)H(s)}$$

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let $F(s) = \frac{1}{1+G(s)H(s)}$

$$E(s) = R(s) \cdot F(s)$$

$$L^{-1} E(s) = L^{-1}[R(s)F(s)]$$

$$e(t) = \int_0^\infty f(T)\, r(t-T)\, dT$$

Expanding $r(t-T)$ using Taylor series-

$$r(t-T) = r(t) - T\dot{r}(t) + \frac{T^2}{2!}\ddot{r}(t) - \frac{T^3}{3!}\dddot{r}(t) + \dots$$

$$e(t) = r(t)\int_0^\infty f(T)\,dT - \dot{r}(t)\int_0^\infty T f(T)\,dT + \frac{\ddot{r}(t)}{2!}\int_0^\infty T^2 f(T)\,dT - \frac{\dddot{r}(t)}{3!}\int_0^\infty T^3 f(T)\,dT \dots$$

~~Dynamic~~ "Dynamic Error Constants"

$K_0 = \int_0^\infty f(T)\,dT \qquad K_1 = -\int_0^\infty T f(T)\,dT$

$$K_2 = \int_0^\infty T^2 f(T)\,dT$$

$$K_3 = -\int_0^\infty T^3 f(T)\,dT$$

$$e(t) = K_0\, r(t) + K_1\, \dot{r}(t) + \frac{K_2}{2!}\, \ddot{r}(t) + \frac{K_3}{3!}\, \dddot{r}(t) + \ldots$$

error series

$$e_{ss} = \lim_{t \to \infty} e(t)$$

To find Dynamic Error Constants -

$$L\, f(T) = F(s) = \int_0^\infty f(t)\, e^{-sT}\, dT$$

$$\lim_{s\to 0} F(s) = \lim_{s \to 0} \int_0^\infty f(T)\, e^{-sT}\, dT = \int_0^\infty f(T)\, dT \Rightarrow K_0$$

$$\rightarrow \frac{d}{ds} F(s) = \lim_{s\to 0} - \int_0^\infty T\, f(T)\, e^{-sT}\, dT \Rightarrow - \int_0^\infty T\, f(T)\, dT \Rightarrow K_1$$

$$K_0 = \lim_{s\to 0} F(s)$$

$$K_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$K_2 = \lim_{s\to 0} \frac{d^2}{ds^2} F(s)$$

$$\vdots \qquad \vdots$$

where $F(s) = \dfrac{1}{1 + G(s)H(s)}$

Relationship between Static & Dynamic error constants

$$G(s)H(s) = \frac{100}{s(s+2)}$$

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I. Static Error Constants -

$$K_p = \lim_{s \to 0} \frac{100}{s(s+2)} = \infty$$

$$K_v = \lim_{s \to 0} \cancel{s} \cdot \frac{100}{\cancel{s}(s+2)} = 50$$

$$K_A = \lim_{s \to 0} s^2 \frac{100}{\cancel{s}(s+2)} = 0$$

II. Dynamic Error Constants -

$$F(s) = \frac{1}{1+G(s)H(s)} = \frac{1}{1+\frac{100}{s(s+2)}}$$

$$K_0 = \lim_{s \to 0} F(s) = \lim_{s \to 0} \frac{1}{1+\frac{100}{s(s+2)}} = \frac{1}{1+\infty} = 0$$

$$\boxed{K_0 = \frac{1}{1+K_p}}$$

$$K_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$\frac{d}{ds} F(s) = \frac{d}{ds} \frac{1}{1+\frac{100}{s(s+2)}} = \frac{d}{ds}\left[\frac{s(s+2)}{s^2+2s+100}\right]$$

$$= \frac{(s^2+2s+100)(2s+2) - s(s+2)(2s+2)}{(s^2+2s+100)^2}$$

$$K_1 = \lim_{s \to 0} \frac{d}{ds} F(s)$$

$$= \frac{100(2) - 0}{(0+0+100)^2} = \frac{200}{100 \times 100} = \frac{1}{50}$$

$K_1 = \frac{1}{K_V}$
$K_2 = \frac{1}{K_A}$

NOTE :
Static & Dynamic error constants need not always be direct reciprocal values.

Q. Find ess for $r(t) = 5 + 2t$

1. Error Series –

$$e_{ss} = \lim_{t \to \infty} \left[K_0 r(t) + K_1 \dot{r}(t) + \frac{K_2}{2!} \ddot{r}(t) + \dots \right]$$

$r(t) = 5 + 2t \Rightarrow K_0 = 0$

$\dot{r}(t) = \cancel{2}\ 2 \Rightarrow K_1 = \frac{1}{50}$

$\ddot{r}(t) = 0$

$$e_{ss} = \lim_{t \to \infty} \left[0 \times (5 + 2t) + \frac{1}{50} \times 2 \right]$$

$e_{ss} = \frac{2}{50}$ units.

2. Error Ratio

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot R(s)}{1 + G(s) H(s)}$$

$r(t) = 5 + 2t$

$R(s) = \frac{5}{s} + \frac{2}{s^2} = \frac{5s+2}{s^2}$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{5s+2}{s^2}}{1 + \frac{100}{s(s+2)}}$$

$$e_{ss} = \lim_{s \to 0} \frac{5s+2}{s + \frac{\cancel{s} \cdot 100}{\cancel{s}(s+2)}} = \frac{2}{50} \text{ units}$$

3. Short cut Method

$r(t) = 5 + 2t$

$$R(s) = \frac{5}{s} + \frac{2}{s^2}$$

$$G(s) = \frac{100}{s(s+2)}$$

$$e_{ss} = 0 + \frac{A}{k}$$

$A = 2$

$$k = k_v = \lim_{s \to 0} s \cdot \frac{100}{s(s+2)} = 50$$

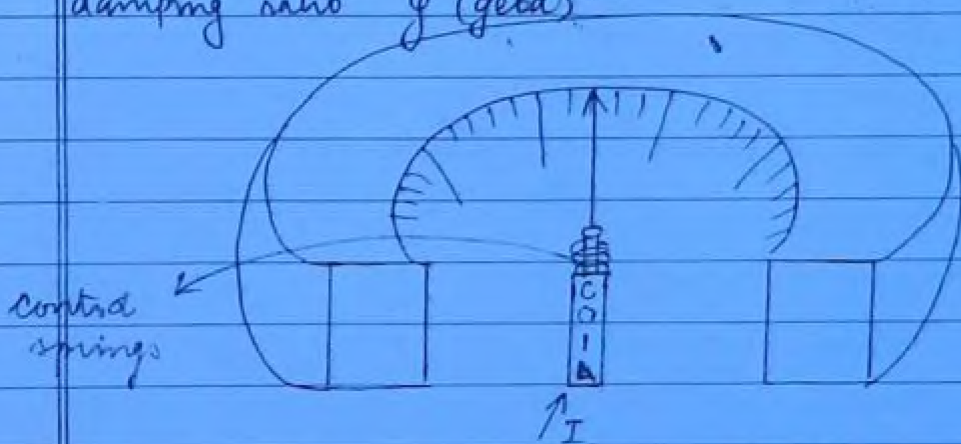
$$e_{ss} = \frac{2}{50} \text{ units}$$

TRANSIENT STATE ANALYSIS -

It deals with the nature of response of a system & depends on the order of the system.

The response of a 2nd order or higher order system exhibits continuous & sustained oscillations about the steady state value of input with a frequency known as undamped natural frequency ω_n rad/s.

These oscillations in the response are damped to the steady state value of input using appropriate damping methods with damping expressed mathematically as the damping ratio ζ (zeta)



Input = Deflecting torque (T_d)

Output = Angular deflection of pointer (θ)

J, B, K

$$T_d = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\theta$$

$$T_d(s) = (Js^2 + Bs + K)\,\theta(s)$$

$$\frac{\theta(s)}{T_d(s)} = \frac{1}{Js^2 + Bs + K} = \frac{1/J}{s^2 + \frac{B}{J}s + \frac{K}{J}}$$

$$s^2 + \frac{B}{J}s + \frac{K}{J} = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\omega_n = \sqrt{\frac{K}{J}} \quad rad/s$$

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$$2\zeta\sqrt{\frac{K}{J}} = \frac{B}{J} \Rightarrow \boxed{\zeta = \frac{B}{2\sqrt{KJ}}}$$

$$\zeta \propto B.$$

Undamped Natural frequency ω_n rad/s.

∴ Damping Ratio ζ [Zeta]

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Effect of Damping on Nature of Response -

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$= \frac{-2\zeta\omega_n s \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

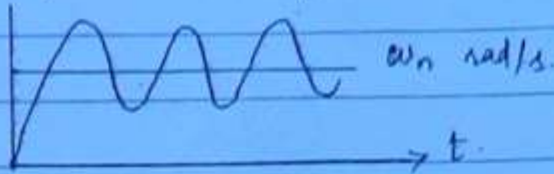
$D = \zeta^2 - 1 = 0 \Rightarrow \zeta = 1$

$D = \zeta^2 - 1 < 0 \Rightarrow \zeta < 1$

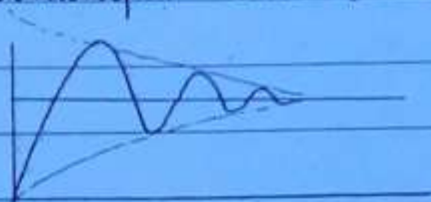
$D = \zeta^2 - 1 > 0 \Rightarrow \zeta > 1$

G3

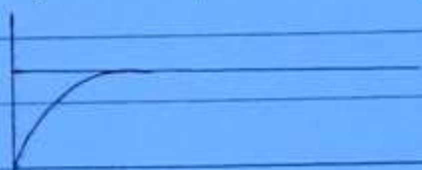
case 1 ⇒ Undamped case ($\zeta = 0$)



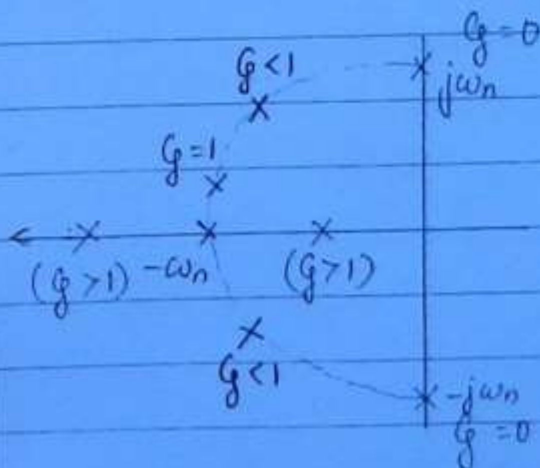
case 2 ⇒ Underdamped case $[0 < \zeta < 1]$



case 3 ⇒ Critically damped case $[\zeta = 1]$



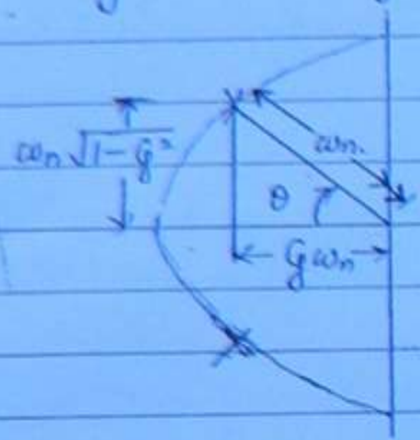
case 4 ⇒ Overdamped case $[\zeta > 1]$



Most of the control systems are defined for $\zeta < 1$ because the response can be analysed using more number of specifications

Characteristics of Underdamped systems -

$$-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$$



(1) $\cos\theta = \frac{\zeta\omega_n}{\omega_n}$ $\quad\theta = \cos^{-1}\zeta$

(2) Damping co-efficient
(or) Damping factor
(or) Actual damping
$\alpha = \zeta\omega_n$

(3) Time constant of under damped response
$$T = \frac{1}{\alpha} = \frac{1}{\zeta\omega_n}$$

(4) Damped Natural frequency
$\omega_d = \omega_n\sqrt{1-\zeta^2}$ rad/s

(5) $s^2 + 2\zeta\omega_n s + \omega_n^2 = (s+\zeta\omega_n)^2 + \omega_d^2$

(6) Damping ratio $= \frac{\text{Actual damping}}{\text{Critical damping}} = \frac{\zeta\omega_n}{\omega_n} = \zeta$

Actual damping $= \zeta\omega_n$

TRANSIENT ANALYSIS (underdamped response)

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Let $R(s) = \frac{1}{s}$

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

$$C(s) = \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{(s + 2\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$= \frac{1}{s} - \frac{(s + \zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} - \frac{\omega_d}{(s+\zeta\omega_n)^2 + \omega_d^2} \cdot \frac{\zeta\omega_n}{\omega_n\sqrt{1-\zeta^2}}$$

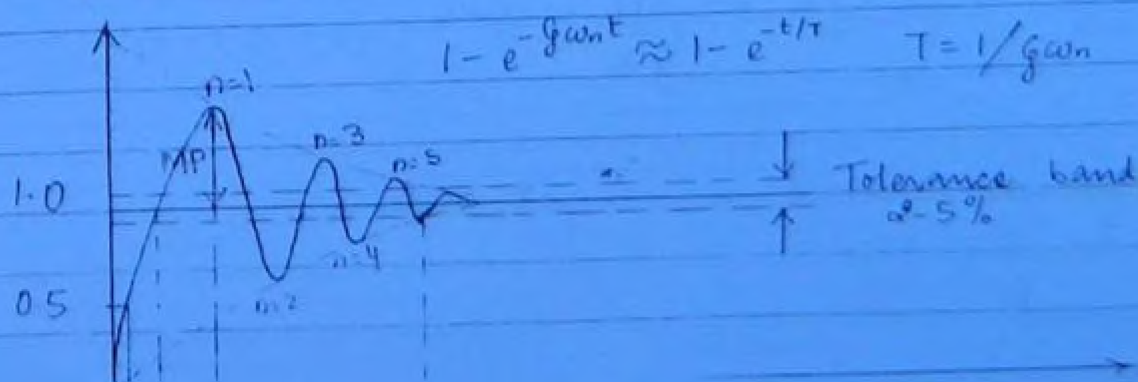
$$C(t) = 1 - e^{-\zeta\omega_n t}\cos\omega_d t - e^{-\zeta\omega_n t}\sin\omega_d t \cdot \frac{\zeta}{\sqrt{1-\zeta^2}}$$

$$= 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\underbrace{\sqrt{1-\zeta^2}}_{B}\cos\omega_d t + \underbrace{\zeta}_{A}\sin\omega_d t\right]$$

$$\boxed{A\sin\omega t + B\cos\omega t = \sqrt{A^2+B^2}\sin\left[\omega t + \tan^{-1} B/A\right]}$$

$$\boxed{C(t) = \underbrace{1}_{\text{steady state}} - \underbrace{\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin\left[\omega_d t + \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right]}_{\text{transient}}}$$

$1 - e^{-\zeta\omega_n t} \approx 1 - e^{-t/T}$ $\quad T = 1/\zeta\omega_n$



Page

1. Delay time (t_d) — Time required by response to reach 50% of final value

$$t_d = \frac{1+0.7\zeta}{\omega_n} \text{ secs} \quad \text{for unit step i/p}$$

2. Rise time (t_r) — 10 to 90% of final value

$$t_r = \frac{\pi - \theta}{\omega_d} \quad \left[\theta = \tan^{-1}\frac{\sqrt{1-\zeta^2}}{\zeta}\right]$$

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3. Peak time (t_p)

$$t_p = \frac{n\pi}{\omega_d} \text{ secs}$$

4. Settling time (t_s)

$$2\% \text{ T.B} \rightarrow 4T \rightarrow 4/\zeta\omega_n \text{ secs.}$$

$$5\% \text{ T.B} \rightarrow 3T \rightarrow 3/\zeta\omega_n \text{ secs.}$$

5. Maximum peak overshoot (M_p)

$$M_p = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cong e^{-\zeta n\pi/\sqrt{1-\zeta^2}}$$

6. Number of cycles

$$\omega_d = 2\pi f_d \Rightarrow f_d = \frac{\omega_d}{2\pi} \left[\frac{\text{cycles}}{\text{sec}}\right]$$

$$2\% \text{ T.B} \rightarrow t_s \times f_d \rightarrow \frac{4 f_d}{\zeta\omega_n} \text{ cycles}$$

$$5\% \text{ T.B} \rightarrow t_s \times f_d \rightarrow \frac{3 f_d}{\zeta\omega_n} \text{ cycles}$$

7. Time interval / period of damped sine

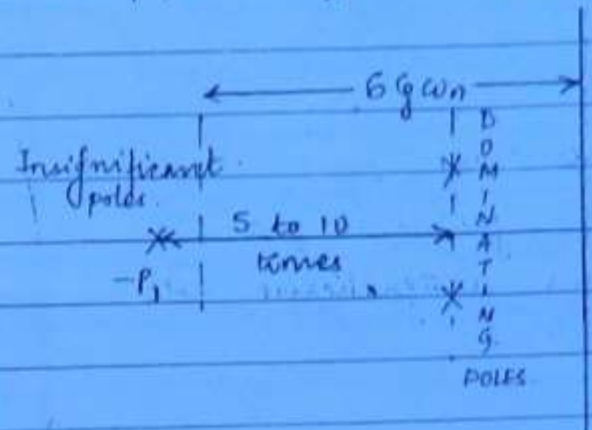
$$T = \frac{1}{f_d} \text{ secs}$$

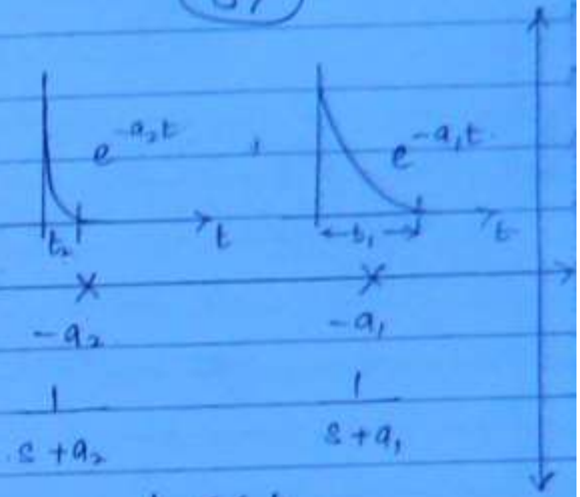
TIME RESPONSE ANALYSIS FOR HIGHER ORDER SYSTEM -

Consider a 3^{rd} order characteristic equation

$$s^3 + ps^2 + qs + k = 0$$

$$(s + p_1)(s^2 + q_1 s + k_1) = 0$$





$$t_2 << t_1$$

The time response of higher order system is obtained by approximating to second order system w.r.t dominant poles. The time domain specifications obtained for approximated second order system are valid for the higher order system also

t_r, t_s etc

all values remain same for higher order & second order systems

CWB chapter 7

5 (a)

chapter 6

22 $T(s) = \dfrac{5}{(s+5)(s^2+s+1)} = \dfrac{1}{(1+s/5)(s^2+s+1)}$

first convert to time constant form $(1 \pm Ts)$ then remove insignificant pole

$= \dfrac{1}{(s^2+s+1)}$ (d)

chapter 3

(14) $$G(s) = \frac{100}{(s+1)(s+100)}$$ 68

One pole is at $s = -1$.
other pole is 100 times away.
After 10 times its insignificant pole.
So $(s+100)$ is an insignificant pole.

$$= \frac{1}{(1+s)(1+\cancel{s/100})}$$

$$= \frac{1}{1+s} = \frac{1}{1+Ts}$$

$T = 1$ sec

$t_s = 4T \Rightarrow 4 \times 1$ sec
$= 4$ secs Ans

ONV 5. $$G(s) = \frac{K(s+2)}{s^3 + \alpha s^2 + 4s + 1}$$

$1 + G(s) = 0$

$$1 + \frac{K(s+2)}{s^3 + a s^2 + 4s + 1} = 0$$

$s^3 + \alpha s^2 + s(4+K) + (1+2K) = 0$

$(s+a)(s^2 + bs + c) = 0$

$\Rightarrow \quad c = \omega_n^2 = 9$

$b = 2\zeta\omega_n = 2 \times 0.2 \times 3 = 1.2$

$\Rightarrow (s+a)(s^2 + 1.2s + 9)$

$s^3 + 1.2s^2 + 9s + as^2 + 1.2as + 9a = 0$

$s^3 + s^2(1.2+a) + s(9+1.2a) + 9a = 0$

$\alpha = 1.2 + a$

$4 + K = 9 + 1.2a$

$2K + 1 = 9a$

$\underline{K = 7}$ $\underline{\alpha = 2.7}$

CONV 4. $G(s) = \dfrac{K}{s(sT+1)}$ $H(s) = 1$

$T.F = \dfrac{G(s)}{1 + H(s)G(s)}$

$= \dfrac{K}{s(sT+1)}$

$Ts^2 + s + K = 0$

$s^2 + \dfrac{s}{T} + \dfrac{K}{T} = 0$

$\omega_n = \sqrt{\dfrac{K}{T}}$ r/s.

$2\zeta\sqrt{\dfrac{K}{T}} = \dfrac{1}{T}$

$\zeta = \dfrac{1}{2\sqrt{KT}}$

Case 1 $\% M_p = 40\%$

$M_p = 0.4$

$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.4$

$\zeta = \zeta_1 = 0.28$

let $K = K_1$

$\dfrac{\zeta_1}{\zeta_2} = \dfrac{1/2\sqrt{K_1 T}}{1/2\sqrt{K_2 T}}$

$\left[\dfrac{0.28}{0.16}\right]^2 = \dfrac{K_2}{K_1}$

$\boxed{K_2 = 3K_1}$

Case 2 $\% M_p = 60\%$ $M_p = 0.6$

$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.6$

SENSITIVITY ANALYSIS -

Let α = A variable that changes its value.

β = A parameter that changes the value of α.

$$S_{\beta}^{\alpha} = \frac{\%\text{ change in } \alpha}{\%\text{ change in } \beta} = \frac{\partial\alpha/\alpha}{\partial\beta/\beta}$$

$$\boxed{S_{\beta}^{\alpha} = \frac{\beta}{\alpha} \cdot \frac{\partial\alpha}{\partial\beta}}$$

Open Loop Control System -

$$R(s) \longrightarrow \boxed{G(s)\,H(s)} \longrightarrow C(s)$$

Let $M(s)$ = O.L.C.S

$\alpha = M(s)$ [O.L.C.S]

$\beta = G(s)$

$$S_{G(s)}^{M(s)} = \frac{G(s)}{M(s)} \cdot \frac{\partial M(s)}{\partial G(s)}$$

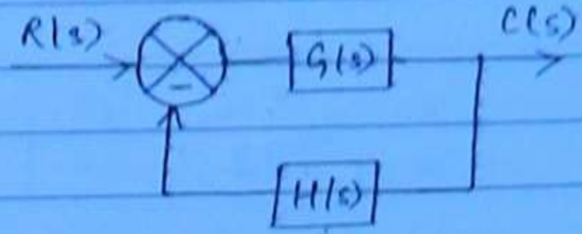
$$M(s) = G(s)\,H(s)$$

$$\frac{G(s)}{M(s)} = \frac{1}{H(s)}$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\, G(s)\,H(s) = H(s)$$

$$\boxed{S_{G(s)}^{M(s)} = \frac{1}{H(s)} \times H(s) = 1}$$

Closed Loop Control System :-



let $M(s) = C.L.C.S$

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

Let $\alpha = M(s)$

$\beta = G(s)$

$$S_{G(s)}^{M(s)} = \frac{G(s)}{M(s)} \frac{\partial M(s)}{\partial G(s)}$$

$$M(s) = \frac{G(s)}{1+G(s)H(s)}$$

$$\frac{G(s)}{M(s)} = 1+G(s)H(s)$$

$$\frac{\partial M(s)}{\partial G(s)} = \frac{\partial}{\partial G(s)}\left[\frac{G(s)}{1+G(s)H(s)}\right]$$

$$\frac{1+G(s)H(s) - G(s)H(s)}{[1+G(s)H(s)]^2} = \frac{1}{[1+H(s)G(s)]^2}$$

$$S_{G(s)}^{M(s)} = 1+G(s)H(s) \times \frac{1}{[1+G(s)H(s)]^2}$$

$$\boxed{S_{G(s)}^{M(s)} = \frac{1}{1+G(s)H(s)}}$$

$\boxed{1+G(s)H(s) = \text{Noise Reduction factor}}$

Open Loop Systems are highly sensitive to internal or external disturbances compared to closed loop systems.

CWB chapter 2

② FA → disturbance in fwd path

Return diff = $1+G(s)H(s)$

$= 1 + A\beta$ (b)

-ve means feedback.

③ $\frac{10\times10\times10}{1+10^3\times B} = 100$ ⇒ $\frac{G(s)}{1+H(s)G(s)}$ ⇒ $\frac{10^3}{1+10^3 B} = 10$

B → feedback

$B = 9\times10^{-3}$ (b)

B → freq dependent & diff for diff freq ranges.

④ $G(s) = \frac{k}{s(s+a)}$

$e_{ss} = \lim_{s\to 0} \frac{s\cdot\frac{1}{s^2}}{1+\frac{k}{s(s+a)}} = \frac{a}{k}$

I $S_k^{e_{ss}} = \frac{k}{e_{ss}} \frac{\partial e_{ss}}{\partial k}$

$e_{ss} = \frac{a}{k}$

$\frac{e_{ss}}{k} = \frac{a}{k^2}$

$$\frac{\partial e_{ss}}{\partial K} = \frac{\partial}{\partial K}\left[\frac{a}{K}\right] = \frac{-a}{K^2}$$

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$$\boxed{S_K^{e_{ss}} = \frac{K^2}{a} \times \frac{-a}{K^2} = -1}$$

II $S_a^{e_{ss}} = \frac{a}{e_{ss}} \cdot \frac{\partial e_{ss}}{\partial a}$

$e_{ss} = \frac{a}{K}$

$\frac{a}{e_{ss}} = K$

$$\frac{\partial e_{ss}}{\partial a} = \frac{\partial}{\partial a}\left[\frac{a}{K}\right] = \frac{1}{K}$$

$$\boxed{S_a^{e_{ss}} = K \times \frac{1}{K} = 1}$$

Ans = $-1, 1$ (b)

(*) Sensitivity when it comes to comparison is a mod value.

In this ques.
sensitivity w.r.t both K & a is same
for the given system

6. $G(s) = \frac{K}{s(s+2)}$ $R(s) = \frac{1}{s}$

$1 + \frac{32}{s(s+2)} = 0$ for $k = 32$.

$s^2 + 2s + 32 = 0$

$\omega_n = \sqrt{32} = 5.65$ rad/s.

$2\zeta \times 5.65 = 2$

$\zeta = 0.176$

$\omega_d = \omega_{d_1} = \omega_n \sqrt{1-\zeta^2}$

$= 5.65 \times \sqrt{1-(0.176)^2}$

$= 5.56$ rad/s

$1 + \frac{16}{s(s+2)} = 0$ for $k = 16$

$s^2 + 2s + 16 = 0$

$\omega_n = \sqrt{16} = 4$ rad/sec

$2\zeta \times 4 = 2$

$\zeta = 0.25$

$\omega_d = \omega_{d_2} = 4\sqrt{1-(0.25)^2}$

$= 3.86$ rad/s.

$\frac{\omega_{d_1}}{\omega_{d_2}} = \frac{5.56}{3.86} = 1.44$ Ans.

chapter 3.

(5) . $L\{c(t)\} = T.F = \dfrac{12.5 \times 8}{(s+5)^2 + 8^2} = \dfrac{100}{s^2+10s+89}$

$s^2+10s+89 = s^2+2\zeta\omega_n s+\omega_n^2$

$\omega_n = \sqrt{89} = 9.5$ r/s.

$2\zeta \times 9.5 = 10 \Rightarrow \zeta = 0.52$ (a)

(8) $G(s) = \dfrac{k}{s(s+4)}$

~~s^2+4s~~ $1 + \dfrac{K}{s(s+4)} = 0$

$s^2 + 4s + K = 0$

$\omega_n = \sqrt{K}$ r/s

$2\zeta\sqrt{K} = 4$

given $\zeta = 0.5$

$2 \times 0.5\sqrt{K} = 4$

$\boxed{K = 16}$ (b)

(3) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{s+1}{s^2+2s+1}$

$x(t) = e^{-t}u(t)$ $X(s) = \dfrac{1}{s+1}$

$Y(s) = \dfrac{s+1}{(s+1)(s^2+2s+2)}$

$Y(s) = \dfrac{1}{s^2+2s+2}$

$Y(s) = \dfrac{1}{(s+1)^2+1^2}$

$y(t) = [e^{-t}\sin t]\, u(t)$

10 $\dfrac{H(s+c)}{(s+a)(s+b)}$

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1) $u(t) \rightarrow 2 + De^{-t} + Ee^{-3t}$

2) $e^{-2t}u(t) \rightarrow Fe^{-t} + Ge^{-3t}$

1) $\dfrac{H(s+c)}{(s+a)(s+b)} = \dfrac{k_1}{s} + \dfrac{k_2}{s+a} + \dfrac{k_3}{s+b}$

$= 2 + De^{-t} + Ee^{-3t}$

$a = 1 \quad b = 3$

$\lim_{s \to 0} \dfrac{s \cdot H(s+c)}{s(s+a)(s+b)} = \lim_{t \to \infty} [2 + De^{-t} + Ee^{-3t}]$

$\dfrac{HC}{ab} = 2 \Rightarrow \dfrac{HC}{1 \times 3} = 2 \Rightarrow HC = 6$

2) $\dfrac{H(s+c)}{(s+2)(s+a)(s+b)}$ → 3 terms are given here

$\boxed{\begin{array}{c} C = 2 \\ H = 3 \end{array}}$ (a)

only 2 constants are present in $u(t)$
so one has to cancel
obv. $c = 2$.

11 $Y(s) = 1 - e^{-5t} - 5te^{-t}$

$\dfrac{d}{dt}$ (step response) = impulse response

Impulse response $= 0 + 5e^{-5t} - 5[-te^{-5t} + e^{-5t}]$

$= 5e^{-5t} + 25te^{-5t} - 5e^{-5t}$

$= 25te^{-5t}$

$TF = \dfrac{25}{\quad} = \dfrac{25}{\quad}$

(16) $H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+2}$

$x(t) = 10u(t) \quad X(s) = \frac{10}{s}$

$Y(s) = \frac{10}{s(s+2)} = \frac{5}{s} - \frac{5}{s+2}$

$y(t) = 5[1 - e^{-2t}]$

$\frac{99}{100} \times 5 = 4.95$

$4.95 = 5[1 - e^{-2t}]$

$\ln e^{-2t} = \ln(0.01) \Rightarrow \cancel{-}2t = \cancel{-}4.6$

$t = 2.3$ secs.

(17) $\frac{d^2y}{dt^2} + \frac{3dy}{dt} + 2y = x(t)$

$(s^2 + 3s + 2)Y(s) = X(s)$

$x(t) = 2u(t) \quad X(s) = \frac{2}{s}$

$Y(s) = \frac{2/s}{s^2+3s+2} = \frac{2}{s(s+2)(s+1)} = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$

$y(t) = (1 - 2e^{-t} + e^{-2t})u(t)$ (a)

(20) $4\frac{d^2C(t)}{dt^2} + 8\frac{dC(t)}{dt} + 16\,C(t) = 16u(t)$

$(4s^2 + 8s + 16)\,C(s) = 16\,U(s)$

$\frac{C(s)}{U(s)} = \frac{16}{4s^2+8s+16} = \frac{4}{s^2+2s+4}$

$\boxed{\omega_n = 2 \text{ r/s}}$ $\quad 2\zeta \times 2 = 2$

$\boxed{\zeta = 0.5}$ (b)

(19) $G(s) = \frac{100(K_P + K_D s)}{s(s+10)}$

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$K_V = 1000 \qquad \zeta = 0.5$.

$1000 = \lim_{s \to 0} s \cdot \frac{100(K_P + K_D s)}{s(s+10)}$

$K_P = 100$

$1 + G(s) = 0$

$1 + \frac{100(K_P + K_D s)}{s(s+10)} = 0$

$s^2 + 10s + 100 K_P + 100 K_D s = 0$

$s^2 + s(10 + 100 K_D) + 100 K_P = 0$

$s^2 + s(10 + 100 K_D) + 100 \times 100 = 0$

$\boxed{\omega_n = 100 \text{ r/s}}$

$2\zeta \times 100 = 10 + 100 K_D$

$\zeta = 0.5$ (given)

$2 \times 0.5 \times 100 = 10 + 100 K_D$

$\boxed{K_D = 0.9}$ (b)

(22) $\cancel{x(t)} = \frac{1}{6} e^{-0.8t} \sin(0.6t)$ = Impulse response

~~Impulse response~~ $= \frac{1}{6}\left[-0.8 e^{-0.8t} \sin 0.6t + e^{-0.8t} \cos 0.6t \cdot 0.6\right]$

$Y(s) = \frac{1}{6} \cdot \frac{(0.6)}{(s+0.8)^2 + (0.6)^2} = 0$

$= \frac{0.1}{s^2 + 1.6s + 1}$

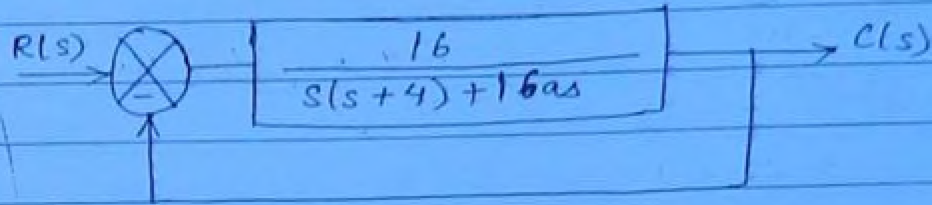
$\boxed{\omega_n = 1} \qquad 2\zeta \times 1 = 1.6$

$\boxed{\zeta = 0.8}$ (b)

CONV 3

(79)

$$\frac{\frac{16}{s(s+4)}}{1+\frac{16as}{s(s+4)}} = \frac{16}{s(s+4)+16as}$$



a) $1+\frac{16}{s(s+4)+16as} = 0$

$s^2 + s(4+16a) + 16 = 0$

$\omega_n = 4$ rad/s

$2\zeta \times 4 = 4 + 16a$

Given % M_p = 1.5%

$M_p = 0.015$

$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.015$

$\zeta = 0.8$

$2 \times 0.8 \times 4 = 4 + 16a$

$a = 0.15$

b) e_{ss} for unit ramp input without a [a = 0]

$G(s) = \frac{16}{s(s+4)}$

$e_{ss} = \lim_{s\to 0} \frac{s \cdot \frac{1}{s^2}}{1+\frac{16}{s(s+4)}}$ $\qquad e_{ss} = \frac{4}{16} = 0.25$ units with $a = 0.15$

$$G(s) = \frac{16}{s(s+4) + 16 \times 0.155} = \frac{16}{s(s+6.4)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{16}{s(s+6.4)}} = \frac{6.4}{16} = 0.4 \text{ units}$$

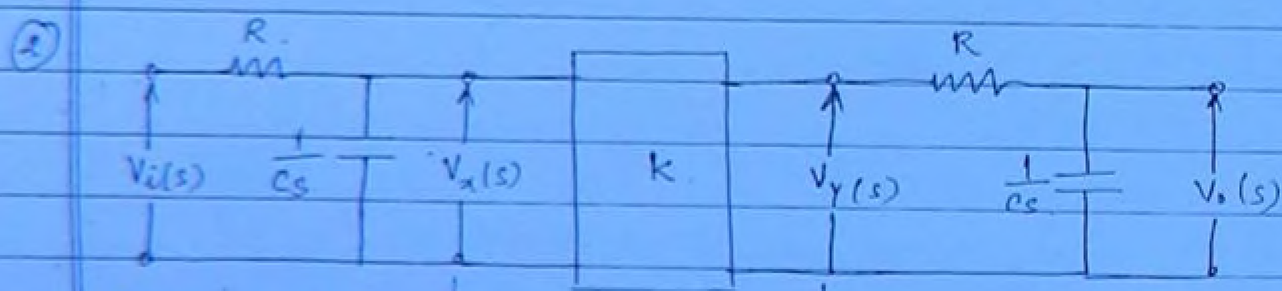
(c) $$G(s) = \frac{K}{s(s+4) + K \times 0.155}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot \frac{1}{s^2}}{1 + \frac{K}{s(s+4) + 0.155 \times K}}$$

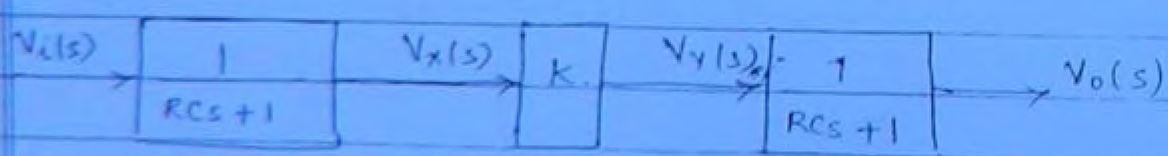
$$0.25 = \frac{4 + 0.15K}{K}$$

$$\boxed{K = 40}$$

(2)



$$\frac{V_x(s)}{V_i(s)} = \frac{1}{RCs+1} \qquad K = \frac{V_y(s)}{V_x(s)} \qquad \frac{V_o(s)}{V_y(s)} = \frac{1}{RCs+1}$$

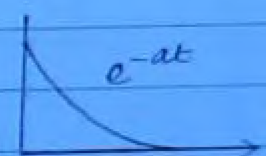
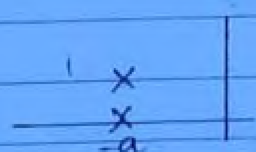
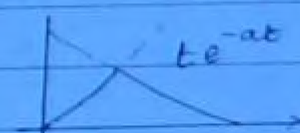
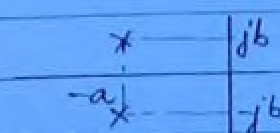
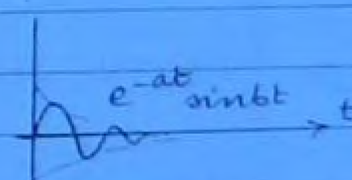
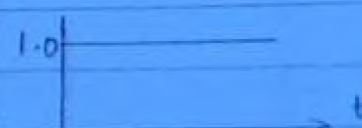
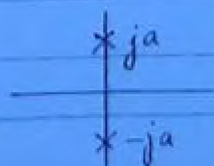
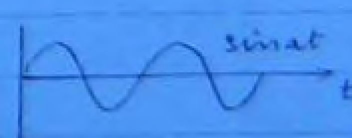


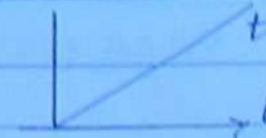
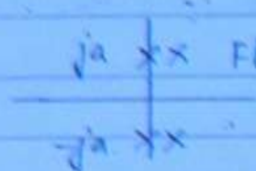
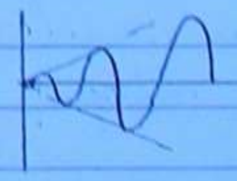
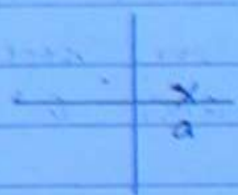
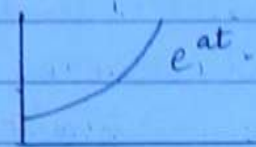
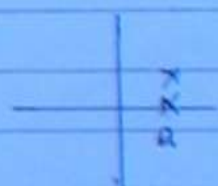
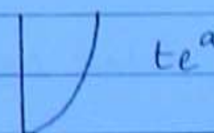
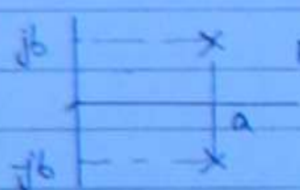
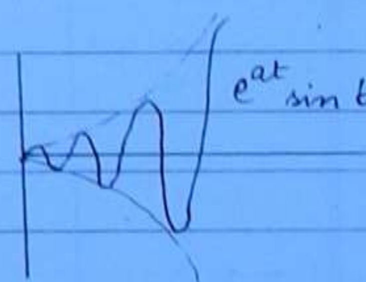
$$\frac{V_o(s)}{} = \frac{K}{} \rightarrow \text{critically damped (d)}$$

STABILITY IN TIME DOMAIN

The stability of LTI system may be defined as when the system is subjected to bounded input the output should be bounded.

BIBO → implies the impulse response of the system should tend to 0 as time approaches infinity.
The stability of the system depends on roots of the characteristic equation $1 + H(s)G(s) = 0$
ie closed loop poles.

	C.L. Pole locations	Stability criteria	Impulse Response
1.	 $F(s) = \frac{1}{s+a}$	Absolutely stable	 e^{-at}
2.	 $F(s) = \frac{1}{(s+a)^2}$	"	 te^{-at}
3.	 $F(s) = \frac{1}{(s+a)^2+b^2}$	"	 $e^{-at}\sin bt$
4.	 $F(s) = \frac{1}{s}$	Marginally (or) Critically stable	
5.	 $F(s) = \frac{1}{s^2+a^2}$	"	 $\sin at$

6.		$F(s) = \frac{1}{s^2}$	unstable	
7.		$F(s) = \frac{1}{(s^2+a^2)^2}$	"	 $t \sin at$
8.		$F(s) = \frac{1}{s-a}$	"	 e^{at}
9.		$F(s) = \frac{1}{(s-a)^2}$	"	 te^{at}
10.		$F(s) = \frac{1}{(s-a)^2+b^2}$	"	 $e^{at} \sin bt$

28-1-13 ROUTH HURWITZ METHOD -

Eg $P(s) = s^4 + 8s^3 + 18s^2 + 16s + 5 = 0$

ROUTH ARRAY →

s^4	1	18	5
s^3	8	16	0
s^2	$b_1 = 16$	$b_2 = 5$	0
s^1	$c_1 = 13.5$	~~b_2~~ 0	0
s^0	$d_1 = 5$	0	0

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$b_1 = \dfrac{8\times 18 - 1\times 16}{\boxed{8}} = 16$ → since denominator always has coefficients from the 1st column, the sign changes of only the first column are considered

$b_2 = \dfrac{8\times 5 - 1\times 0}{8} = 5$

$c_1 = \dfrac{16\times 16 - 8\times 5}{16} = 13.5$

$c_2 = \dfrac{13.5\times 5 - 16\times 0}{13.5} = 5$

CWB chapter 4

(11) *

s^5	1	2	3
s^4	1	2	15
s^3	~~0~~ ε	-12	0
s^2	$\dfrac{2\varepsilon+12}{\varepsilon}$	15	0
s^1	$\dfrac{-24\varepsilon-144-15\varepsilon^2}{2\varepsilon+12}$	0	0
s^0	15	0	0

$\varepsilon \rightarrow$ small +ve no.

To check for sign changes

(i) $\lim\limits_{\varepsilon\to 0} \dfrac{2\varepsilon+12}{\varepsilon} = \dfrac{2(0)+12}{0} = +\infty$

(ii) $\lim\limits_{\varepsilon\to 0} \dfrac{-24\varepsilon-144-15\varepsilon^2}{2\varepsilon+12} = \dfrac{-144}{12} = -12$

Two sign changes

$+\infty \rightarrow -12$

$-12 \rightarrow +15$

⇒ Two C.L. poles in RHS of s-plane (unstable) — (b)

Difficulty - 1

When the first element of any row is zero while the rest of the row has atleast one non-zero term, in such cases substitute a small +ve number 'ϵ' in place of zero & evaluate the rest of the Routh Array in terms of ϵ. Check for sign changes in the first column by taking limit $\epsilon \to 0$ to comment on stability.

(2)

s^6	1	8	20	16	
s^5	2	12	16	0	
s^4	2	12	16	0	
s^3	8	24	0	0	→ Abrupt end (system now can be either marginally stable or unstable)
s^2	6	16	0	0	
s^1	2.6	0	0	0	
s^0	16	0	0	0	

Aux eq^n → $A(s) = 2s^4 + 12s^2 + 16$

$$\frac{d}{ds} A(s) = 8s^3 + 24s$$

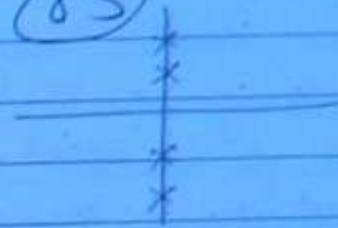
The roots of A(s) = C.L pole lying on [illegible]

Roots of $A(s) = \frac{-12 \pm \sqrt{144 - 8 \times 16}}{4}$

$= -2, -4$

$(s^2+2)(s^2+4) = 0$

$s = \pm j1.4, \pm j2$



$\therefore$ marginally stable (as no roots are repeating)

(10) *

s^5	1	4	2
s^4	1	2	1
s^3	$\cancel{0}$ 4	$\cancel{0}$ 4	0
s^2	1	1	0
s^1	$\cancel{0}$ 2	0	0
s^0	1	0	0

$\rightarrow A_1(s) = s^4 + 2s^2 + 1$

$\frac{d}{ds} A_1(s) = 4s^3 + 4s$

$\rightarrow A_2(s) = s^2 + 1$

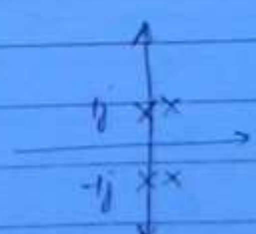
$\frac{d}{ds} A_2(s) = 2s$

for stability check only take roots of $A_1(s)$

Roots of $A_1(s) \Rightarrow \frac{-2 \pm \sqrt{4-4}}{2} = -1, -1$

$(s^2+1)(s^2+1) = 0$

$s = \pm j, \pm j$



(unstable)

Difficulty - 2

When Routh - Array ends abruptly, construct an auxiliary equation A(s), differentiate it to get new co-efficients and evaluate the root of the Routh Array. Check for multiplicity of auxiliary equation roots on $j\omega$ axis to comment on stability.

RELATIVE STABILITY ANALYSIS USING ROUTH ARRAY -

$t_2 << t_1$

$\times$	$\times$
$-a_2$	$-a_1$
$\frac{1}{s+a_2}$	$\frac{1}{s+a_1}$
sys 2	sys 1

→ Both sys 1 & 2 are absolutely stable systems.

→ Sys 2 is said to be relatively more stable than sys 1 because $t_2 << t_1$

Eg $P(s) = s^3 + 7s^2 + 25s + 39 = 0$

To check whether the roots are lying more -vely w.r.t -1?

$s + 1 = z$

$s = z - 1$

$P(z) = (z-1)^3 + 7(z-1)^2 + 25(z-1) + 39 = 0$

$P(z) = z^3 + 4z^2 + 14z + 20 = 0$

z^3	1	14
z^2	4	20
z^1	9	0
z^0	20	0

Shortcuts for such problems

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Difficulty - 3 -

Relative stability analysis using Routh Array is not feasible for higher order polynomials because it involves shifting of origin of s-plane more negatively.

Shortcut

Substitute the given pt. (eg $s+1=0$ mean $s=-1$) in the polynomial assuming the pt makes it stable

→ If LHS > 0 all roots lie on LS of s-plane

→ If LHS = 0 one root lies on the shifted axis line, & rest of the roots to the left.

eg in prev ques $s=-1$ in $P(s)$ gives $\neq 0$

∴ All 3 roots lie on LS of s-plane

Conditionally Stable Systems -

(6)

s^4	1	3	K
s^3	2	2	0
s^2	2	k	0
s^1	$\frac{4-2k}{2}$	0	0
s^0	k	0	0

1) $\frac{4-2k}{2} > 0$

$k < 2$

2) $K > 0$

$\boxed{0 < K < 2}$

At $k = K_{max} = 2$

∴ s^1 row $= 0$

$A(s) = 2s^2 + k$

$= 2s^2 + 2 = 0$

$s = \pm j$

(8) $G(s) = \dfrac{K}{(s^2+2s+2)(s+2)}$ (58)

$1 + G(s) = 0$

$1 + \dfrac{K}{(s^2+2s+2)(s+2)} = 0$

$(s^2+2s+2)(s+2) + K = 0$

$s^3 + 4s^2 + 6s + (4+K) = 0$

s^3	1	6
s^2	4	$4+K$
s^1	$\dfrac{24-(4+K)}{4}$	0
s^0	$4+k$	0

1) $\dfrac{24-(4+K)}{4} > 0$

$K < 20$

2) $4 + K > 0 \Rightarrow k > -4$

$\boxed{-4 < k < 20}$

$k = K_{max} = 20$

$A(s) = 4s^2 + (4+k)$

$= 4s^2 + (4+20) = 0$

$s^2 = -6$

$s = \pm j\sqrt{6}$

$j\omega$

$\omega = \omega_{max}\ \sqrt{6}$ r/s

Short cut method - (only for cubic polynomials)

$$s^3 + 4s^2 + 6s + (4+k) = 0$$ (89)

$$1(4+k) < 6(4)$$
$$1(4+k) < 24$$
$$k < 20$$

EXTERNAL PRODUCTS < INTERNAL PRODUCTS (Stable)

EXTERNAL PRODUCTS = INTERNAL PRODUCTS (Marginally Stable)

EXTERNAL PRODUCTS > INTERNAL PRODUCTS (Unstable)

$$K = K_{max} = 20$$

$$A(s) = 4s^2 + (4+k) = 0 \quad \text{[EVEN PART OF POLYNOMIAL]}$$
$$4s^2 + (4+20) = 0$$
$$4s^2 + 24 = 0$$
$$s^2 = \pm j\sqrt{6}$$ $j\omega$ $\quad \omega = \omega_{max} = \sqrt{6}$ r/s

(12) $$G(s) = \frac{K(s+1)}{s^3 + as^2 + 2s + 1}$$

$$1 + \frac{K(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + 2s + 1 + K(s+1) = 0$$
$$s^3 + as^2 + s(2+k) + (1+k) = 0$$

$$1(1+K) < a(2+K)$$ Stable

$A(s) = as^2 + (k+1) = 0$

$s^2 = -\frac{(k+1)}{a}$

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$s^2 = -\frac{(k+1)(k+2)}{(k+1)}$

$s = \pm j\sqrt{k+2}$

$\omega = \sqrt{k+2}$

$2 = \sqrt{k+2}$

$\Rightarrow \quad k = 2$

(b)

$a = \frac{2+1}{2+2} = 3/4$

CONV of k_I of 2 variables

1. $1 + \frac{10(K_p s + k_I)}{s(s^2 + s + 20)} = 0$

$s^3 + s^2 + 20s + 10(K_p s + k_I) = 0$

$s^3 + s^2 + s(20 + 10K_p) + 10K_I = 0$

s^3	1	$20 + 10K_p$
s^2	1	$10k_I$
s^1	$20 + 10K_p - 10k_I$	0
s^0	$10k_I$	0

1) $10k_I > 0$

$\boxed{k_I > 0}$

2) $20 + 10K_p - 10k_I > 0$

$\boxed{K_p > \frac{10k_I - 20}{10}}$

K_p & K_I are dependent on each other ∴ we're to choose those values of K_p & K_i for which there is no sign change in 1st column of relation is satisfied conv eg or 2 variables

3. $$G(s) = \frac{K(s+2)^2}{s(s^2+1)(s+4)}$$

$$1 + \frac{K(s+2)^2}{s(s^2+1)(s+4)} = 0$$

$$s(s^2+1)(s+4) + K(s+2)^2 = 0$$

$$s^4 + 4s^3 + s^2(1+K) + s(4+4K) + 4K = 0$$

s^4	1	$(1+K)$	$4K$
s^3	4	$(4+4K)$	0
s^2	$\cancel{0}\ \epsilon$	$4K$	0
s^1	$\frac{(4+4K)\epsilon - 16K}{\epsilon}$	0	0
s^0	$4K$	0	0

1) $4K > 0 \Rightarrow \boxed{K > 0}$

2) $\frac{(4+4K)\epsilon - 16K}{\epsilon} > 0 \Rightarrow \boxed{\epsilon > \frac{16K}{4+4K}}$

cant put $\epsilon \to 0$ here as theres a condition on ϵ for stability.

LIMITATIONS -

1. All the co-efficients of the polynomials must be real.

2. The sign changes in 1st column of Routh Array determines poles of RHS in s-plane but not their locations.

(5) O.L system.

$$G(s) = \frac{1}{s^3+1.5s^2+s-1}$$

(92)

$$\cancel{1+\frac{1}{s^3+1.5s^2+s-1}} = 0$$

$$s^3+1.5s^2+s-1 = 0$$

s^3	1	1
s^2	1.5	-1
s^1	2.5/1.5	0
s^0	-1	0

Unstable

C.L system

$$H(s) = 20(s+1)$$

C.L poles $= 1+H(s)G(s) = 0$

$$1+\frac{20(s+1)}{s^3+1.5s^2+s-1} = 0$$

$$s^3+1.5s^2+s(1+20)+19 = 0$$

$$s^3+1.5s^2+21s+19 = 0$$

s^3	1	21
s^2	1.5	19
s^1	8.3	
s^0	19	

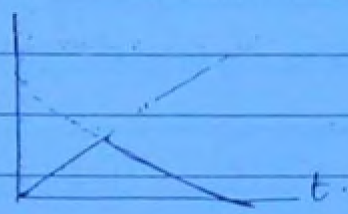
Stable (c).

(93)

(5) Impulse Response $= \frac{d}{dt}\left[1 - e^{-t}(1+t)\right]$

$= 0 + e^{-t} - \left[-te^{-t} + e^{-t}\right]$

$= te^{-t}$ Stable → (b)



(9) $1 + \frac{k(s-2)^2}{(s+2)^2} = 0$ $\quad G(s) = k \times \frac{s-2}{(s+2)^2} \quad H(s) = s-2$

$(s+2)^2 + k(s-2)^2 = 0$

$s^2 + 4 + 4s + k(s^2 + 4 - 4s) = 0$

$s^2(1+k) + s(4-4k) + (4+4k) = 0$

s^2	$1+k$	$4+4k$
s^1	$4-4k$	0
s^0	$4+4k$	0

1) $1+k > 0 \Rightarrow k > -1$

2) $4-4k > 0 \Rightarrow k < 1$

$\boxed{\begin{array}{c} -1 < k < 1 \\ 0 < k < 1 \end{array}}$ Ans (c)

ROOT LOCUS

The root locus is defined as locus of closed loop poles obtained when system gain K is varied from 0 to ∞. It determines the relative stability of the system w.r.t variation in system gain K.

Angle & Magnitude Conditions

Angle condition → is used for checking whether any point is lying on root locus or not & also the validity of root locus shape for closed loop poles

For negative feedback systems

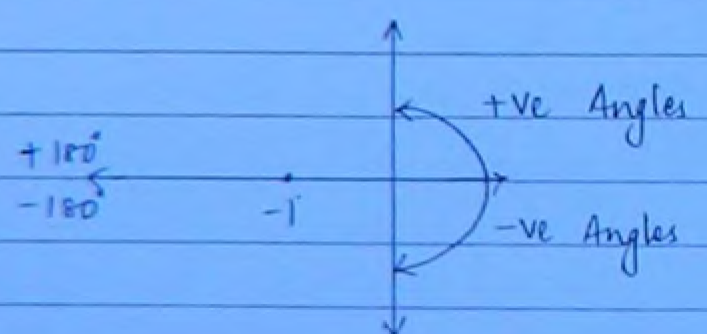
$$1 + G(s)\,H(s) = 0$$

$$G(s)\,H(s) = -1$$

$$= -1 + 0j$$

$$= 180^\circ - \tan^{-1}\left[\frac{0}{1}\right] = 180^\circ \approx \pm 180^\circ$$

$$\boxed{\approx [2q+1]\,180^\circ}$$



Angle condition may be stated as, for a point to lie on root locus, the angle evaluated at that point must be an odd multiple of ± 180°

Page

Magnitude condition → is used for finding the magnitude of system gain K at any point on the root locus.

95

$$1 + G(s) H(s) = 0$$
$$G(s) H(s) = -1$$

$$|G(s) H(s)| = \sqrt{(-1)^2 + 0^2} = 1$$

$$\boxed{|G(s) H(s)| = 1}$$

CWB chapter 5

(6) $s_1 = -3 + 4j \qquad s_2 = -3 - 2j$

$$G(s) H(s) = \frac{K}{(s+1)^4}$$

$$G(s) H(s)\Big|_{s = s_1 = -3+4j} = \frac{K}{[-3+4j+1]^4} = \frac{K}{[-2+4j]} = \frac{0^\circ}{[116^\circ] \times 4}$$
$$= -464^\circ$$

$$G(s) H(s)\Big|_{s = s_2 = -3-2j} = \frac{K}{[-3-2j+1]^4} = \frac{K}{[-2-2j]} = \frac{0^\circ}{[-135^\circ] \times 4}$$
$$= +\underbrace{540^\circ}_{\text{odd multiple of } 180} = (3 \times 180^\circ)$$

s_2 lies on root locus, s_1 does not (b)

(9) $G(s) = \dfrac{K}{s(s^2+7s+12)}$

$s = -1+1j$

(96)

$$G(s)\Big|_{s=-1+1j} = \frac{K}{(-1+j)[(-1+j)^2+7(-1+j)+12]} = \frac{K+0j}{(-1+j)(5+5j)}$$

$$= \frac{0^\circ}{(135^\circ)(45^\circ)} = -180^\circ$$

$$|G(s)|_{s=-1+j} = 1 \Rightarrow \frac{\sqrt{K^2+0^2}}{\sqrt{(-1)^2+1^2}\sqrt{5^2+5^2}} = 1 \Rightarrow \frac{K}{\sqrt{2}\sqrt{50}} = 1$$

$$\Rightarrow K = \sqrt{100} \Rightarrow \boxed{K=10} \text{ (d)}$$

Rules of RL are applicable only to OPTF but final RL satisfies CLTF

CONSTRUCTION RULES OF ROOT LOCUS –

Rule 1 → The root locus is symmetrical about real axis ($G(s)H(s) = -1$)

Rule 2 → Let

P = no. of Open loop poles

Z = no. of open loop zeros

and $P > Z$

Then,

No of branches of RL = P

No of branches terminating at zeros = Z

No. of branches terminating at infinity = $P - Z$

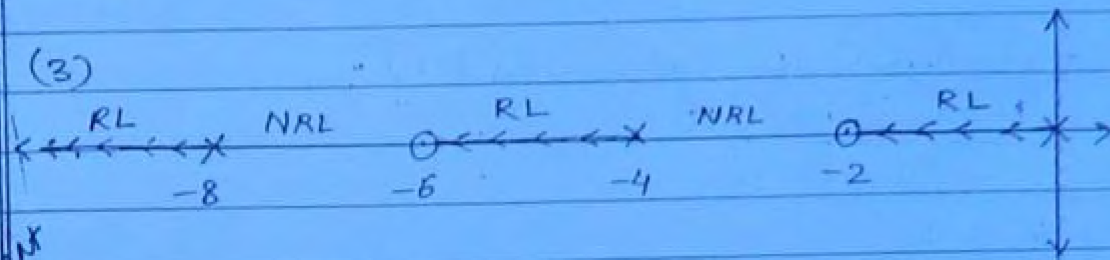
we assume there's a zero at infinity.

Rule-3 → A point on real axis is said to be on root locus if to the right side of this point the sum of open loop poles and zeros is odd eg.

$$G(s) = \frac{k(s+2)(s+6)}{s(s+4)(s+8)}$$

(2) : $P = 3$ $\quad Z = 2$ $\quad P - Z = 1$

(3)



CLTF

Closed Loop T.F.

$$\frac{G(s)}{1+G(s)} = \frac{\dfrac{K(s+2)(s+6)}{s(s+4)(s+8)}}{1 + \dfrac{K(s+2)(s+6)}{s(s+4)(s+8)}}$$

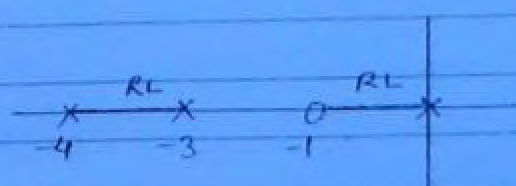
$$= \frac{k(s+2)(s+6)}{s(s+4)(s+8) + k(s+2)(s+6)}$$

C.L. poles $= s(s+4)(s+8) + K(s+2)(s+6) = 0$

when $k = 0$

C.L poles $= 0, -4, -8$

(5)



(c)

Rule 4 → Angle of Asymptotes

The P − Z branches will terminate at infinity along certain straight lines known as asymptotes of root locus. Therefore, number of asymptotes = P − Z

Angle of asymptotes is given by:

$$\boxed{\theta = \frac{[2q+1]\,180^\circ}{P-Z}}$$

$q = 0, 1, 2, 3 \ldots\ldots$

eg. $P - Z = 2$.

$$\theta_1 = \frac{[2(0)+1]\,180}{2} = 90^\circ$$

$$\theta_2 = \frac{[2(1)+1]\,180^\circ}{2} = 270^\circ$$

12 $\underbrace{s(s+4)(s^2+2s+1)} + k(s+1) = 0$

$$1 + \frac{k(s+1)}{s(s+4)(s^2+2s+1)} = 0$$

$$1 + G(s)\,H(s) = 0$$

$$G(s)\,H(s) = \frac{k(s+1)}{s(s+4)(s^2+2s+1)}$$

$P = 4 \qquad Z = 1 \qquad P - Z = 3$.

$$\theta_1 = \frac{[2(0)+1]\,180}{3} = 60^\circ$$

$$\frac{2\pi}{P-Z} = \frac{2\pi}{3} = \underline{120^\circ}$$

$$\theta_2 = \frac{[2(1)+1]\,180^\circ}{3} = 180^\circ$$

$$\theta_3 = \frac{[2(2)+1]\,180^\circ}{3} = 300^\circ$$

Rule 5 → Centroid

It is the intersection point of asymptotes on the real axis. It may or may not be a part of RL

$$\text{Centroid} = \frac{\sum \text{Real part of open loop poles} - \sum \text{Real part of open loop zeros}}{P - Z}$$

(10) $s^3 + 5s^2 + (K+6)s + k = 0$

$s^3 + 5s^2 + sK + 6s + k = 0$

$\underbrace{s^3 + 5s^2 + 6s} + k(s+1) = 0$

$$1 + \frac{k(s+1)}{s^3+5s^2+6s} = 0$$

$1 + G(s)H(s) = 0$

$$G(s)H(s) = \frac{k(s+1)}{s^3+5s^2+6s} = \frac{k(s+1)}{s(s^2+5s+6)} = \frac{k(s+1)}{s(s+3)(s+2)}$$

Poles $= 0 + j0$

$-2 + j0$

$-3 + j0$

$\underline{}$

-5

Zeros $= -1 + j0$

$\underline{}$

-1

$P = 3 \quad Z = 1$

$P - Z = 2$

$$\text{Centroid} = \frac{-5-(-1)}{2} = -2 = [-2, 0] \quad (C)$$

Rule 6 → Break Away Points

They are those points where multiple roots of the characteristic equation occur

Procedure -

1. Construct $1 + G(s)H(s) = 0$
2. Write k in terms of s
3. Find $\frac{dk}{ds} = 0$
4. The roots of $\frac{dk}{ds} = 0$ will give B.A points
5. To test valid B.A points substitute in step (2)
   If $k = +ve \Rightarrow$ Valid B.A point.

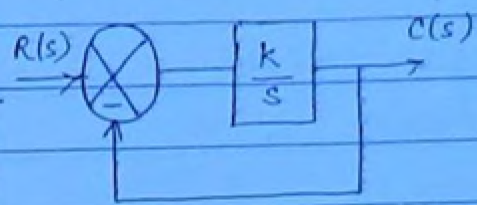
General Predictions about B.A points -

1. The branches of RL either approach or leave the B.A points at an angle of $\pm \frac{180^\circ}{n}$
   where n = no. of branches approaching or leaving B.A point.

2. The complex conjugate path for the branches of RL approaching or leaving a B.A point is a circle

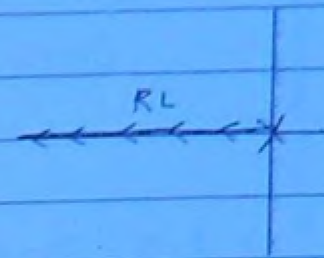
3. Whenever there are 2 adjacently placed poles on the real axis with the section of real axis between them as a part of RL, then there exists a breakaway point between the adjacently placed poles

FIRST ORDER SYSTEM

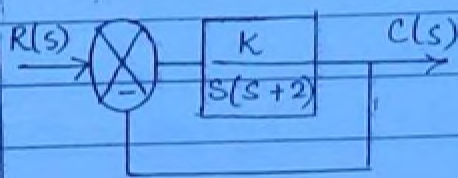


$$\frac{C(s)}{R(s)} = \frac{k}{s+K}$$

$$G(s) = \frac{k}{s}$$



SECOND ORDER SYSTEM

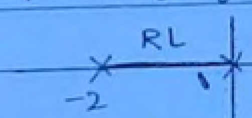


$$\frac{C(s)}{R(s)} = \frac{K}{s^2+2s+K}$$

$$G(s) = \frac{K}{\cancel{2}s(s+2)}$$

2) $P = 2 \quad Z = 0 \quad P - Z = 2$

3)



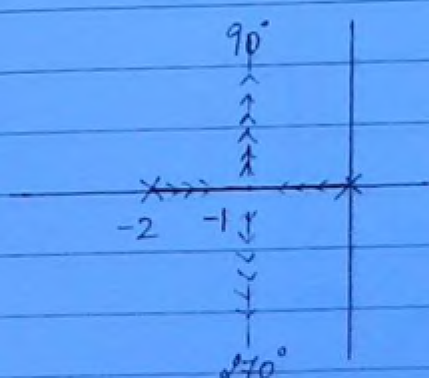
4) $\theta_1 = 90^\circ \quad \theta_2 = 270^\circ$

5) Centroid $= \frac{0 + (-2) - 0}{2} = -1$

6) B.A point $\rightarrow$ $s^2 + 2s + K = 0$

$K = -s^2 - 2s$

$\frac{dK}{ds} = -2s - 2 = 0 \qquad \boxed{s = -1}$



RL gets shifted to right

Effect of adding poles to a T.F — 102

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

2) $P = 3 \quad Z = 0 \quad P - Z = 3$

3) 

4) $\theta_1 = 60° \quad \theta_2 = 180° \quad \theta_3 = 300°$

5) $\frac{0 + (-2) + (-4) - 0}{3} = -2$

6) B.A point

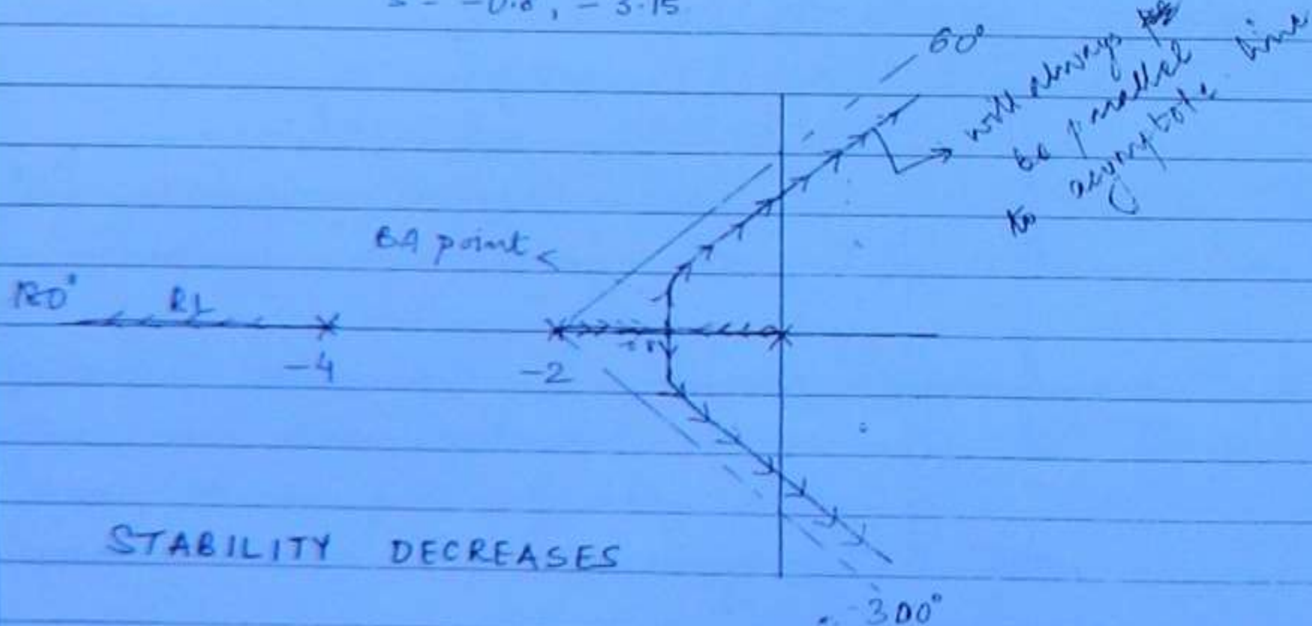
$s^3 + 6s^2 + 8s + K = 0$

$K = -s^3 - 6s^2 - 8s$

$\frac{dK}{ds} = -3s^2 - 12s - 8 = 0$

$\Rightarrow \quad 3s^2 + 12s + 8 = 0$

$s = -0.8, -3.15$



STABILITY DECREASES

RL gets shifted to left.

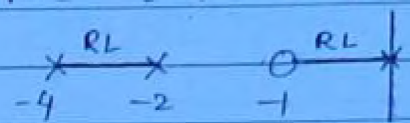
Effect of adding zeros to a T.F.

103

$$G(s) = \frac{K(s+1)}{s(s+2)(s+4)}$$

2) $P = 3 \quad Z = 1 . \quad P - Z = 2$

3)



4) $\theta_1 = 90^\circ \quad \theta_2 = 270^\circ$

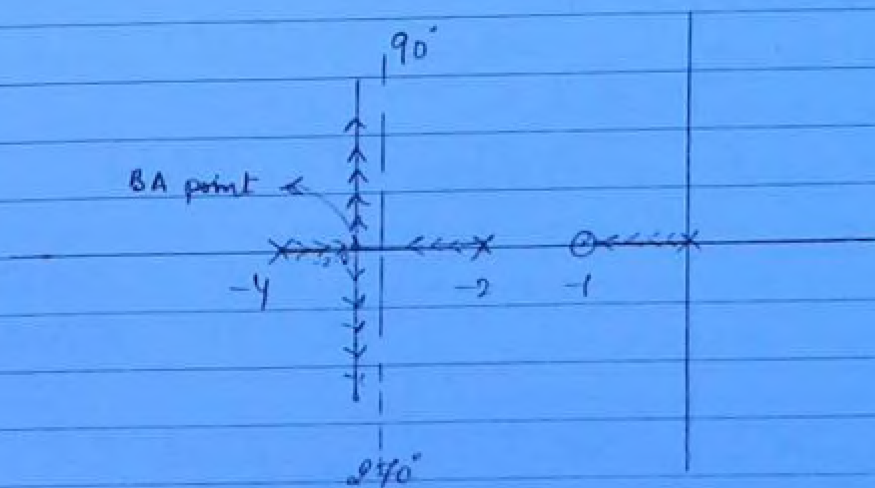
5) Centroid $= \dfrac{0 + (-2) + (-4) - (-1)}{2} = -2.5$

6) B.A point $\Rightarrow$

$$s^3 + 6s^2 + 8s + Ks + K = 0$$

$$K = \frac{-s^3 - 6s^2 - 8s}{s+1} = \frac{-s(s^2+6s+8)}{s+1}$$

$$\frac{dK}{ds} = 0 \Rightarrow s = -2.8$$



STABILITY INCREASES.

$Z > P$ never possible practically

The zeros also exist at infinity on RL remember these

General Predictions about B.A point (contd...)

4 Wherever there are 2 adjacently placed zeros on x-axis with the section between them as a part of RL, then there exists a breakaway point between the adjacently placed zeros.

$$G(s) = \frac{K(s+2)}{s(s+1)}$$

2) $P = 2 \quad Z = 1 \quad P - Z = 1$

5) RL: $-\infty$ to -2 (zero at -2); RL: -1 to 0

6) B.A points

$$s(s+1) + K(s+2) = 0$$

$$K = \frac{-s^2 - s}{s+2}$$

$$\frac{dK}{ds} = 0 \Rightarrow \frac{(s+2)(-2s-1) - (-s^2-s)(1)}{(s+2)^2} = 0$$

$$-2s^2 - s - 4s - 2 + s^2 + s = 0$$

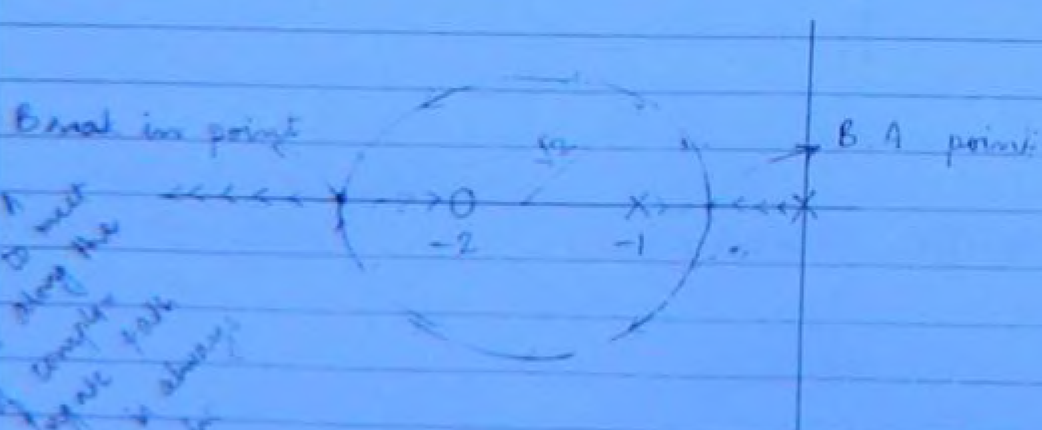
$$-s^2 - 4s - 2 = 0$$

$$s^2 + 4s + 2 = 0$$

$$s = \frac{-4 \pm \sqrt{16-8}}{2} = -2 \pm \sqrt{2}$$

(centre) (radius)

$$= -0.6,\ -3.4$$



Since the B.A points have to meet along the complex path is always circular

Proof of path being a circle 105

$$\frac{K(s+b)}{s(s+a)}$$

let $s = x + jy$

$$\frac{K[x+jy+b]}{[x+jy][x+jy+a]} = \frac{K[(x+b)+jy]}{x^2+jxy+ax+jxy-y^2+jay}$$

$$= \frac{K[(x+b)+jy]}{[x^2+ax-y^2]+j[2xy+ay]}$$

$$\tan^{-1}\left(\frac{y}{x+b}\right) - \tan^{-1}\left[\frac{2xy+ay}{x^2+ax-y^2}\right] = 180^\circ$$

Taking Tan on both sides

$$\frac{y}{x+b} - \left[\frac{2xy+ay}{x^2+ax-y^2}\right] = 0$$

$$x^2+ax-y^2-[(2x+a)(x+b)] = 0$$

$$x^2+ax-y^2-[2x^2+2xb+ax+ab] = 0$$

$$-x^2-y^2-2xb-ab = 0$$

$$x^2+2xb+y^2 = -ab$$

$$x^2+2xb+b^2+y^2 = -ab+b^2$$

$$(x+b)^2+y^2 = b(b-a)$$

centre $= -b, 0$

Radius $= \sqrt{b(b-a)}$

B.A points = Centre ± Radius

Rule - 7 Intersection of RL with Imaginary Axis
~~Intersection of~~
Roots of auxiliary equation A(s) at $K = K_{max}$ from Routh Array gives the intersection of Root Locus with Imaginary axis.

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

7) $s^3 + 6s^2 + 8s^1 + K = 0$

s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	0

$\rightarrow \frac{48-K}{6} > 0 \Rightarrow K < 48$

$\rightarrow K > 0$

$$\boxed{0 < K < 48}$$

At $K = K_{max} = 48$

$A(s) = 6s^2 + K = 0$

$6s^2 + K = 0$

$s = \pm j\sqrt{8} = \pm j2.8$

Shortcut Method -

$$G(s) = \frac{K}{s(s+a)(s+b)}$$

Intersection of RL with $j\omega$ axis $= \pm j\sqrt{ab}$

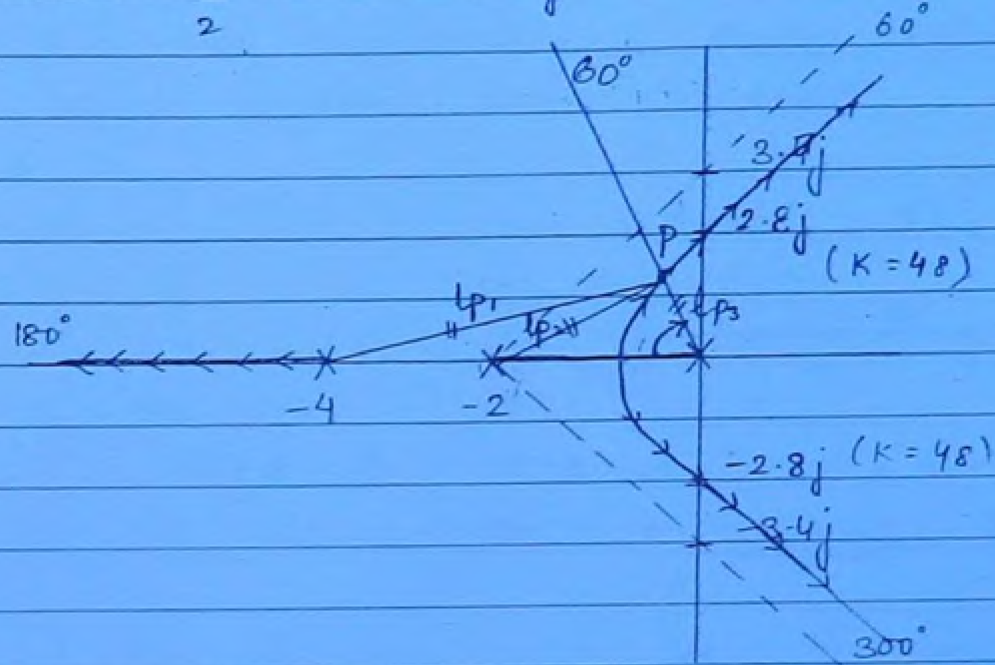
Intersection of Asymptotes with jω Axis

107

$\tan\theta = \frac{y}{x}$ $\quad\quad$ $y = \tan 60 \times 2$

$y = \sqrt{3} \times 2 = 3.4$

$\tan 60^\circ = \frac{y}{2}$ $\quad\quad$ $= \pm j\,3.4$



Q Find K for $\zeta = 0.5$ from RL?

$\theta = \cos^{-1}\zeta$

$\theta = \cos^{-1} 0.5 = 60^\circ$ (+60° from -ve X-axis in ~~anti~~ clockwise dirn)

$$\boxed{K = \frac{\text{Product of vector lengths of poles}}{\text{Product of vector lengths of zeros}} = \frac{l_{p_1} \times l_{p_2} \times l_{p_3}}{1}}$$

In exam draw to scale these lengths & find out the value of K.

Rule 8 - Angle of Departure & Arrival -

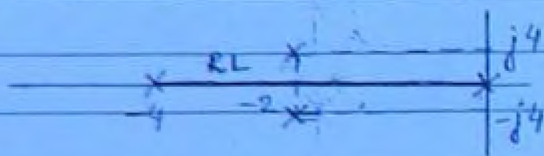
Angle of Departure is obtained when complex poles terminate at infinity

Angle of Arrival is obtained at complex zeros.

(11) $G(s)H(s) = \dfrac{K}{s(s+4)(s^2+4s+20)}$

2) $P = 4$ $Z = 0$ $P - Z = 4$

3)



4) $\theta_1 = 45°$ $\theta_2 = 135°$ $\theta_3 = 225°$ $\theta_4 = 315°$

5) Centroid $\dfrac{0+(-2)+(-2)+(-4)-0}{4} = -2$

6) B.A points

Short cut method

Average value of real poles $= \dfrac{0+(-4)}{2} = -2$ ⇒ equal to real part of complex poles. So there are 3 B A pts one b/w the region b/w poles 0 & −4 & other 2 due to complex poles

$\left|\text{avg value of real poles}\right| \times x$ = constant term of complex root

So there are 3 B A pts

$|-2| \times x = 10$

$2x = 10$

$x = 5$

$x \geq 5$ → One real & 2 complex ~~poles~~ B.A pts

$x < 5$ → 3 real B.A points

↓

comes [illegible] pole

⑧ Real poles ⇒ $s = 0, -4$

avg value $= \frac{0+(-4)}{2} = -2$

$|-2| x = 5 \Rightarrow x = 2.5$

$x < 5$

∴ 3 B.A pts real

(cont...) $s^4 + 8s^3 + 36s^2 + 80s + K = 0$

$K = -s^4 - 8s^3 - 36s^2 - 80s$

$\frac{dK}{ds} = 0 \Rightarrow 4s^3 + 24s^2 + 72s + 80 = 0$

$\Rightarrow -2, -2 \pm j2.45$

NOTE: To check the validity of complex B.A points use angle condition.

7)

s^4	1	36	K
s^3	8	80	0
s^2	26	K	0
s^1	$\frac{2080-8K}{26}$	0	0
s^0	K	0	0

$\rightarrow \frac{2080-8K}{26} > 0 \Rightarrow K < 260$

$\rightarrow K > 0$

$\boxed{0 < K < 260}$ $K = K_{max} = 260$

$A(s) = 26s^2 + K$

$= 26s^2 + 260 = 0$

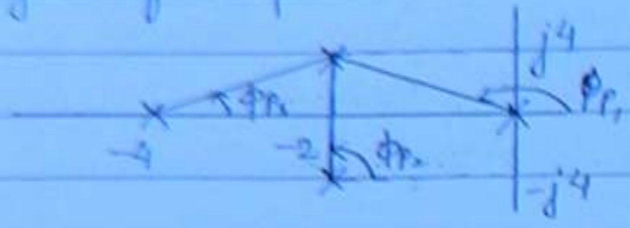
$s = \pm j3.16$

Intersection of asymptotes with $j\omega$ axis

$Y = \tan 45 \times 2 = 2 = \pm j2$

(110)

8) Angle of Departure -

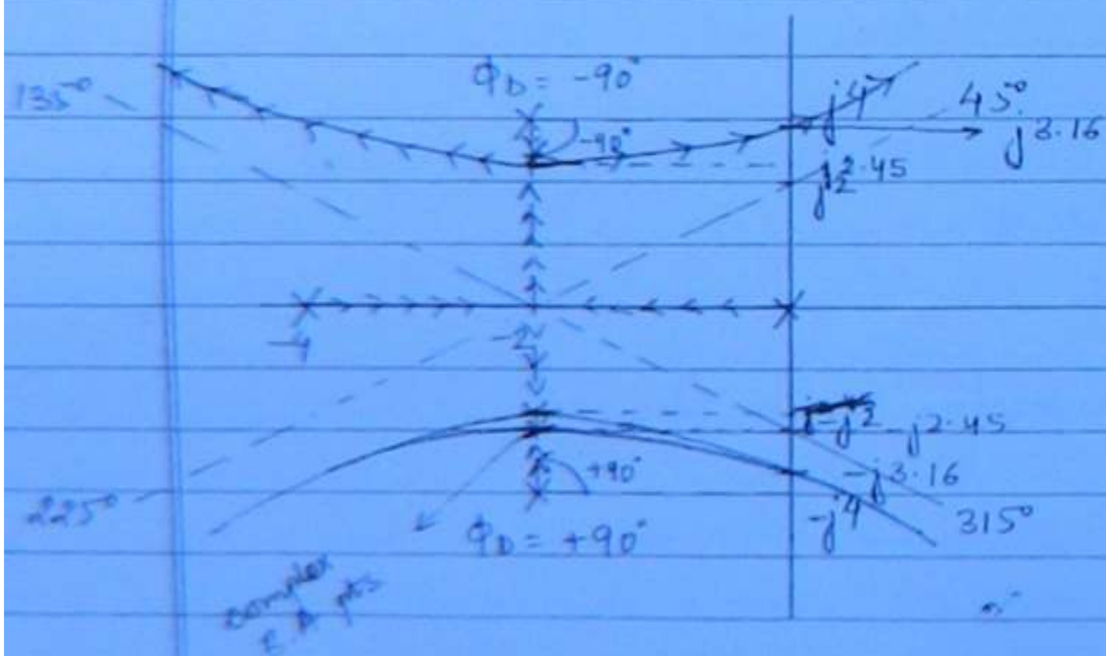


$$\phi_{P_1} = 180^\circ - \tan^{-1}\left[\frac{4-0}{0-(-2)}\right]$$
$$= 180^\circ - \tan^{-1} 2$$
$$= 180 - 63.4$$
$$= 116.6^\circ$$

$$\phi_{P_2} = 90^\circ$$
$$\phi_{P_3} = \tan^{-1}\left[\frac{4-0}{2-(-4)}\right] = 63.4$$

$$\boxed{\begin{aligned}\phi_D &= 180^\circ + \phi \\ \phi &= \Sigma\phi_Z - \Sigma\phi_P\end{aligned}}$$

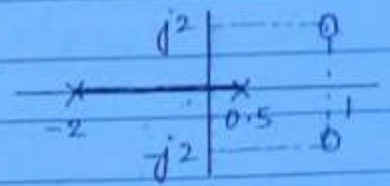
$$\phi = 0 - [116.6 + 90 + 63.4]$$
$$= -270^\circ$$
$$\phi_D = 180^\circ + \phi = 180 + (-270^\circ) = -90$$



CONV 3. $G(s) = \dfrac{K(s^2-2s+5)}{(s+2)(s-0.5)}$ (TN)

2) $P = 2$ $\quad Z = 2$ $\quad P - Z = 0$

3)



6) B.A points

$(s+2)(s-0.5) + K(s^2-2s+5) = 0$

$K = \cancel{\dfrac{-(s^2-2s+5)}{(s+2)(s-0.5)}} = \dfrac{-s^2+1.5s+1}{s^2-2s+5}$

$\dfrac{dK}{ds} = 0 \Rightarrow 3.5s^2 - 12s - 5.5 = 0$

$= \boxed{-0.4}, 3.6$ ✓

7) $s^2(1+K) + s(1.5-2K) + (5K-1) = 0$

s^2	$1+K$	$5K-1$
s^1	$1.5-2K$	0
s^0	$5K-1$	0

→ $1+K > 0 \Rightarrow K > -1$

→ $1.5-2K > 0 \Rightarrow K < 0.75$

→ $5K-1 > 0 \Rightarrow K > 0.2$

put K = -1 in the co-eff of Routh Array column 1 there will be a sign change so for system stability

$\boxed{0.2 < K < 0.75}$

$K = K_{max} = 0.75$

$A(s) = (1+K)s^2 + (5K-1)$

$= (1+0.75)s^2 + [5\times0.75-1] = 0$

$s = \pm j1.25$

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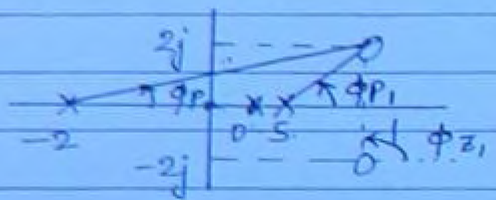
8) Angle of Arrival

$$\boxed{\begin{aligned}\phi_A &= 180^\circ - \phi \\ \phi &= \Sigma\phi_z - \Sigma\phi_P\end{aligned}}$$

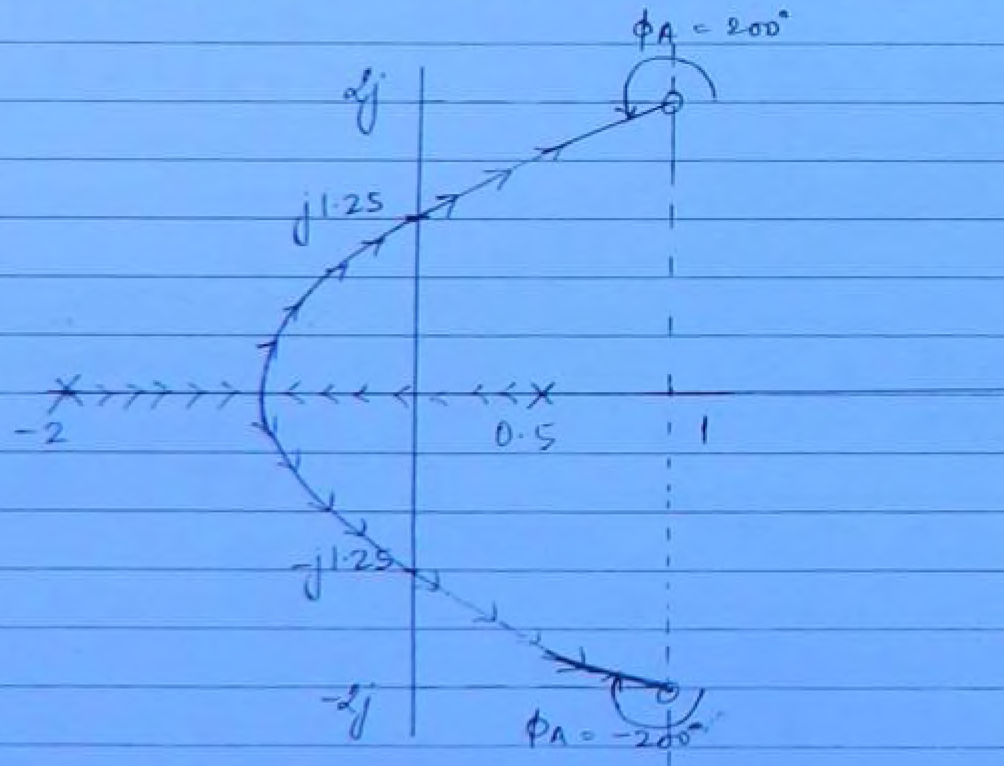


$\phi_{z_1} = 90^\circ$ $\quad \phi_{P_1} = Tan^{-1}\left[\frac{2-0}{1-0.5}\right]$ $\quad \phi_{P_2} = Tan^{-1}\left[\frac{2-0}{1-(-2)}\right]$

$\phi_{P_1} = 76^\circ$ $\quad \phi_{P_2} = 34^\circ$

$\phi = 90^\circ - [76^\circ + 34^\circ] = -20^\circ$

$\phi_A = 180^\circ + 20^\circ = 200$



(1) $G(s) = \frac{K(s+a)}{s^2(s+b)}$ (113)

Check for RL is always Routh Array.

$$s^3 + bs^2 + Ks + aK = 0$$

s^3	1	K
s^2	b	aK
s^1	$\frac{bK - aK}{b}$	0
s^0	aK	0

1) $aK > 0 \Rightarrow \boxed{K > 0}$

2) $\frac{bK - aK}{b} > 0 \Rightarrow \boxed{K > 0}$

$$\boxed{K > 0}$$

$K = K_{mar} = 0$

$A(s) = bs^2 + aK = 0$

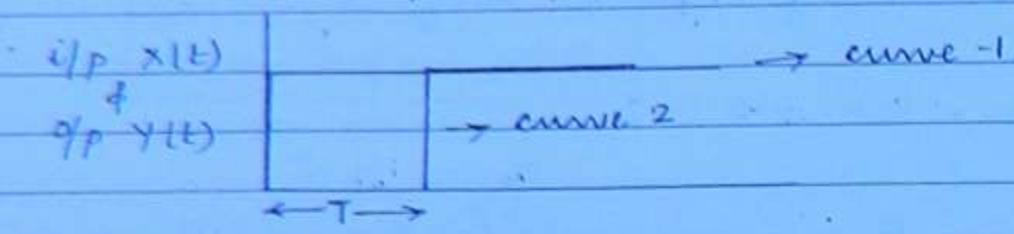
$bs^2 + 0 = 0$

$\boxed{s = 0}$ (C)

The system is stable for $K > 0$

so there is no way RL will cross Im axis

ANALYSIS OF SYSTEMS HAVING DEAD TIME (or) TRANSPORTATION LAG



For curve-1

$$o/p \; Y(t) = i/p \; X(t)$$

For curve-2

$$o/p \; Y(t) = X(t-T)$$

Applying L.T.

$$Y(s) = e^{-Ts} X(s)$$

$$\frac{Y(s)}{X(s)} = e^{-Ts}$$

I. Time Domain Approximation
[T.D Analysis, RH, RL]

$$Y(t) = X(t-T) = X(t) - T\dot{X}(t) + \frac{T^2}{2!}\ddot{X}(t) - \dots$$

$$Y(t) = X(t) - T\dot{X}(t)$$

$$Y(s) = X(s) - Ts\,X(s)$$

$$= X(s)[1-Ts]$$

$$Y(s) = X(s)e^{-Ts}$$

$$\boxed{e^{-Ts} \cong 1 - Ts}$$

eg $$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)}$$

1. Dead Time → is one of the forms of non-linearities.

2. It is approximated as a zero in RHS of s-plane

3. Those Transfer functions having poles or zeros in RHS of s-plane are known as Non-minimum Phase fn^s.

Non Minimum Phase Functions

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$$\cancel{\angle F(s)\big|_{\omega\to\infty} \neq -[P-Z]}\ 90^\circ$$

$$\angle F(s)\big|_{\omega\to\infty} \neq -[P-Z]\,90^\circ$$

eg

$$G(s) = \frac{K(1-s)}{s(s+3)}$$

$$G(j\omega) = \frac{(1-j\omega)(K+j0)}{(0+j\omega)(j\omega+3)}$$

$$\angle G(j\omega) = \frac{[0^\circ][-\tan^{-1}\omega]}{[90^\circ][\tan^{-1}\omega/3]}$$

$$= -90^\circ - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\omega$$

$$\angle G(j\omega)\big|_{\omega\to\infty} = -90^\circ - 90^\circ - 90^\circ$$

$$= -270^\circ$$

4. LTI transfer functions must be ~~non~~-minimum phase functions ie poles & zeros must lie in LHS of s-plane.

eg

$$G(s) = \frac{K(1+s)}{s(s+3)}$$

$$G(j\omega) = \frac{(K+j0)(1+j\omega)}{(0+j\omega)(j\omega+3)}$$

$$\angle G(j\omega) = \frac{[0^\circ][+\tan^{-1}\omega]}{[90^\circ][\tan^{-1}\omega/3]}$$

$$= -90^\circ - \tan^{-1}\frac{\omega}{3} + \tan^{-1}\omega$$

$$\angle G(j\omega)\Big|_{\omega=\infty} = -90^\circ - 90^\circ + 90^\circ$$

$$= -90^\circ$$

$\therefore$ Given function is a minimum phase function

— x —

$$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)}$$

Since 's' cannot be −ve
(1−s) should be expressed
as −(s−1)

$$1 + G(s)H(s) = 0 \quad \text{char. eq}^n$$

$$1 + \left[\frac{-K(s-1)}{s(s+3)}\right] = 0$$

$$1 - G(s)H(s) = 0$$

$$G(s)H(s) = 1$$

Complementary RL or Inverse RL
(CRL) (IRL)

$$1 - G(s)H(s) = 0$$

Angle condition

$\angle G(s)H(s) = 0° = \pm [2q]\, 180°$

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Magnitude condition

$|G(s)H(s)| = 1$

CONSTRUCTION RULES OF CRL -

Rule 1 → The CRL is symmetrical about real axis

$[\angle G(s)H(s) = 1]$

Rule 2 → Same as RL

Rule 3 → ~~For~~ A points on real axis is said to be on ~~root~~ CRL if to the right side of this point the sum of ~~pole~~ open loop poles & zeros is even

Rule 4 → Angle of Asymptotes

$$\theta = \frac{[2q]\, 180°}{P - Z}$$

$q = 0, 1, 2, 3 \ldots$

Rule 5 → Centroid

Same as RL

Rule 6 → Break Away Points

Same as RL

Rule 7 → Intersection of CRL with $j\omega$ axis
Same as RL

Rule 8 → Angle of Departure & Arrival

$$\phi_D = 0^\circ + \phi$$
$$\phi_A = 0^\circ - \phi$$

where

$$\phi = \Sigma\phi_z - \Sigma\phi_p$$

CWB chapter 5

8) $$G(s) = \frac{Ke^{-s}}{s(s+3)} = \frac{K(1-s)}{s(s+3)} = \frac{-K(s-1)}{s(s+3)}$$

2) $P = 2$ $Z = 1$ $P - Z = 1$

3) 

6) B.A points

$$1 + \frac{K(1-s)}{s(s+3)} = 0$$

$$s(s+3) + K(1-s) = 0$$

$$K = \frac{-s^2 - 3s}{1-s}$$

$$\frac{dK}{ds} = 0$$

$$\frac{(1-s)(-2s-3) - [(-s^2-3s)(-1)]}{(1-s)^2} = 0$$

$$-2s - 3 + 2s^2 + 3s - s^2 - 3s = 0$$

$$s^2 - 2s - 3 = 0$$

1) $s^2 + s(3-k) + K = 0$ (119)

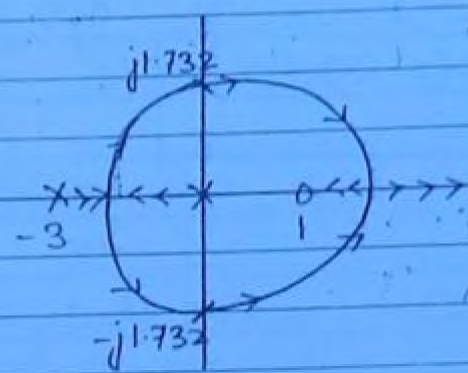
s^2	1	K
s^1	3-K	0
s^0	K	0

$3-K = 0 \Rightarrow K_{mar} = 3$

$A(s) = s^2 + K = 0.$

$s^2 + 3 = 0$

$s = \pm j1.732$



7) Damping can be determined by ξ value or location of poles

→ If poles are on Im axis - undamped

→ If poles are on real axis of s plane & repeating → critically damped

→ If poles are in 2nd & 3rd quad of s plane → underdamped

→ If poles are on real axis & unequal → overdamped

For $0 \leq K < 1$ → poles are on real axis & unequal → Overdamped

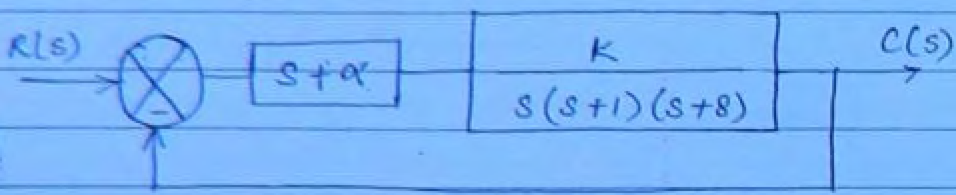
At $k = 1$ → poles repeated → marginally stable

$k > 5$ → pole on Real axis & unequal

Ans (c) Since equality cannot have equal to hence option (d) is incorrect

ROOT CONTOURS

Root Contours are multiple root locus diagrams obtained by varying multiple parameters in a Transfer function drawn on same s-plane

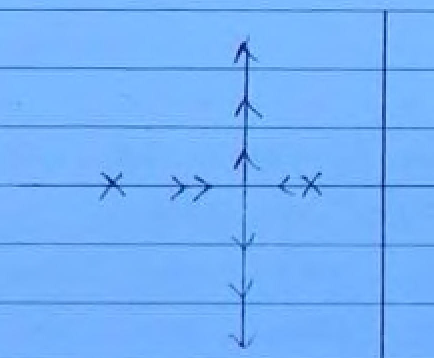


$$G(s) = \frac{K(s+\alpha)}{s(s+1)(s+8)}$$

Case 1 → Let $\alpha = 0$

$$G(s) = \frac{K.\cancel{s}}{\cancel{s}(s+1)(s+8)}$$

$$= \frac{K}{(s+1)(s+8)}$$



Case 2 → $1 + \frac{Ks + K\alpha}{s(s+1)(s+8)} = 0$

$$\underbrace{s(s+1)(s+8) + Ks} + K\alpha = 0$$

$$1 + \frac{K\alpha}{s(s+1)(s+8) + Ks} = 0$$

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = \frac{K\alpha}{s(s+1)(s+8) + Ks}$$

Put $K = 1$

$$= \frac{\alpha}{s(s^2+9s+9)}$$

Q Find B.A pts for $k = 10$?

sol. $G(s)H(s) = \dfrac{10\alpha}{s(s+1)(s+8)+10s}$

Let $10\alpha = k'$

$= \dfrac{k'}{s(s^2+9s+18)}$

$1 + G(s)H(s) = 0$

$1 + \dfrac{k'}{s(s^2+9s+18)} = 0$

$s^3 + 9s^2 + 18s + k' = 0$

$k' = -s^3 - 9s^2 - 18s$

$\dfrac{dk'}{ds} = 0 \Rightarrow 3s^2 + 18s + 18 = 0$

FREQUENCY DOMAIN ANALYSIS

When any system is subjected to sinusoidal input, the output is also sinusoidal having different magnitude & phase angle but same input frequency ω rad/sec.

$$\xrightarrow[A\sin\omega t]{r(t)} \boxed{\text{SYSTEM}} \xrightarrow[B\sin(\omega t \pm \phi)]{c(t)}$$

Frequency response analysis implies varying ω from 0 to ∞ & observing corresponding variations in the magnitude & phase angle of the response.

Let $F(s) = \dfrac{C(s)}{R(s)} = T.F$

Put $s = j\omega$

$F(j\omega)$ = Sinusoidal T.F

≈ Sinusoidal response

$$\boxed{\begin{array}{l} F(j\omega) = |F(j\omega)| \angle F(j\omega) \\ \qquad |F(j\omega)| = \sqrt{(r.p)^2 + (i.p)^2} \qquad \angle F(j\omega) = \tan^{-1}\left[\dfrac{i.p}{r.p}\right] \end{array}}$$

CWB chapter 6

(20) $$x(t) \rightarrow \boxed{\frac{1}{s+1}} \rightarrow y(t)$$

For $x(t) = \sin t$ find $y(t)$.

$$F(s) = \frac{Y(s)}{X(s)} = \frac{1}{s+1}$$

$$F(j\omega) = \frac{1+j0}{1+j\omega}$$

$$|F(j\omega)| = \frac{\sqrt{1^2+0^2}}{\sqrt{1^2+\omega^2}} = \frac{1}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \frac{\tan^{-1} 0/1}{\tan^{-1} \omega/1} = -\tan^{-1}\omega$$

$$F(j\omega) = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

Given $x(t) = \sin t$

$\approx \sin \omega t \Rightarrow \omega = 1\ r/s$

$$F(j\omega) = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\boxed{y(t) = \frac{1}{\sqrt{2}} \sin(t - 45^\circ)}$$

Q. $$G(s) = \frac{(s^2+9)(s+2)}{(s+3)(s+5)(s+7)}$$ The steady state sinusoidal response will become zero at a freq of

a) 2 r/s b) 3 r/s c) 4 r/s d) 5 r/s

sol $$G(j\omega) = \frac{(-\omega^2+9)(j\omega+2)}{(j\omega+3)(j\omega+5)(j\omega+7)} = 0$$

$$(-\omega^2+9)(j\omega+2) = 0$$

FREQUENCY RESPONSE PLOTS

I POLAR PLOT

Absolutely values of $|F(j\omega)|$ Vs ω
& $\angle F(j\omega)$ (degrees)

II BODE PLOT

db (decibel) values of $|F(j\omega)|$ Vs $\log \omega$ → logarithmic scale.
[$20 \log |F(j\omega)|$]
& $\angle F(j\omega)$ (degrees)

to be able to calibrate ω value from small values to very large power values
$\omega = 1$ to 10^9

$$\boxed{\begin{aligned} \text{Power} &= VI \rightarrow 2 \text{ variables so not considered in } v \text{ domain} \\ &= I^2 R \longrightarrow 10 \log (I^2 R) = 20 \log I + 10 \log R \\ & \qquad\qquad\qquad\qquad [Mx + C] \\ &= V^2/R \end{aligned}}$$

Frequency Response Analysis of 2nd order system -

$$F(s) = \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$F(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

Put $s = j\omega$

$$F(j\omega) = \frac{1}{\frac{(j\omega)^2}{\omega_n^2} + \frac{2\zeta j\omega}{\omega_n} + 1}$$

* $$\boxed{F(j\omega) = \frac{1}{1 - \left[\frac{\omega}{\omega_n}\right]^2 + 2j\zeta\frac{\omega}{\omega_n}}}$$ → use this to find out values of ω_n & ζ from eqn given in ques

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$$|F(j\omega)| = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}$$

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}$$

Asymptotic Approximations

$$-20\log\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

Case 1 → LOW FREQ

$1 \gg (\omega/\omega_n)^2$

$-20\log\sqrt{1} = 0\ db$

Case 2 → HIGH FREQ

$(\omega/\omega_n)^2 \gg 1$

$$-20\log\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2\right]^2}$$

$-20\log(\omega/\omega_n)^2$

$-40\log(\omega/\omega_n)$ — 1

$-40\log\omega + 40\log\omega_n$

$[Mx + C]$

Slope $(m) = -40\ \frac{db}{dec}$

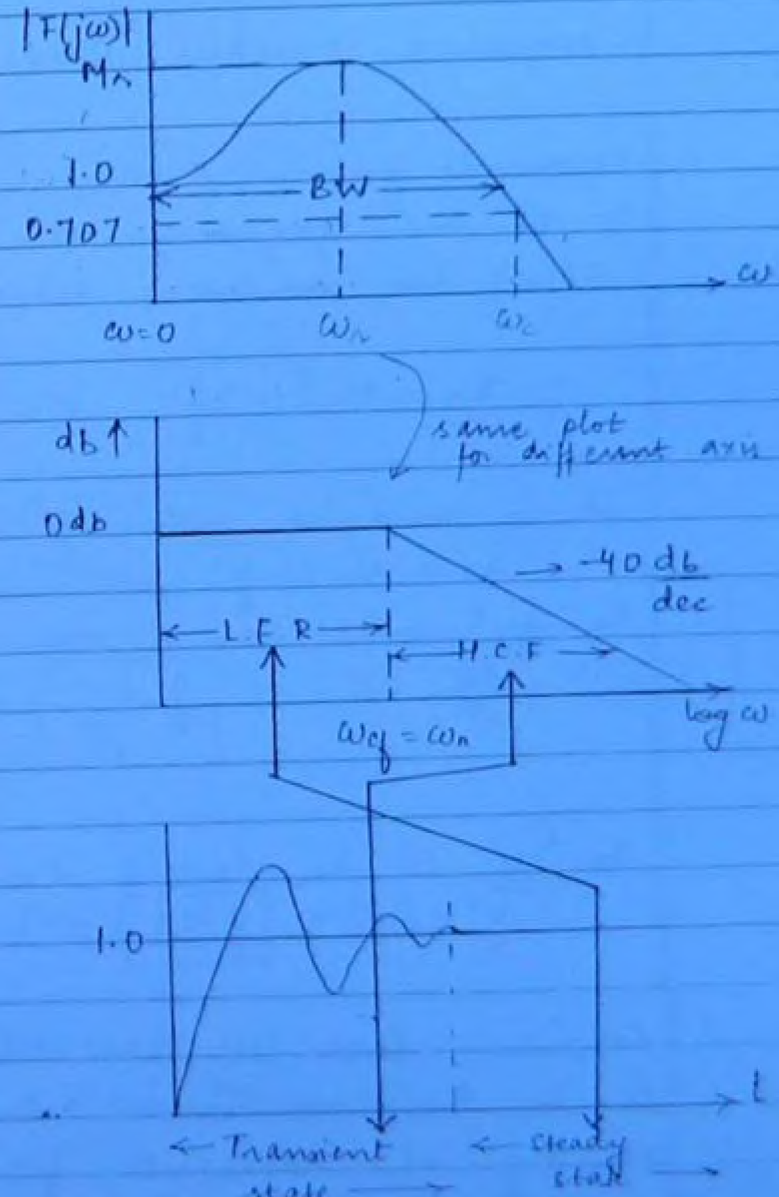
Corner frequency (ω_{cf})

$0 = -40\log(\omega/\omega_n)$

$\log\omega/\omega_n = 0$

$\omega/\omega_n = \log^{-1}0 = 1$

$\omega = \omega_{cf} = \omega_n$ r/s



Error at ω_{cf}

At $\omega = \omega_{cf} = \omega_n$.

$$= -20 \log \sqrt{(1-1)^2 + (2\xi)^2}$$

FREQUENCY DOMAIN SPECIFICATIONS -

Resonant frequency (ω_r)

It is the frequency at which magnitude of $F(j\omega)$ has maximum value

$$\omega_r = \omega_n \sqrt{1 - 2\xi^2} \quad r/s$$

It is correlated with

$$\omega_d = \omega_n \sqrt{1 - \xi^2} \quad r/s$$

For 'ω_r' to be Real & +ve

$$2\xi^2 < 1 \Rightarrow \boxed{\xi < \frac{1}{\sqrt{2}}}$$

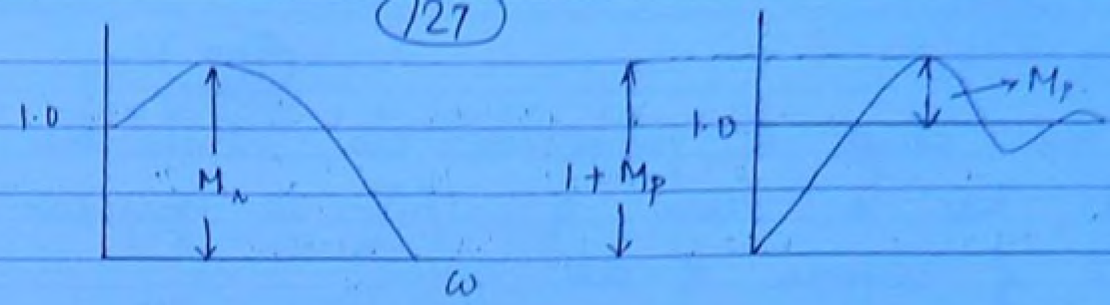
For 'ω_d' to be Real & +ve

$$\xi^2 < 1 \Rightarrow \boxed{\xi < 1}$$

Resonant Peak (or) Peak Magnitude (M_r)

It is the maximum value of magnitude occuring at resonant frequency ω_r

$$M_r = \frac{1}{\quad}$$



$\zeta < \frac{1}{\sqrt{2}}$ $M_r > 1$

$\zeta = \frac{1}{\sqrt{2}}$ $M_r = 1$

$\zeta > \frac{1}{\sqrt{2}}$ No M_r

CWB chap 6

(21) $M(j\omega) = \dfrac{100}{100 + 10\sqrt{2}j\omega - \omega^2}$

$= \dfrac{1}{1 - \frac{\omega^2}{100} + \frac{10\sqrt{2}j\omega}{100}}$

$= \dfrac{1}{1 - \left(\frac{\omega}{10}\right)^2 + j\frac{\sqrt{2}\omega}{10}}$ $\left[\dfrac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}}\right]$

$2\zeta\frac{\omega}{\omega_n} = \frac{\sqrt{2}\omega}{10}$

$\zeta = \frac{1}{\sqrt{2}}$; M_p $(M_r) = 1$ (b)

Peak mag^n means M_r value.
They have represented it as M_p

3 Bandwidth (B.W):
It is the range of frequencies over which the magnitude has a value of $1/\sqrt{2}$. It indicates the speed of response of the system.

Wider B.W ⇒ Faster response

$$BW \propto \frac{1}{t_r} \quad \text{where } t_r = \text{rise time}$$

4. Cut-off Frequency (ω_c)
It is the frequency at which the magnitude has a value of $1/\sqrt{2}$. It indicates the ability of the system to distinguish signal from noise

$$\boxed{B.W \text{ (or) } \omega_c = \omega_n\sqrt{1-2\zeta^2+\sqrt{4\zeta^4-4\zeta^2+2}}} \text{ r/s}$$

Frequency Domain Approximation of Dead Time or Transportation Lag -

$$F(s) = \frac{Y(s)}{X(s)} = e^{-Ts}$$

$$F(j\omega) = e^{-j\omega T} = \cos\omega T - j\sin\omega T$$

$$|F(j\omega)| = \sqrt{(\cos\omega T)^2 + (\sin\omega T)^2} = 1$$

$$\angle F(j\omega) = \tan^{-1}\left[-\frac{\sin\omega T}{\cos\omega T}\right] = \tan^{-1}[-\tan\omega T] = -\omega T \text{ (radians)}$$

$$\boxed{e^{-j\omega T} \cong 1 \angle -\omega T \text{ (radians)}}$$

$\pi \rightarrow 180^\circ$

$-\omega T \rightarrow ?$

$$\Rightarrow \frac{-\omega T \times 180}{\pi}$$

$$= -57.3\ \omega T \text{ (degrees)}$$

$$\boxed{e^{-j\omega T} \cong 1 \angle -57.3\omega T \text{ (degrees)}}$$

STABILITY FROM FREQUENCY RESPONSE PLOTS -

$$1 + G(s)H(s) = 0$$

$$G(s)H(s) = -1$$

Put $s = j\omega$

$$G(j\omega)H(j\omega) = -1 + j0$$

(critical point)

Stability Criteria

1. Gain crossover frequency (ω_{gc})

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{gc}} = 1 \text{ or } 0\,db$$

2. Phase crossover frequency (ω_{pc})

$$\angle G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = -180^\circ$$

8. Gain Margin (G.M)
It is the "allowable gain"

$$|G(j\omega)H(j\omega)|_{\omega=\omega_{pc}} = X \qquad G.M = \frac{1}{X} \qquad G.M(db) = 20\log\left(\frac{1}{X}\right)$$

9. Phase Margin (P.M)
It is the "allowable phase lag"

$$\angle G(j\omega)H(j\omega)\Big|_{\omega=\omega_{gc}} = \phi$$

$$PM = 180^\circ + \phi$$

STABLE $\Rightarrow$ GM & PM = +ve
$\Rightarrow \omega_{gc} < \omega_{pc}$

UNSTABLE $\Rightarrow$ GM & PM = −ve
$\Rightarrow \omega_{gc} > \omega_{pc}$

MARGINALLY STABLE $\Rightarrow$ GM = PM = 0
$\Rightarrow \omega_{gc} = \omega_{pc}$

GM & PM for second order system –

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2 - \omega_n^2}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s} = \frac{\omega_n^2}{s(s + 2\zeta\omega_n)}$$

$G(j\omega)$

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$$|G(j\omega)| = \frac{\omega_n^2}{\omega\sqrt{\omega^2 + 4\zeta^2\omega_n^2}}$$

$$\angle G(j\omega) = -90^\circ - \tan^{-1}\left(\frac{\omega}{2\zeta\omega_n}\right)$$

At $\omega = \omega_{gc}$

$$\frac{\omega_n^2}{\omega\sqrt{\omega^2 + 4\zeta^2\omega_n^2}} = 1$$

$$\omega_n^4 = \omega^2(\omega^2 + 4\zeta^2\omega_n^2)$$

$$\omega^4 + \omega^2 4\zeta^2\omega_n^2 - \omega_n^4 = 0$$

$$\frac{-4\zeta^2\omega_n^2 \pm \sqrt{16\zeta^4\omega_n^4 + 4\omega_n^4}}{2}$$

$$-2\zeta^2\omega_n^2 \pm \omega_n^2\sqrt{4\zeta^4 + 1}$$

$$\omega^2 = -2\zeta^2\omega_n^2 + \omega_n^2\sqrt{4\zeta^4 + 1}$$

$$\omega = \boxed{\omega_{gc} = \omega_n\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}} \text{ r/s}$$

$$\angle G(j\omega)\Big|_{\omega = \omega_{gc}} = \phi$$

$$\phi = -90^\circ - \tan^{-1}\left(\frac{\cancel{\omega_n}\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}{2\zeta\cancel{\omega_n}}\right)$$

$$PM = 180^\circ + \phi$$

$$\boxed{PM = 90^\circ - \tan^{-1}\left[\frac{\sqrt{-2\zeta^2 + \sqrt{4\zeta^4 + 1}}}{2\zeta}\right]}$$

At $\omega = \omega_{pc} = \infty$ r/s $\quad \angle G(j\omega) = -90° - 90°$

$= -180°$

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$|G(j\omega)|_{\omega=\omega_{pc}} = \infty = X = 0$

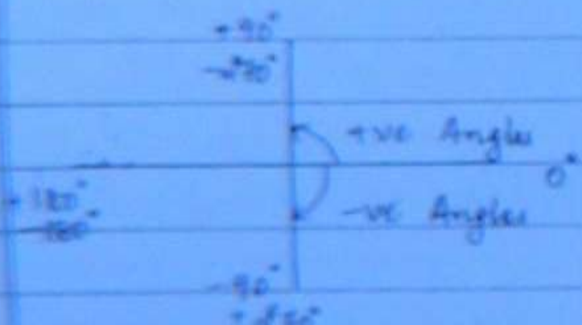
$$G.M = \frac{1}{X} = \frac{1}{0} = \infty$$

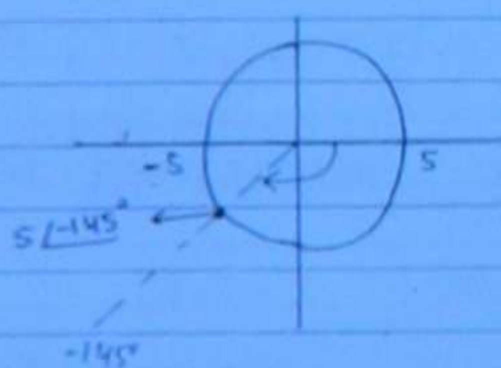
$\boxed{\omega_{pc} = \infty \text{ r/s}}$ $\quad \boxed{G.M = \infty}$

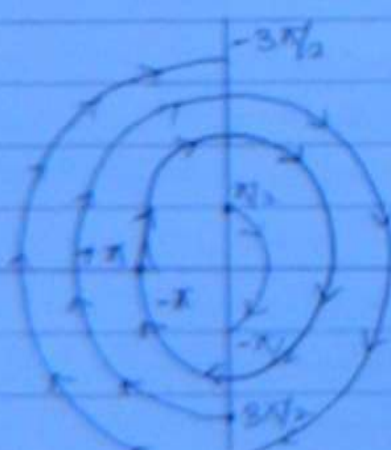
POLAR PLOT

It is a plot of absolute values of magnitude & phase angle in degrees of open loop transfer function. $G(j\omega)\,H(j\omega)$ vs ω drawn on Polar co-ordinates

Polar Coordinates $\qquad$ eg $5\angle -145°$







$G(s) = \frac{1}{s+1}$ Always convert to time constant form

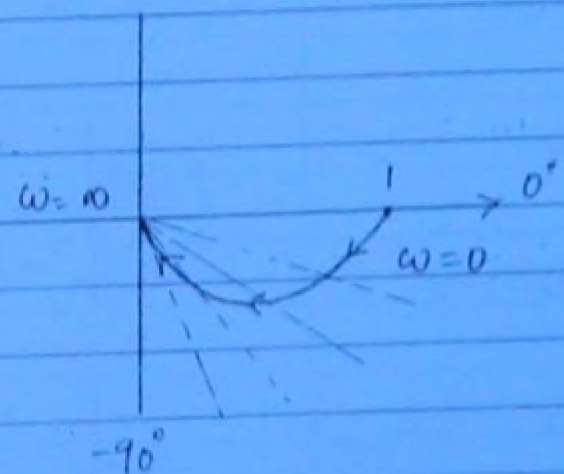
$G(s) = \frac{1}{1+s}$ Type = 0 , Order = 1 system. 133

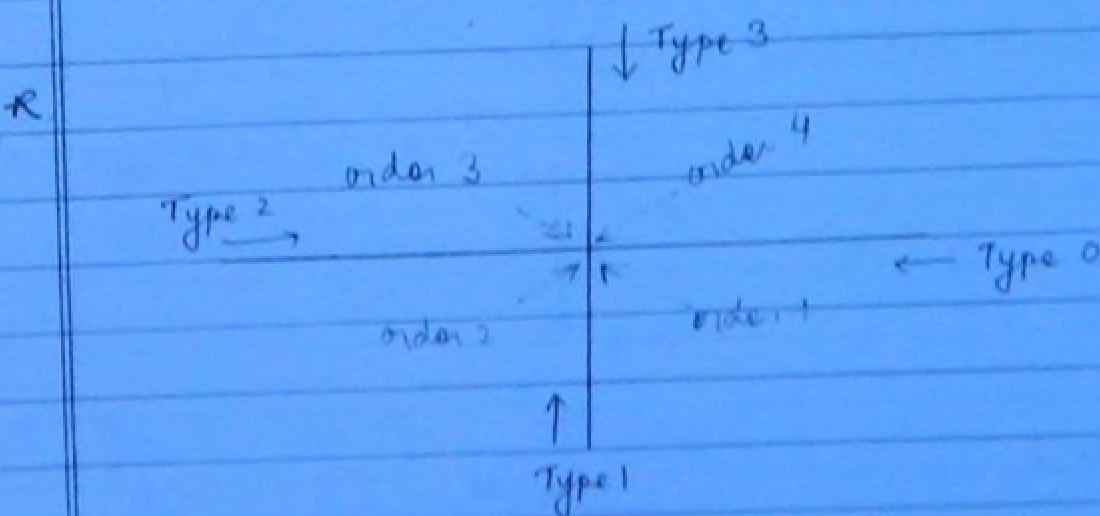
$G(j\omega) = \frac{1}{1+j\omega}$

$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$

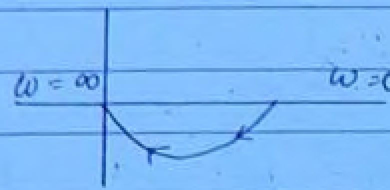
$\angle G(j\omega) = -\tan^{-1}\omega$

ω	0		∞
$\lvert G(j\omega)\rvert$	1		0
$\angle G(j\omega)$	0°		-90°





General shapes of Polar Plots -

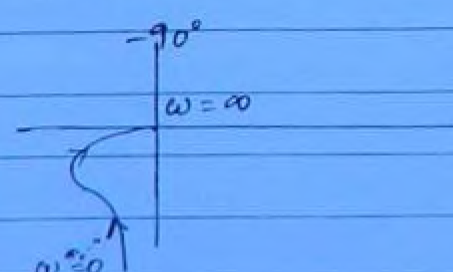
Type / Order	Polar Plot
1. Type 0 / Order 1 $G(s) = \frac{1}{1+Ts}$	
2. Type 0 / Order 2 $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)}$	
3. Type 0 / Order 3 $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)}$	
4. Type 0 / Order 4 $G(s) = \frac{1}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)}$	

5. Type 1 / Order 1

$$G(s) = \frac{1}{s} = \frac{1}{j\omega} = \frac{1}{\omega} \angle -90^\circ$$

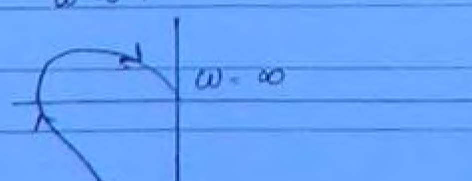
6. Type 1 / Order 2

$$G(s) = \frac{1}{s(1+T_1 s)}$$



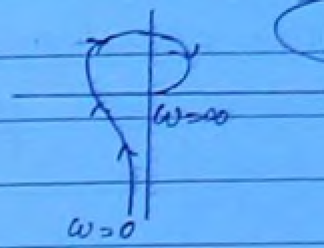
7. Type 1 / Order 3

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)}$$



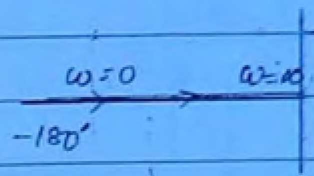
8. Type 1 / Order 4

$$G(s) = \frac{1}{s(1+T_1 s)(1+T_2 s)(1+T_3 s)}$$



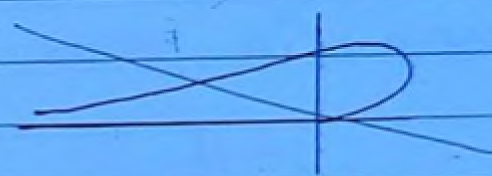
9. Type 2 / Order 2

$$G(s) = \frac{1}{s^2} = \frac{1}{(j\omega)^2} = \frac{1}{\omega^2}\angle -180^\circ$$



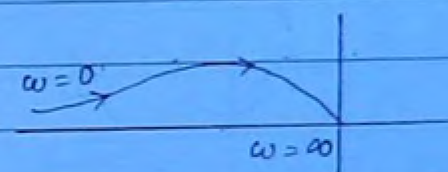
~~10. Type 2 / Order 3~~

$$\cancel{G(s) = \frac{1}{s^2(1+Ts)}}$$



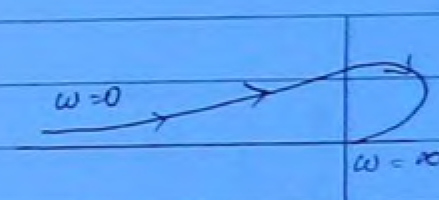
10. Type 2 / Order 3

$$G(s) = \frac{1}{s^2(1+Ts)}$$



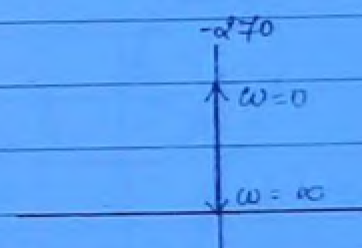
11. Type 2 / Order 4

$$G(s) = \frac{1}{s^2(1+T_1 s)(1+T_2 s)}$$



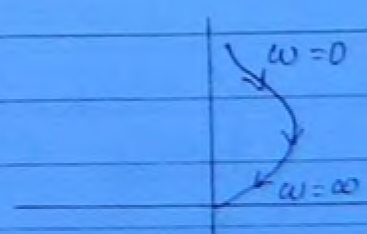
12. Type 3 / Order 3

$$G(s) = \frac{1}{s^3} = \frac{1}{(j\omega)^3} = \frac{1}{\omega^3}\angle -270^\circ$$

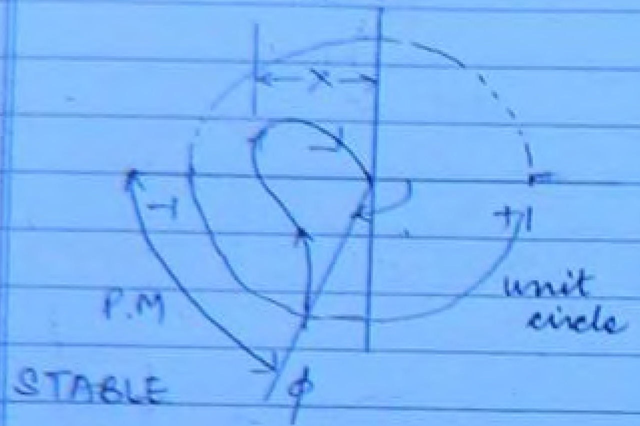


13. Type 3 / Order 4

$$G(s) = \frac{1}{s^3(1+Ts)}$$



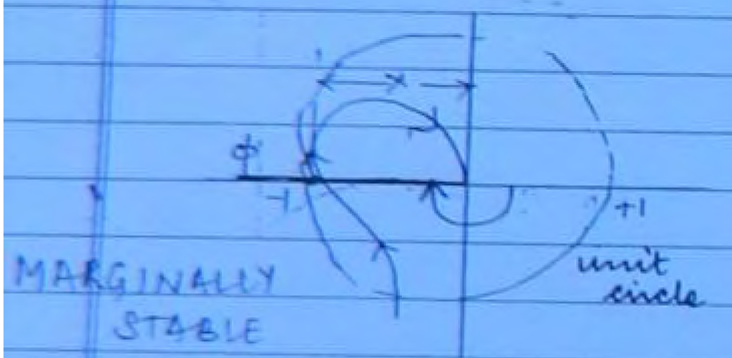
STABILITY FROM POLAR PLOTS -



$G.M = \frac{1}{x}$ $\quad$ $G.M\,(dB) = 20 \log \frac{1}{x} = +ve$

$PM = \phi - [-180^\circ] = 180^\circ + \phi$

$= +ve$

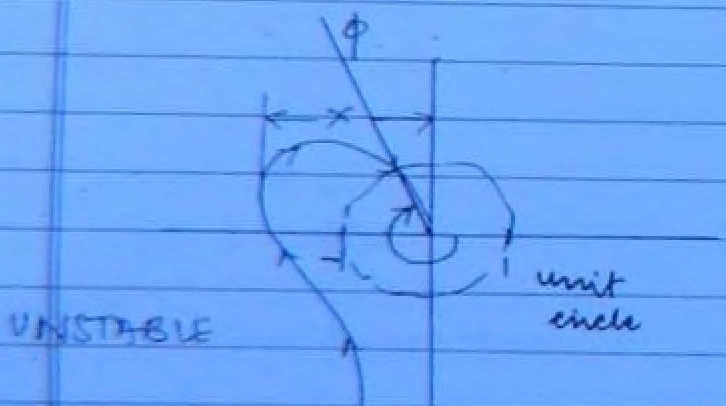


$x = 1$ $\quad$ $G.M = \frac{1}{x} = \frac{1}{1}$

$G.M\,(db) = 20 \log 1 = 0\,db$

$\phi = -180^\circ$

$PM = 180 - 180 = 0^\circ$



$GM = \frac{1}{x}$ $\quad$ $GM\,(db) = 20 \log \frac{1}{x} = -ve$

$PM = 180^\circ + \phi = -ve$

Concept of Enclosement & Encirclement

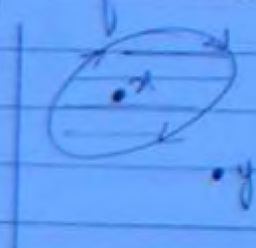


fig (i)

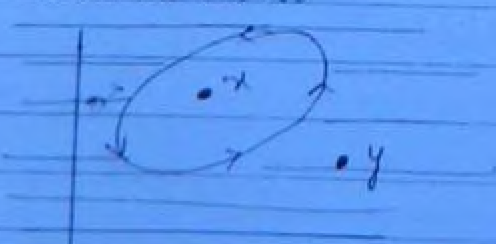


fig (ii)

A point is said to be enclosed by a contour if it lies to the right side of the direction of the contour.

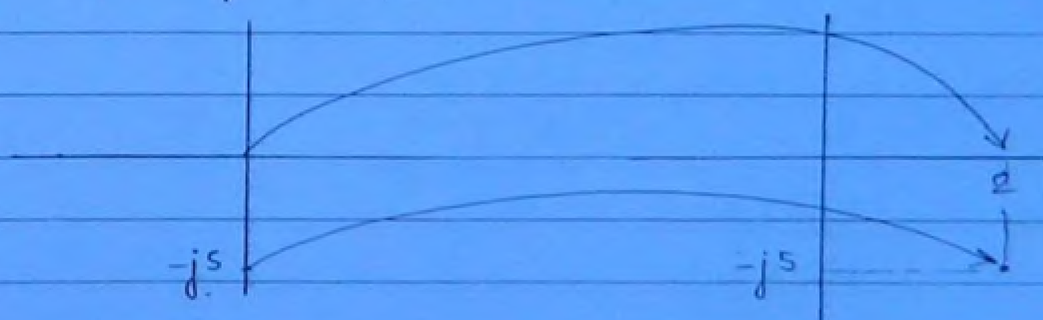
A point is said to be encircled if the contour is a closed path.

In fig ② pt y is said to be enclosed whereas point x is said to be encircled in anticlockwise direction.

In Polar plots if the critical point $-1 + j0$ is not enclosed then the system is said to be stable.

THEORY OF NYQUIST PLOTS

Principle of Mapping



S - plane

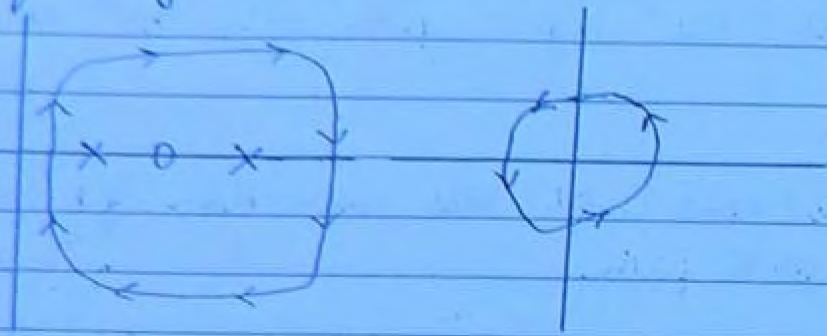
P(s) - plane

$P(s) = s + 2$

$P(0) = 0 + 2 = 2$

$P(-j5) = -j5 + 2$

Principle of Arguments



s-plane $\qquad$ P(s) - plane

$$P(s) = \frac{(s-b)}{(s-a)(s-c)}$$

$N = P - Z$

N = No. of encirclements
= +ve (Anticlockwise)
= -ve (clockwise)

P = No. of poles in RHS of s-plane

Z = No. of zeros in RHS of s-plane

Nyquist Stability Criteria -

$$G(s)H(s) = \frac{K(s \pm z_1)}{s(s \pm p_1)}$$

$$1 + G(s)H(s) = 0$$

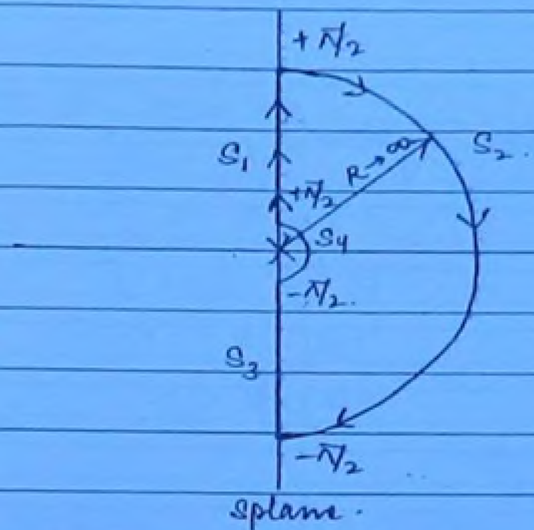
$$1 + \frac{K(s \pm z_1)}{s(s \pm p_1)} = 0$$

$$\frac{s(s \pm p_1) + K(s \pm z_1) \rightarrow \text{C.L. poles}}{s(s \pm p_1) \rightarrow \text{O.L poles}} = 0 \quad \text{——} \quad (1)$$

P = No of Open loop poles in RHS of s-plane

Z = No of closed loop poles in RHS of s-plane

NYQUIST PATH



→ To map S_1
Polar plot

→ To map S_2
Put $s = \lim_{R\to\infty} Re^{j\theta}$ $[\theta = +\pi/2 \to -\pi/2]$

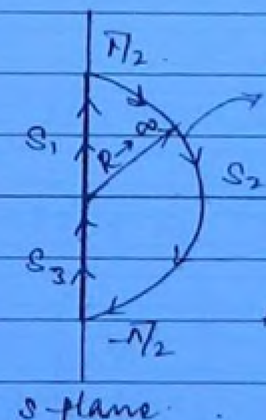
→ To map S_3
Inverse polar plot

→ To map S_4
Put $s = \lim_{R\to 0} Re^{j\theta}$ $[\theta = -\pi/2 \to \pi/2]$

$$G(s) = \frac{10}{(s+2)(s+4)}$$

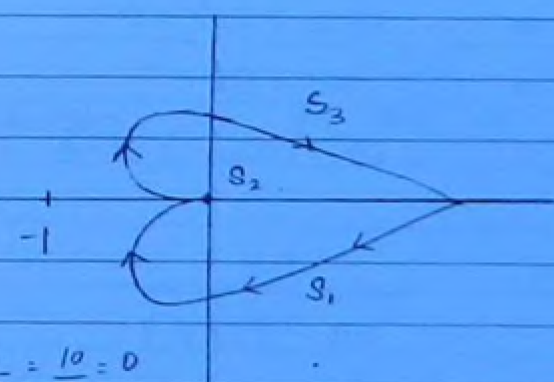
$N = P - Z$

$0 = 0 - Z \Rightarrow \boxed{Z = 0}$ STABLE



This is at ∞. $\underbrace{(s+2)(s+4)}$

∞ dominates not 2 or 4 so in Nyquist they meet at origin eqn $\frac{10}{s^2} = \frac{10}{\infty} = 0$



$$G(s) = \frac{10}{(s+2)(s+4)}$$

To map S_2

$$G(s) = \frac{10}{s^2} = \frac{10}{\lim_{R\to\infty}(Re^{j\theta})^2} = \frac{10}{\lim_{R\to\infty} R^2 e^{j2\theta}} = 0e^{-j2\theta}$$

$$G(s) = \frac{100}{(s+2)(s+4)(s+8)}$$

calculate ω_{pc}

Short-cut method (for 3rd order system)

$$\frac{100}{(-\omega^2 + 6j\omega + 8)(j\omega + 8)}$$

$$\Rightarrow \frac{100}{-j\omega^3 - 8\omega^2 - 6\omega^2 - 48j\omega + 8j\omega + 64}$$

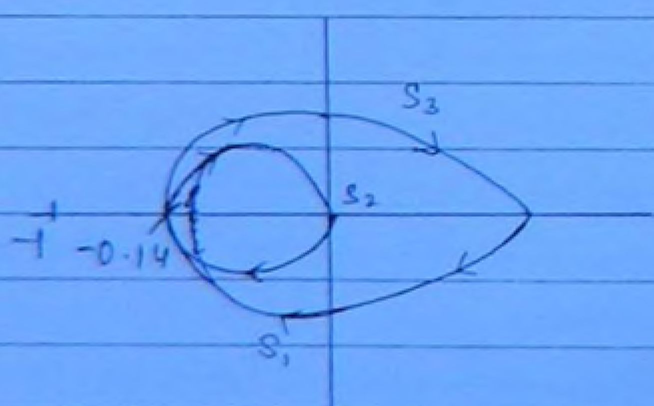
$$\Rightarrow \frac{100}{64 - 14\omega^2 + j[56\omega - \omega^3]}$$

$\Rightarrow \quad 56\omega - \omega^3 = 0$

$\Rightarrow \quad \omega^2 = 56$

$\Rightarrow \quad \omega = \omega_{pc} = \sqrt{56} = 7.4 \ r/s.$

$$\Rightarrow \frac{100}{64 - 14(7.4)^2} = -0.14.$$



$N = P - Z$

$0 = 0 - Z \quad \Rightarrow Z = 0$

STABLE

Date
Page

chapter 6.

CONV

(4) (ii) $G(s) = \dfrac{s+2}{(s+1)(s-1)}$ (T4)

sol

→ To map S_1

Polar plot.

$$\frac{2[1+0.5s]}{1[1+s](-1)[1-s]}$$

$$= \frac{-2(1+0.5j\omega)}{(1+j\omega)(1-j\omega)}$$

$-180°$

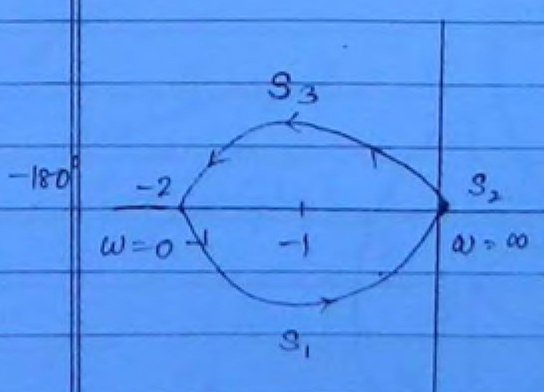
$$|G(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}\sqrt{1+\omega^2}} = \frac{2\sqrt{1+(0.5\omega)^2}}{1+\omega^2}$$

$$\angle G(j\omega) = \frac{(-180°)(\tan^{-1}0.5\omega)}{(\tan^{-1}\omega)(-\tan^{-1}\omega)}$$

NOTE : $-$ve Gain $(-K + j0)$ contributes $-180°$ for all ω

$-180° + \tan^{-1} 0.5\omega$

ω	0		∞
$\lvert G(j\omega)\rvert$	2		0
$\angle G(j\omega)$	$-180°$		$-90°$



$N = P - Z$

Q. Without constructing Nyquist Plot find the no. of encirclements about critical pt ?

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Sol: Nyquist Plot in freq domain is equivalent to Routh Array in time domain for stability check
(Z of nyquist = no. of sign changes in RA)

$$1 + G(s) = 0$$

$$1 + \frac{(s+2)}{(s+1)(s-1)} = 0$$

$$s^2 - s + s - 1 + s + 2 = 0$$

$$s^2 + s + 1 = 0$$

s^2	1	1	1
s^1	1	1	0
s^0	1	1	0

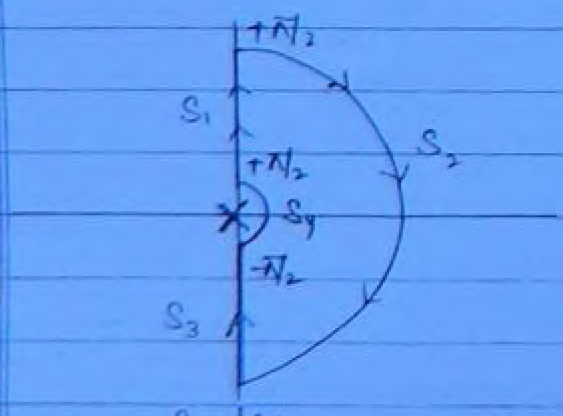
$$N = P - Z$$

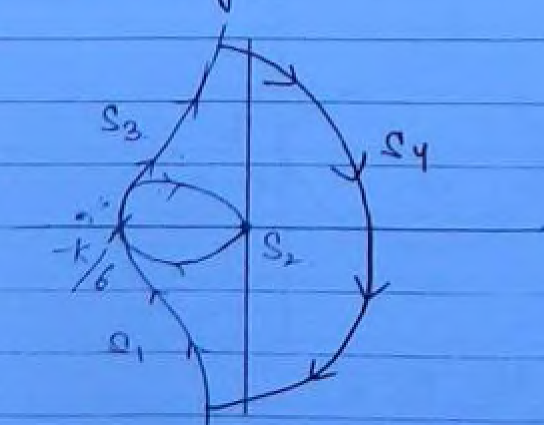
$$N = 1 - 0$$

$$N = 1$$

Q. $G(s) = \frac{K}{s(s+1)(s+2)}$ Find the range of K for stability ?

Sol:





To map S_4 ⇒

$$G(s) = \frac{K}{s(1)(2)} = \frac{0.5K}{s} = \frac{0.5K}{\lim_{R \to 0} Re^{j\theta}} = \infty e^{-j\theta} \qquad [\theta = -\pi/2 \to \pi/2]$$

$$\infty e^{j\pi/2} \to \infty e^{-j\pi/2}$$

$$\frac{K}{j\omega(-\omega^2 + 3j\omega + 2)}$$

$$\frac{K}{-3\omega^2 + j[2\omega - \omega^3]}$$

$$2\omega - \omega^3 = 0$$

$$\omega^2 = 2$$

$$\Rightarrow \omega = \omega_{pc} = \sqrt{2} \text{ r/s}$$

$$\frac{K}{-3(\sqrt{2})^2} = \frac{-K}{6}$$

$$\frac{-K}{6} = -1 \quad \Rightarrow \quad K_{max} = 6$$

case 1) $K > 6$

put $K = 12$ the pt is -2; -1 is encircled 2 times in clockwise dir

$$-2 = 0 - Z$$

$$Z = 2$$

unstable

case 2) $K < 6$

$$0 = 0 - Z$$

$$Z = 0$$

stable

(is distance) no -ve taken as we always

$$\underline{X} = \frac{K}{6}$$

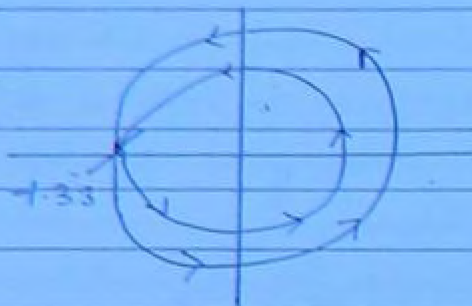
$$GM = \frac{1}{X} = \frac{6}{K}$$

$GM \propto \frac{1}{K}$ GM ↓ K ↑   stability ↓

K ↑ stability ↓

(27) $G(s) = \dfrac{k(s+3)(s+5)}{(s-2)(s-4)}$

For $k = 1$.



$-1.33\,k = -1$

$K_{max} = \dfrac{1}{1.33} = 0.75$

case 1) $k < \dfrac{1}{1.33}$

$0 = 2 - Z$

$Z = 2$

unstable

case 2) $k > \dfrac{1}{1.33}$

$2 = 2 - Z$

2 encirclement in anti clkwise dirⁿ

$Z = 0$

stable.

$\therefore\ k > \dfrac{1}{1.33}$ for stability.

(26) Given $P = 0$

the critical pt inside R-I

R - I

$1 - 1 = 0 - Z$

$Z = 0$

Stable $\longrightarrow$

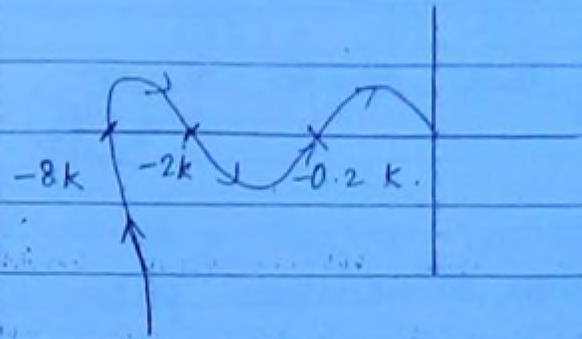
R - II

$-2 = 0 - Z$

$Z = 2$

Unstable

(18)



1) $-0.2k = -1$

$K_{max} = 5$

$k < 5$

stable (cuz -1 is not enclosed in RHS of contour)

2) $-2k = -1$

$K_{max} = \frac{1}{2}$

$k > \frac{1}{2}$ Stable cuz (for same reason)

3) $-8k = -1$

$K_{max} = \frac{1}{8}$

$k < \frac{1}{8}$ Stable.

$\frac{1}{2} < k < 5$ ff $k < \frac{1}{8}$ (b)

Effect of adding zeros on the shape of Polar or Nyquist plots. (T46)

1. For minimum phase or non-minimum phase functions of any type, when zeros are present check for intersection of polar plot with −ve Real axis.

2. For type 2 & type 3 standard open loop functions when zeros are added before the location of poles then the polar plot intersects the negative real axis as many times as there are zeros. dont consider poles at origin.

$$G(s) = \frac{1}{s^2(1+s)(1+2s)}$$

$$-180^\circ - \tan^{-1}\omega - \tan^{-1}2\omega$$

$\omega = 0$ $|G(j\omega)| = \infty$ $\angle G(j\omega) = -180^\circ$

$\omega = \infty$ $|G(j\omega)| = 0$ $\angle G(j\omega) = -360^\circ$

To map s_4.

$$G(s) = \frac{1}{s^2} = \frac{1}{\lim_{R \to 0} R^2 e^{j2\theta}} = \infty e^{-j2\theta}$$

$$[\theta = -\pi/2 \to \pi/2]$$

$$= \infty e^{j\pi} \to \infty e^{-j\pi}$$

go to −180° from 0° & draw the plot from −180° to [illegible] (ccl)

$$-2 = 0 - Z$$

$$\boxed{Z = 2}$$

unstable

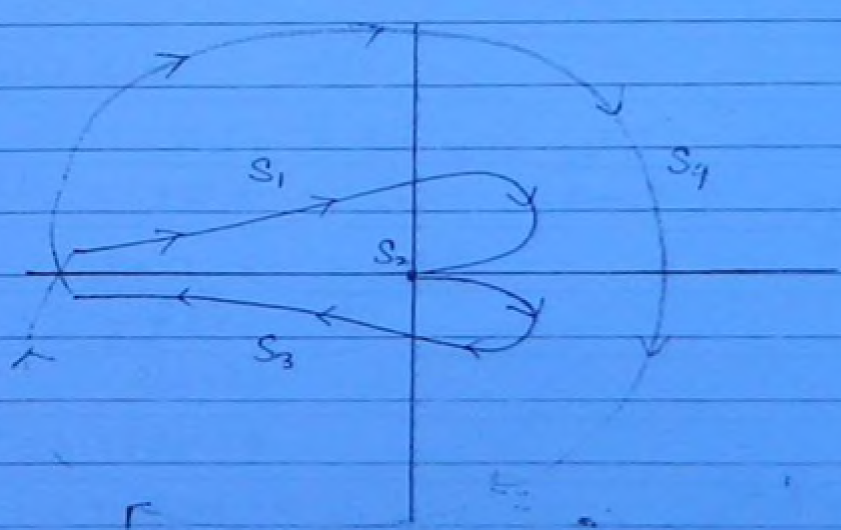
$$G(s) = \frac{1+4s}{s^2(1+s)(1+2s)}$$ T47

$-180° - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega$

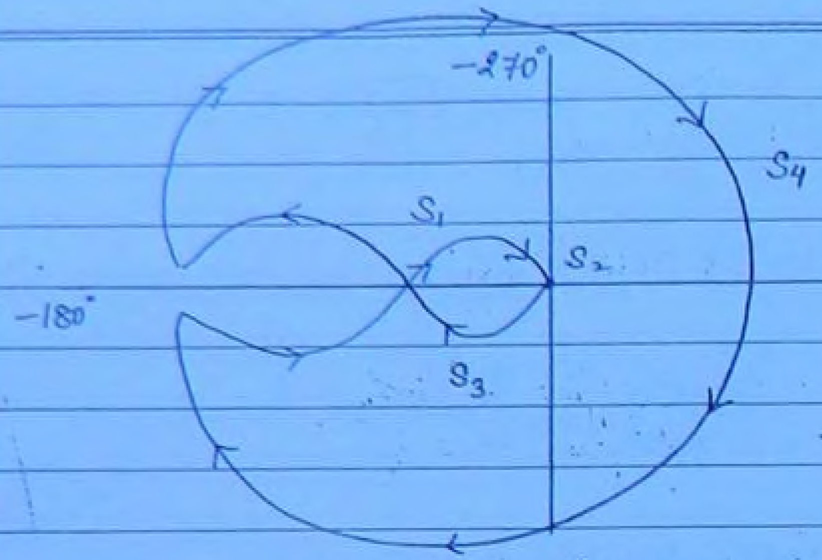
$\omega = 0 \quad |G(j\omega)| = \infty \quad \angle G(j\omega) = -180°$
$\omega = \infty \quad |G(j\omega)| = 0 \quad \angle G(j\omega) = -270°$

$$-180° - \tan^{-1}\omega - \tan^{-1}2\omega + \tan^{-1}4\omega = -180°$$
$$\tan^{-1}4\omega = \tan^{-1}\omega + \tan^{-1}2\omega$$
$$4\omega = \frac{\omega + 2\omega}{1 - 2\omega^2}$$
$$4 - 8\omega^2 = 3 \Rightarrow \omega = \omega_{pc} = \frac{1}{\sqrt{8}} \text{ r/s.}$$

$$x = \frac{\sqrt{1 + (4/\sqrt{8})^2}}{\left(\frac{1}{\sqrt{8}}\right)^2 \sqrt{1 + \left(\frac{1}{\sqrt{8}}\right)^2} \sqrt{1 + \left(\frac{2}{\sqrt{8}}\right)^2}} = 10.6$$



Nyquist for $G(s) = \frac{1}{s^2(1+s)(1+2s)}$



(3)

$$\frac{1}{j\omega\left[1 + j\omega(T_1+T_2) - \omega^2 T_1 T_2\right]}$$

$$\Rightarrow \frac{1}{-\omega^2(T_1+T_2) + j\left[\omega - \omega^3 T_1 T_2\right]}$$

$$\omega - \omega^3 T_1 T_2 = 0$$

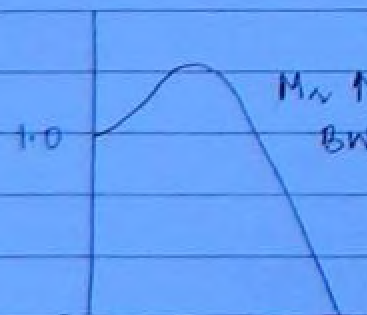
$$1 - \omega^2 T_1 T_2 = 0$$

$$\omega = \omega_{pc} = \frac{1}{\sqrt{T_1 T_2}} \text{ r/s.}$$

$$\frac{1}{-\left[\frac{1}{\sqrt{T_1 T_2}}\right]^2 (T_1+T_2)} \Rightarrow \frac{-T_1 T_2}{T_1 + T_2} \Rightarrow X = \frac{+T_1 T_2}{T_1 + T_2}$$

$$GM = \frac{1}{X} = \frac{T_1 + T_2}{T_1 T_2}$$

(11)



$M_r \uparrow$, $BW \downarrow$ → Less stable system

(9) $\% M_p = 50\%$

$T = 0.2$ secs

(149)

$M_p = 0.5$

$e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 0.5$

$\zeta = 0.215$

$f_d = \frac{1}{T} \Rightarrow f_d = \frac{1}{0.2} = 5\ Hz.$

$\omega_d = 2\pi f_d$

$\omega_d = 2\pi(5) = 31.41\ r/s.$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$31.41 = \omega_n\sqrt{1-(0.215)^2}$

$\omega_n = 32.16\ r/s.$

$\omega_r = \omega_n\sqrt{1-2\zeta^2}$

$\omega_r = 32.16\sqrt{1-2(0.215)^2}$

$\omega_r = 30.63\ r/s.$

(13) $G(s)H(s) = \frac{2\sqrt{3}}{s(s+1)}$

$PM = 100\,\zeta$ for order 2 systems

$1 + \frac{2\sqrt{3}}{s(s+1)} = 0$

$s^2 + s + 2\sqrt{3} = 0$

$s^2 + s + 3.46 = 0$

$\omega_n = \sqrt{3.46} = 1.86\ r/s$

$2\zeta \times 1.86 = 1$

$\zeta = 0.27$

Q The value of a to give PM = 45°

$$G(s) = \frac{as+1}{s^2}$$

a) $\sqrt{2}$ 1.414 b) $\frac{1}{\sqrt{2}}$ 0.707 c) $\sqrt{\sqrt{2}}$ 1.18 d) $\frac{1}{\sqrt{\sqrt{2}}}$ 0.84

150

Sol

$$1 + G(s) = 0$$

$$1 + \frac{as+1}{s^2} = 0$$

$$s^2 + as + 1 = 0$$

$$\omega_n = 1 \text{ r/s}$$

$$2\xi \times 1 = a$$

$$\xi = \frac{a}{2}$$

$$PM = 45°$$

$$45° = 100\,\xi$$

$$\xi = \frac{45}{100} = 0.45$$

$$\xi = 0.45 = \frac{a}{2}$$

$\underline{\underline{a = 0.9}}$

Ans (d)

(14)

$$G(s) = \frac{1}{s(s^2+s+1)}$$

$$G(j\omega) = \frac{1}{j\omega(1-\omega^2+j\omega)}$$

$$|G(j\omega)| = \frac{1}{\omega\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$\angle G(j\omega) = -90° - \tan^{-1}\left(\frac{\omega}{1-\omega^2}\right)$$

At $\omega = \omega_{pc} = 1 \text{ r/s}$

$$|G(j\omega)|\Big|_{\omega=\omega_{pc}=1} = x = \frac{1}{1\sqrt{(1-1)^2+1^2}} = 1$$ 151

$$GM = \frac{1}{x} = \frac{1}{1} = 1 \qquad GM(db) = 20\log 1 = 0\ db.$$

(17) Poles = 0.01 Hz ~~1 Hz~~ ~~80 Hz~~

Zeros ~~5 Hz~~ ~~100 Hz~~ ~~200 Hz~~

due to dominant pole effect

$$f = 20\ Hz \Rightarrow \omega = 2\pi \times 20$$
$$= 125\ r/s$$

$$F(s) \propto \frac{1}{1+Ts} \quad \angle F(j\omega) = -\tan^{-1}\omega T$$

At max^m value of $\omega = \infty$, $\angle F(j\omega) = -90°$

for any higher value of ω $\angle F(j\omega) \approx -90°$ (a)

(19) $$G(s) = \frac{3e^{-2s}}{s(s+2)}$$

$$G(j\omega) = \frac{3e^{-2j\omega}}{j\omega(j\omega+2)}$$

$$|G(j\omega)| = \frac{3\times 1}{\omega\sqrt{\omega^2+4}}$$

$$\angle G(j\omega) = -90° - \tan^{-1}\frac{\omega}{2} - 57.3\times 2\omega$$

At $\omega = \omega_{gc}$

$$\frac{3}{\omega\sqrt{\omega^2+4}} = 1$$

$$9 = \omega^2(\omega^2+4) \Rightarrow \omega^4 + 4\omega^2 - 9 = 0$$

$$\frac{-4 \pm \sqrt{16+36}}{2}$$

-2 ± 3.6

$1.6\ ,\ -5.6$

$\omega^2 = 1.6$

$\omega = \omega_{gc} = \sqrt{1.6} = 1.26\ r/s$

$\angle G(j\omega) \Big|_{\omega = \omega_{gc} = 1.26\ r/s} = \phi.$

$\phi = -90^\circ - \tan^{-1}\left(\frac{1.26}{2}\right) - 57.3 \times 2 \times 1.26$

$\phi = -267.5$

$PM = 180^\circ + \phi = 180^\circ - 267.5 = -87.5^\circ$

Between ω_{pc} & ω_{gc}

PM & GM will be +ve

or PM & GM will be -ve

or both will be 0

to ~~make~~ satisfy

$\omega_{pc} > \omega_{gc}$

$\omega_{pc} = \omega_{gc}$

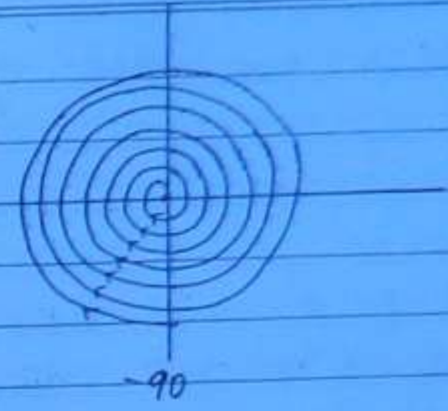
$\omega_{pc} < \omega_{gc}$

PM = -ve

$\therefore$ GM = -ve (d)

To map Polar Plot

ω	0	∞
$\lvert G(j\omega)\rvert$	∞	0
$\angle G(j\omega)$	$-90°$	$-\infty$



To determine stability for systems with dead time, Polar / Nyquist plots is not a good idea for it as it cuts -ve Real axis ∞ times

so $X = ?$

Thus find PM ; GM.

(28)

$$G(s) = \frac{e^{-Ts}}{s(s+1)}$$

$$G(j\omega) = \frac{e^{-j\omega T}}{j\omega(j\omega+1)}$$

$$\angle G(j\omega) = \frac{-\omega T}{[\pi/2][\tan^{-1}\omega]}$$

$$\Rightarrow -\frac{\pi}{2} - \tan^{-1}\omega - \omega T$$

At $\omega = \omega_1$ $\quad \angle G(j\omega) = 0$

$$\Rightarrow -\frac{\pi}{2} - \omega_1 T - \tan^{-1}\omega_1 = 0$$

$$-\tan^{-1}\omega_1 = \frac{\pi}{2} + \omega_1 T$$

$$-\omega_1 = \tan\left[\frac{\pi}{2} + \omega_1 T\right]$$

(30) ABCD are plots of Type 2 order 3

$\downarrow$

Phase $= -90 \times 3$
$= -270°$

A & B are stable around critical point
3, 4 plots have well defined mag^n & phase so A, B belong to 3, 4

$\xi \propto \frac{1}{\sqrt{K}}$ As $K \uparrow$ $\xi \downarrow$ ∴ $M_r \uparrow$

eg $s^2 + s + K = 0$

$\omega_n = \sqrt{K}$ r/s.

$2\xi\sqrt{K} = 1$

$\xi = \frac{1}{2\sqrt{K}}$

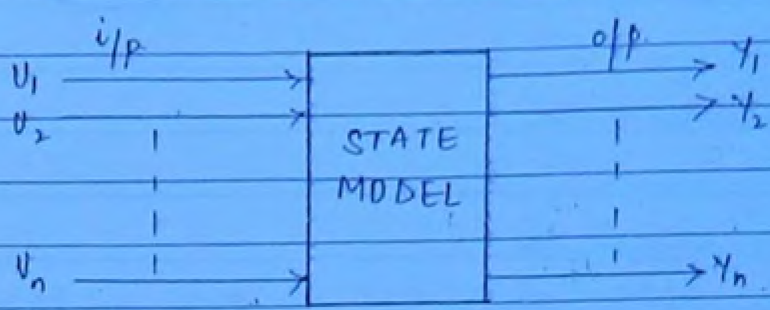
$\xi \propto \frac{1}{\sqrt{K}}$

A. $K\downarrow$ $\xi\uparrow$ —(4)
$M_r \downarrow$

B. $K\uparrow$ $\xi\downarrow$ —(3)
$M_r \uparrow$

C. Marginally stable —(2)
→ oscillatory
→ $\xi = 0$ $M_r = \infty$

STATE SPACE ANALYSIS



1. STATE EQUATION

$$\dot{X}(t) = A X(t) + B U(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} \\ A \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \\ B \\ \\ \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

2. OUTPUT EQUATION

$$Y(t) = C X(t) + D U(t)$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \\ C \\ \\ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} \\ D \\ \\ \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

TYPE-1 To obtain S.M from differential eq^n

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 10u$$

$$(s^3 + 4s^2 + 6s + 8)\,Y(s) = 10\,U(s)$$

To define state variables.

Let $Y = x_1$ $\quad \frac{dY}{dt} = \dot{x}_1 = x_2$

$\frac{d^2Y}{dt^2} = \dot{x}_2 = x_3$ $\quad \frac{dY_3}{dt^3} = \dot{x}_3$

$\dot{x}_1 = x_2$

$\dot{x}_2 = x_3$

$\dot{x}_3 = 10u - 4x_3 - 6x_2 - 8x_1$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -8 & -6 & -4 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}}_{B} u(t)$$

O/p Equation

$Y = x_1$

$$Y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(Bush/Companion form)

CWB chapter 8

8 $\frac{Y(s)}{U(s)} = \frac{1}{1s^4 + 5s^3 + 8s^2 + 6s + 3}$ ← ascending order of sign

↳ co-eff of highest order term = 1

sol

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

(9) $$\frac{d^2Y}{dt^2} + \frac{7dY}{dt} + 9Y = \frac{2du}{dt} + u$$

$$(s^2 + 7s + 9)\, Y(s) = (2s+1)\, U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{2s+1}{s^2+7s+9}$$

Phase Variable Method

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2+7s+9} (2s+1) \leftarrow \text{reverse order}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Type 2 To obtain T.F from State Model

$$\dot{X}(t) = A\,X(t) + B\,U(t)$$
$$Y(t) = C\,X(t) + D\,U(t)$$

Applying L.T

$$s\,X(s) - X(0) = A\,X(s) + B\,U(s)$$
$$Y(s) = C\,X(s) + D\,U(s)$$

In T.F. $X(0) = 0$

$$sX(s) - AX(s) = BU(s)$$

$$X(s)[sI - A] = BU(s)$$

$$X(s) = [sI - A]^{-1} \cdot B\,U(s)$$

$$Y(s) = \{C[sI - A]^{-1} B + D\} U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \underbrace{C[sI - A]^{-1} B + D}_{\text{Transfer Matrix}}}$$

Q11. $\dot{x} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \qquad y = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T x$

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \Rightarrow \begin{bmatrix} s+1 & 0 \\ 0 & s+2 \end{bmatrix}$$

$$\text{Adj}(sI - A) = \begin{bmatrix} (s+2) & 0 \\ 0 & (s+1) \end{bmatrix}$$

$$|sI - A| \Rightarrow (s+1)(s+2)$$

$$(sI - A)^{-1} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix}$$

$$(sI - A)^{-1} \cdot B = \begin{bmatrix} \frac{1}{s+1} & 0 \\ 0 & \frac{1}{s+2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} \\ 0 \end{bmatrix}$$

$$C[sI - A]^{-1} B \Rightarrow [1 \quad 1] \begin{bmatrix} \frac{1}{s+1} \end{bmatrix}$$

(3) $\dot{x}(t) = -2x(t) + 2u(t)$

$y(t) = 0.5\,x(t)$

sol

$$sX(s) = -2X(s) + 2U(s)$$

$$Y(s) = 0.5\,X(s)$$

$$(s+2)\,X(s) = 2U(s)$$

$$X(s) = \frac{2}{s+2}\,U(s)$$

$$Y(s) = \frac{0.5 \times 2}{s+2}\,U(s)$$

$$\boxed{\frac{Y(s)}{U(s)} = \frac{1}{s+2}}$$

TYPE - 3 Stability for S.M

$$\frac{Y(s)}{U(s)} = C(sI - A)^{-1}B + D$$

$$\Rightarrow C\,\frac{\operatorname{Adj}(sI-A)}{|sI-A|}\,.B + D$$

$$\Rightarrow \frac{C.\operatorname{Adj}(sI-A).B + |sI-A|\,D}{|sI-A|}$$

∝ Zeros = $C.\operatorname{Adj}(sI-A)\,B + |sI-A|.D = 0$

∝ Poles $(1 + G(s)H(s)) = 0 = |sI - A| = 0$

$\Rightarrow$ char eqⁿ $= |sI - A| = 0$

EIGEN VALUES OF SYSTEM MATRIX [A] = C.L poles

CONV 2. $\dot{X} = \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r.$

(160)

$C = \begin{bmatrix} -17 & -5 \end{bmatrix} X + \begin{bmatrix} 1 \end{bmatrix} r.$

$SI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -20 & -9 \end{bmatrix}$

$= \begin{bmatrix} s & -1 \\ 20 & s+9 \end{bmatrix}$

$Adj(sI - A) = \begin{bmatrix} s+9 & -20 \\ 1 & s \end{bmatrix}$

$= \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix}$

$Adj(sI - A) B = \begin{bmatrix} s+9 & 1 \\ -20 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ s \end{bmatrix}$

$C \cdot Adj(sI - A) B \Rightarrow \begin{bmatrix} -17 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} = -17 - 5s$

$|sI - A| \Rightarrow s(s+9) + 20$

$|sI - A| D \Rightarrow [s^2 + 9s + 20] [1]$

Zeros $= -17 - 5s + s^2 + 9s + 20 = 0$

$s^2 + 4s + 3 = 0$

$-1, -3.$

Poles $= |sI - A| = 0$

$s^2 + 9s + 20 = 0$

$-4, -5$

(7)

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$u = [-0.5 \quad -3 \quad -5] x + V$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [0.5 \quad -3 \quad -5] x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & 1 & 2 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & -3 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} V$$

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -2 & -3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} V$$

$$sI - A \Rightarrow \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 2 & s+3 \end{bmatrix}$$

$$|sI - A| = s[s(s+3)+2] + 1[0] + 0 = 0$$

$$= s[s^2 + 3s + 2] = 0$$

$0, -1, -2$ Ans

Marginally stable

TYPE 4 -

STATE CONTROLLABILITY -

To control the state variables

$\Rightarrow Q_c = [B \quad AB \quad A^2B \; . \; . \; . \quad A^{n-1}B]$

STATE OBSERVABILITY -

To measure the state variables:

$\Rightarrow Q_o = [C^T \quad A^TC^T \quad (A^T)^2C^T \; . \; . \; . \; (A^T)^{n-1}C^T]$

$|Q_c| \neq 0 \qquad |Q_o| \neq 0$

KALMAN'S TEST

(2)
$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} U$$

$$Y = [1 \quad 1] X$$

$$Q_c = [B \quad AB]$$

$$AB = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$Q_c = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} = -1 \qquad \text{controllable}$$

$$Q_o = [C^T \quad A^TC^T]$$

$$A^TC^T = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$Q_o = \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix} = -4 \qquad \text{observable}$$

(10) $Q_0 = [C^T \quad A^T C^T]$

$$A^T C^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} b \\ 2b \end{bmatrix}$$

$$Q_0 = \begin{vmatrix} b & b \\ 0 & 2b \end{vmatrix} = 2b^2 \neq 0$$

Ans (c)

TYPE - 5 Solution of State Equation

$$\dot{x}(t) = A x(t) + B u(t)$$

a) Free response $[u(t) = 0]$

$$\dot{x}(t) = A x(t)$$

$$\dot{x}(t) - A x(t) = 0 \quad \text{--- (1)}$$

$$x(t) = K e^{At}$$

Applying L.T to equation (1)

$$s X(s) - x(0) - A X(s) = 0$$

$$s X(s) - A X(s) = x(0)$$

$$X(s)[sI - A] = x(0)$$

$$X(s) = (sI - A)^{-1} x(0)$$

$\phi(s) = (sI - A)^{-1}$ = Resolvant Matrix

$$x(t) = \{ L^{-1} (sI - A)^{-1} \} x(0)$$

$$x(t) = e^{At} \quad K$$

b) Forced Response -

$$sX(s) - X(0) = AX(s) + BU(s)$$ (164)

$$sX(s) - AX(s) = X(0) + BU(s)$$

$$X(s)[sI - A] = X(0) + BU(s)$$

$$X(s) = [sI - A]^{-1} X(0) + (sI - A)^{-1} B U(s)$$

$$X(t) = \{ L^{-1} (sI - A)^{-1} \} X(0) + L^{-1} [(sI - A)^{-1} B U(s)]$$

(4) $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ $X(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$$sI - A = \begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}$$

$$\mathrm{Adj}(sI - A) = \begin{bmatrix} s-1 & 1 \\ 0 & s-1 \end{bmatrix}$$

$$= \begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}$$

$$|sI - A| = (s-1)^2$$

$$(sI - A)^{-1} = \phi(s) = \begin{bmatrix} \frac{1}{s-1} & 0 \\ \frac{1}{(s-1)^2} & \frac{1}{s-1} \end{bmatrix}$$

S.T.M $\phi(t)$ $\phi(t) = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$

Property of S.T.M

At $t = 0$ $e^{A0} = \phi(0) = I$

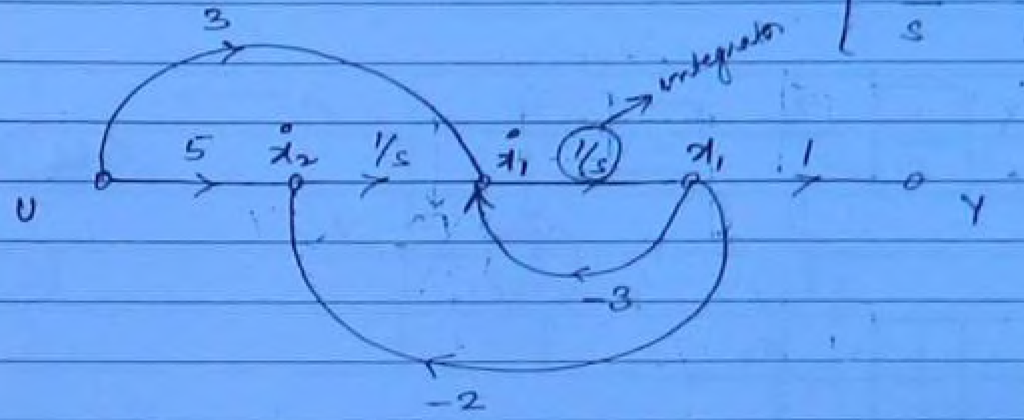
$X(t) = \begin{bmatrix} e^t & 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \Rightarrow X(t) = [\, e^t \,]$ Ans

TYPE 6 - STATE DIAGRAMS

TBS

1. Observable canonical form

$$\frac{Y(s)}{U(s)} = \frac{3s+5}{s^2+3s+2} = \frac{\frac{3}{s} + \frac{5}{s^2}}{1 - \left[\frac{-3}{s} - \frac{2}{s^2}\right]}$$



$\dot{x}_1 = -3x_1 + x_2 + 3u$

$\dot{x}_2 = -2x_1 + 5u$

$Y = x_1$

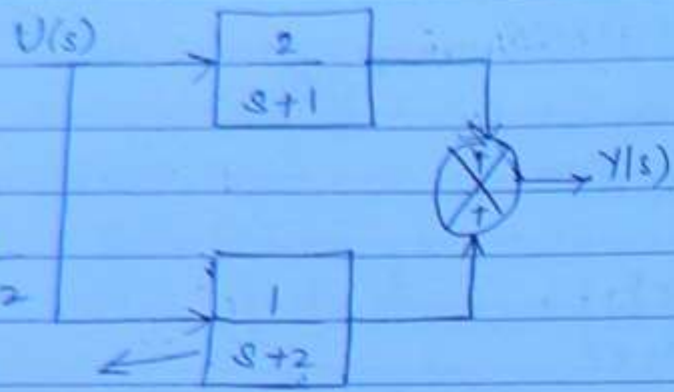
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 3 \\ 5 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

2. Controllable canonical form

$$\frac{Y(s)}{U(s)} = \frac{2}{s+1} + \frac{1}{s+2}$$

$$Y(s) = \frac{2\,U(s)}{s+1} + \frac{U(s)}{s+2}$$



$$\frac{G(s)}{1+H(s)G(s)} = \frac{1}{s+1}$$

suppose $H(s)=1$

$$G(s) = \frac{N}{D-N} = \frac{1}{s+1-1} = \frac{1}{s}$$

$$G(s) = \frac{1}{s} \quad H(s) = 1$$

$$\frac{G(s)}{1+H(s)G(s)} = \frac{1}{s+2}$$

$$\frac{1+HG}{G} = s+2$$

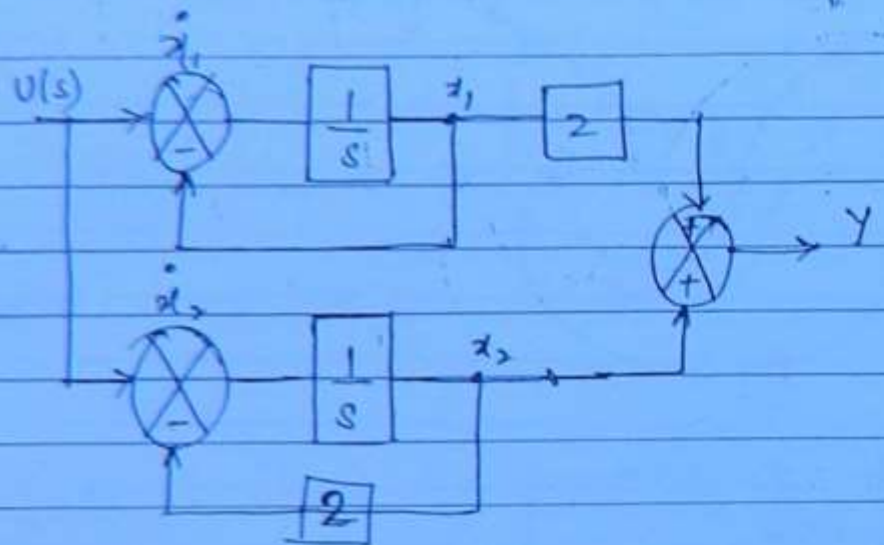
$$1 = G(s+2-H)$$

$$G = \frac{1}{s+2-H}$$

put $H=2$

$$G = \frac{1}{s}$$

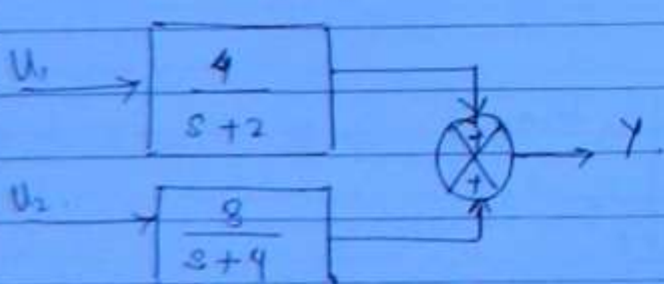
(166)



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q



$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

No. of states = No. of energy storing elements.

12 No. of energy storing elements = 3. (167)

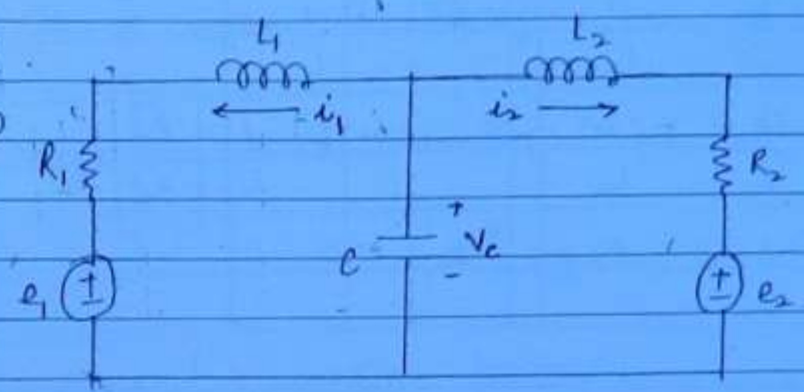
Since C_1 & C_2 are in ||

Initially they will have same v/g across them

So min. no. of states = 2.

CONV'S

1(a)



Only Inductive n/w → KVL

Only Capacitive n/w → KCL

Both → Apply Both

KVL in loop ①

$$L_1 \frac{di_1}{dt_1} + i_1 R_1 + e_1 - V_c = 0$$

$$\frac{di_1}{dt_1} = -i_1 \frac{R_1}{L_1} - \frac{e_1}{L_1} + \frac{V_c}{L_1} \rightarrow ①$$

KVL to loop ②

$$L_2 \frac{di_2}{dt} + i_2 R_2 + e_2 - V_c = 0$$

$$\frac{di_2}{dt} = \frac{-R_2}{L_2} i_2 + \frac{V_c}{L_2} - \frac{e_2}{L_2} \rightarrow ②$$

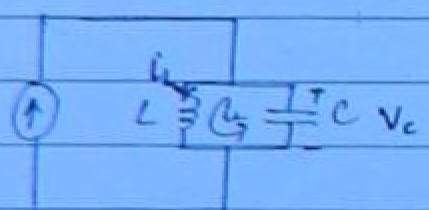
KCL at node V_c

$$i_1 + i_2 + C\frac{dV_c}{dt} = 0$$

$$\frac{dV_c}{dt} = \frac{-i_1}{C} - \frac{i_2}{C} \quad \text{--- (3)}$$

$$\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \\ \frac{dV_c}{dt} \end{bmatrix} = \begin{bmatrix} \frac{-R_1}{L_1} & 0 & \frac{1}{L_1} \\ 0 & \frac{-R_2}{L_2} & \frac{1}{L_2} \\ \frac{-1}{C} & \frac{-1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ V_c \end{bmatrix} + \begin{bmatrix} \frac{-1}{L_1} & 0 \\ 0 & \frac{1}{L_2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

(b)



$$i(t) = i_L + i_C$$

$$i(t) = i_L + C\frac{dV_c}{dt}$$

$$\frac{dV_c}{dt} = \frac{-i_L}{C} + \frac{i(t)}{C} \longrightarrow (1)$$

$$L\frac{di_L}{dt} - V_c = 0 \Rightarrow \frac{di_L}{dt} = \frac{V_c}{L} \quad \text{--- (2)}$$

$$\begin{bmatrix} \frac{dV_c}{dt} \\ \frac{di_L}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -1/C \\ 1/L & 0 \end{bmatrix} \begin{bmatrix} V_c \\ i_L \end{bmatrix} + \begin{bmatrix} 1/C \\ 0 \end{bmatrix} i(t)$$

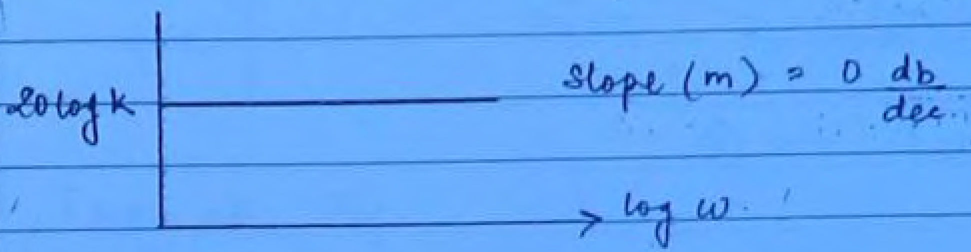
BODE PLOTS

1. System Gain ($\pm K$)

$$F(j\omega) = \pm K + 0j$$

$$|F(j\omega)| = \sqrt{(\pm K)^2 + 0^2} = K$$

Its db value is $20 \log K$.



2. Internal / Derivative factors

[Poles / Zeros at origin]

$(S)^{\pm n}$

$$F(j\omega) = (j\omega)^{\pm n} = (0 + j\omega)^{\pm n}$$

$$|F(j\omega)| = \left[\sqrt{0^2 + \omega^2}\right]^{\pm n} = [\omega]^{\pm n}$$

Its db value $= 20 \log (\omega)^{\pm n}$

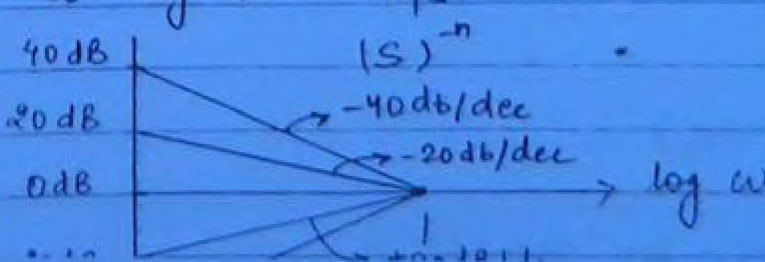
$$= \cancel{\pm 20 \log} = \underbrace{\pm 20 \times n}_{M} \log \underbrace{\omega}_{x}$$

$$\text{Slope } (m) = \pm 20 \times n \ \frac{db}{dec}$$

$$\pm 20 \times n \log \omega = 0 \text{ db}$$

$$\log \omega = 0$$

$$\omega = \log^{-1} 0 = 1 \ r/s$$



3 First Order Factor -

$$(1 \pm Ts)^{\pm 1}$$

$$F(j\omega) = (1 \pm j\omega T)^{\pm 1}$$

$$|F(j\omega)| = \left[\sqrt{1+(\omega T)^2}\right]^{\pm 1}$$

Its db value

$$20 \log \left[\sqrt{1+(\omega T)^2}\right]^{\pm 1}$$

$$\pm 20 \log \sqrt{1+(\omega T)^2} \longrightarrow (1)$$

Asymptotic Assumptions -

case 1 → LOW FREQ

$$1 \gg (\omega T)^2$$

$$\pm 20 \log \sqrt{1} = 0 \, db$$

case 2 → HIGH FREQ

$$(\omega T)^2 \gg 1$$

$$\pm 20 \log \sqrt{(\omega T)^2}$$

$$\pm 20 \log \omega T \longrightarrow (2)$$

$$\Rightarrow \pm 20 \log \omega \pm 20 \log T$$

$$[M \quad x \quad + \quad C]$$

Slope $(m) = \pm 20$ db / sec

Corner frequency (ωcf)

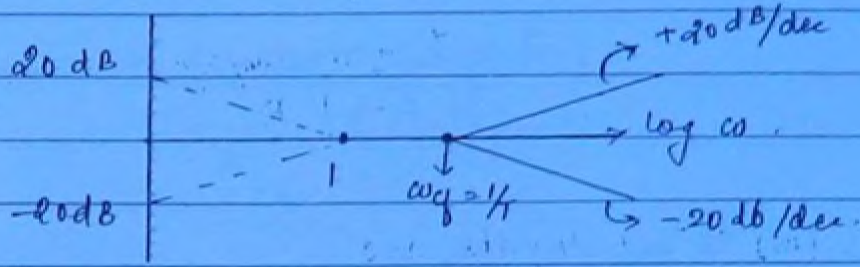
$$0 = \pm 20 \log \omega T$$

$\log \omega T = 0 \quad \Rightarrow \quad \omega T = \log^{-1}(0) = 1$

eg. $[s \pm 2]^{\pm 1}$

$[1 \pm s/2]^{\pm 1}$

$T = 1/2 \Rightarrow \omega_g = \frac{1}{T} = 2\ r/s.$



Error at ω_g

At $\omega = \omega_g = \frac{1}{T}$

$$\pm 20 \log \sqrt{1 + \left(\frac{1}{T} \times T\right)^2}$$

$$\pm 20 \log \sqrt{2} = \pm 3\,dB$$

4. Quadratic factor -

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)^{\pm 1}$$

$$\left[\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1\right]^{\pm 1}$$

Put $s = j\omega$.

$$\left[1 - \left(\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n}\right]^{\pm 1}$$

db value Magnitude

$$\mp 20 \log \sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}$$

(172)

0 db
L.F.R

$\mp 40 \log \omega/\omega_n$
H.F.R.

Slope (m) = $\mp$ 40 log db/dec.

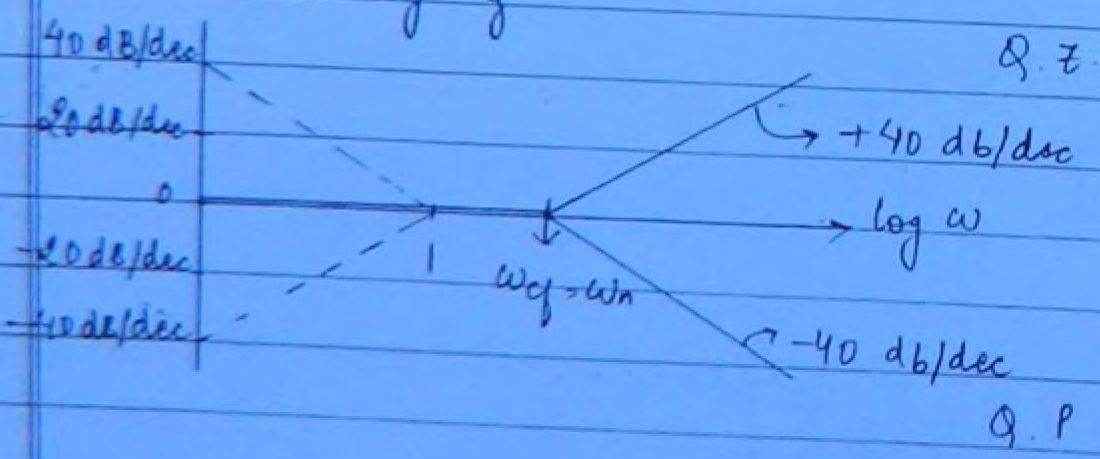
→ Corner freq
$\omega_{cf} = \omega_n$ r/s.

eg $(s^2 + 4s + 25)^{\pm 1}$

$\omega_n^2 = 25$

$\omega_{cf} = \omega_n = 5$ r/s.

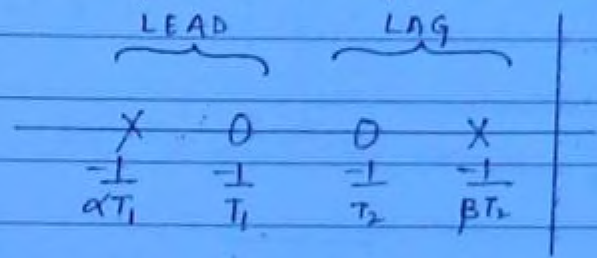
→ Error at corner freq ω_{cf}
$\mp 20 \log 2\zeta$



BODE PLOT FOR LAG LEAD or LEAD LAG COMPENSATOR

$$F(s) = \frac{V_0(s)}{V_i(s)} = \frac{\alpha(1+T_1 s)(1+T_2 s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}$$

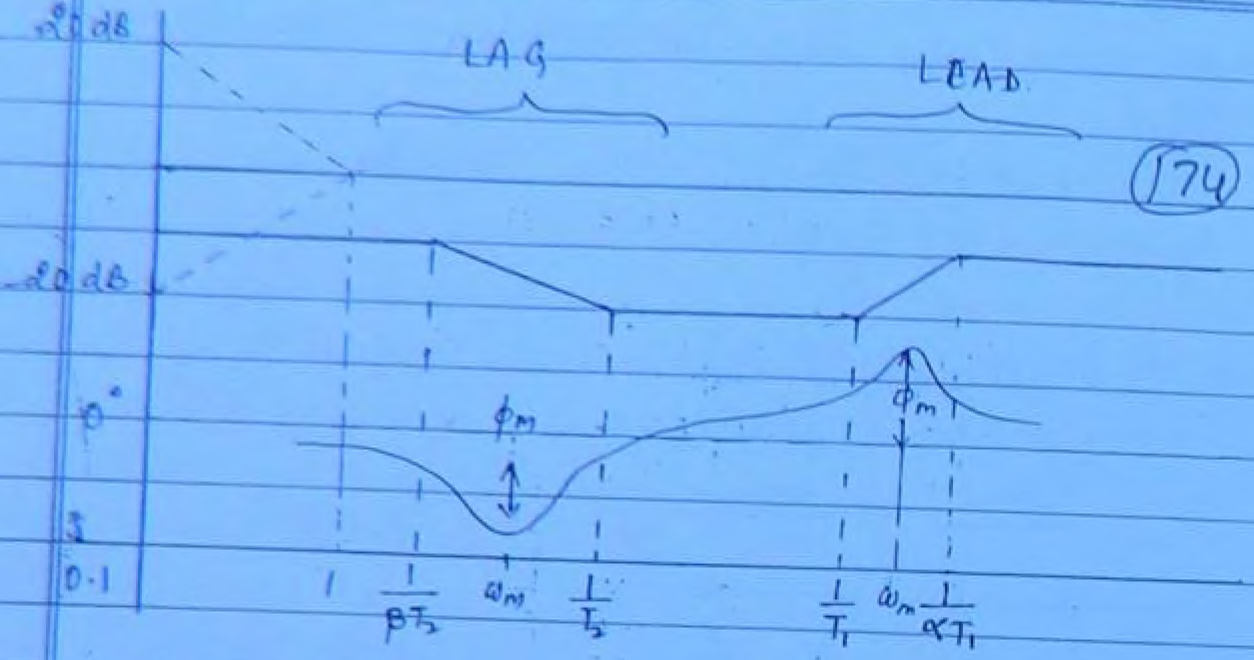
$$F(j\omega) = \frac{\alpha(1+j\omega T_1)(1+j\omega T_2)}{(1+j\omega\alpha T_1)(1+j\beta\omega T_2)}$$



$$\angle F(j\omega) = \tan^{-1}\omega T_1 - \tan^{-1}\omega\alpha T_1 + \tan^{-1}\omega T_2 - \tan^{-1}\omega\beta T_2$$

MAG^n TABLE

	Factor	c.f.	Mag^n
gain	$K=\alpha$	–	$20\log\alpha$
P or Z at origin	$(j\omega)^{\pm n}$	– 0	Nil
ω_c placed in ascending order	$\frac{1}{1+j\omega\beta T_2}$	$\frac{1}{\beta T_2}$	$-20\ \frac{db}{dec}$
			(-20)
	$1+j\omega T_2$	$\frac{1}{T_2}$	$+20\ \frac{dB}{dec}$
			(0)
	$1+j\omega T_1$	$\frac{1}{T_1}$	$+20\ \frac{dB}{dec}$
			$(+20)$



GP of $\frac{1}{\beta T_2}$ & $\frac{1}{T_2}$ $= \omega_m$

$$\omega_m = \sqrt{\frac{1}{\beta T_2} \times \frac{1}{T_2}} = \frac{1}{T_2\sqrt{\beta}}$$

$$\omega_m = \sqrt{\frac{1}{T_1} \times \frac{1}{\alpha T_1}} = \frac{1}{T_1\sqrt{\alpha}}$$

Characteristics of Phase Lead Compensator -

1. It improves transient state characteristics of the system

2. In terms of filtering property, its a high pass filter

3. In terms of controllers it is similar to P + D controller

4. Lead compensate contributes phase lead, ∴ response will be faster & rise time will ↓ & BW ↑

5. It increases the stability of ~~adds~~ system / adding a

$P = f(e)$

1. Proportional Mode

$P \propto e$

$P = K_p e$

K_p = proportional gain

2. Integral Mode (or) Reset Mode

$\frac{dP}{dt} \propto e$

$\frac{dP}{dt} = K_I e$

K_I = Integral scaling.

$P = K_I \int e \, dt$

Defining Reset Time

$T_i = \frac{1}{K_I}$ $\quad$ $P = \frac{1}{T_i} \int e \, dt$

3. Derivative (or) Rate Mode

$P \propto \frac{de}{dt}$

$P = K_D \frac{de}{dt}$

K_D = Rate constant

Defining "Rate Time"

$T_d = K_D$

$P = T_d \frac{de}{dt}$

P + D Controller

$$P = K_p e + K_p T_d \frac{de}{dt}$$

$$P(s) = \{K_p (1 + T_d s)\} E(s)$$

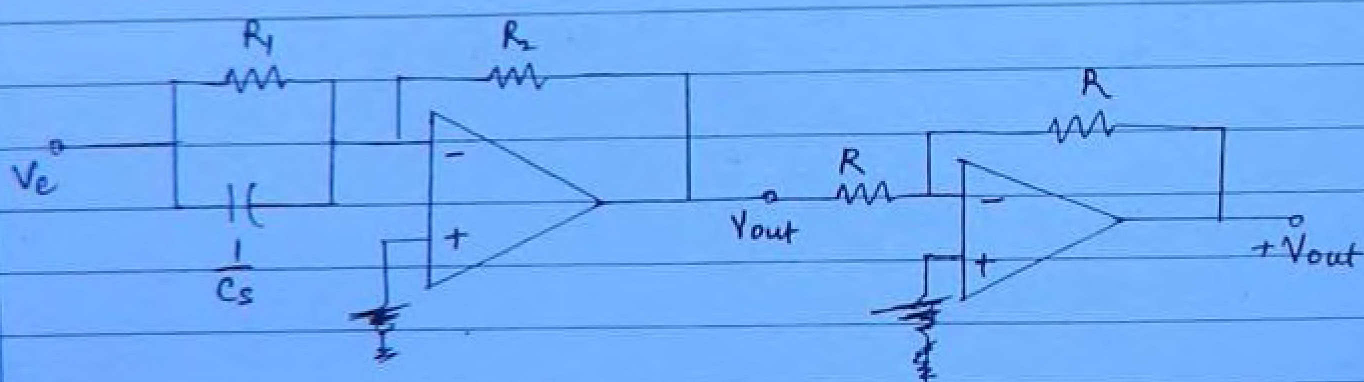
$$E(s) \longrightarrow \boxed{K_p(1 + T_d s)} \longrightarrow P(s)$$

let $e = \sin \omega t$

$$P = K_p \sin \omega t + K_p T_d \frac{d \sin \omega t}{dt}$$

$$P = K_p \sin \omega t + \omega K_p T_d \cos \omega t$$

$$P = \sqrt{(K_p)^2 + (\omega K_p T_d)^2} \sin\left[\omega t + \tan^{-1} \omega T_d\right]$$



$$\frac{V_e - V_1}{\frac{R_1}{R_1 C s + 1}} = \frac{V_1 - V_{out}}{R_2}$$

$$\Rightarrow \frac{V_e [R_1 C s + 1]}{R_1} \cdot R_2 = -V_{out}$$

$$\Rightarrow -V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} R_1 C s V_e$$

$$\Rightarrow +V_{out} = \frac{R_2}{R_1} V_e + \frac{R_2}{R_1} R_1 C \frac{dV_e}{dt}$$

ch 3 CONV

2. $$G(s) = \frac{K[K_p + K_D s]}{s(Ts+1)}$$

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1) Without P+D controller

$$G(s) = \frac{K}{s(Ts+1)}$$

Type 1 / order 2

with P+D controller

$$G(s) = \frac{K(K_p + K_D s)}{s(Ts+1)}$$

Type 1 / order 2

2. With P controller only.

$$1 + \frac{KK_p}{s(Ts+1)} = 0$$

$$Ts^2 + s + KK_p = 0$$

$$s^2 + \frac{1}{T}s + \frac{KK_p}{T} = 0$$

$$\omega_n = \sqrt{\frac{KK_p}{T}} \ r/s \qquad 2\zeta\sqrt{\frac{KK_p}{T}} = \frac{1}{T}$$

$$\zeta = \frac{1}{2\sqrt{KK_pT}}$$

3. With P+D controller

$$1 + \frac{K(K_p + K_D s)}{s(Ts+1)} = 0$$

$$Ts^2 + s + KK_D s + KK_p = 0$$

$$s^2 + s\frac{(1+KK_D)}{T} + \frac{KK_p}{T} = 0$$

$$\omega_n = \sqrt{KK_D} \ r/s$$

$$\alpha^{2}\mathcal{G}\sqrt{\frac{KK_p}{T}} = \frac{1+KK_D}{T}$$

$$\frac{4\mathcal{G}^2 KK_p}{\cancel{T}} = \frac{(1+KK_D)^2}{T^{\cancel{2}}}$$

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$$\boxed{\mathcal{G} = \frac{1+KK_D}{\alpha^{2}\sqrt{KK_p T}}}$$

—— x ——

contd...

6. The Phase Lead Compensator shifts gain crossover frequency to higher values where the designed phase margin is acceptable. Hence, it is effective when the slope of the uncompensated system near the gain crossover frequency is low.

7. The maximum phase lead occurs at geometric mean of the 2 corner frequencies

$$\angle F(j\omega) = \phi = \tan^{-1}\omega T - \tan^{-1}\omega\alpha T$$

$$\tan\phi = \tan[\tan^{-1}\omega T - \tan^{-1}\omega\alpha T]$$

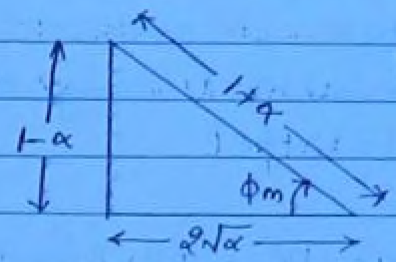
$$\tan\phi = \frac{\omega T - \omega\alpha T}{1+(\omega T)^2.\alpha} \Rightarrow \frac{\omega T[1-\alpha]}{1+[\omega T]^2.\alpha}$$

At $\omega = \omega_m = \dfrac{1}{T\sqrt{\alpha}}$ $\quad\phi = \phi_m$

ω_m: max phase lead angle freq at which occurs

$$\tan\phi_m = \frac{\frac{1}{T\sqrt{\alpha}}.T\sqrt{1-\alpha}}{1+\left[\frac{1}{\cancel{T}}.\cancel{T}\right]^2.\alpha} = \frac{1-\alpha}{2\sqrt{\alpha}}$$

$$\phi_M = \tan^{-1}\left[\frac{1-\alpha}{2\sqrt{\alpha}}\right]$$



$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}$$

$$\phi_m = \sin^{-1}\left[\frac{1-\alpha}{1+\alpha}\right].$$

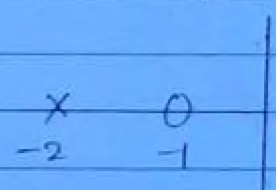
$$\sin\phi_M(1+\alpha) = 1-\alpha$$

$$\alpha\sin\phi_M + \alpha = 1-\sin\phi_M$$

$$\alpha(1+\sin\phi_M) = 1-\sin\phi_M$$

$$\alpha = \frac{1-\sin\phi_M}{1+\sin\phi_M}$$

Polar Plot



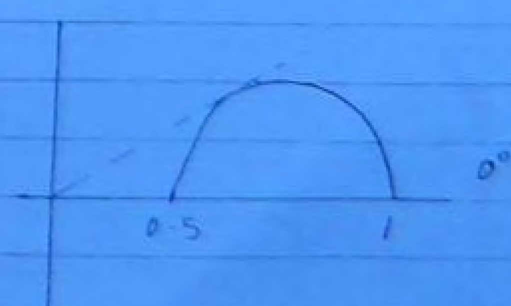
$$F(s) = \frac{s+1}{s+2} = \frac{0.5(1+s)}{(1+0.5s)}$$

$$F(j\omega) = \frac{0.5(1+j\omega)}{(1+0.5j\omega)}$$

$$|F(j\omega)| = \frac{0.5\sqrt{1+\omega^2}}{\sqrt{1+(0.5\omega)^2}}$$

$$\angle F(j\omega) = \tan^{-1}\omega - \tan^{-1}0.5\omega$$

ω	0	∞
$\lvert G(j\omega)\rvert$	0.5	1
$\angle G(j\omega)$	0°	0°



Characteristics of Phase Lag Compensator.

1. It improves the steady state characteristics of the system only. ie it eliminates steady state error between input & output.

2. In terms of filtering property it is similar to low pass filter.

3. It shifts the gain crossover frequency to lower values

4. The maximum phase lag occurs at the geometric means of the two corner frequencies

$$\angle F(j\omega) = \phi = \tan^{-1}\omega T - \tan^{-1}\omega\beta T$$

$$\tan\phi = \tan\left[\tan^{-1}\omega T - \tan^{-1}\omega\beta T\right]$$

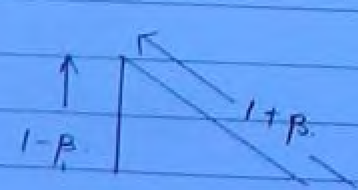
$$\tan\phi = \frac{\omega T - \omega\beta T}{1+(\omega T)^2\beta} \Rightarrow \frac{\omega T[1-\beta]}{1+(\omega T)^2\beta}$$

At $\omega = \omega_m = \dfrac{1}{T\sqrt{\beta}}$ $\quad \phi = \phi_M$

$$\tan\phi_m = \frac{\dfrac{1}{T\sqrt{\beta}}\, T\,[1-\beta]}{1+\left[\dfrac{1}{T\sqrt{\beta}}\, T\right]^2\beta} = \frac{1-\beta}{2\sqrt{\beta}}$$

maximum phase lag angle at [illegible]

$$\phi_M = \tan^{-1}\frac{1-\beta}{2\sqrt{\beta}}$$



$$\sin\phi_M = \frac{1-\beta}{1+\beta}$$

$$\phi_M = \sin^{-1}\frac{1-\beta}{1+\beta}$$

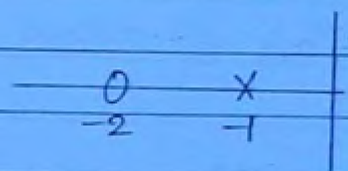
$$\sin\phi_M(1+\beta) = 1-\beta$$

$$\beta\sin\phi_M + \beta = 1 - \sin\phi_M$$

$$\beta(1+\sin\phi_M) = 1-\sin\phi_M$$

$$\boxed{\beta = \frac{1-\sin\phi_M}{1+\sin\phi_M}}$$

Polar Plot



$$F(s) = \frac{s+2}{s+1}$$

$$= \frac{2\,[1+0.5s]}{1+s}$$

$$F(j\omega) = \frac{2\,(1+0.5j\omega)}{1+j\omega}$$

$$|F(j\omega)| = \frac{2\sqrt{1+(0.5\omega)^2}}{\sqrt{1+\omega^2}}$$

$$\angle F(j\omega) = \tan^{-1}0.5\omega - \tan^{-1}\omega$$

ω	0	∞
$\lvert F(j\omega)\rvert$	2	1
$\angle F(j\omega)$	0°	0°

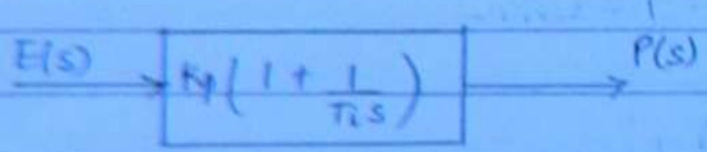
Date
Page
[P+I]

5. Its similar to Proportional plus Integral controller

P + I Controller (182)

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt$$

$$P(s) = \left\{ K_p \left[1 + \frac{1}{T_i s} \right] \right\} E(s)$$

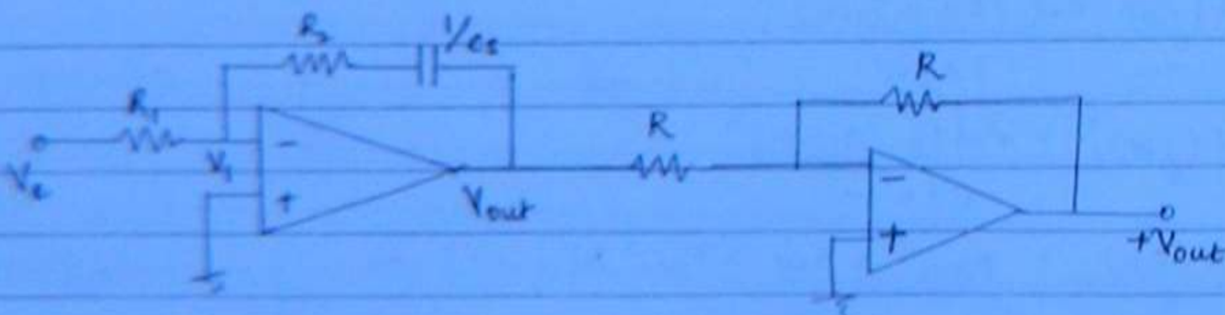


→ Let $e = \sin \omega t$

$$P = K_p \sin \omega t + \frac{K_p}{T_i}\int \sin \omega t\,dt$$

$$P = K_p \sin \omega t + \left(\frac{-K_p}{\omega T_i}\right) \cos \omega t$$

$$P = \sqrt{(K_p)^2 + \left(\frac{K_p}{\omega T_i}\right)^2}\, \sin\left(\omega t - \tan^{-1}\frac{1}{\omega T_i}\right)$$



$$\frac{V_e - V_1}{R_1} = \frac{V_1 - V_{out}}{\frac{R_2 Cs + 1}{Cs}}$$

$$-V_{out} = \frac{V_e[R_2Cs+1]}{R_1Cs} = \frac{V_e[R_2C/s]}{R_1C/s} + \frac{V_e}{R_1Cs} = \frac{R_2}{R_1}V_e + \frac{R_2}{R_1}\cdot\frac{1}{R_2C}\cdot\frac{V_e}{s}$$

$$+V_{out} = \frac{R_2}{R_1}\cdot V_e + \frac{R_2}{R_1}\cdot\frac{1}{R_2C}\int V_e\,dt$$

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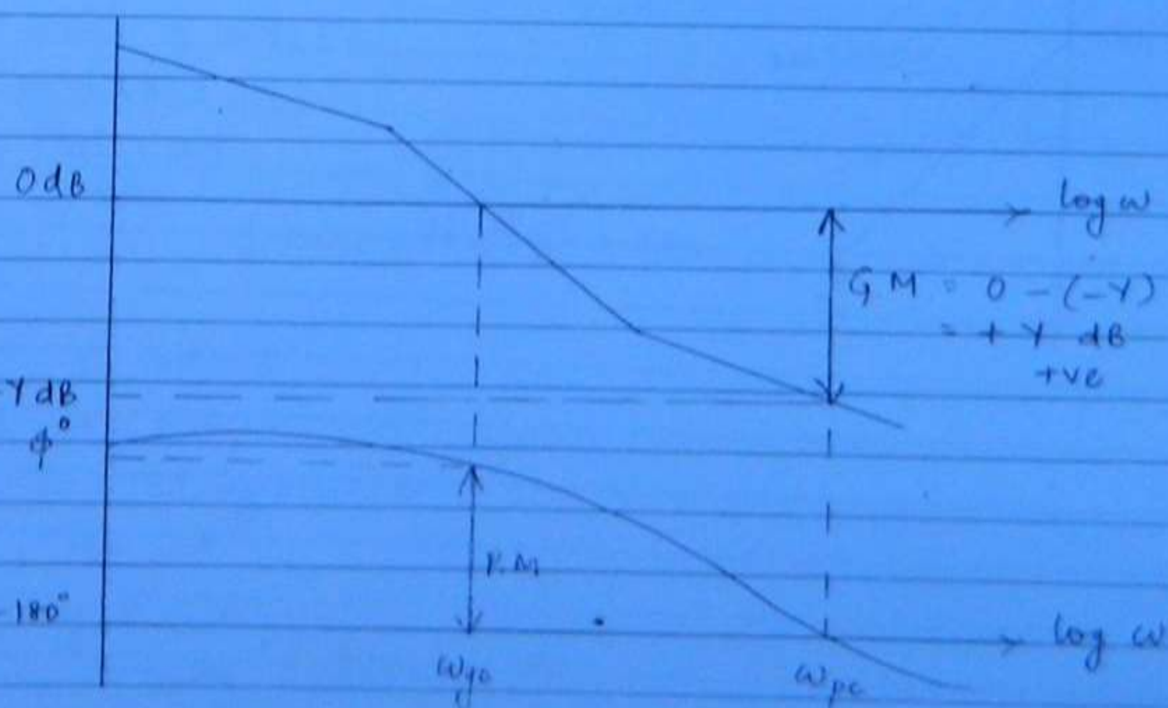
$$\boxed{K_p = \frac{R_2}{R_1} \qquad T_i = R_2C}$$

6. It increases rise time. It reduces B.W.

7. It reduces stability (addition of pole)

8. A PI controller increases the type & order of the system by 1.

TO FIND GM & PM FROM BODE PLOT.

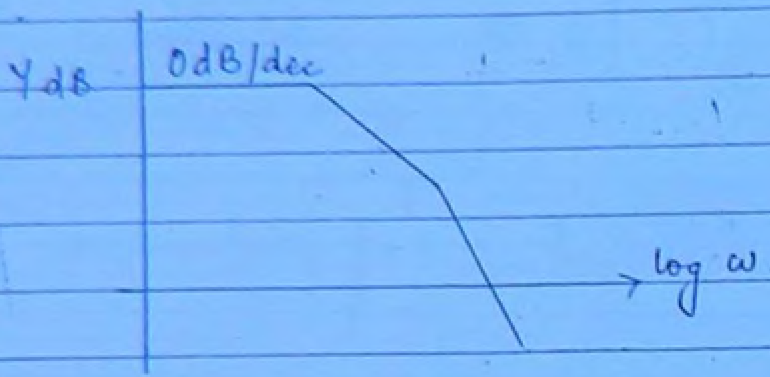


PM = φ − (−180°)
= φ + 180°

Date ______
Page ______

TO FIND STATIC ERROR CONSTANTS FROM BODE MAGNITUDE PLOT -

(184)

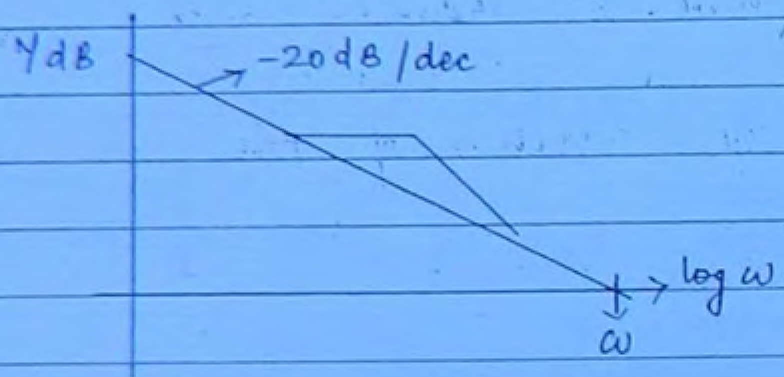


TYPE 0 SYSTEM

$K_V = K_A = 0$

$20 \log K_p = Y$

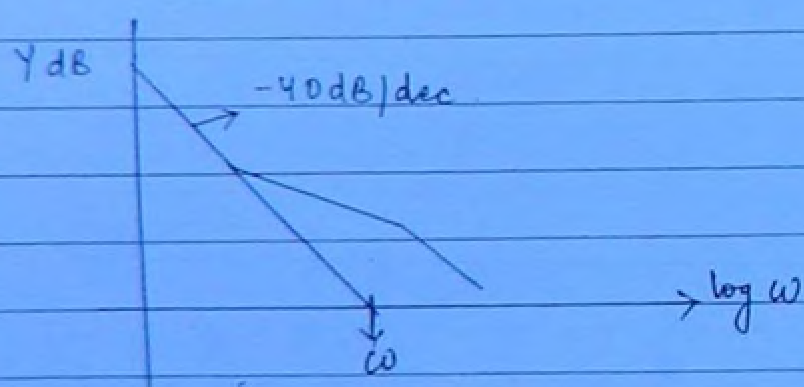
$K_p = \log^{-1}[Y/20]$



TYPE -1 SYSTEM

$K_p = \infty \quad K_A = 0$

$K_V = \omega$



TYPE -2 SYSTEM

$K_p = K_V = \infty$

$K_A = \omega^2$

INVERSE BODE PLOT -

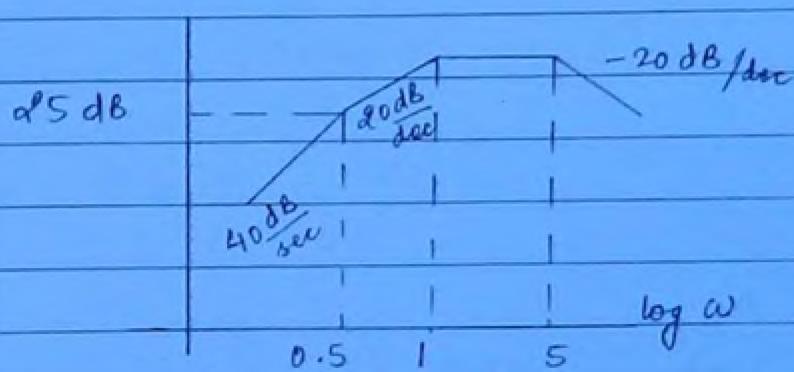
1. Observe the starting slope. This will give the information of poles or zeros occuring at origin.

2. From the starting slope write the equation

$Y = MX + C$

$C = 20 \log K$

3. At every corner frequency, observe the ~~changing slop~~ change in slope. This will give the information of first or higher order factors.



$$G(s) = \frac{70\,(s^2)}{(1+2s)\,(1+s)\,(1+0.2s)}$$

$Y = MX + C$

At $\omega = 0.5$

$25 = 40 \log 0.5 + C$

$C = 37$

$20 \log K = 37$

$K = \log^{-1}(37/20) = 70$

Chapter 6

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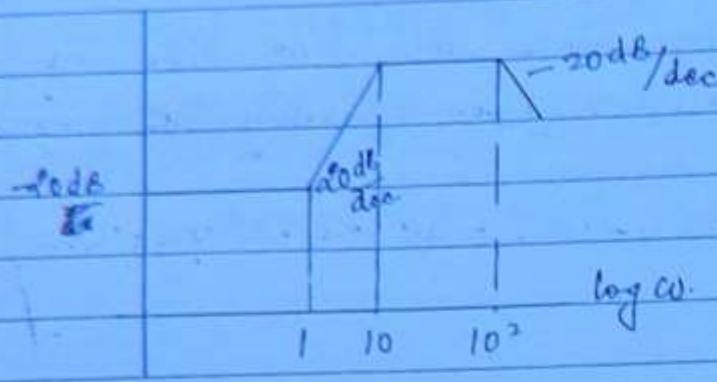


$$G(s) = \frac{0.1(1+s)}{(1+0.1s)(1+0.01s)}$$

At $\omega = 1$

$Y = MX + C$

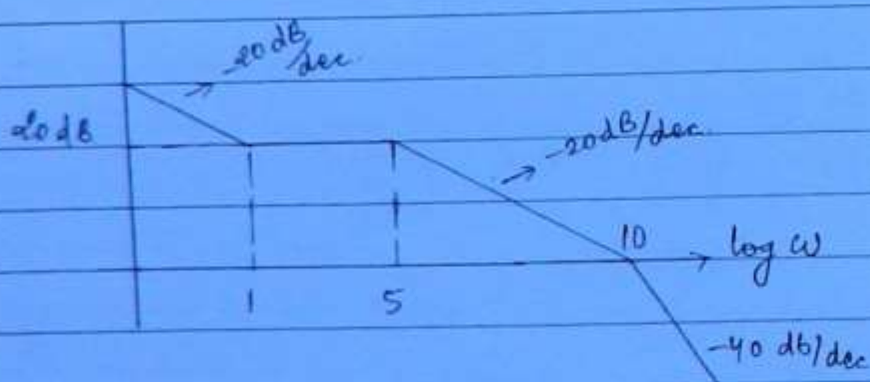
$-20 = 0 \times \log 1 + C$

$C = -20$

$20 \log k = -20$

$k = \log^{-1}(-1) = 0.1$

Q



$$G(s) = \frac{10(1+s)}{s(1+0.2s)(1+0.1s)}$$

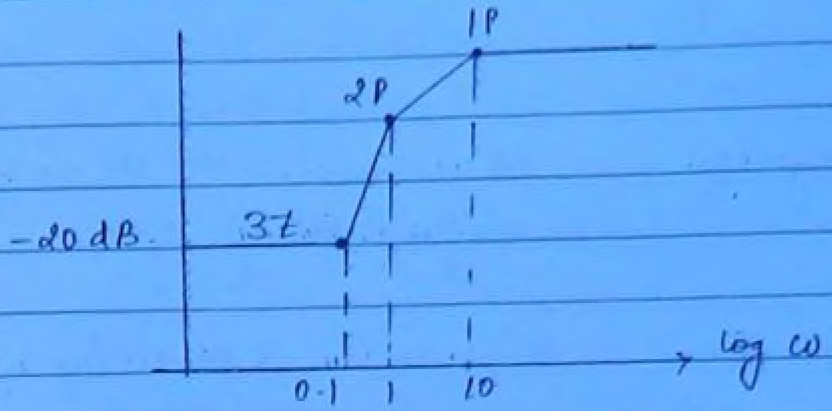
At $\omega = 1$

$Y = MX + C$

$20 = -20 \log 1 + C$

$C = 20$

Q



$$G(s) = \frac{0.1\,(1+10s)^3}{(1+s)^2\,(1+0.1s)}$$

At $\omega = 0.1$

$Y = Mx + C$

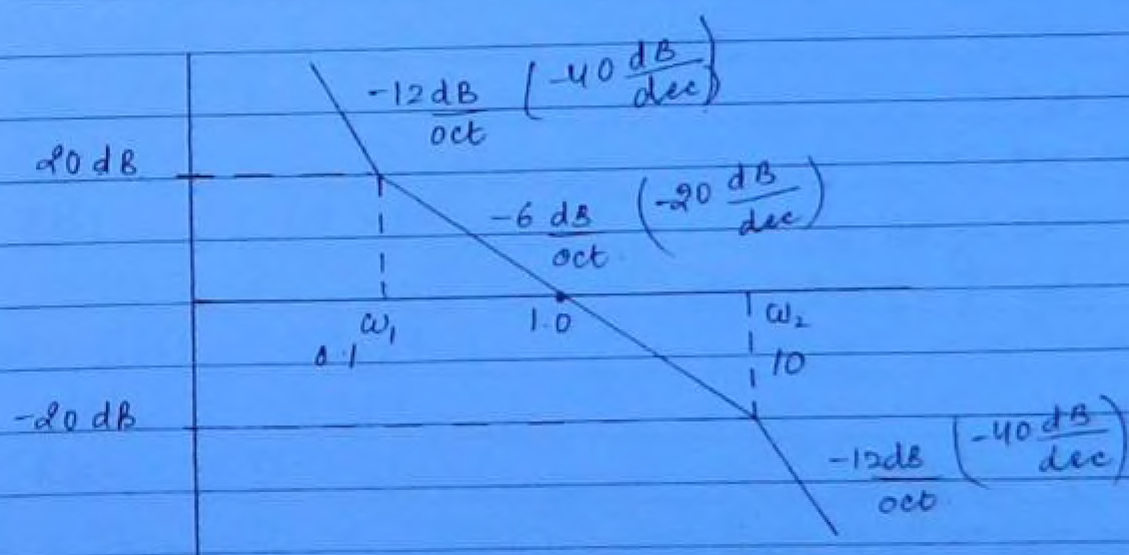
$-20 = 0 \times \log 0.1 + C$

$C = -20$

$20 \log K = -20$

$K = 0.1$

Imp Q



$$G(s) = \frac{0.1\,(1+10s)}{s^2(1+0.1s)}$$

DECADE SCALE

$\omega_2 = 10\omega_1$

dB value = $\pm 20 \times n \log \omega$
Magnitude

Slope (m) = $\pm 20 \times n \log 10$
$= \pm 20 \times n \ \frac{dB}{dec}$

OCTAVE SCALE

$\omega_2 = 2\omega_1$

dB value = $\pm 20 \times n \log \omega$
magnitude

Slope (m) = $\pm 20 \times n \log 2$
$= \pm 6 \times n \ \frac{dB}{oct}$

$$\pm 6 \times n \ \frac{dB}{oct} \approx \pm 20 \times n \ \frac{dB}{dec}$$

Second Line choose a line which passes through a known frequency.

At $\omega = 1$

$Y = MX + C$

$0 = -20 \log 1 + C$

$C = 0$

At $\omega = \omega_1$

$Y = MX + C$

$20 = -20 \log \omega_1 + 0$

$\omega_1 = \log^{-1}(-1) = 0.1 \ r/s$

At $\omega = \omega_2$

$-20 = -20 \log \omega_2 + 0$

$\omega_2 = \log^{-1}(1) = 10 \ r/s$

First line

At $\omega = \omega_1 = 0.1$

$$Y = MX + C$$

$$20 = -40 \log 0.1 + C$$

$$C = -20$$

$$20 \log K = -20$$

$$K = 0.1$$

M & N CIRCLES

1. The nicol's chart gives information about closed loop frequency response of the system.

2. It consists of magnitudes & phase angles of closed loop system represented as a family of circles known as M & N circles.

3. M CIRCLES

Let the CLTF

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

Let $G(s) = X + jY$

$$\frac{C(s)}{R(s)} = \frac{X + jY}{(1+X) + jY}$$

The magnitude (M) -

$$M = \frac{\sqrt{X^2 + Y^2}}{\sqrt{(1+X)^2 + Y^2}}$$

$$M^2 = \frac{X^2 + Y^2}{(1+X)^2 + Y^2}$$

$$M^2X^2 - X^2 + M^2Y^2 - Y^2 + 2XM^2 + M^2 = 0$$

$$X^2[M^2-1] + Y^2[M^2-1] + 2XM^2 + M^2 = 0 \quad \text{—— (1)}$$

In eq (1) If $M = 1$

$$2X + 1 = 0$$

→ It represents a st. line passing through $-\frac{1}{2}, 0$.

→ If $M \neq 1$ eqn (1) represents a family of circles.

$$x^2 + y^2 + 2x\frac{M^2}{M^2-1} + \frac{M^2}{M^2-1} = 0$$

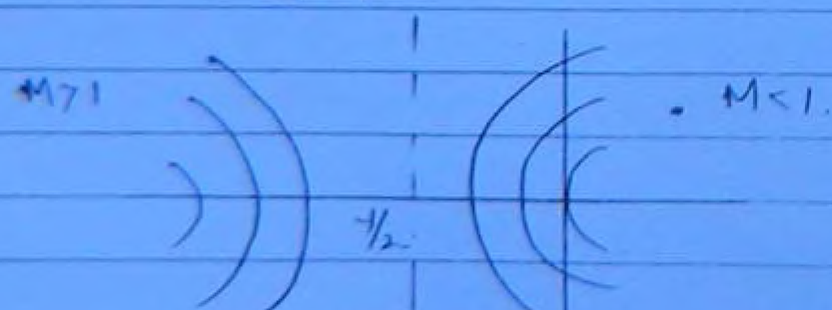
$$x^2 + y^2 + 2x\frac{M^2}{M^2-1} + \underbrace{\frac{M^2}{M^2-1} + \frac{M^2}{(M^2-1)^2}}, = \frac{M^2}{(M^2-1)^2}$$

$$X^2 + 2X\frac{M^2}{M^2-1} + \frac{M^4}{(M^2-1)^2} + Y^2 = \frac{M^2}{(M^2-1)^2}$$

$$\boxed{\left[X + \frac{M^2}{M^2-1}\right]^2 + Y^2 = \frac{M^2}{(M^2-1)^2}}$$

Centre → $-\frac{M^2}{M^2-1}, 0$

Radius → $\frac{M}{M^2-1}$



4. N - circles

let α = Phase angle of C.L. system

N = Tanα represents a family of circles

$$\alpha = \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)$$

$$N = \tan\alpha = \tan\left[\tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{y}{1+x}\right)\right]$$

$$N = \frac{\frac{y}{x} - \frac{y}{1+x}}{1 + \frac{y^2}{x(1+x)}} \Rightarrow x^2 + x + y^2 - \frac{y}{N} = 0$$

Adding term $\frac{1}{4} + \left(\frac{1}{2N}\right)^2$ on b.s.

$$x^2 + x + \frac{1}{4} + y^2 - \frac{y}{N} + \left(\frac{1}{2N}\right)^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2$$

$$\boxed{\left[x + \frac{1}{2}\right]^2 + \left[y - \frac{1}{2N}\right]^2 = \frac{1}{4} + \left(\frac{1}{2N}\right)^2}$$

Centre $= \frac{-1}{2}, \frac{1}{2N}$

Radius $= \sqrt{\frac{1}{4} + \left(\frac{1}{2N}\right)^2}$

For diff values of N all N circles intersect the real axis b/w -1 and origin only

5) $x^2 + 2.25x + y^2 + 1.125 = 0$

$$x^2 + 2x \cdot \frac{M^2}{M^2-1} + y^2 + \frac{M^2}{M^2-1} = 0$$

$$\frac{M^2}{M^2-1} = 1.125$$

$M^2 = 1.125 M^2 - 1.125$

$0.125 M^2 = 1.125$

$\boxed{M = 3}$ Ans (c)

