346

346

-: HAND WRITTEN NOTES:-

OF

1

ELECTRONICS & COMMUNICATION ENGINEERING

-: SUBJECT:-

COMMUNICATION SYSTEM

4

2

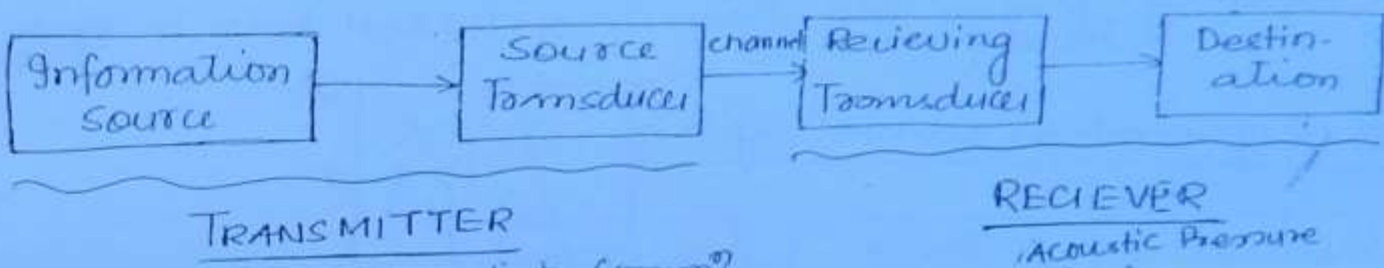
wwww.gatenotes.in

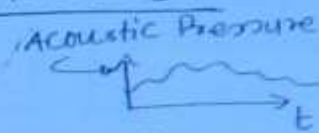
Defn:

communication is the process of transformation of Information from source to Destination or from transmitter to Reciever.

③

* Basic Block diagram of Commn System: (wired commn system)





→ Preferable for short dist. commn

Note:

Voice → 300Hz – 3.5KHz {variation of Acoustic Pressure with time}. (subset)

Audio → 20Hz – 20KHz

Video → 0 to 4.5MHz {variation of light intensity with time}

Information Source:

× Information source is the source of Information

Source Transducer: Source xducer converts a physical signal to electrical signal equivalent. Eg Mic

channel:

channel is the medium through which signal is transmitted from one place to another.

Note:

1. wired commn system, is preferred for short distance commn: where the channel will be:

a) co-axial cable

b) Parrallel wire.

c) Twisted pair etc.

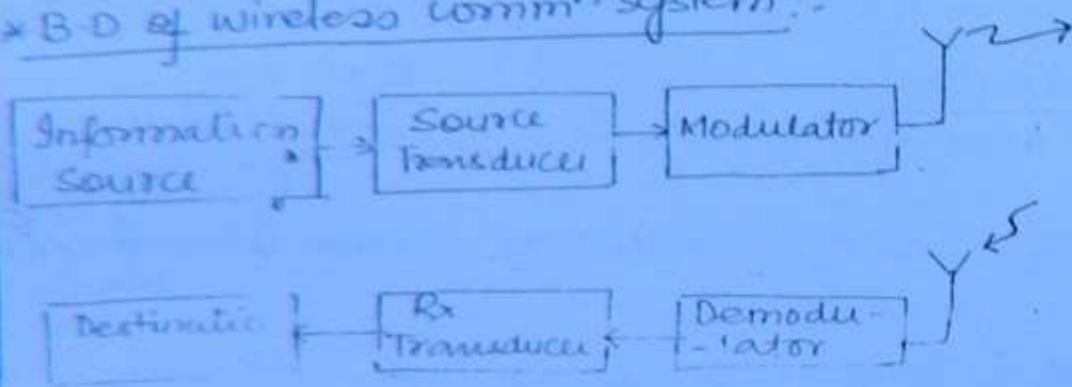
> For long distance commⁿ wireless commⁿ is preferred where the channel will be free space

Receiver ...

. It converts electrical signal as physical signal

Eg. loudspeaker.

* B.D of wireless commⁿ system :-

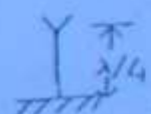


> For transmission of a signal to very much long distance through free space, modulation has to be used.

* Need of Modulation :-

1. Reducing of Antenna height : for faithful transmission, (i.e. transmission for long distance) the height of Antenna from the ground should be λ/4

$\boxed{h_t = \lambda/4.}$



$\boxed{\lambda = \frac{v}{f}}$

Note :- As, only EM wave travel in free space and travels with speed of light. So

$$\boxed{\lambda = c/f}$$

$$\text{So, } \boxed{h_t = \frac{c}{4f} \quad , h_t \propto \frac{1}{f}}$$

let $f = 15KHz$, So, $h_t = 5Km$, which is not possible

Hence frequency has to be increased and hence modulation is introduced.

x.

$15KHz \rightarrow \boxed{\text{Modulator}} \xrightarrow{1MHz}$; $h_t = \dfrac{c}{4f} = \dfrac{3\times10^8}{4\times10^6}$

$h_t = 75m$, practically possible

5

Note :-

1. Transmitting Antenna converts electrical signal as EM signal and will travel with the velocity of light

so,

$$\boxed{\begin{aligned}\lambda &= c/f \\ h_t &= c/4f\end{aligned}}$$

2. Modulation is the Process of increasing the frequency of the signal to reduce Antenna height

a) $f = 15KHz$ $\xrightarrow[\text{modulation}]{\text{After}}$ b) $f = 1MHz$

$h_t = c/4f$ $\qquad$ $h_t = c/4f$

$\boxed{h_t = 5000 mtr}$ $\qquad$ $\boxed{h_t = 75 Mtr}$

2. Multiplexing :- It is the process of transmission of multiple no. of signal through a single channel at the same time

× Generally, without modulation, Multiplexing is not possible.

× The lowest Possible frequency contained by a signal has to be taken into Reference to decide Antenna height.

Note :-

1. Modulation is used in wired comm^n system for Multiplexing

2.

* FOURIER TRANSFORM:

* Fourier Transform is a mathematical tool used to find the frequencies contained by given time domain signal

As,

$$x(t) \xrightarrow{F.T} X(f)$$

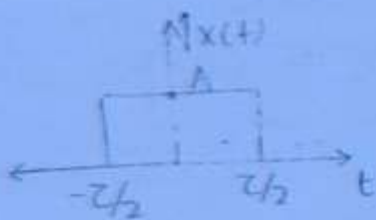
and mathematically,

$$\boxed{X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt}$$

→ Fourier xform is performed by the spectrum Analyser and has 2 modes:

1) magnitude plot (b/w $|X(f)|$ & f)
2) phase plot (b/w $\angle X(f)$ & f).

let



$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} A e^{-j2\pi f t}\, dt$$

$$= \frac{A}{-j2\pi f}\left[e^{-j2\pi f t}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{-A}{j2\pi f}\left[e^{-j2\pi f \times \tau/2} - e^{-j2\pi f \times -\tau/2}\right]$$

$$= \frac{-A}{j2\pi f}\left[e^{-j\pi f \tau} - e^{j\pi f \tau}\right]$$

$$= \frac{A}{j2\pi f}\left[e^{j\pi f \tau} - e^{-j\pi f \tau}\right]$$

$$= \frac{A}{\pi f}\left[\frac{e^{j\pi f \tau} - e^{-j\pi f \tau}}{2j}\right]$$

Note:

1. The Rectangular Pulse covers the frequency from +∞ to ... but only +ve frequency is taken into consideration

2. Signal B.W is defined as:

$$\boxed{\text{Signal B.W} = \text{Highest +ve freq}^n - \text{lowest +ve freq}^n}$$

Now,
In above case,

$$\text{Signal B.W} = +\infty - 0 = +\infty.$$

3. Channel Bandwidth is defined as the Range of frequency which the channel allows to pass without any distortion or attenuation.

Generally,

Co-axial cable = 0 to 600 MHz → B.W = 600MHz

Parallel wire = 0 to 200 MHz → B.W = 200MHz

optical fibre = GHz → B.W = In GHz

Also,

$$\boxed{\text{Channel B.W} \geq \text{Signal B.W}}$$

4. In the above example, the B.W of signal was ∞. Hence no channel supports its transmission. Hence either has to be performed:

1) channel to be designed of B.W = ∞. ✗

2) Signal B.W reduced to some finite value. (Band limiting)

5. For proper xmission of signal without any loss,

$$\boxed{\text{Channel B.W} \geq \text{Signal B.W}}$$

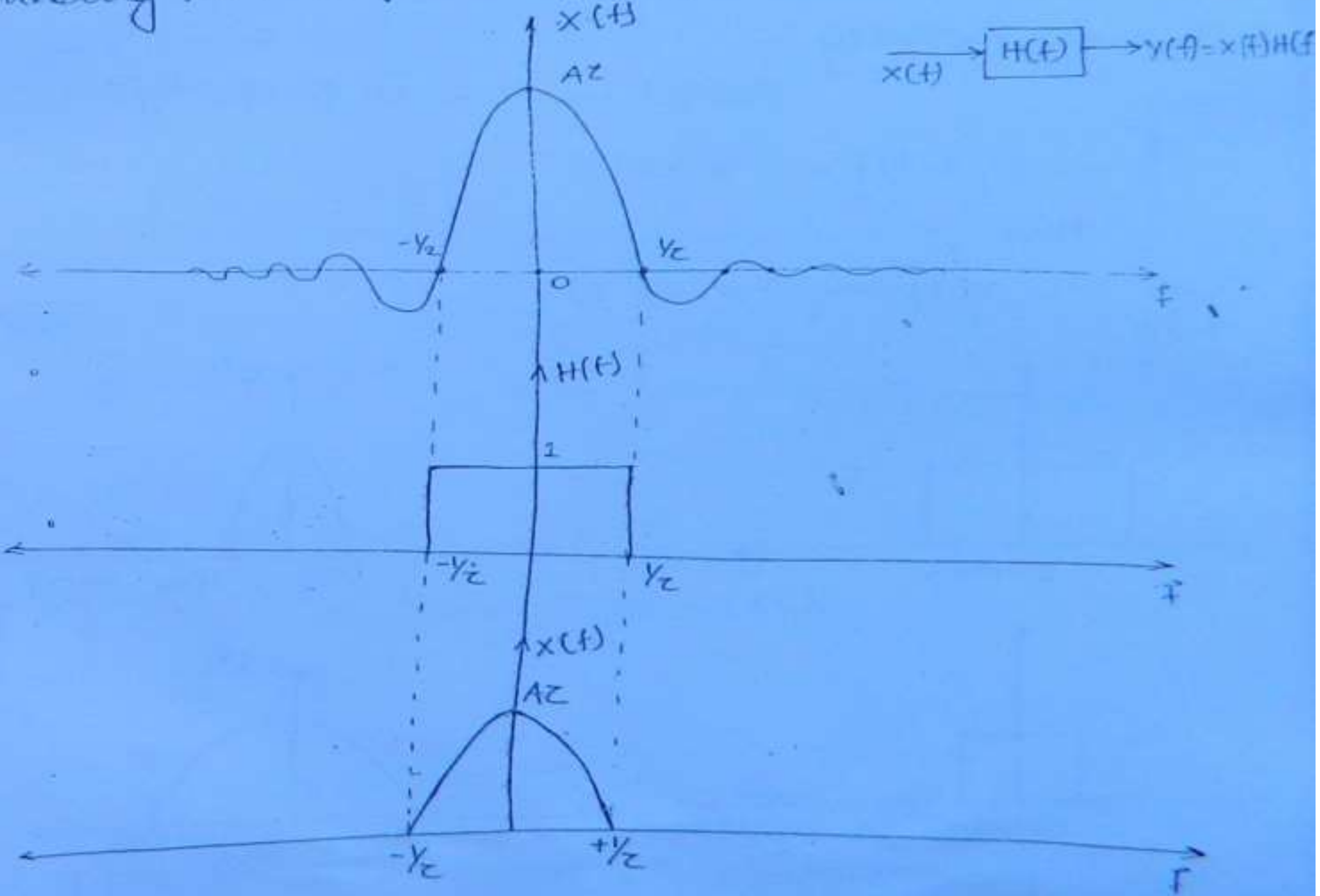
Since B.W of above signal is ∞. a channel having B.W ∞ is needed for transmission, but for practical channel, B.W will be finite.

(9)

Note:

1. The process of decreasing the B.W of a signal from ∞ to some finite value is called as "BAND LIMITING PROCESS".

2. The frequency ranging from $-1/\tau$ to $+1/\tau$ contains 95 to 99% of the strength of the total signal. Hence we have to eliminate the Rest. This is done by passing the signal through a L.P.F.



* Generally for a signal, most of the strength will be retained by low frequencies, strength of high frequencies go on decreasing and finally becomes zero.

2. For Band limiting a signal, all the significant frequencies should be Retained and Insignificant frequencies has to be eliminated.

10

3. For Band limiting, generally the signal will be passed through proper LPF

NOTE:

1. To use the channel B.W efficiently, we generally xmit Significant frequencies only.

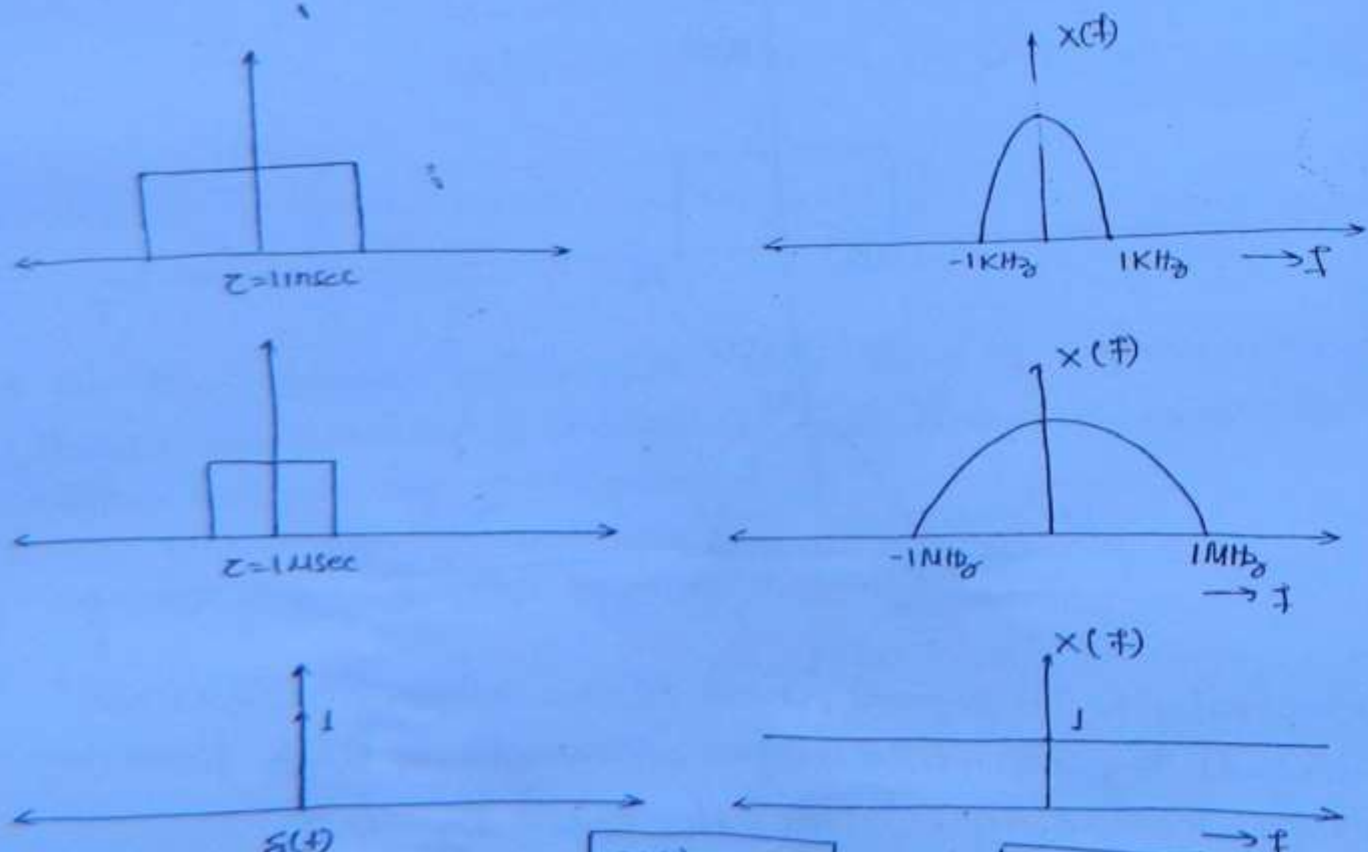
Properties of FOURIER TRANSFORM:

1. Duality Property: According to this property:

$$F\{x(t)\} \longleftrightarrow X(f)$$

then.

$$X(t) \longleftrightarrow x(-f)$$



$\boxed{\delta(t) \longleftrightarrow 1}$ $\boxed{B.W = \infty}$

Note:

1. $\boxed{\delta(t) = \infty, \; t = 0}$
$\quad = 0 \; ; \; t \neq 0$ $\Rightarrow \boxed{\int_{-\infty}^{\infty} \delta(t)\, dt = 1}$

(11)

2. $\boxed{2\delta(t) = \infty \; : \; t = 0}$
$\quad = 0 \; ; \; t \neq 0$ $\Rightarrow \boxed{\int_{-\infty}^{\infty} 2\delta(t)\, dt = 2}$

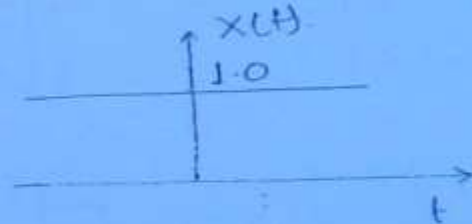
Now,

$\underset{x(t)}{\delta(t)} \longleftrightarrow k\, X(f)$

by duality property.

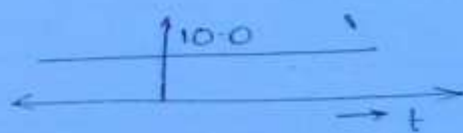
$\underset{x(t)}{1} \longleftrightarrow \underset{x(-f)}{\delta(-f)} = \delta(f)$

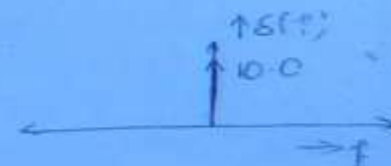
1. So, $\boxed{1 \longleftrightarrow \delta(f)}$



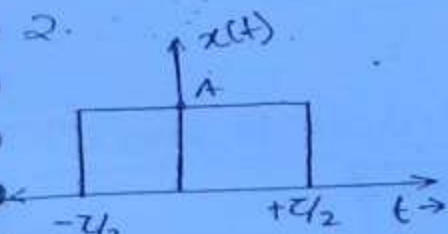
Also,

$10 \longleftrightarrow 10\,\delta(f)$





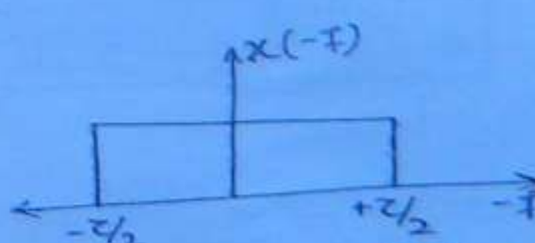
2.



$\xleftrightarrow{F.T} \overset{X(f)}{A\tau\, \mathrm{sinc}(f\tau)}$

So, $x(t) \xleftrightarrow{F.T} x(-f)$.

So, $A\tau\,\mathrm{sinc}(t\tau) \longleftrightarrow$



$\therefore$ (It is even funcⁿ)

Frequency shifting property:

According to this property

if $\boxed{x(t) \longleftrightarrow X(f)}$

then

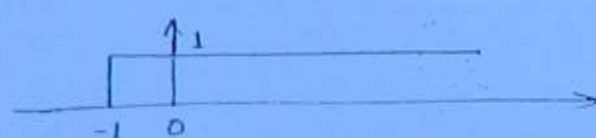
$$\boxed{\begin{array}{l} e^{j2\pi f_0 t} x(t) \longleftrightarrow X(f-f_0) \\ e^{-j2\pi f_0 t} x(t) \longleftrightarrow X(f+f_0) \end{array}}$$

let

$u(t) = 1 ; t \geq 0$
$\quad = 0 ; t < 0$

$u(t-1) = 1 ; t \geq 1$
$\quad 0 ; t < 1$

$u(t+1) = 1 ; t \geq -1$
$\quad 0 ; t < -1$

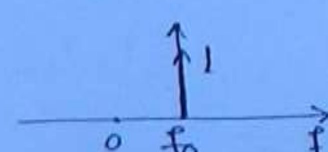


Note:

$e^{j2\pi f_0 t} \longleftrightarrow ?$

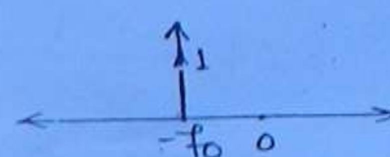
As, $\underset{\curvearrowright x(t)}{1} \longleftrightarrow \delta(f) \rightarrow X(f)$

$$\boxed{e^{j2\pi f_0 t} \cdot 1 \longleftrightarrow \delta(f-f_0)}$$



Also,

$$\boxed{e^{-j2\pi f_0 t} \longleftrightarrow \delta(f+f_0)}$$

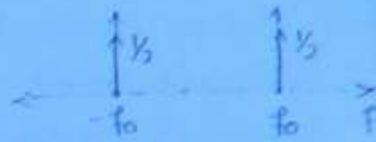


Also,

$$\cos 2\pi f_0 t = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \longleftrightarrow \frac{1}{2}\left\{\delta(f-f_0) + \delta(f+f_0)\right\}.$$

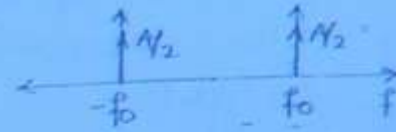
SO,

1. $\cos 2\pi f_0 t \longleftrightarrow \dfrac{\delta(f-f_0)+\delta(f+f_0)}{2}$

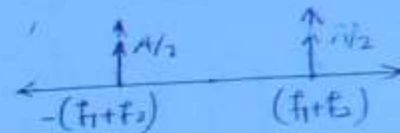


(13)

2. $A\cos 2\pi f_0 t \longleftrightarrow \dfrac{A}{2}\left\{\delta(f-f_0)+\delta(f+f_0)\right\}$



3. $A\cos 2\pi (f_1+f_2)t \longleftrightarrow \dfrac{A}{2}\left\{\delta(f-(f_1+f_2))+\delta(f+(f_1+f_2))\right\}$



3. Modulation Property:-

It states that:

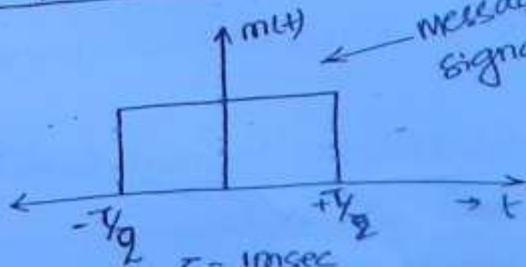
If $\boxed{x(t) \longleftrightarrow X(f)}$

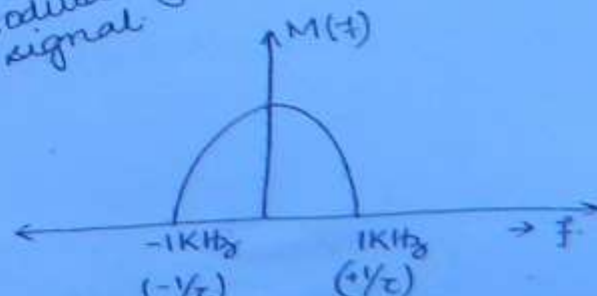
then $\boxed{x(t)\cos 2\pi f_0 t \longleftrightarrow \dfrac{X(f-f_0)+X(f+f_0)}{2}}$

$x(t)\left\{\dfrac{e^{j2\pi f_0 t}+e^{-j2\pi f_0 t}}{2}\right\}$

Concept of Modulation:-

message signal or modulating signal





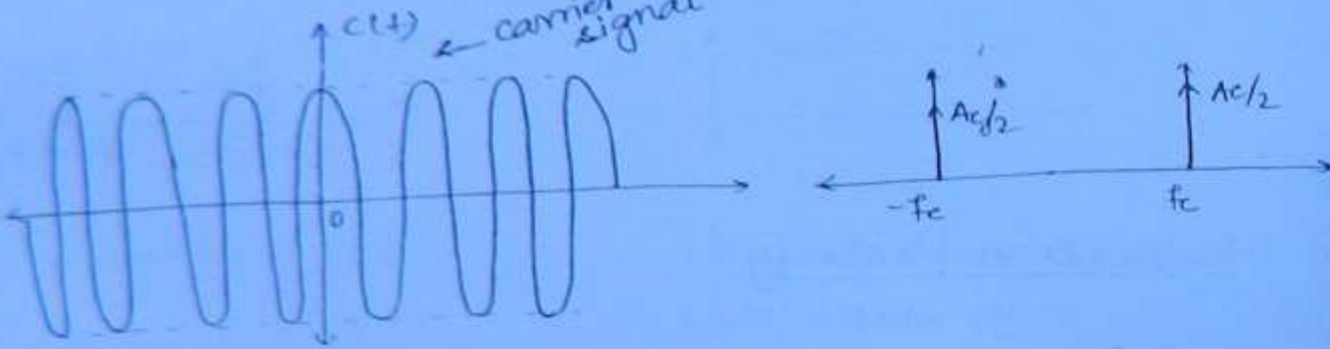
→ If a signal contain all the significant low frequency then such signals are called as "BASE BAND SIGNAL".

→ These Base band signals have significant low frequencies hence they require huge Antenna heights, which is impossible to construct. Hence the process of Modulation is introduced such that the frequency is increased to Reduce Antenna heights.

(14)

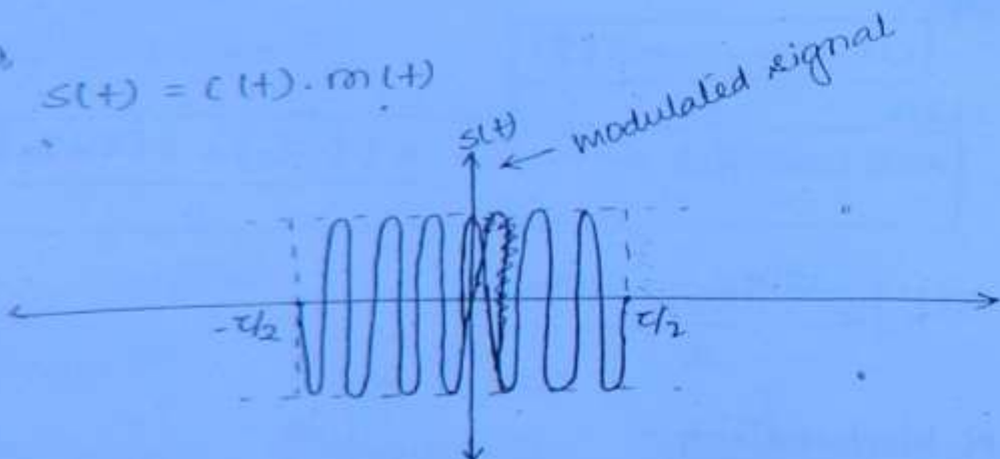
so, let

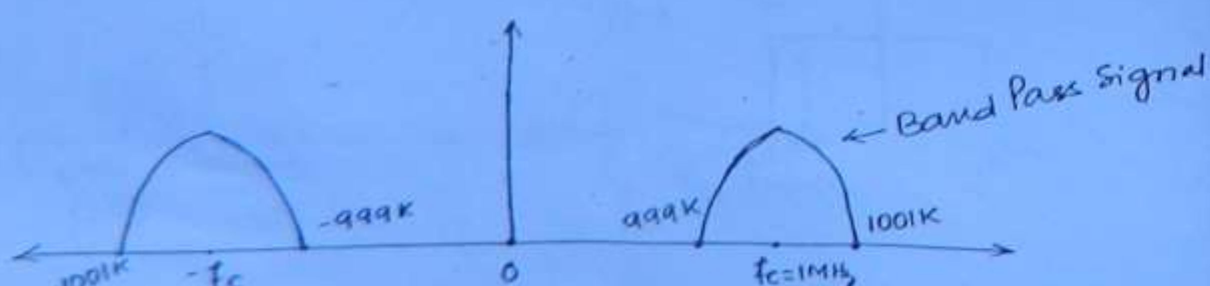
Carrier Signal; $c(t) = A_c \cos 2\pi f_c t$; $f_c = 1MHz$
$= 1000KHz$.



So,

$s(t) = c(t) \cdot m(t)$





→ If a Signal contain only significant high frequencies then such signal are called BAND PASS SIGNAL.

Note:

A Base Band Signal can't be transmitted faithfully as it requires huge Antenna, but a Band Pass signal can be transmitted faithfully.

15

× By MODULATION ;-

i) the signal is translated from low frequency Region to high frequency Region.

ii) the Base Band Signal, is converted as Band Pass signal.

iii) A wide Band Signal, becomes narrow Band signal.

Note:

If ;

$$\boxed{\frac{\text{Highest Frequency}(+ve)}{\text{lowest Frequency}(+ve)} >>> 1 \leftarrow \text{wide Band Signal}}$$

$$\boxed{\frac{\text{Highest +ve Frequency}}{\text{lowest +ve Frequency}} \approx 1 \leftarrow \text{Narrow Band Signal}}$$

* Concept of Demodulation :-

→ Process of Recieving back the message signal, from the modulated signal.

→ Demodulation will be done at the Reciever.

× Classification of modulation :

- Single tone modulation (Single freq^n msg signal modulation)
- Multitone modulation (Multi freq^n msg signal modulation)

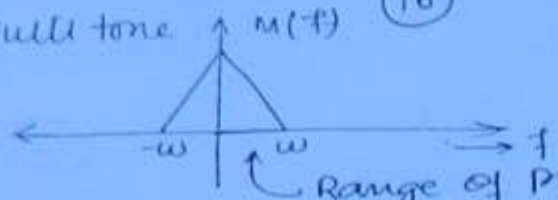
eg.

1. $m(t) = A_m \cos 2\pi f_m t$ ← Single tone.

(16)

2. $m(t) \longleftrightarrow M(f)$ ← Multi tone

(Single message containing Range of freq^n)



or

$m(t) \longleftrightarrow A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t$

(group of messages having diff. freq^n)

Note :-

Modulation is basically of two types:

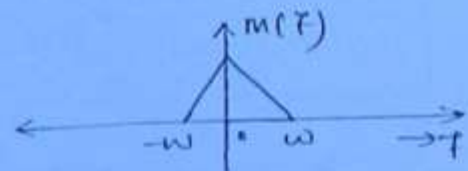
i) If the message signal having single freq^n then the corresponding frequency is called as SINGLE TONE MODULATION.

eg $m(t) = A_m \cos 2\pi f_m t$

ii) If the message signal having multiple freq^n then the corresponding modulation is called as MULTI TONE MODULATION.

eg

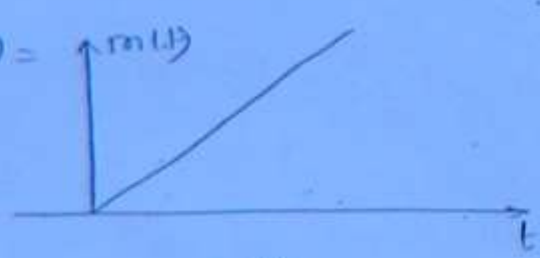
$m(t) \longleftrightarrow M(f)$

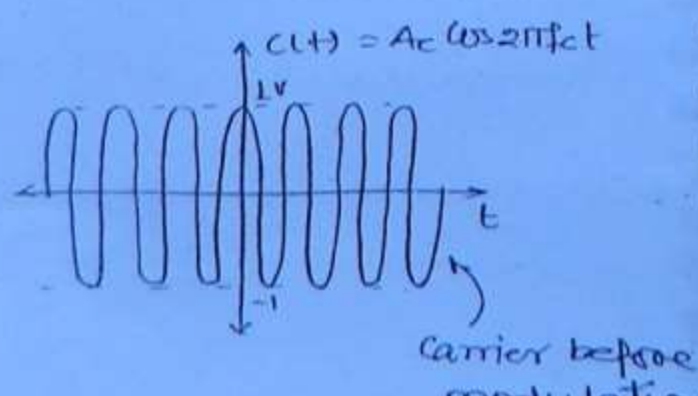


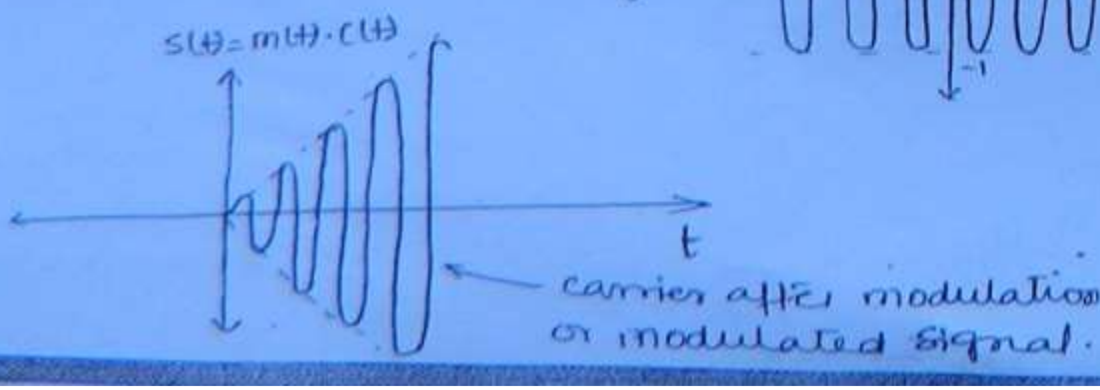
or

$m(t) = A_{m_1} \cos 2\pi f_{m_1} t + A_{m_2} \cos 2\pi f_{m_2} t$

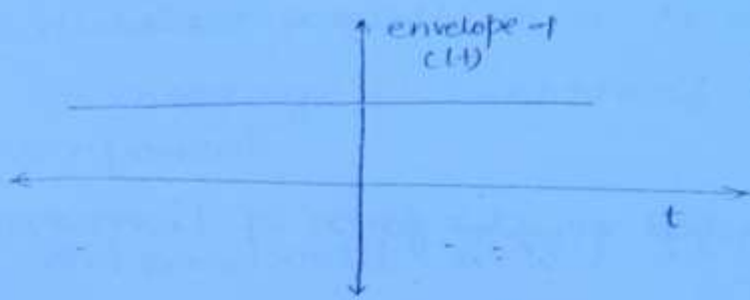
eg $m(t) =$



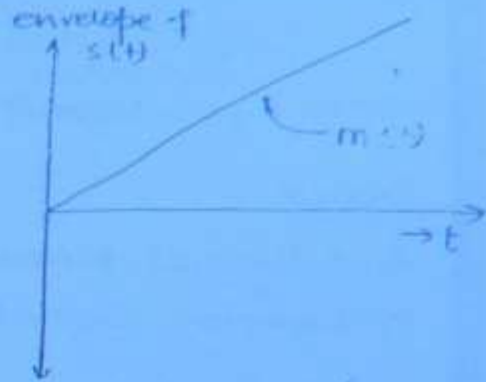




envelope: A line which touches all the +ve peak of the signal is called as ENVELOPE



17



Note :-

The above corresponds to Amplitude modulation where the message signal is stored in the form of Amplitude variations of carrier after modulation or in the form of envelope of the modulated signal

* MODULATION:

It is the process in which one of the Parameter (Amplitude, frequency or phase) of the carrier signal will be varied linearly in accordance with message signal amplitude variations

* AMPLITUDE MODULATION:-

Def^n: It is the process in which Amplitude of the carrier signal will be changed (varied) linearly in accordance with message signal Amplitude variations

Assume,

$m(t)$ = msg signal

$A_c \cos 2\pi f_c t$ = carrier signal $c(t)$.

*General exp of AM signal:

$$\boxed{S_{AM}(t) = A_c\{1 + K_a m(t)\} \cos 2\pi f_c t}$$

K_a = Amplitude sensitivity of AM modulator.

So,

$S_{AM}(t) = A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$ (18)

carrier signal — modulated signal

Disad:

Additional power is wasted in the form of transmitting of carrier signal.

Advantage:

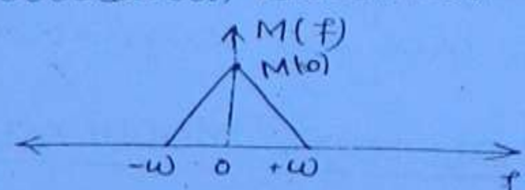
Due to the additional carrier signal, the demodulation of the AM signal becomes easier and cheaper.

Note:-

* AM signal consists of Additional carrier along with the Actual modulated signal

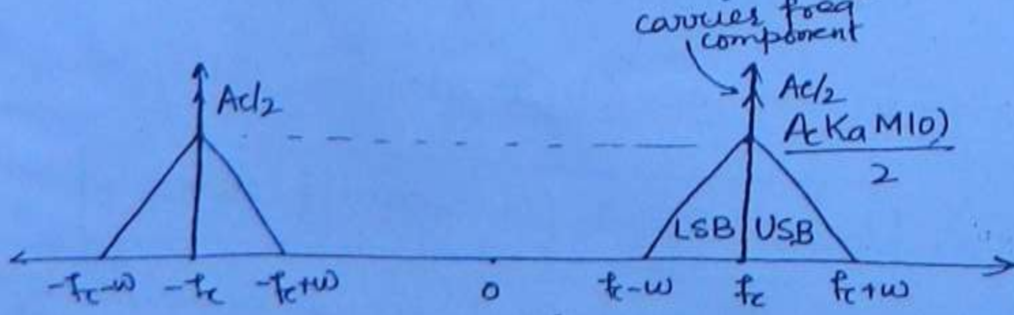
Because of this additional carrier transmitter power will be wasted but demodulation becomes simple.

$m(t) \longleftrightarrow M(f)$



$c(t) = A_c \cos 2\pi f_c t \longleftrightarrow$

$S_{AM}(t) \longleftrightarrow$



Note:

The original message signal is contained in the modulated signal ie in the USB & LSB

19

2. No message is contained in the frequency (carrier) component

3. AM Bandwidth = $(f_c + \omega) - (f_c - \omega)$

$= 2\omega$

So,

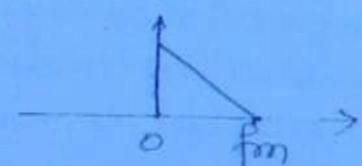
$$\boxed{\text{AM Bandwidth} = 2 \times \text{msg signal B.W}}$$

* Importance of -ve frequency:

let only +ve frequency be considered

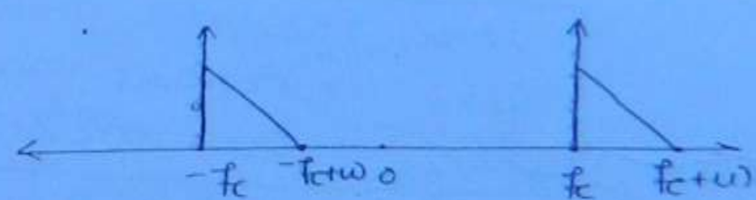
Hence,

$m(t) \longleftrightarrow M(f)$



So,

$S_{AM}(t) \longleftrightarrow$



B.W = ω

originally B.W of AM = 2ω

Hence needed the -ve frequency in the discussion

* AM spectrum consists of:

1) carrier frequency component existing at f_c.
2) USB existing above f_c.
3) LSB existing below f_c.

Note:

The Actual msg signal will be Retained by only Side Bands

* The B.W of each of the sideband is equal to ω ie equal to the msg B.W

* SINGLE TONE AM:

let,

$$m(t) = A_m \cos 2\pi f_m t$$

&

$$S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$$

Substituting we get:-

$$S_{AM}(t) = A_c\{1 + K_a \cdot A_m \cos 2\pi f_m t\} \cos 2\pi f_c t$$

where,

$$\boxed{\mu = K_a \cdot A_m = \text{modulation index of AM.}}$$

$$\boxed{\mu \times 100\% = \%\text{ of modulation or depth of modulation}}$$

→ The physical significance of depth of modulation is th the content of message signal that is stored in th carrier signal is called as 'depth of modulation'.

$\mu < 1$ → under modulation	✓	Generally used.
$\mu = 1$ → critical modulation	✓	
$\mu > 1$ → over modulation	✗	

demodulation of AM signal becomes difficult.

* To what extent the carrier signal is modulated by the msg signal is specified by MODULATION INDEX.

* over modulation is not preferred because demodulati becomes complex.

*** so,

$$\boxed{S_{AM}(t) = A_c\{1 + \mu \cos 2\pi f_m t\}\cos 2\pi f_c t} \quad \text{--- (1)}$$

on expanding we get:-

$$S_{AM}(t) = A_c \cos 2\pi f_c t + A_c \mu \cos 2\pi f_m t \cos 2\pi f_c t \text{ --- (2)}$$

Two
XXXX

$$\boxed{S_{AM}(t) = A_c\cos 2\pi f_c t + \frac{A_c\cdot\mu}{2}\cos 2\pi(f_c+f_m)t + \frac{A_c\cdot\mu}{2}\cos 2\pi(f_c-f_m)t}$$

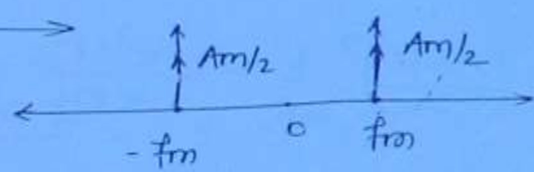
* Frequency Analysis of Single tone AM:

(21)

As,

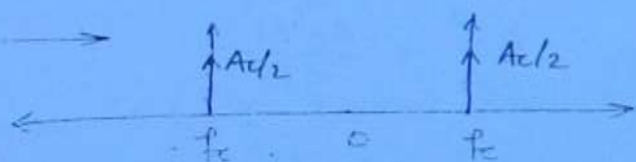
$$\boxed{S_{AM}(t) = \underbrace{A_c\cos 2\pi f_c t}_{\text{carrier component}} + \underbrace{\frac{A_c\cdot\mu}{2}\cos 2\pi(f_c+f_m)t}_{\text{USB}} + \underbrace{\frac{A_c\cdot\mu}{2}\cos 2\pi(f_c-f_m)t}_{\text{LSB}}}$$

$m(t) = A_m\cos 2\pi f_m t \longleftrightarrow$

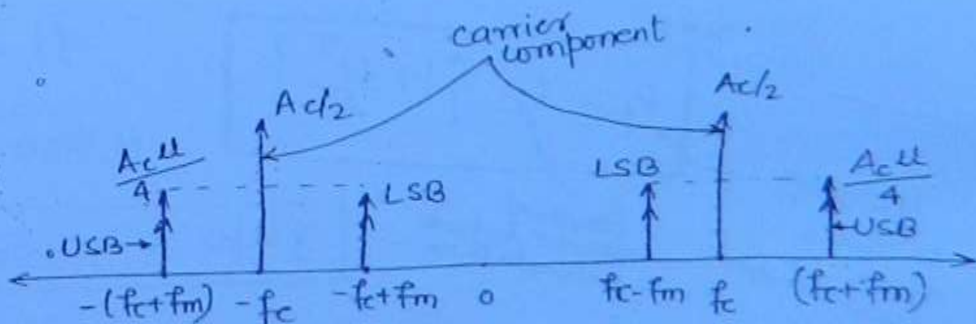


{only single +ve frequency (fm). Hence B.W = n}

$c(t) = A_c\cos 2\pi f_c t \longleftrightarrow$



$S_{AM}(t) \longleftrightarrow$



So, $\boxed{\text{AM Bandwidth} = (f_c+f_m) - (f_c-f_m) = 2f_m}$

So, f_m = frequency of msg signal.

So, $\boxed{\text{AM Bandwidth} = 2\times\text{frequency of msg signal}}$

* POWER OF AM SIGNAL:-

The total power of AM signal is given as:

$$P_t = P_c + P_{USB} + P_{LSB}$$

$x(t) = V_m$; $x(t) = V_m \cos 2\pi f t$

$\left\{ P_{dc} = \frac{V_m^2}{R} \quad P_{ac} = \frac{V_{rms}^2}{R} \right.$

$V_{rms} = V_m/\sqrt{2}$

$P_{ac} = \frac{V_m^2}{2R} \}$

Now,

$$P_c = \left(\frac{A_c}{\sqrt{2}}\right)^2 \cdot \frac{1}{R} = \frac{A_c^2}{2R}$$

$$P_{USB} = \left(\frac{A_c \mu}{2}\right)^2 \cdot \frac{1}{2R} = \frac{A_c^2 \mu^2}{8R}$$

$$P_{LSB} = \left(\frac{A_c \mu}{2}\right)^2 \cdot \frac{1}{2R} = \frac{A_c^2 \mu^2}{8R}$$

So,

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{4R}$$

$$P_t = \frac{A_c^2}{2R}\left\{1 + \frac{\mu^2}{2}\right\}$$

So, ***

$$\boxed{P_t = P_c \left\{1 + \frac{\mu^2}{2}\right\}}$$

Power of carrier after modulation

Power of carrier before modulation.

Now,

$$P_t = P_c + \frac{P_c \mu^2}{2} = P_c + P_{SB}$$

where, $\boxed{P_{SB} = \frac{P_c \mu^2}{2}}$; $\boxed{P_{USB} = P_{LSB} = \frac{P_c \mu^2}{4}}$

* The Power of carrier is independent of μ.

*** The Side Band power depends on μ, and as the μ is increased, the P_{SB} also increases (Power of the modulated signal & total power of AM signal increases).

Case 1:
$\mu = 0$; no modulation

Hence, $\boxed{P_t = P_c}$ (23)

as, $S_{AM}(t) = A_c \cos 2\pi f_c t$

Case 2:

$\mu = 1$; total modulation
(100% modulation)

Hence, $P_t = \frac{3}{2} P_c$

$$\boxed{P_t = 1.5 P_c}$$

And, $S_{AM}(t) = A_c \cos 2\pi f_c t + A_c \cos 2\pi f_m t * \cos 2\pi f_c t$

Note:
As μ increases from 0 to 1, total AM power is increased by 50%.

Now, $P_c = \frac{2}{3} P_t$

$P_c = 0.666 P_t$

So, $\boxed{P_c = 66.66\% \text{ of } P_t}$

Now, as,

$P_t = P_c + P_{SB}$

$P_t = \frac{2}{3} P_t + P_{SB}$

$P_{SB} = \frac{1}{3} P_t$

$$\boxed{P_{SB} = 33.33\% \text{ of } P_t}$$

CONCLUSION:

1. If $\mu=0 \Rightarrow P_c = 100\%$ of $P_t \Rightarrow P_{SB} = 0\%$ of P_t (24)

2. If $\mu=1 \Rightarrow P_c = 66.66$ of $P_t \Rightarrow P_{SB} = 33.33\%$ of P_t.

Note:-
In the efficient power distribution case, ie $\boxed{\mu=1}$
still 66.66% of P_t; is wasted in the form of transmission of additional carrier. This is the biggest draw back of AM.

* Modulation efficiency (η):
* It specifies share of SideBand power in total power

eg
$\eta = 0.1 \longrightarrow 10\%$ of P_{SB} in P_t.
$\eta = 0.3 \longrightarrow 30\%$ of P_{SB} in P_t

Mathematically

$$\boxed{\eta = \frac{P_{SB}}{P_t}}$$

so,

$$\eta = \frac{\frac{P_c \mu^2}{2}}{P_c\left\{1+\frac{\mu^2}{2}\right\}}$$

***

$$\boxed{\eta = \frac{\mu^2}{2+\mu^2}}$$

Case 1:-
$\mu=0$

for $\mu=0$; $\eta=0 \Rightarrow P_{SB} = 0\%$ of P_t

$P_c = 100\%$ of P_t

$\eta = 0.11 \Rightarrow P_{SB} = 11\%$ of P_t

$P_c = 89\%$ of P_t

25

Case 3: ($\mu = 0.707$)

$\eta = 0.2 \Rightarrow P_{SB} = 20\%$ of P_t

$P_c = 80\%$ of P_t

Case 4: ($\mu = 1$)

$\eta = 0.33 \Rightarrow P_{SB} = 33.3\%$ of P_t

$P_c = 66.7\%$ of P_t

Note: let

$P_t = P_c + P_{SB}$

$50W = 30W + 20W$
$\quad\quad 60\% \quad 40\%$

$60W = 30W + 30W$
$\quad\quad 50\% \quad 50\%$

$\mu \uparrow$

* P_c is independent of μ and share of carrier power in total power decreases as 'μ' increases.

* An unmodulated AM transmitted power is given 500 W. Find AM transmitted power with 100% modulation?

Soln: Given $P_t = 500W$, $\mu = 0 \Rightarrow P_c = 500W$ & $P_{SB} = 0$

$\mu = 1$; ~~$P_t = P_c + P_{SB}$~~
~~$= 500 +$~~

~~$P_c = \frac{3}{2} P_t$~~

~~$P_c = \frac{3}{2} \times 500$~~

~~$= 750$ watt.~~

$P_t = P_c \left\{1 + \frac{\mu^2}{2}\right\}$

$\boxed{P_t = 500\left\{1 + \frac{1}{2}\right\} = 750W}$

Note:

Q. For an AM signal total sideBand power is given by 100W with 50% of modulation. Find total AM transmitted power.

(26)

Soln:

$P_{SB} = 100W \quad ; \mu = \frac{1}{2}$

$P_t = ?$

$$P_{SB} = \frac{P_c \mu^2}{2}$$

$$P_c = \frac{P_{SB} \cdot 2}{\mu^2} = 100 \times 2 \times 4$$

$$P_c = 800$$

So, $P_t = P_c + P_{SB}$

$$\boxed{P_t = 900W}$$

Q2. For an AM each of the S.B power is given by 2KW and carrier power is given by 8KW. Find % of modulation.

Soln:

$P_c = 8KW$

$P_{SB} = 4KW \quad \{P_{SB} = P_{USB} + P_{LSB} = 2+2\}$

So,

$$P_{SB} = \frac{P_c \mu^2}{2}$$

$$\mu^2 = \frac{P_{SB} \times 2}{P_c}$$

$$\mu^2 = \frac{4 \times 2}{8}$$

100% modulation → $\boxed{\mu = 1}$ Ans

Q3. A carrier of $10\cos 2\pi \times 10^6 t$ is amplitude modulated by a message signal of $4\cos 4\pi \times 10^3 t$ with 50% of modulation. Antenna Resistance is given by 5Ω.

1. Find all the parameters of AM
2. Plot AM spectrum and Identify the spectral components

Q7

Sol:- Given:

$$c(t) = 10\cos 2\pi \times 10^6 t = A_c \cos 2\pi f_c t.$$

$$m(t) = 4\cos 4\pi \times 10^3 t = A_m \cos 2\pi f_m t$$

So, $A_m = 4\,V$; $f_m = 2\times 10^3 = 2\,KHz$

$A_c = 10\,V$; $f_c = 10^6 = 1\,MHz$

So, for single tone modulation

$B.W = 2 f_m$

$B.W = 2 \times 2K$

$\boxed{B.W = 4\,KHz}$ Ans

And, $\mu = 0.5$; $R = 5\,\Omega$

So, $\boxed{P_c = \frac{A_c^2}{2R} = \frac{10^2}{2\times 5} = 10\,W}$ Ans

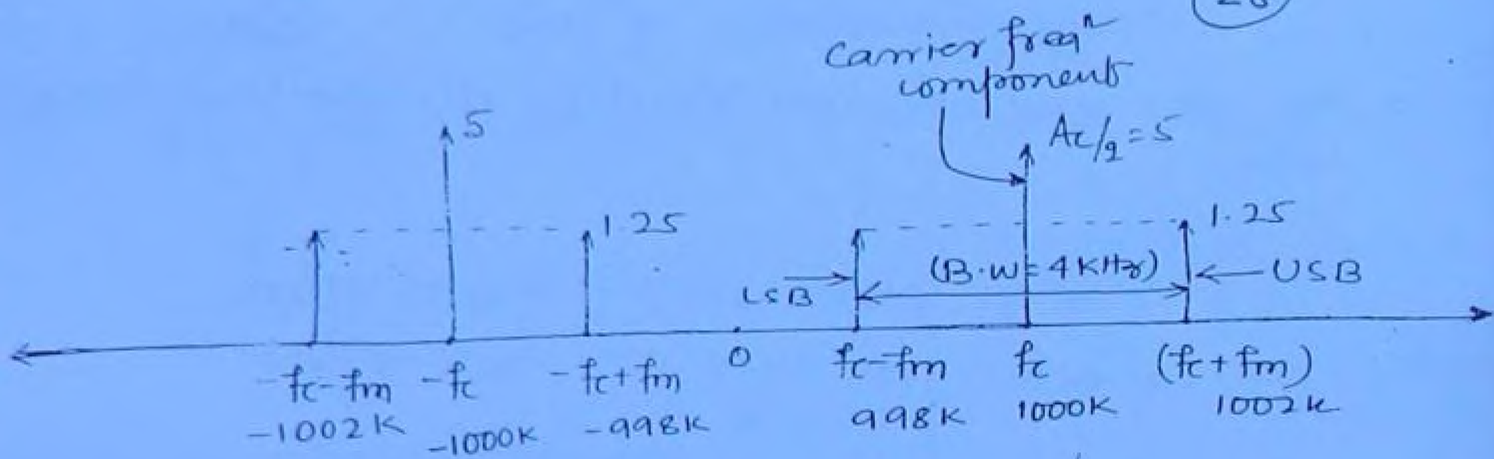
$$P_t = P_c\left\{1 + \frac{\mu^2}{2}\right\} = 10\left\{1 + \frac{0.5^2}{2}\right\}$$

$\boxed{P_t = 11.25\,W}$ Ans

So, $\boxed{P_{SB} = P_t - P_c = 1.25\,W}$ Ans

$\boxed{P_{USB} = P_{LSB} = P_{SB}/2 = 0.625\,W}$ Ans

And, $\boxed{\eta = \frac{P_{SB}}{P_t} = \frac{\mu^2}{2+\mu^2} = 0.11 = 11\%}$ Ans



Q4. A carrier of $10\cos 8\pi \times 10^5 t$ is amplitude modulated by a msg signal of $6\cos \pi \times 10^4 t$. i) find all the parameters of AM.

ii) Plot spectral components & spectrum.

Soln: Given:

$$c(t) = 10\cos 8\pi \times 10^5 t = A_c \cos 2\pi f_c t$$

$$A_c = 10; \quad f_c = 4\times 10^5 = 400\,KHz.$$

$$m(t) = 6\cos \pi \times 10^4 t = A_m \cos 2\pi f_m t$$

$$A_m = 6 \quad ; \quad f_m = 5\,KHz.$$

Now, As μ is not given. So take :-

$$\boxed{\mu = \frac{A_m}{A_c} = \frac{6}{10} = 0.6}$$

Now, $P_c = \cancel{\frac{A_c^2}{2}} \frac{A_c^2}{2R} = \frac{10^2}{2\times 1}$ {As Antenna Resistance is not given. Hence $R = 1\Omega$}

$$\boxed{P_c = 50\,W.}$$

$$P_t = P_c\left\{1 + \frac{\mu^2}{2}\right\}$$

$$P_t = 50\left\{1 + \frac{0.6^2}{2}\right\}$$

$$\boxed{P_t = 59\,W}$$

AM Bandwidth = $2 f_m$

AM B.W = 2×5K

$\boxed{AM\ B.W = 10K}$ Ans

(29)

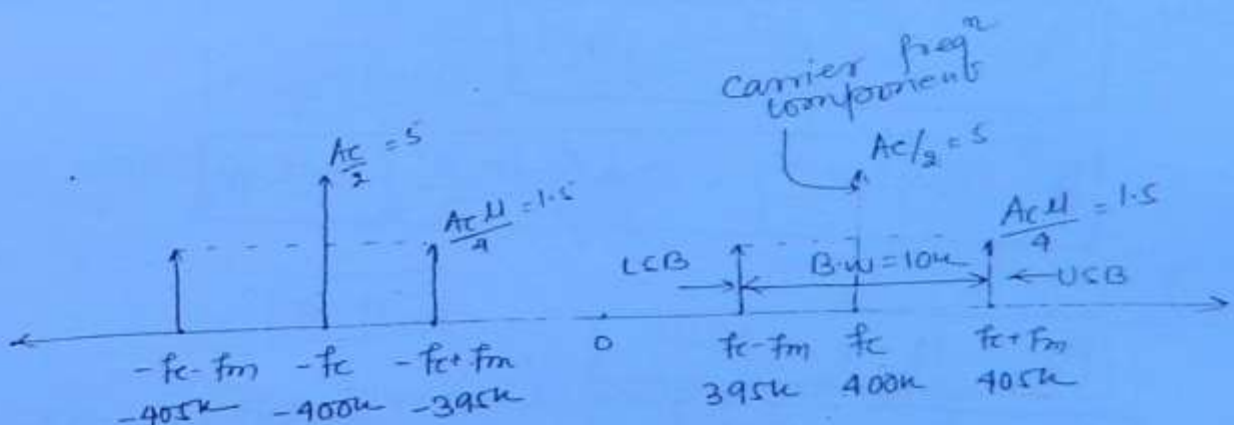
Now, $\boxed{P_{SB} = P_t - P_c = 9W}$ Ans

$\boxed{P_{USB} = P_{LSB} = 4.5W}$ Ans

And, $\eta = \frac{\mu^2}{2+\mu^2} = \frac{0.6^2}{2+0.6^2}$

$\boxed{\eta = 0.15}$ Ans

spectrum :



Q5. An AM signal is given by

$s(t) = 4\cos 3200\pi t + 10\cos 4000\pi t + 4\cos 4800\pi t$

Find all the parameters of AM and plot the spectrum.

Sol^n :- As,

$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m)t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)t$

Comparing we get :-

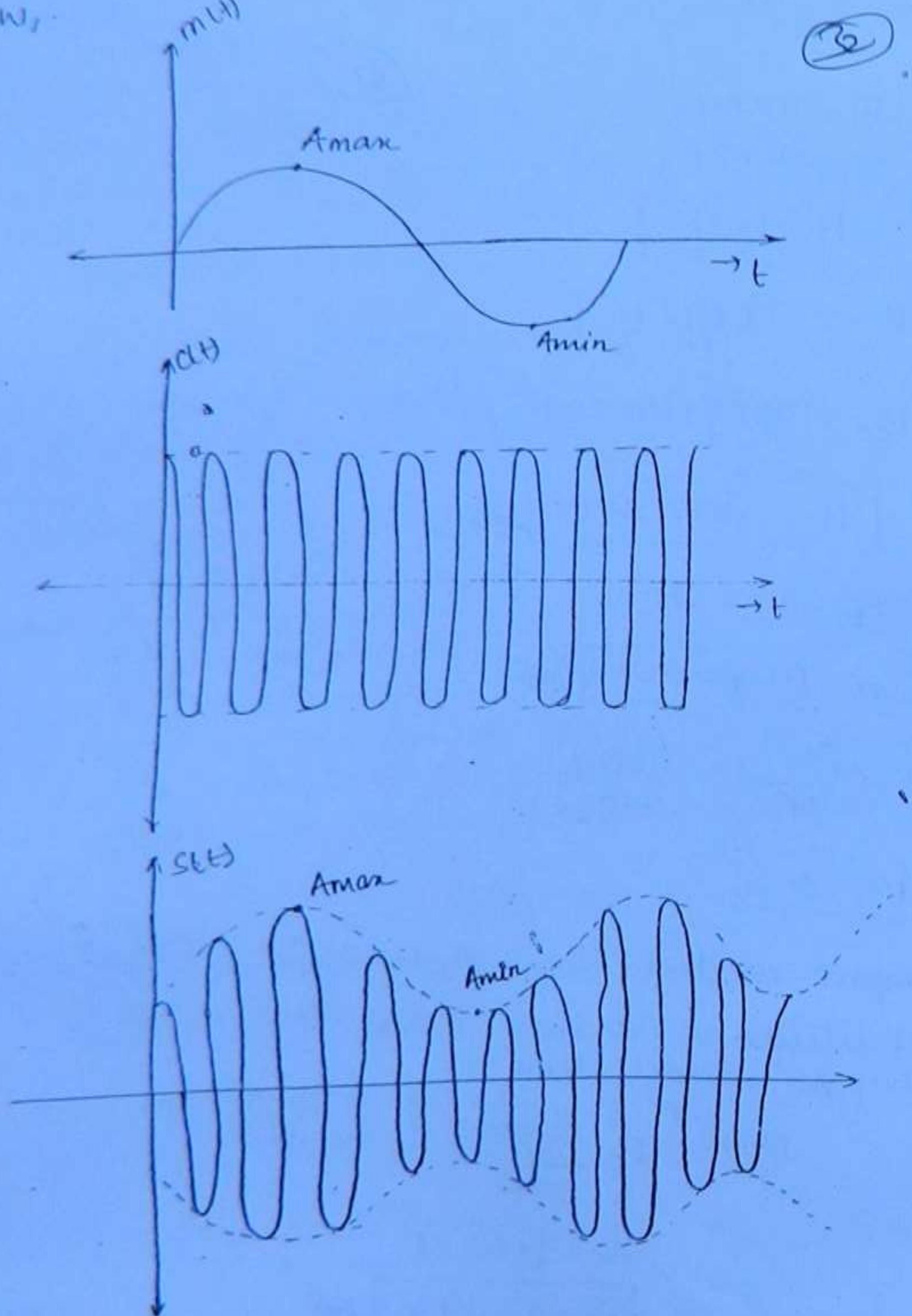
$S_{AM}(t) = 10\cos 4000\pi t + 4\cos 4800\pi t + 4\cos 3200\pi t$

{As the value of magnitude remain same for USB & LSB}

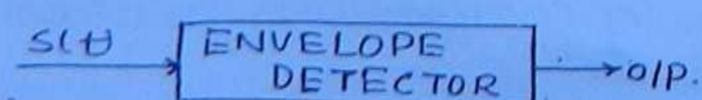
→ The peak amplitude of the carrier after modulation is not const but it varies in accordance to the msg signal.

Now,

2



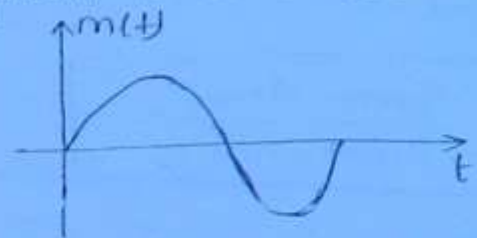
Note:- If the above signal is fed to the input of the envelope detector for it's demodulation as in the fig. shown below:-



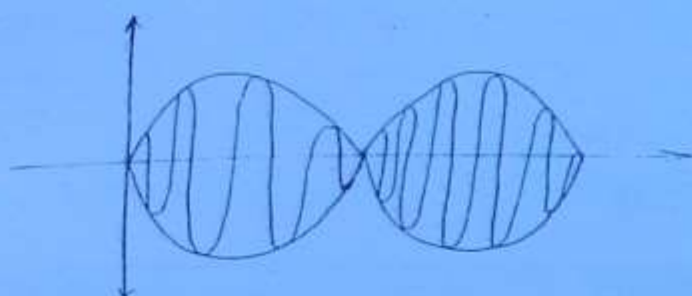
The O/P of the envelope [illegible]

33

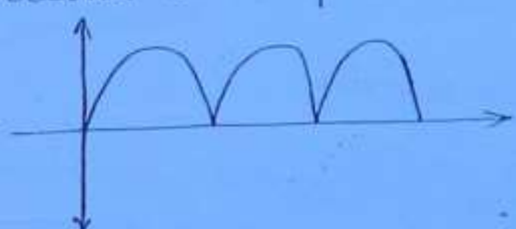
The above signal when shifted by some DC value, the waveform obtained resembles to that of the original signal. Its waveform is as shown below:-



Now, if the wave of the AM transmitted signal is as follows:-



And if this above signal is passed through the ED the O/P obtained is as follows:-



← This signal doesn't correspond to the original message signal and by any means it cannot be converted to the original message signal. Hence the above waveform is not the AM transmitted wave.

Note:-

$\boxed{\text{Max}^m \text{ peak of AM signal, } A_{max} = A_c\{1+\mu\}.}$

$\boxed{\text{Min}^m \text{ peak of AM signal; } A_{min} = A_c\{1-\mu\}}$

$\Big\{$ ∵ max^m value of the $\cos 2\pi f_m t = 1$

$-1 \leq \cos 2\pi f_m t \leq 1$

So, $A_{max} = A_c(1+\mu) = 12.64\,(1+0.707) = 21.5$ volts.

$A_{min} = A_c(1-\mu) = 12.64\,(1-0.707) = 3.7$ volts.

Also,

$$A_{max} + A_{min} = 2A_c$$

So,

$$\boxed{A_c = \frac{A_{max}+A_{min}}{2}}$$

(34)

substituting the value of A_c in above eqn of A_{max} & A_{min} we get:

$$\boxed{\mu = \frac{A_{max}-A_{min}}{A_{max}+A_{min}}}$$

Q7 The peak voltage of AM signal varies from 2 volts to 10 volts. Find μ, P_t & η?

Soln: Given:

$A_{max} = 10$ Volts

$A_{min} = 2$ Volts

So,

$$\boxed{\mu = \frac{A_{max}-A_{min}}{A_{max}+A_{min}} = \frac{10-2}{10+2} = \frac{8}{12} = 0.66}$$ Ans

$$A_c = \frac{A_{max}+A_{min}}{2} = \frac{2+10}{2} = 6 \text{ volts}$$

$$P_c = \frac{A_c^2}{2R} = \frac{6^2}{2\times 1} = 18W$$

$$P_t = P_c\left\{1+\frac{\mu^2}{2}\right\}$$

$$\boxed{P_t = 18\left\{1+\frac{0.66^2}{2}\right\} = 22W}$$ Ans

$$\eta = \frac{\mu^2}{2+\mu^2} = \frac{0.66^2}{2+0.66^2}$$

$$\boxed{\eta = 18.1}$$ Ans

× MULTI-TONE AMP. MODULATION.

Assume,

$$m(t) = A_{m1}\cos 2\pi f_{m1}t + A_{m2}\cos 2\pi f_{m2}t$$

and,

$$S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t \qquad (35)$$

So,

$$S_{AM}(t) = A_c\{1 + K_a A_{m1}\cos 2\pi f_{m1}t + K_a A_{m2}\cos 2\pi f_{m2}t\}\cos 2\pi f_c t$$

let,

$$K_a A_{m1} = \mu_1$$

$$K_a A_{m2} = \mu_2$$

So,

$$\boxed{S_{AM}(t) = A_c\{1 + \mu_1\cos 2\pi f_{m1}t + \mu_2\cos 2\pi f_{m2}t\}\cos 2\pi f_c t}$$

on expanding we get:

$$S_{AM}(t) = A_c\cos 2\pi f_c t + A_c\mu_1\cos 2\pi f_{m1}t\cos 2\pi f_c t + A_c\mu_2\cos 2\pi f_{m2}t\cos 2\pi f_c t$$

$$S_{AM}(t) = \underbrace{A_c\cos 2\pi f_c t}_{\text{carrier frequency component}} + \underbrace{\frac{A_c\mu_1}{2}\cos 2\pi(f_c + f_{m1})t}_{USB1} + \underbrace{\frac{A_c\mu_1}{2}\cos 2\pi(f_c - f_{m1})t}_{LSB1}$$

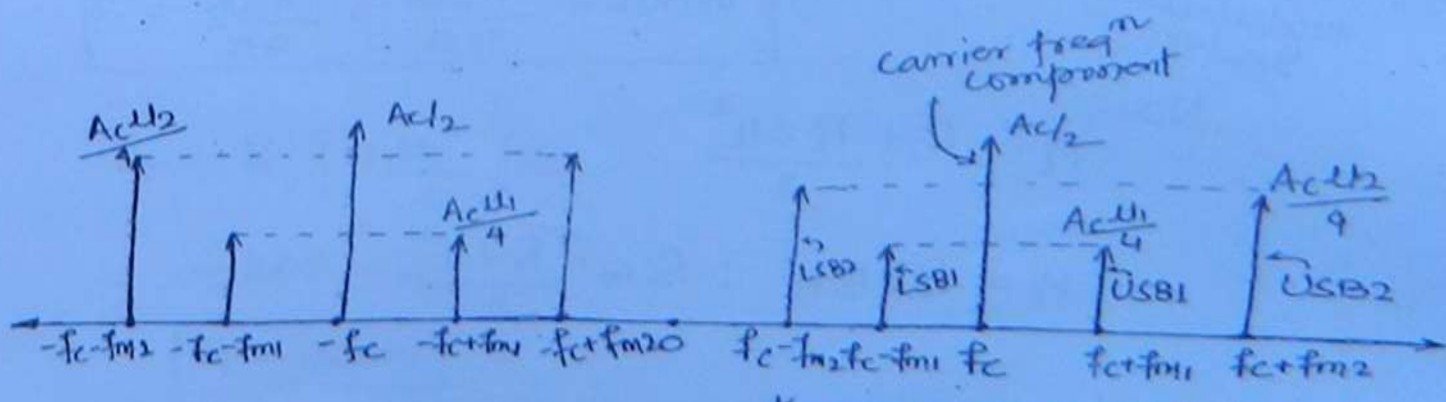
$$+ \underbrace{\frac{A_c\mu_2}{2}\cos 2\pi(f_c + f_{m2})t}_{USB2} + \underbrace{\frac{A_c\mu_2}{2}\cos 2\pi(f_c - f_{m2})t}_{LSB2}$$

Spectrum: For plotting spectrum let us assume

$$f_{m2} > f_{m1}$$

$$\& \quad \mu_2 > \mu_1$$

So,



$BW = 2f_{m2}$
$= 2 \times$ max^m freq^n of msg. signal.

* TOTAL POWER OF AM (multitone).

The total power (P_t) is given as:

(36)

$$P_t = P_c + P_{SB}$$

$$P_t = P_c + P_{USB}(\text{total}) + P_{LSB}(\text{total})$$

$$\boxed{P_t = P_c + P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}}$$

Now,

$$P_c = \frac{A_c^2}{2R} \; ; \; P_{USB1} = \frac{\left(\frac{A_c \mu_1}{2}\right)^2}{2R} = \frac{A_c^2 \mu_1^2}{8R} = P_{LSB1}$$

$$P_{USB2} = \frac{\left(\frac{A_c \mu_2}{2}\right)^2}{2R} = \frac{A_c^2 \mu_2^2}{8R} = P_{LSB2}.$$

So,

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu_1^2}{4R} + \frac{A_c^2 \mu_2^2}{4R}$$

$$P_t = \frac{A_c^2}{2R}\left\{1 + \frac{\mu_1^2 + \mu_2^2}{2}\right\}.$$

So, ***

$$\boxed{P_t = P_c\left\{1 + \frac{\mu_t^2}{2}\right\}}$$

where,

$$\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$$

Total modulation index.

Now,

$$P_t = P_c + \frac{P_c \mu_t^2}{2}$$

$$P_t = P_c + P_{SB} \; ; \; P_{SB} = \frac{P_c \mu_t^2}{2}$$

Now,

$$\eta = \frac{P_{S.B}}{P_t}$$

$$\eta = \frac{\frac{P_c \mu_t^2}{2}}{P_c\left\{1+\frac{\mu_t^2}{2}\right\}}$$

(37)

So, ***

$$\boxed{\eta = \frac{\mu_t^2}{2+\mu_t^2}}$$

Q1. A carrier of $20\cos 4\pi \times 10^6 t$ is amp. modulated by a msg signal of having frequencies 10KHz, 20KHz with modulation indexes 0.6 & 0.8 respt. Find all the parameters and plot the spectrum?

Soln: Given:

$$A_c \cos 2\pi f_c t = 20\cos 4\pi \times 10^6 t$$

So $A_c = 20$, $f_c = 2000\,KHz$

$f_{m1} = 10\,KHz$; $f_{m2} = 20\,KHz$

$\mu_1 = 0.6$; $\mu_2 = 0.8$

So, $\mu_t = \sqrt{\mu_1^2+\mu_2^2} = \sqrt{0.36+0.64} = 1$

Now,

$$B.W = 2 f_{max}$$

$$= 2 \times 20K$$

$$\boxed{BW = 40\,KHz}$$

And,

$$\boxed{P_c = \frac{A_c^2}{2R} = \frac{400}{2\times 1} = 200\,W}$$

$$P_t = P_c\left\{1+\frac{\mu_t^2}{2}\right\}$$

$$= 200 \times 3/2$$

$$\boxed{P_t = 300\,W}$$

$P_{SB} = P_t - P_c$
$= 100\,W$

$\boxed{P_{USB} = P_{LSB} = 50W}$

38

$$\boxed{P_{USB1} = \frac{A_c^2 \mu_1^2}{8R} = 18W = P_{LSB1}}$$

$$\boxed{P_{USB2} = 32W = P_{LSB2}}$$

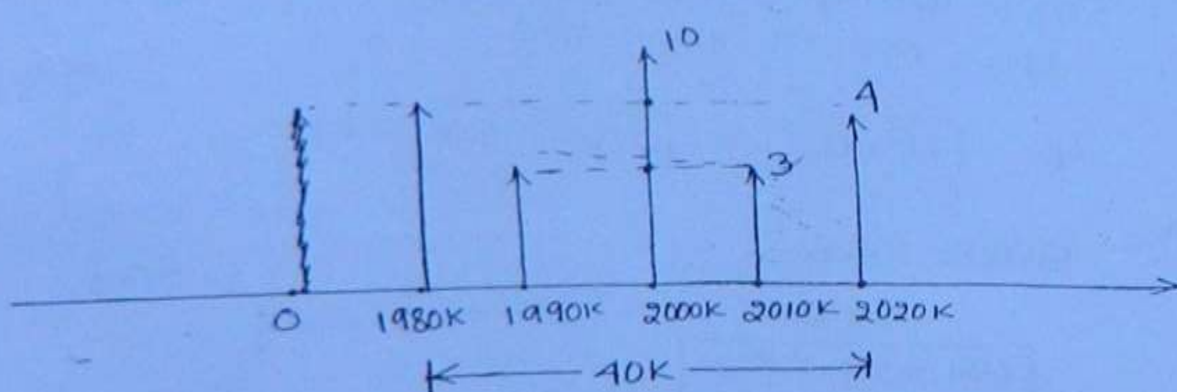
And

$$\eta = \frac{\mu_t^2}{2+\mu_t^2}$$

$$\eta = \frac{1}{1+2}$$

$$\boxed{\eta = 0.33 = 33.3\%}$$

Spectrum



2. An AM signal is given by

$$s(t) = \{20 + 4\cos 8\pi \times 10^4 t + 8\cos \pi \times 10^5 t\} \cos w_c t$$

Find μ_t, P_t, B.W & η?

Sol:- As, $s(t) = 20\{1 + 4/20 \cos 8\pi \times 10^4 t + 8/20 \cos \pi \times 10^5 t\} \cos w_c t$.

$S_{AM}(t) = A_c\{1 + \mu_1 \cos 2\pi f_{m1} t + \mu_2 \cos 2\pi f_{m2} t\} \cos 2\pi f_c t$

so, $A_c = 20V$, $\mu_1 = 4/20$, $\mu_2 = 8/20$.
$f_{m1} = 40K$; $f_{m2} = 50KHz$.

So, $\mu_1 = \frac{4}{20} = 0.2$; $\mu_2 = 0.4$

~~$\mu_2 = \frac{8}{20}$~~

B.W = 2 fmax

$\boxed{BW = 100K}$ Ans

39

$\boxed{\mu_t = \sqrt{0.2^2 + 0.4^2} = 0.44.}$ Ans

$\boxed{P_c = \frac{A_c^2}{2R} = \frac{400}{2 \times 1} = 200W}$ Ans

$$P_t = P_c \left\{ 1 + \frac{\mu_t^2}{2} \right\}$$

$$= 200 \left\{ 1 + \frac{0.44^2}{2} \right\}$$

$\boxed{P_t = 220W}$ Ans

And, $\eta = \frac{\mu_t^2}{2 + \mu_t^2} = \frac{0.44^2}{2 + 0.44^2}$

$\boxed{\eta = 0.09 = 9\%}$ Ans

CURRENT RELATIONS IN AM:-

As,

$$P_t = P_c \left\{ 1 + \frac{\mu_t^2}{2} \right\}.$$

$$I_t^2 R = I_c^2 R \left\{ 1 + \frac{\mu_t^2}{2} \right\}$$

PSU'S ***

$$\boxed{I_t^2 = I_c^2 \left\{ 1 + \frac{\mu^2}{2} \right\}}$$

So, PSU'S ***

$$\boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}}$$

I_t = total Antenna Current

I_c = Carrier current.

→ I_c is Independent of "μ"

× VOLTAGE RELATIONS IN AM

As.

$$P_t = P_c\left\{1+\frac{\mu^2}{2}\right\}$$

(40)

$$\frac{V_t^2}{R} = \frac{V_c^2}{R}\left\{1+\frac{\mu^2}{2}\right\}$$

$$\boxed{V_t = V_c\sqrt{1+\frac{\mu^2}{2}}}$$

Q1. An unmodulated AM transmitter current is given by 5A. Find AM transmitter current with 50% of modulation.

Solⁿ: Given:

$\mu = 0.5$

$I_c = 5A$

So,

$$I_t = I_c\sqrt{1+\frac{\mu^2}{2}}$$

{∵ $\mu = 0$, $I_t = I_c = 5A$} {as modulation index changes, I_c doesn't changes}

$$I_t = 5\sqrt{1+\frac{0.5^2}{2}}$$

$$I_t = 5\sqrt{1+\frac{0.25}{2}}$$

$$\boxed{I_t = 5.32A}$$

Q2. An AM transmitter current is given by 10A with 40% of modulation. Find AM transmitter current with 80% of modulation.

Solⁿ: Given, $I_t = 10A$

∵ as $\mu = 0.4$; $I_c = 9.6A$ $\left\{10 = I_c\sqrt{1+\frac{0.16}{2}} \quad I_c = 9.6A\right\}$.

So,

$$I_t = I_c\sqrt{1+\frac{\mu^2}{2}}$$

$$= 9.6\sqrt{1+\frac{0.8^2}{2}} = 9.6\sqrt{1+0.32}$$

$$\boxed{I_t = 11.05A}$$

*CURRENT RELATION FOR MULTI TONE ...

As,

$$P_t = P_c\left[1+\frac{\mu_t^2}{2}\right]$$

41

So,

$$\boxed{I_t = I_c\sqrt{1+\frac{\mu_t^2}{2}}} \quad ; \quad \boxed{\mu_t = \sqrt{\mu_1^2+\mu_2^2}}$$

Q1. An Unmodulated AM transmitter power is given by 10KW when the carrier is modulated by single sinusoidal message signal transmitter power becomes 13.5KW. Find AM transmitter power if the carrier is simultaneously modulated by 2nd message signal with 60% of modulation.

a) 12KW b) 15 KW c) 20KW d) 22.5 KW

Solⁿ: ∵ $\mu = 0$

So, $P_t = P_c = 10KW$

Now

$$P_t = P_c\left\{1+\frac{\mu_1^2}{2}\right\}.$$

$$\frac{13.5}{10} = 1+\frac{\mu_1^2}{2}$$

$$\frac{\mu_1^2}{2} = 0.35 \Rightarrow \mu_1^2 = 0.7$$

Now, $\mu_t = \sqrt{\mu_1^2+\mu_2^2} = \sqrt{0.7+0.36} \approx 1.$

So,

$$P_t = P_c\left(1+\frac{\mu_t^2}{2}\right\}.$$

$$P_t = 10\times\frac{3}{2}$$

$$\boxed{P_t = 15KW}$$ Ans

Q2 An AM transmitter current is given by 10A with carrier is modulated by single sinusoidal message signal with 40% of modulation with the carrier is simultaneously modulated by 2nd message signal, AM transmitter current is increased to 10.5 A. Find % of modulation due to 2nd message signal?

(42)

Soln:

$$I_t = I_c\sqrt{1+\frac{\mu_1^2}{2}}$$

$$10 = I_c\sqrt{1+0.08}$$

$$I_c = \frac{10}{\sqrt{1+0.08}}$$

$$I_c = 9.624\ A$$

NOW,

$$10.5 = 9.624\sqrt{1+\frac{\mu_t^2}{2}}$$

$$\left(\frac{10.5}{9.624}\right)^2 = 1+\frac{\mu_t^2}{2}$$

$$\frac{\mu_t^2}{2} = 0.1903$$

$$\mu_t = \sqrt{0.3806} = \sqrt{\mu_1^2+\mu_2^2}$$

$$\mu_1^2+\mu_2^2 = 0.3806.$$

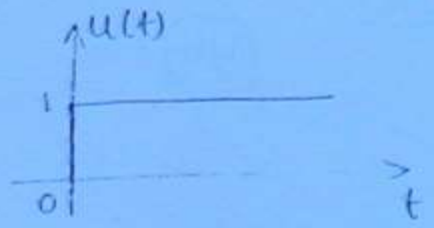
$$\mu_2^2 = 0.3806-0.16 \Rightarrow \mu_2 = \sqrt{0.2206}$$

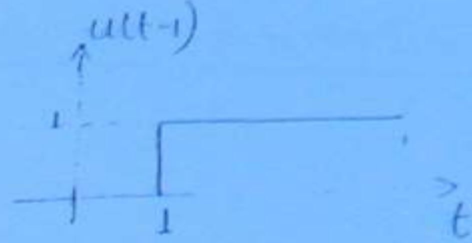
$$\boxed{\mu_2 = 0.47 = 47\%.}$$

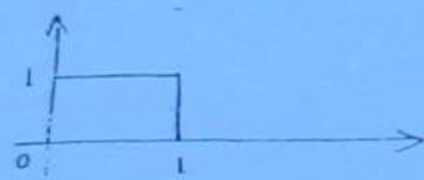
Note :-

$x(t) = u(t) - u(t-1)$

43







So,

$x(t) * x(t)$

Now,

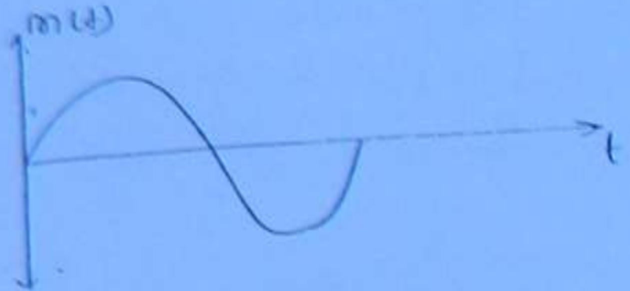
$\{u(t) - u(t-1)\} * \{u(t) - u(t-1)\}$

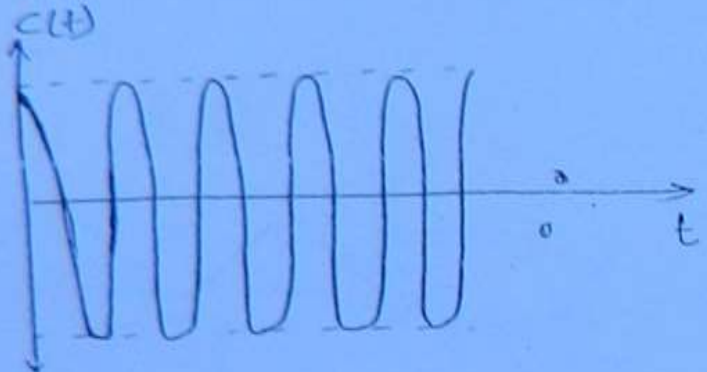
$= r(t) - r(t-1) - r(t-1) + r(t-2)$

$= r(t) - 2r(t-1) + r(t-2)$

Under, critical and over modulation:-

let,





i) Under modulation:-

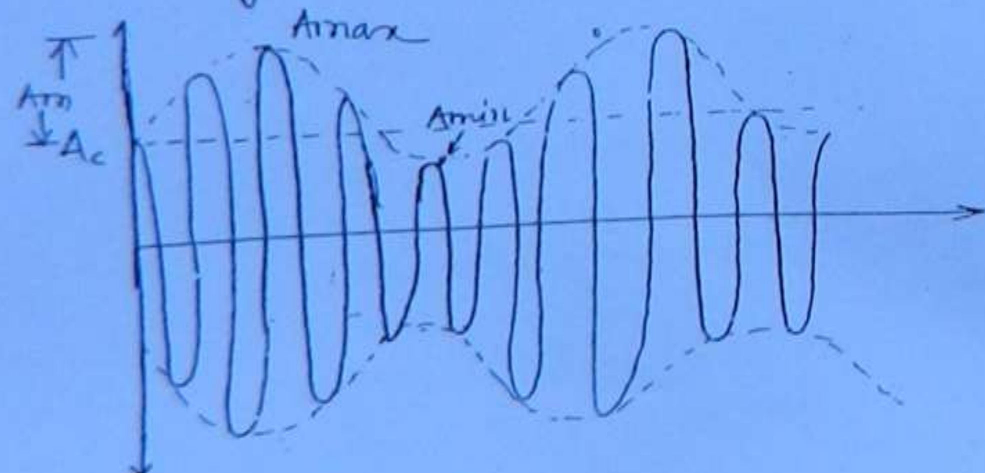
For under modulation

$$\mu < 1$$

$$\mu = \frac{A_m}{A_c} < 1 \Rightarrow A_m < A_c$$

$$A_{min} = A_c \{1 - \mu\} = +ve$$

So, the waveform is:-



ii) Critical modulation:

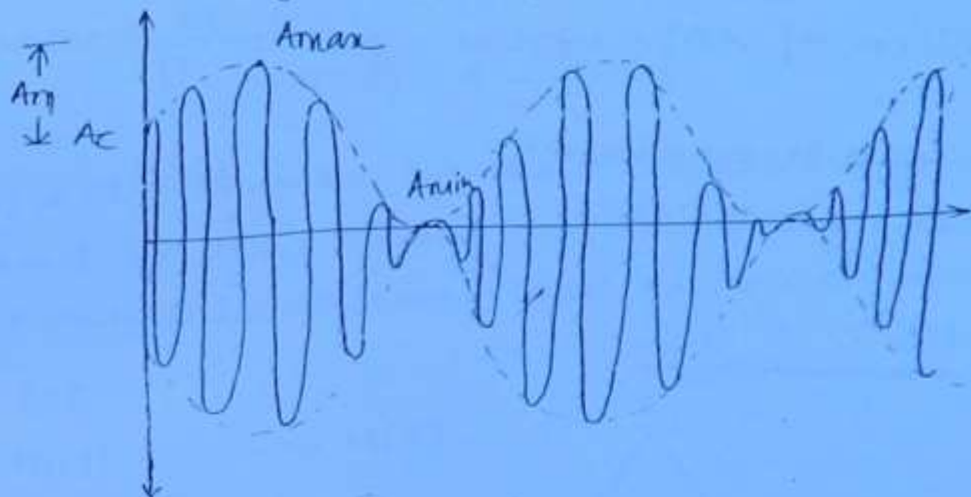
$u = 1$

$\frac{A_m}{A_c} = 1$ (45)

$A_m = A_c$

So, $A_{min} = A_c\{1-u\} = 0$

So, the waveform is given as:-



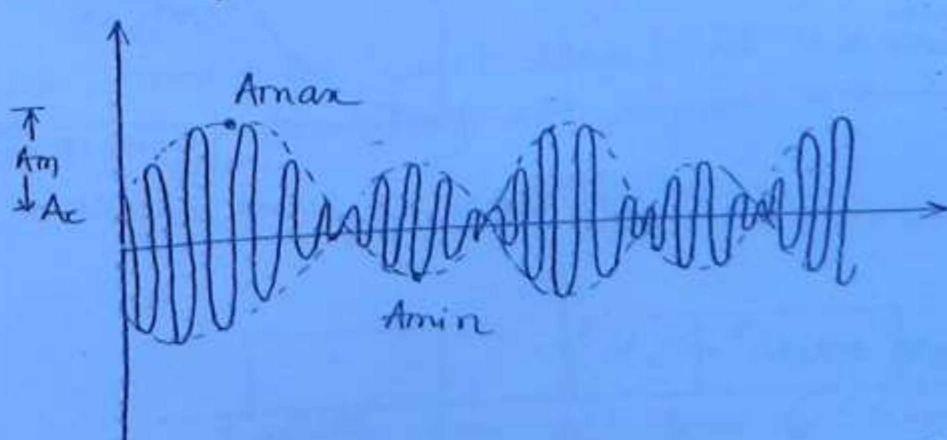
iii) Over modulation:-

$u > 1$

$\frac{A_m}{A_c} > 1 \Rightarrow A_m > A_c$

So, $A_{min} = A_c\{1-u\} = -ve$

So, the waveform is given as:-



Note:

* In under and critical modulation, message signal is preserved in the form of the envelope, so demodulation becomes simple

46

* In overmodulation, the message signal is not stored in the form of the envelope, so demodulation becomes complex.

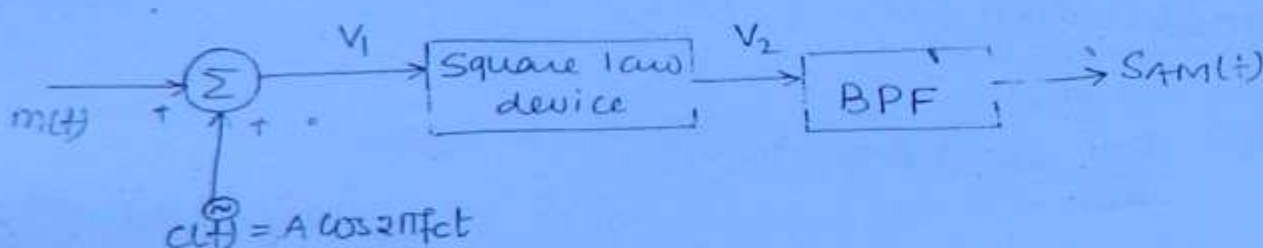
* Generation of AM signal :-

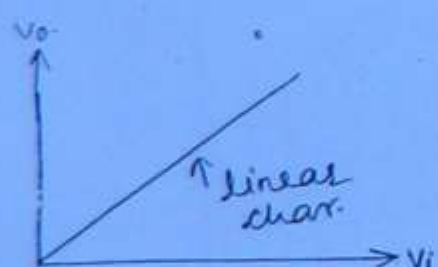
For the generation of AM signal, following modulators are used:

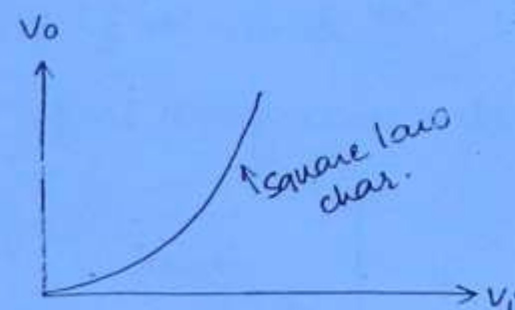
1) Square law modulator.
2) Switching modulator

* SQUARE-LAW MODULATOR:

Block diagm:





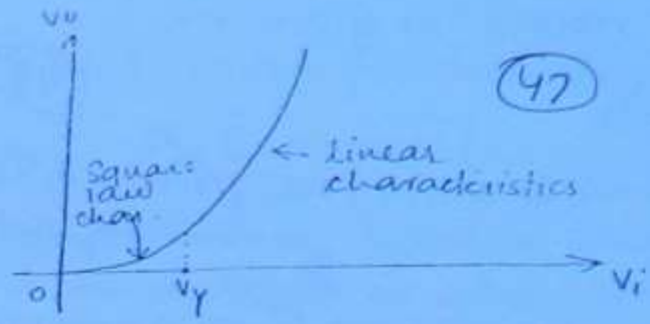


* The Relation b/w Vo & Vi for square law device is given as:

$$V_o = a_0 V_i + a_1 V_i^2 + a_2 V_i^3 + \dots$$

where, $a_0, a_1, a_2 \dots \rightarrow$ square law constant.

Note:



(47)

When the applied voltage $\boxed{V_i < V_\gamma}$ (diode), then the diode exhibits square law characteristics and if $\boxed{V_i > V_\gamma}$ (diode) it then exhibits linear characteristics.

So,

Input of the diode (Square law device) is:-

$$V_I = m(t) + c(t)$$

$$= m(t) + A_c \cos 2\pi f_c t$$

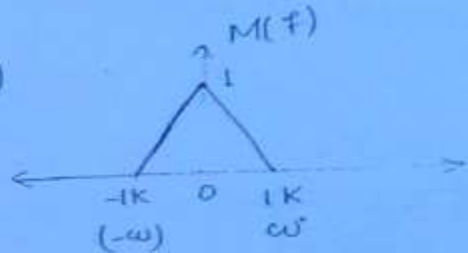
$\times$ m(t) & c(t) should be such that the peak voltage of V_I must be less than cutin voltage of diode, so that diode exhibits Square law characteristics.

Now let

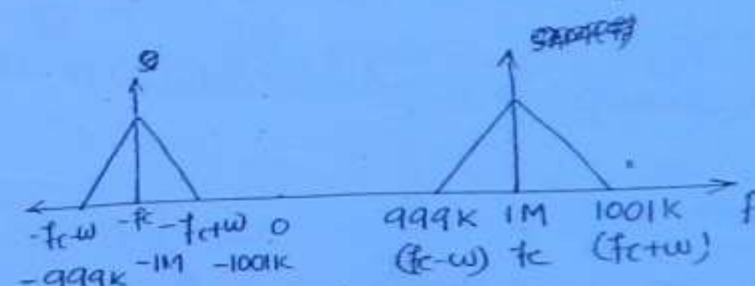
$m(t) \longleftrightarrow M(f)$

$f_c = 1MHz$

$= 1000K$

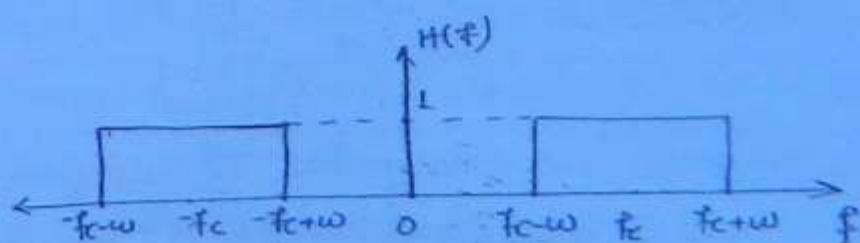


So, $S_{AM}(t)$



The BPF should be such that it has to allow only the frequency Band of AM signal.

So,



The O/P of square law device is given as

(48)

$$(SLD)_{O/P} = V_2 = a_1 V_1 + a_2 V_1^2$$

$$= a_1 \{ m(t) + A_c \cos 2\pi f_c t \} + a_2 \{ m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2A_c m(t) \cos 2\pi f_c t \}$$

Now,

$$(BPF)_{O/P} = a_1 A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$m^2(t) = m(t) \times m(t)$

$\updownarrow$

$M(f) * M(f)$

$-\omega \quad \omega \quad * \quad -\omega \quad \omega \quad = \quad -2\omega \quad 2\omega$

$\cos^2 2\pi f_c t = \dfrac{1 + \cos 4\pi f_c t}{2}$ (d.c)

So

$$(BPF)_{O/P} = a_1 A_c \cos 2\pi f_c t + 2a_2 A_c m(t) \cos 2\pi f_c t$$

$$\boxed{(BPF)_{O/P} = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t) \right] \cos 2\pi f_c t = S_{AM}(t)}$$

Comparing with standard AM signal.

$$A_c' \{ 1 + K_a m(t) \} \cos 2\pi f_c t$$

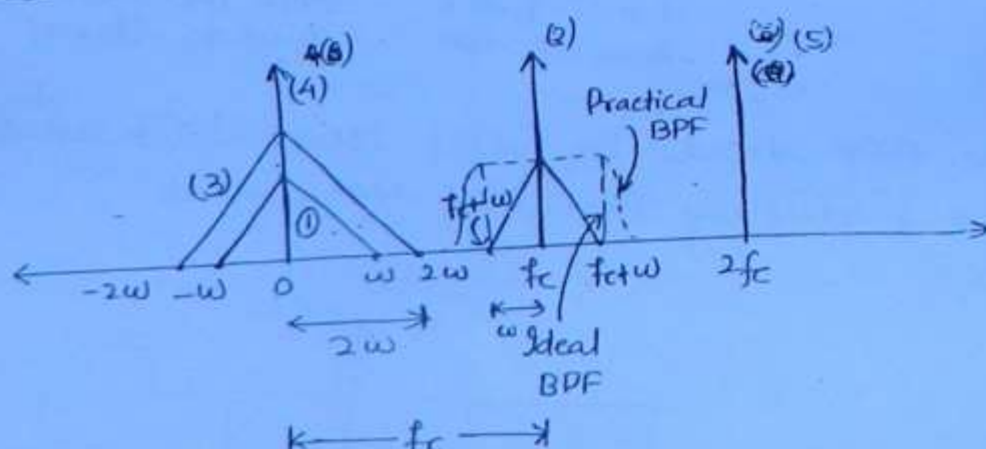
where, $A_c' = a_1 A_c$

** $\boxed{K_a = \dfrac{2a_2}{a_1}}$

Now,

$$V_2 = a_1 \{ \overset{①}{m(t)} + \overset{②}{A_c \cos 2\pi f_c t} \} + a_2 \{ \overset{④}{A_c^2 \cos^2 2\pi f_c t} + \overset{③}{m^2(t)} + \overset{⑤}{2A_c m(t) \cos 2\pi f_c t} \}$$

$V_2(t) \longleftrightarrow$



$\boxed{f_c >>> 3\omega}$ ← To avoid overlapping of desired & undesired signal.

Note :-

To avoid undesired frequencies at BPF output

$$\boxed{f_c >>> 3\omega}$$

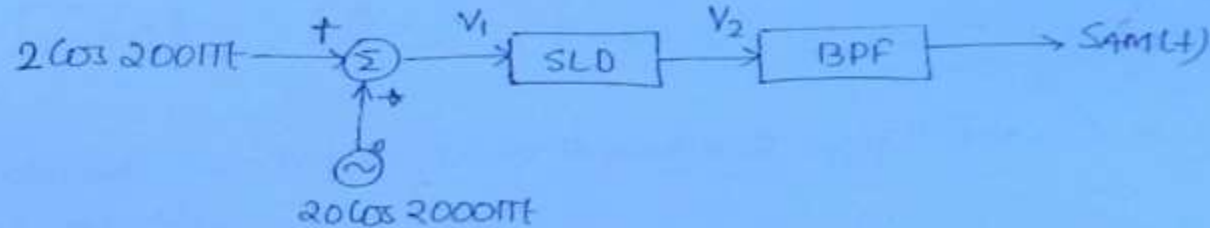
(99)

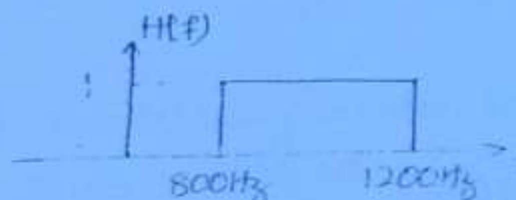
Q! For the following Square law modulator, where the square law device is characterised by

$$V_2 = V_1 + 0.1 V_1^2$$

and pass Band of BPF will be from 800Hz to 1200Hz find all the parameters of Resulting AM Signal.

Sol:





Sol^n: $V_2 = (2\cos 200\pi t + 20\cos 2000\pi t) + 0.1(2\cos 200\pi t + 20\cos 2000\pi t)^2$

$$= 2.0\cos 200\pi t + 20\cos 2000\pi t + 0.1\{4\cos^2 200\pi t + 400\cos^2 2000\pi t + 80\cos 200\pi t \cdot \cos 2000\pi t\}$$

$$V_2 = (\overset{\times}{2\cos 200\pi t} + \overset{\checkmark}{20\cos 2000\pi t} + \overset{\times}{0.4\underbrace{\cos^2 200\pi t}_{(1+\cos 400\pi t/2)}} + \overset{\times}{40\cos^2 2000\pi t}$$

$$+ 8\cos 200\pi t\cos 2000\pi t) \checkmark \; (\cos(2200\pi t) + \cos(1800\pi t))$$

So,

$$(BPF)_{o/p} = SAM(t) = 20\cos 2000\pi t + 8\cos 200\pi t\cos 2000\pi t$$

$$\boxed{SAM(t) = 20\{1 + 0.4\cos 200\pi t\}\cos 2000\pi t}$$

above signal is single tone AM. so

$$SAM(t) = A_c\{1 + \mu\cos 2\pi f_m t\}\cos 2\pi f_c t$$

So,
$A_c = 20\,V$; $\mu = 0.1$; $f_m = 100\,Hz$, $f_c = 1000\,Hz$

objective

$\mu = ?$

as, $\mu = K_a A_m$

and $K_a = \frac{2a_2}{a_1}$

by comparing with $V_2 = V_1 + 0.1 V_1^2$

$V_2 = a_1 V_1 + a_2 V_1^2$

$a_1 = 1$; $a_2 = 0.1$

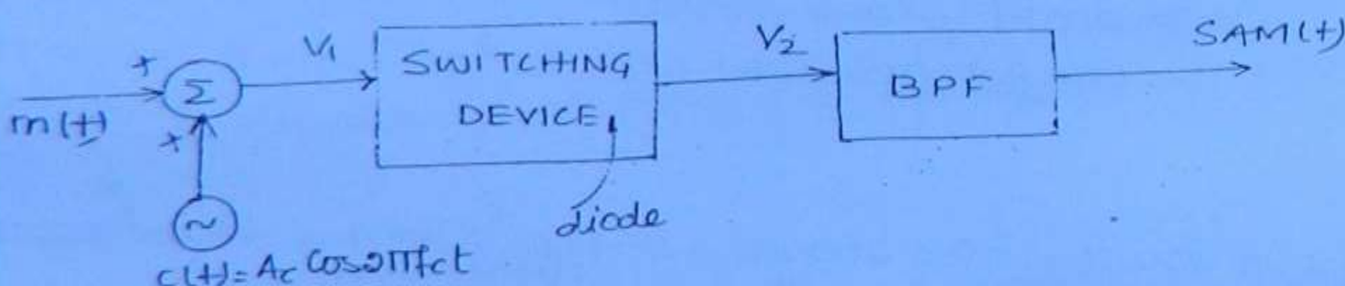
So, $K_a = \frac{2 \times 0.1}{1} = 0.2$

$\mu = 0.2 \times 20$

$\boxed{\mu = 0.4}$

* SWITCHING MODULATOR:

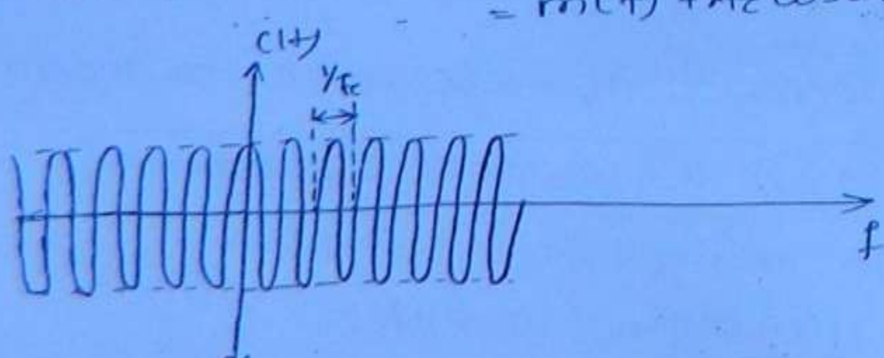
Block diag^{m}:



Analysis :-

I/P of diode $\Rightarrow V_1 = m(t) + c(t)$

$= m(t) + A_c \cos 2\pi f_c t$



* The strength of Practical message signal will be generally of having less strength and carrier signal is generated with high strength. So the operation of the diode is mainly controlled by carrier signal. (57)

1. when $c(t)$ is +ve, diode is forward Biased ie S.C then $\boxed{V_2 = V_1}$

2. when $c(t)$ is -ve, diode is Reverse Biased ie O.C then $\boxed{V_2 = 0}$

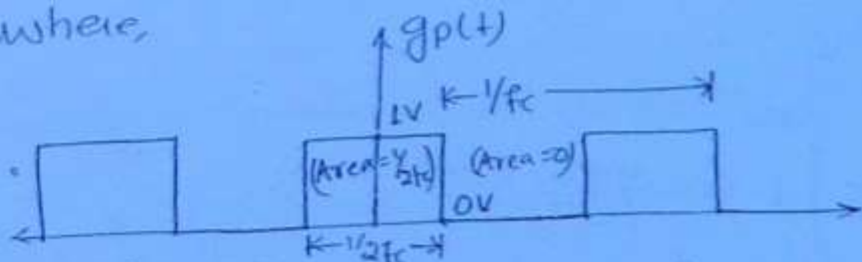
* O/P of diode switches b/w V_1 and 0 with the time interval of $1/f_c$.

Note:

∵ The O/P of switching device, ie V_2 is switching continuously b/w V_1 & 0, so plotting its spectrum is not easy. Hence to plot its spectrum. let

$$V_2 = V_1 \times g_p(t) \quad \text{---- (1)}$$

where,



The analysis of Periodic signal is done by Fourier series. Hence by Trigonometric FS we have:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\} \; ; \omega_0 = 2\pi/T \quad \text{---- (2)}$$

∴ $g_p(t)$ is even, hence $b_n = 0$.

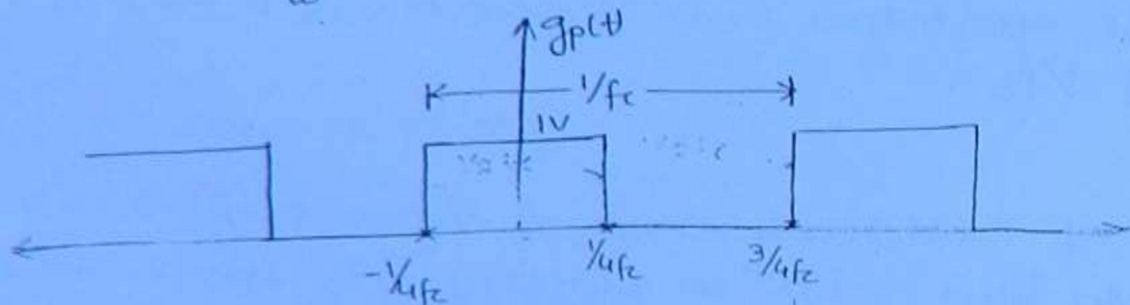
So, $g_p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$

Now $a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} f(t)\,dt = \frac{\int_{t_0} f(t)\,dt \text{ (Area)}}{T}$ (52)

$$a_0 = \frac{1/2f_c}{1/f_c}$$

$$\boxed{a_0 = 1/2}$$

$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} f(t)\cos n\omega_0 T\,dt \quad ; \omega_0 = 2\pi/T = 2\pi f_c$$



So,

$$a_n = \frac{2}{1/f_c}\int_{-1/4f_c}^{3/4f_c} g_p(t)\cos n\,2\pi f_c t\,dt$$

$$a_n = \frac{2}{1/f_c}\int_{-1/4f_c}^{1/4f_c} 1\cdot\cos 2\pi n f_c t\,dt + 0$$

$$= 2f_c \times \left.\frac{\sin 2\pi n f_c t}{2\pi n f_c}\right|_{-1/4f_c}^{1/4f_c}$$

$$= \frac{1}{n\pi}\left\{\sin 2\pi n f_c \times \frac{1}{4f_c} - \sin 2\pi n f_c \times -\frac{1}{4f_c}\right\}$$

$$= \frac{1}{n\pi}\left\{\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right\}$$

$$\boxed{a_n = \frac{2}{n\pi} \sin n\pi/2}$$

(53)

So, from equation (i) we get:

$$g_p(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\pi/2 \cos n\, 2\pi f_c t$$

and from equation (1) we get

$$V_2 = V_1 \cdot g_p(t)$$

$$= (m(t) + A_c \cos 2\pi f_c t)\left\{\frac{1}{2} + \frac{2}{\pi}\cos 2\pi f_c t - \frac{2}{3\pi}\cos 2\pi(3f_c)t + \frac{2}{5\pi}\cos 2\pi(5f_c)t + \cdots\right\}$$

* V_2 is then passed through a BPF. So, its O/P is given as:

$$\boxed{(BPF)_{O/P} = S_{AM}(t) = \left(\frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t\right)}$$

$$\boxed{S_{AM}(t) = \frac{A_c}{2}\left\{1 + \frac{4}{\pi A_c} m(t)\right\} \cos 2\pi f_c t}$$

Comparing with standard equation of AM we get:-

$$A_c'\left\{1 + K_a m(t)\right\} \cos 2\pi f_c t$$

So,

$$A_c' = \frac{A_c}{2}$$

*** & $\boxed{K_a = \frac{4}{\pi A_c}}$

× DEMODULATION OF AM SIGNAL:

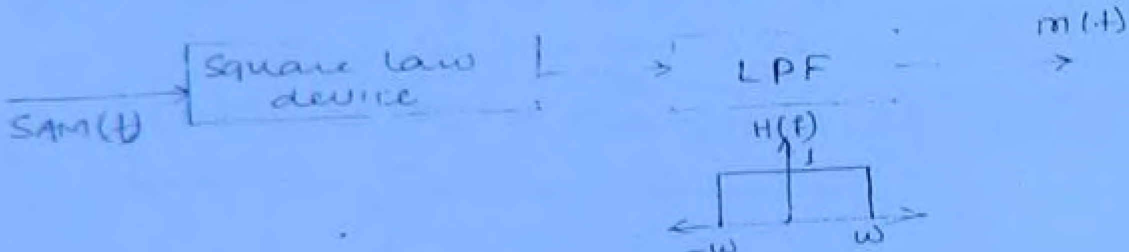
The demodulators that are used for demodulation of AM are:

1) Square law demodulator
2) Envelope detector (diode detector) $\Big\}\ \mu \le 1$
3) Synchronous detector $\}$ any value of μ.

× Synchronous detector is complex and square law demodulator will have some drawbacks. so generally for demodulation of AM ; envelope detector will be used

× SQUARE LAW DEMODULATOR:

B·Diag^m



As,

$$S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + K_a m(t)\cos 2\pi f_c t \cdot A_c$$

The O/P of the Square law device may be given as:

$$(SLD)_{o/p} \Rightarrow V_2 = a_1 S_{AM}(t) + a_2\{S_{AM}(t)\}^2$$

$$= a_1\{A_c \cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t\}$$

$$+ a_2\{A_c^2 \cos^2 2\pi f_c t + K_a^2 m^2(t)\cos^2 2\pi f_c t \cdot A_c^2 + 2K_a A_c^2 m(t)\cos^2 2\pi f_c t\}.$$

✓(but blocked by blocking capacitor)

So,

$$V_2 = \Big[a_1 A_c \cos 2\pi f_c t + a_1 A_c K_a m(t)\cos 2\pi f_c t + \frac{a_2 A_c^2}{2}(1 + \cos 4\pi f_c t) \times$$

✓(but only Part of)

$$+ \frac{a_2 A_c^2 K_a^2 m^2(t)}{2}(1 + \cos 4\pi f_c t) + \frac{a_2\, 2A_c^2 K_a m(t)}{2}(1 + \cos 4\pi f_c t)\Big].$$

The signal v_2 is given to LPF and its O/P is given as following:-

$$(LPF)_{O/P} = \underbrace{\frac{a_2 A_c^2 K_a^2 m^2(t)}{2}}_{\text{unwanted signal or Noise}} + \underbrace{\frac{a_2 \cdot 2 A_c^2 K_a m(t)}{2}}_{\text{expected signal}}$$

If $\frac{S}{N} >>>> 1$; $m(t)$ can be Reconstructed (almost in the Range of 100 or 1000)

$\frac{S}{N} < 1$; $m(t)$ can't be Reconstructed

Now,

$$\frac{S}{N} = \frac{\cancel{a_2} \cancel{A_c^2} \cancel{K_a} m(t)}{\frac{\cancel{a_2} \cancel{A_c^2} K_a^2 m^2(t)}{2}}$$

$$\frac{S}{N} = \frac{2}{K_a m(t)}$$

Now let $m(t) = A_m \cos 2\pi f_m t$

So,

$$\frac{S}{N} = \frac{2}{K_a \cdot A_m \cos 2\pi f_m t}$$

$$\frac{S}{N} = \frac{2}{\mu \cos 2\pi f_m t}$$

As, $-1 \le \cos 2\pi f_m t \le 1$

So, $\boxed{\frac{S}{N} = \frac{2}{\mu \times 1}}$ $\{\cos 2\pi f_m t = 1\}$.

Now $\frac{S}{N} \propto \frac{1}{\mu}$ $\Rightarrow$ To get $\frac{S}{N} = 100 \Rightarrow \mu = 0.02$

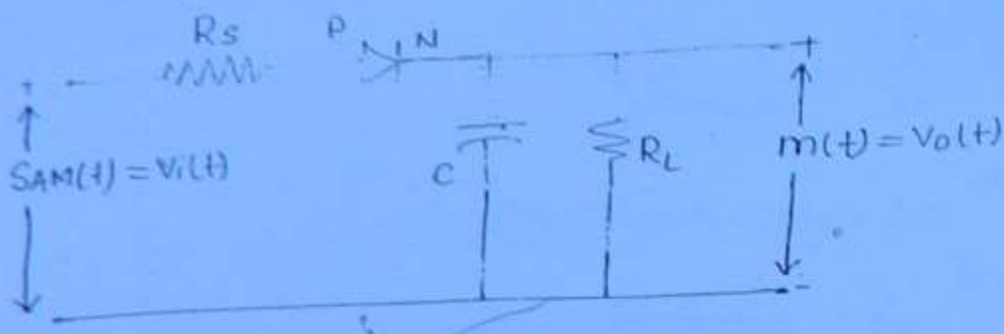
For such small value of μ, η will be worstly affected ie low share of SideBand Power in AM signal.

* Hence in the square law demodulator either of the two i.e. μ or η has to be sacrificed. It means that if μ is increased P_{SB} is increased but S/N is reduced and vice versa.

* To get high value of S/N, μ should be very small. For small values of μ, modulation efficiency will be very much low (worstly affected). But for efficient power distribution, η should be max^m. So this method is practically not preferred for AM demodulation

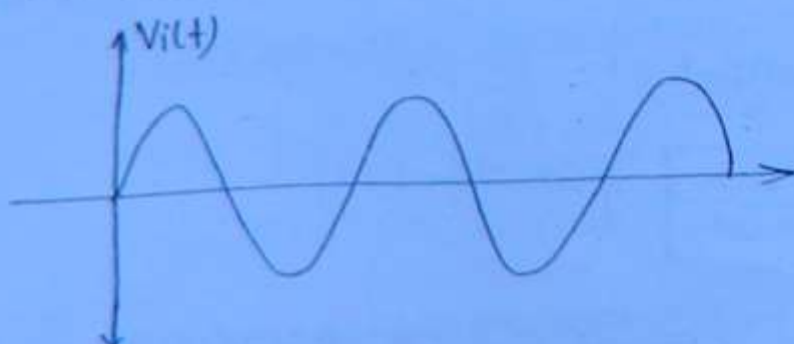
*ENVELOPE DETECTOR:

Ckt diagram:



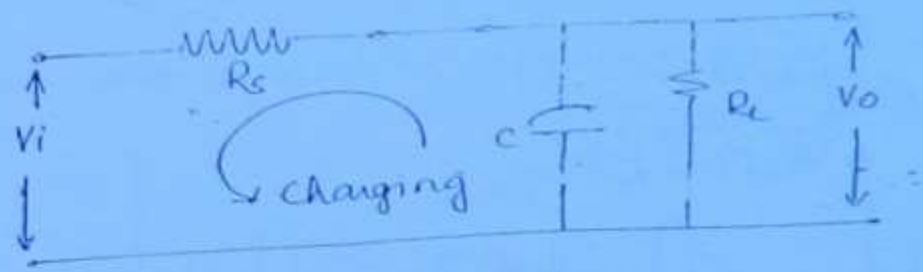
→ Envelope detector is very simple and cheaper.

→ The +ve envelope of applied signal will be produced at the o/p so it is called as ENVELOPE DETECTOR.

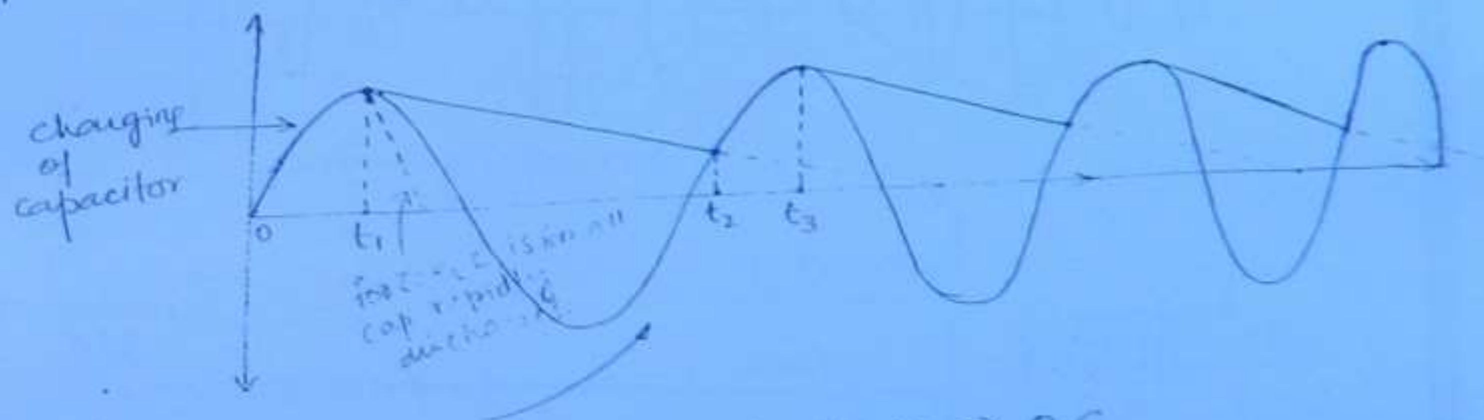


assume that there is [illegible]

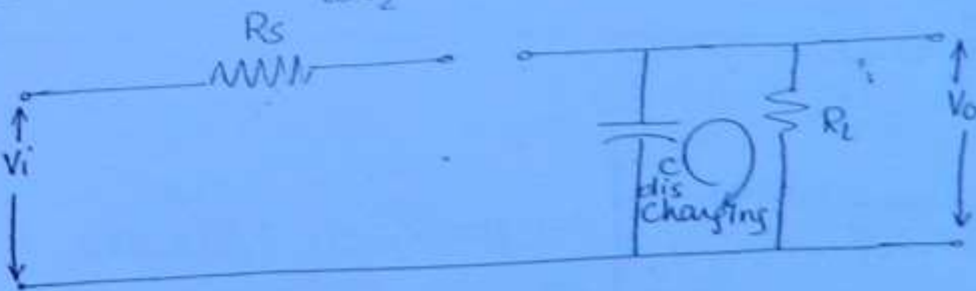
1:- For $t = 0^+$; P > N ⇒ FB diode ⇒ SC



Note :-
The time const $= R_S \cdot C$ should be very small. Hence capacitor rapidly changes to input voltage.



Case 2 : For $t = t_1^+$; to t_2 P < N ⇒ RB diode ⇒ O.C



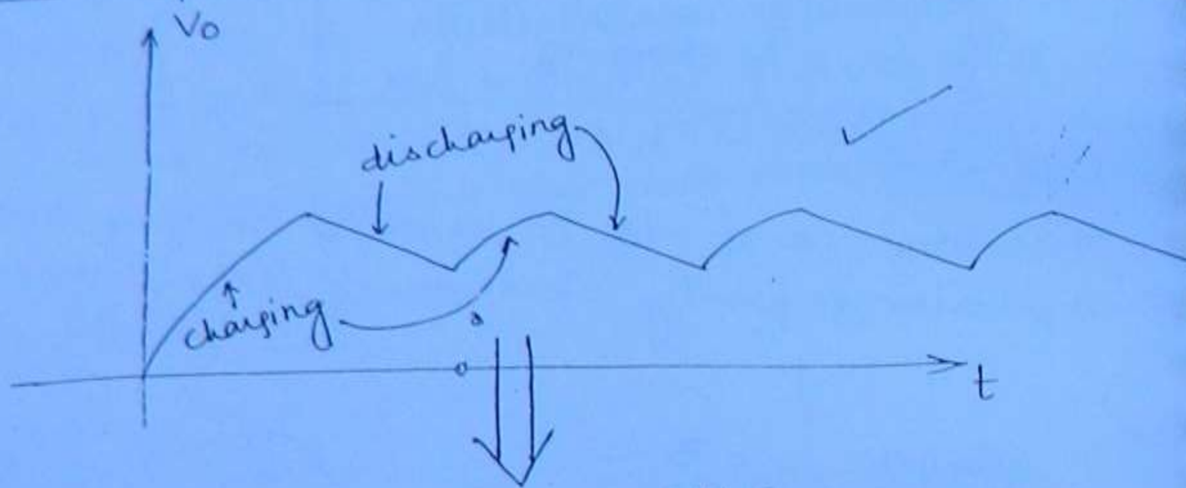
If $\tau = R_L \cdot C$ is very small, the capacitor rapidly discharges to zero.
If $\tau = R_L \cdot C$ is very high, the capacitor discharges slowly.

Note :-
$\tau = R_L \cdot C$; should be very high then capacitor discharges very slowly.

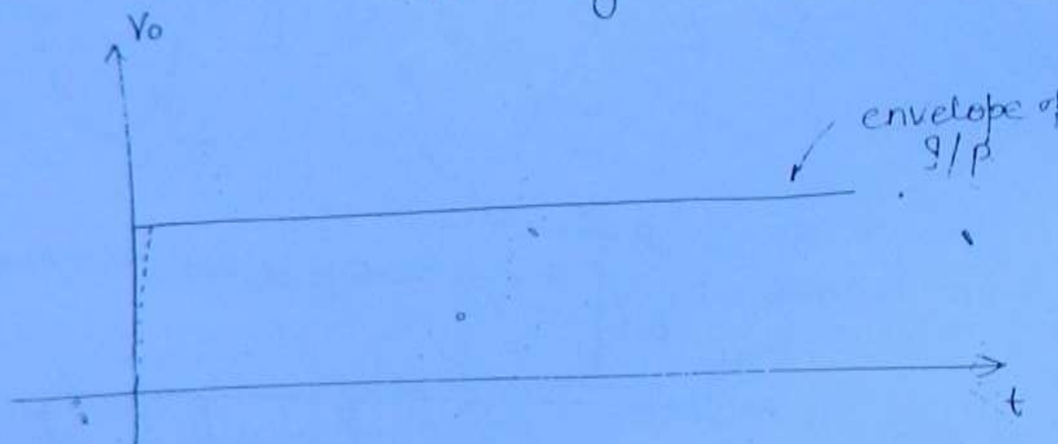
Case 3: for t_2 to t_3 → PN diode = RB
C changes to ...

The curve shown at back

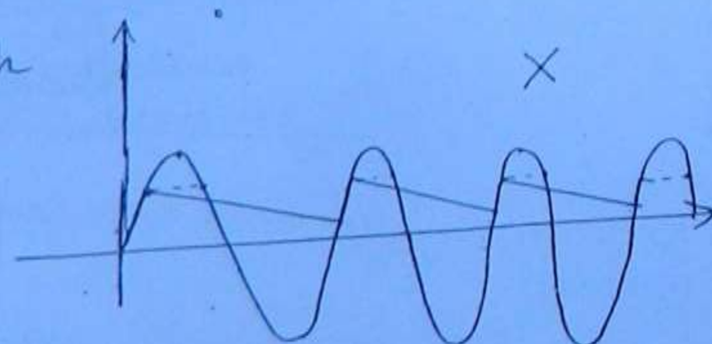
Voltage across capacitor:



Smoothing Filter



Note:- If $R_s \cdot C$ = very high

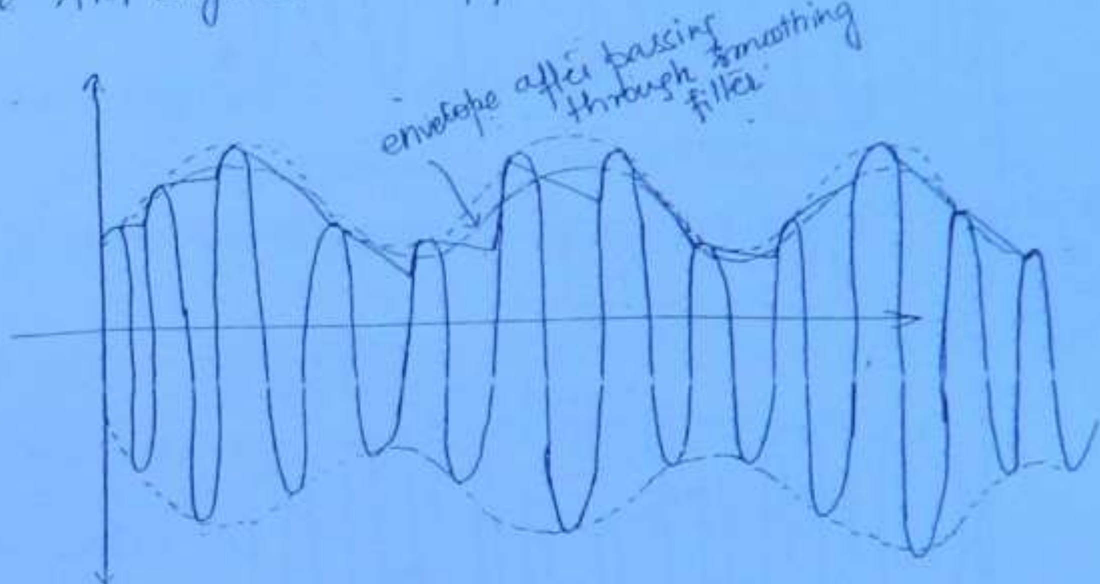


If $R_s \cdot C$ is very high, even before capacitor voltage reaches to peak voltage of input, diode becomes R and capacitor will be discharged.

→ To make capacitor voltage to follow the envelope of Input $R_s \cdot C$ should be very small and $R_L \cdot C$ should be high.

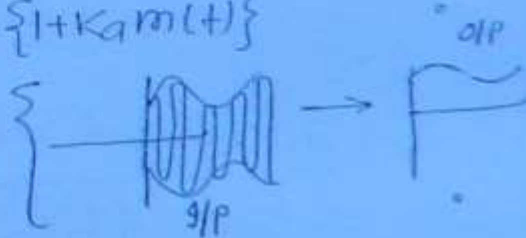
Note:

If the AM signal is applied to the diode detector then:



Analysis:

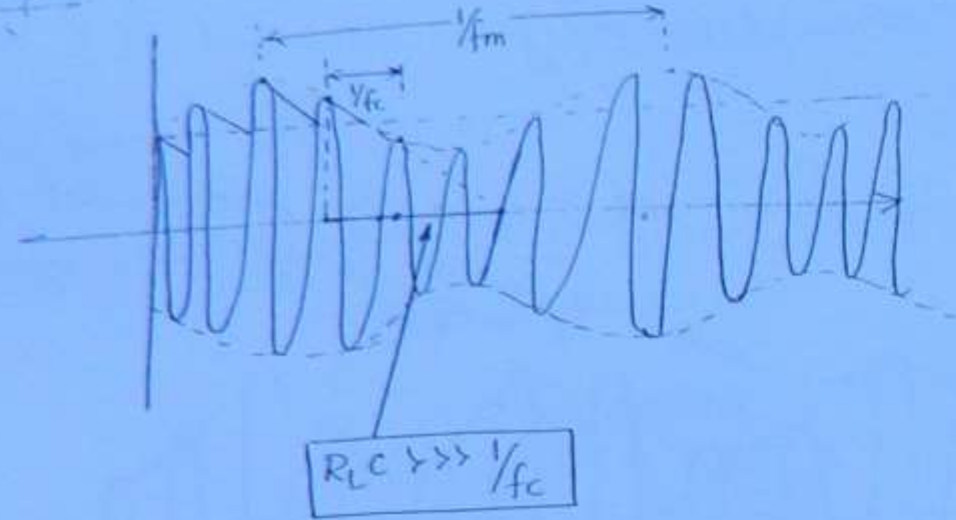
ED I/P		ED O/P
1. $V_m \cos 2\pi f_m t$	$\longrightarrow$	V_m
2. $s(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$	$\longrightarrow$	$A_c\{1 + K_a m(t)\}$
3. $A\cos 2\pi f_0 t + B\sin 2\pi f_0 t$	$\longrightarrow$	$\sqrt{A^2+B^2}$

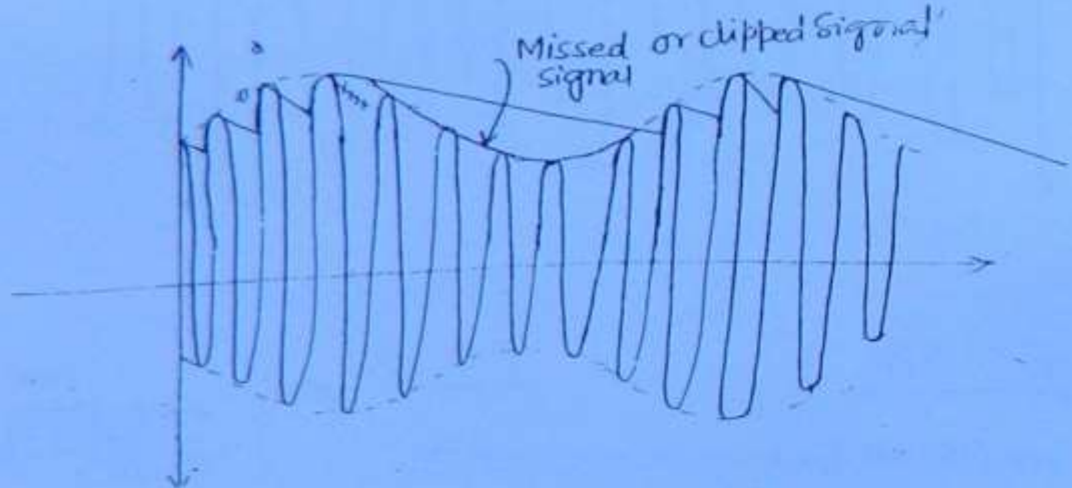


$$= \left\{\sqrt{A^2+B^2} \times \cos(2\pi f_c t + \Phi)\right\}$$

$$\Phi = \tan^{-1} B/A$$

Analysis





CASE OF DIAGONAL CLIPPING

∴ To avoid diagonal clipping

~~$R_L C >>> 1/f_m$~~ $\boxed{R_L C <<< 1/f_m}$

* Proper choice of $R_L C \Rightarrow \boxed{\frac{1}{f_c} <<< R_L \cdot C <<< 1/f_m}$

Q1. The carrier signal of 1 MHz is Amplitude modulated by message signal of frequency 2 KHz. The proper choice of $R_L \cdot C$ for envelope detection should be

a) 1 μsec b) 500 μsec c) 200 μsec d) 0.5 μsec

Sol^n: $f_c = 1\,MHz \Rightarrow 1/f_c = 1\mu sec$

$f_m = 2K \Rightarrow \frac{1}{f_m} = 0.5\,msec = 500\,\mu sec.$

61

$$\frac{1}{f_c} <<< R_L C <<< \frac{1}{f_m}$$

Q2. An AM signal is given by

$$s(t) = A_c \cos w_c t + 2\cos w_c t \cos w_m t$$

For proper envelope detection, the min^m value of A_c should be:

a) 0.5 volts b) 2 volts c) 1 volts d) 2.5 volts.

Sol^n: comparing with AM signal:

$$s(t) = A_c\left\{1 + \frac{2}{A_c}\cos w_m t\right\}\cos 2\pi f_c t$$

$$S_{AM}(t) = A_c\left\{1 + \mu \cos w_m t\right\}\cos 2\pi f_c t$$

So, $\cancel{\mu = \frac{2}{A_m}}$ $\mu = \frac{2}{A_c}$

$\cancel{\frac{A_m}{A_c} = \frac{2}{A_m}}$ for envelope detection $\mu \le 1$.

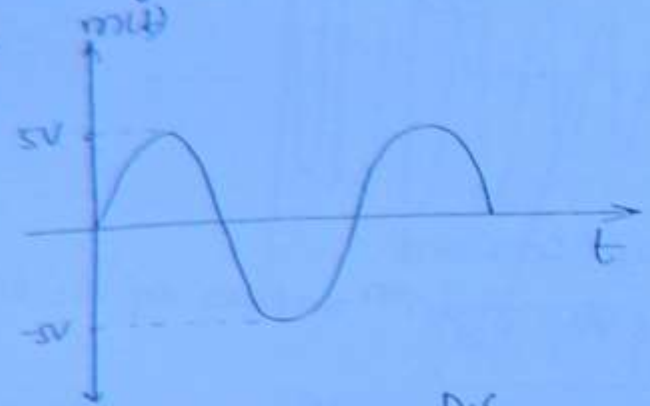
$$\Rightarrow \frac{2}{A_c} \le 1$$

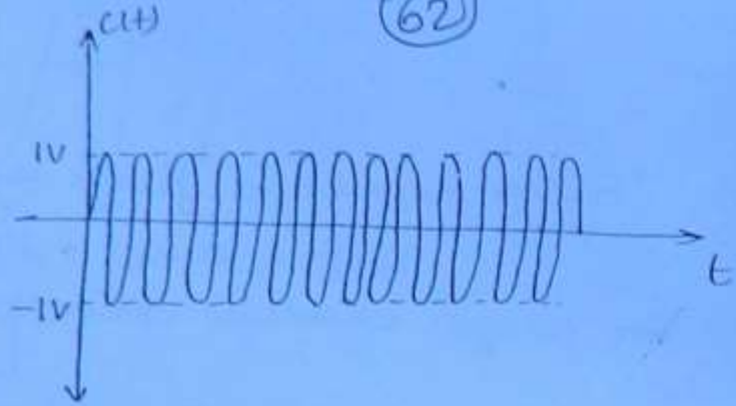
$$\boxed{A_c \geq 2}$$ Ans.

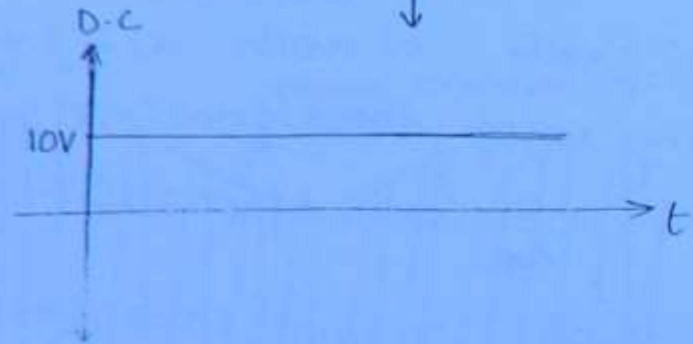
Hence, the min^m value of A_c is 2 volts.

*Analysis:

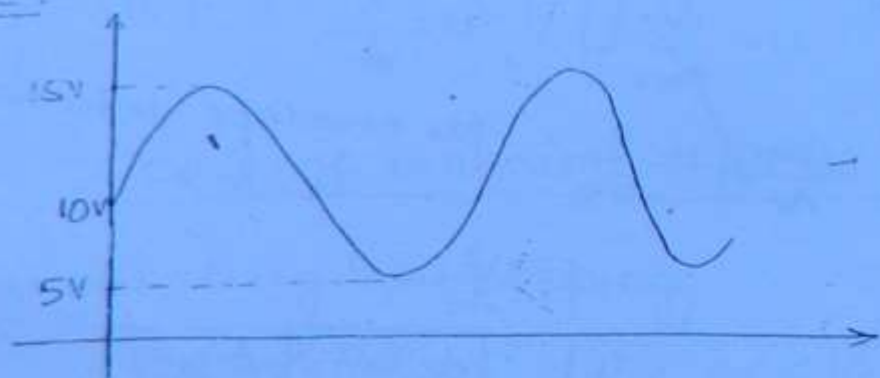
i.



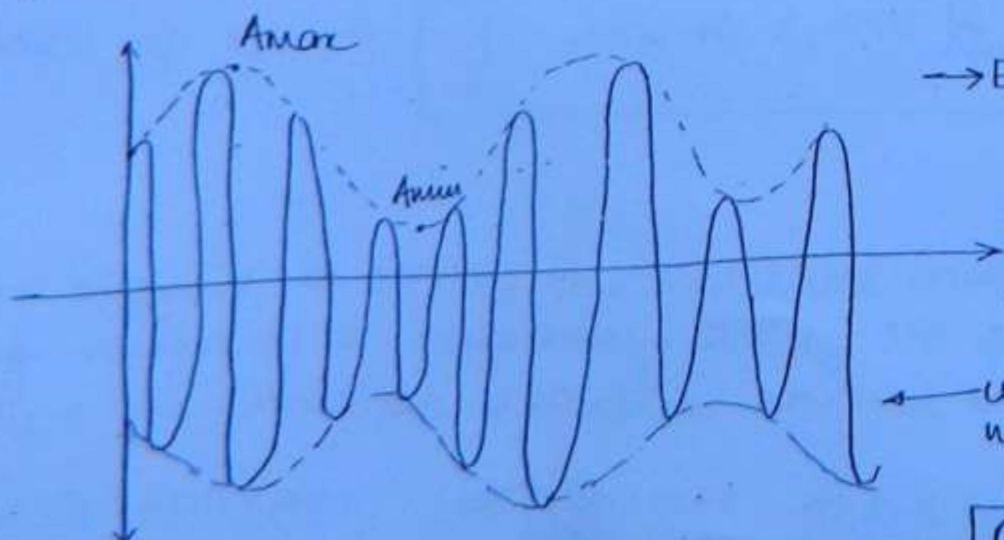




m(t)+DC:



Now, $S_{AM}(t) = \{m(t) + DC\} c(t) = \{m(t) + DC\} \cos 2\pi f_c t$



→ ED O/P = m(t)+DC

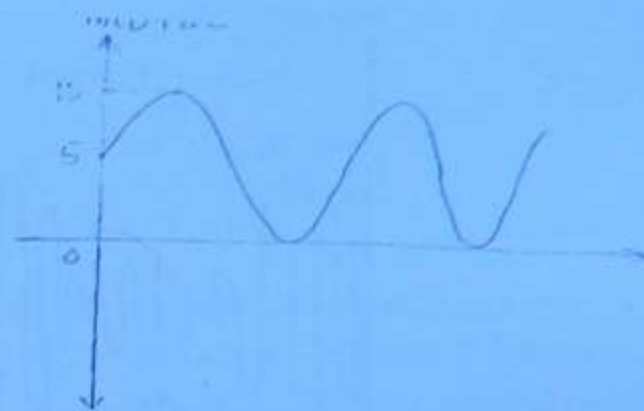
∴ Amin = +ve

under modulated signal

$\boxed{(\mu<1)}$

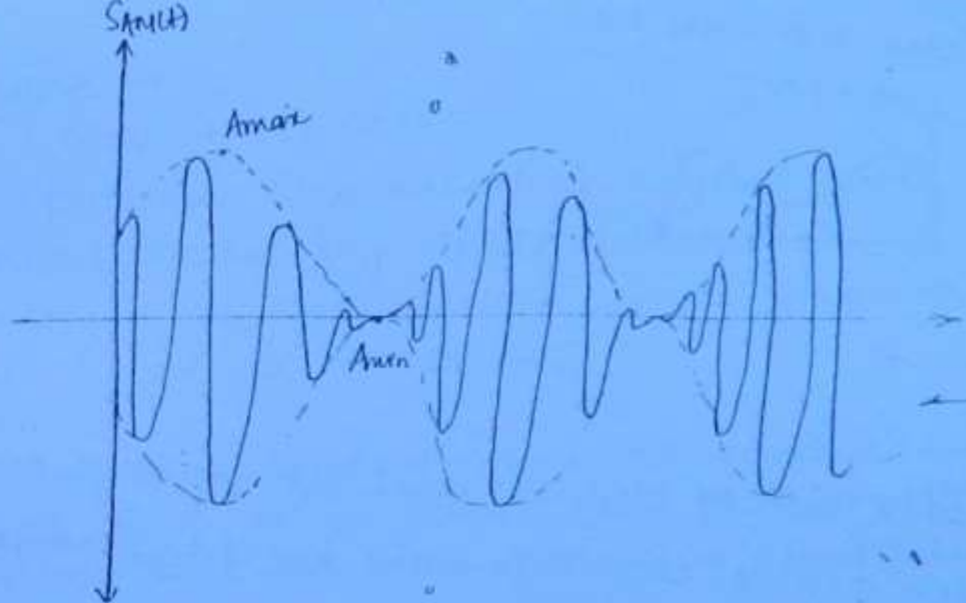
(ii)

63



So,

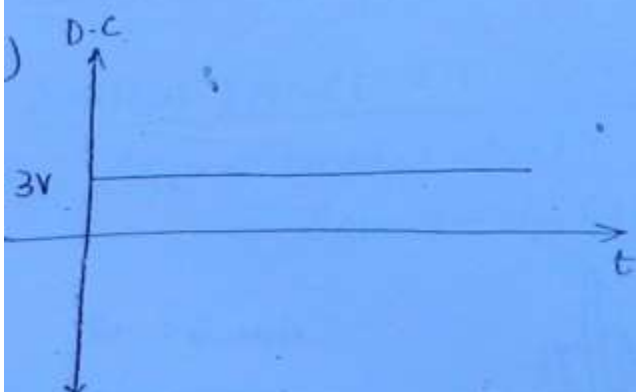
$$S_{AM}(t) = \{m(t) + DC\} \cos 2\pi f_c t$$

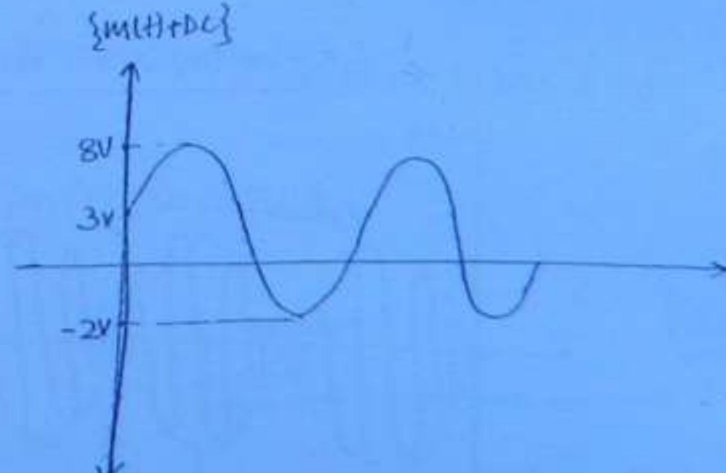


$\because A_{min} = 0$

← Critical modulation

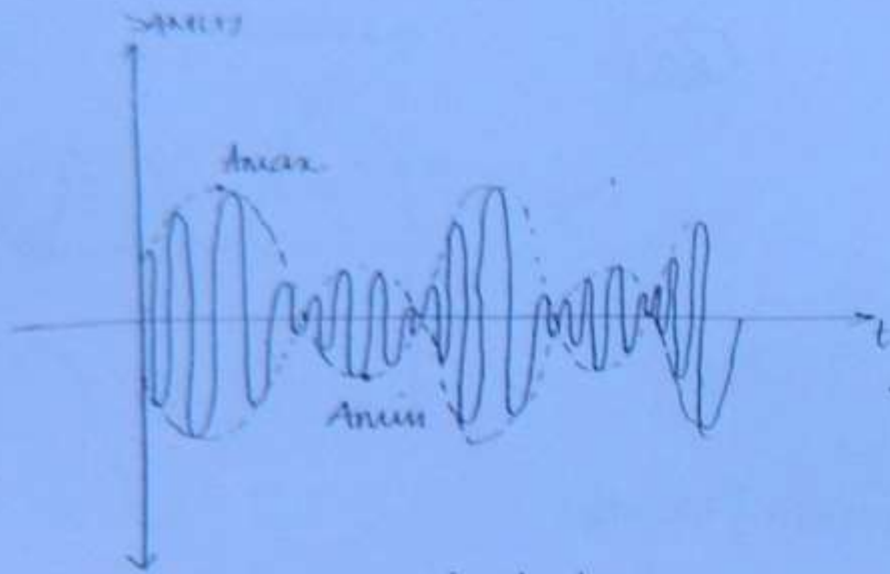
$\boxed{M=1}$





So,

$$S_{AM}(t) = \{m(t) + DC\} \cos 2\pi f_c t$$

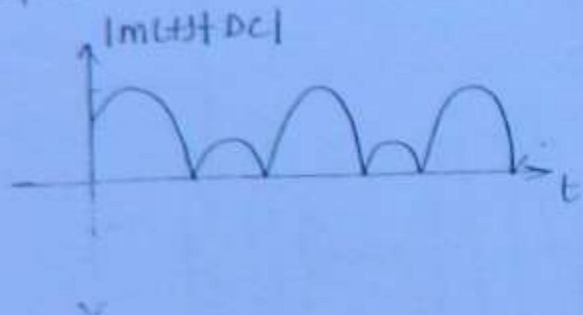


$\because A_{min} = -ve$

← overmodulated

$\boxed{\mu > 1}$

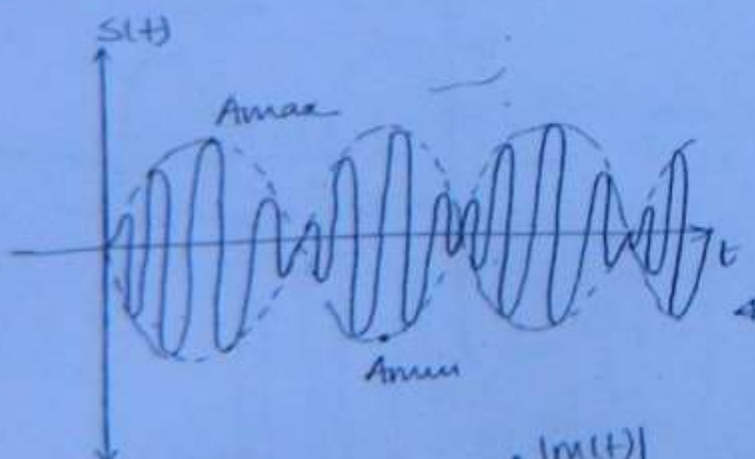
The O/P of the ED will be:-



Note:

1. for $\mu \le 1$, the O/P of the ED will be $A_c\{1+K_a m(t)\}$
2. for $\mu > 1$, the O/P of the ED will be $|A_c\{1+K_a m(t)\}|$

iv. $s(t) = m(t)\cdot c(t) = m(t)\cos 2\pi f_c t$

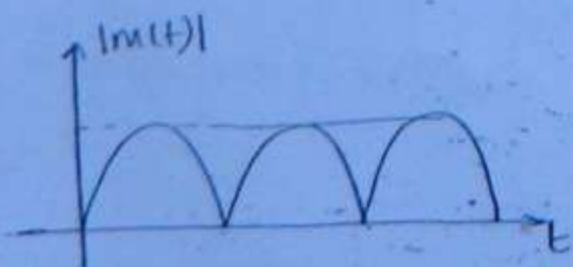


$\because A_{min} = -ve$

← overmodulated

$\boxed{\mu > 1}$

O/P of ED is given as:-



NOTE :-

1. The O/P of ED in case (i) & (iii) is totally +ve.

and, $S_{AM}(t) = \{m(t) + DC\} c(t)$

$= m(t)c(t) + D.C \times c(t)$

(63)

If the DC term is not added in the modulation of the signal the term $D.C \times c(t)$ would not be obtained Hence the AM signal obtained would be over modulated and it can't be demodulated back to obtain m(t) by ED. Hence the ED fails in its demodulation.

2. For case iv ie

$s(t) = m(t) \cdot c(t)$

it contains the message signal m(t), but it can't be demodulated by diode detector.

This scheme of $s(t) = m(t) \cdot c(t)$ is given a specific name called as DSB-SC

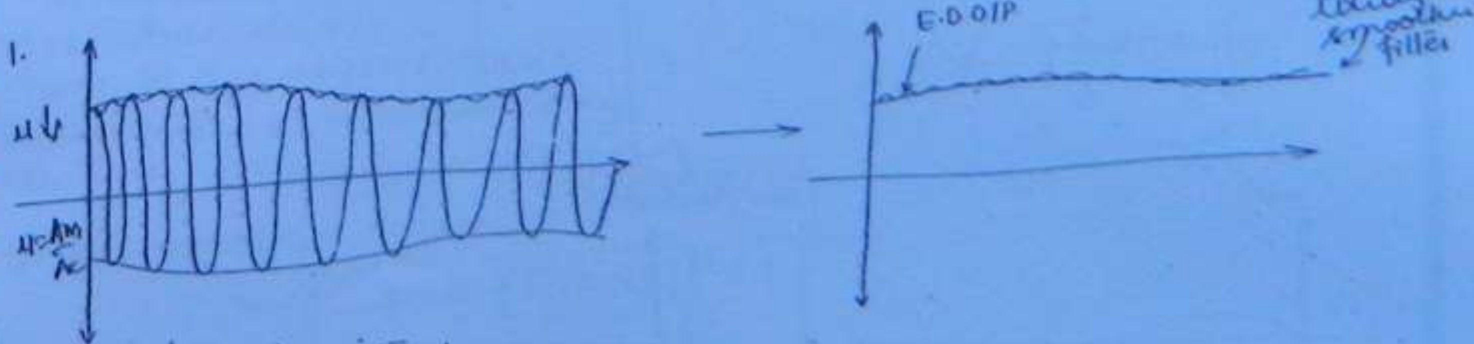
3. Case IV corresponds to DSB-SC modulation. The advantage of DSB-SC is that the Transmitter power will be saved but it's drawback is that its Demodulation becomes complex

* IMPORTANCE OF Ka (amp. sensitivity) :-

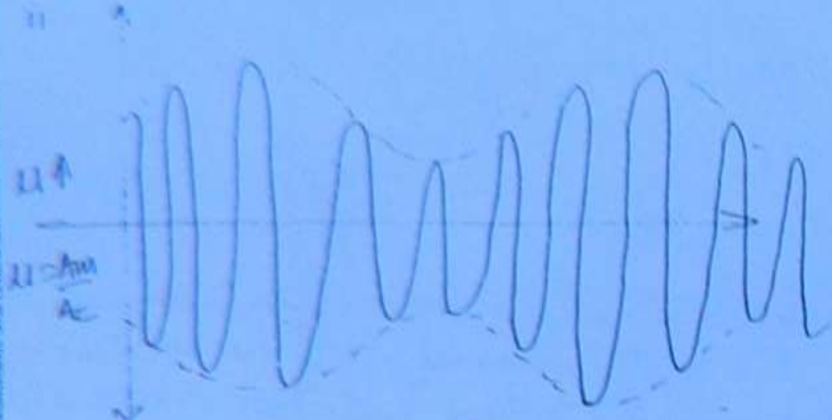
As, we know that :-

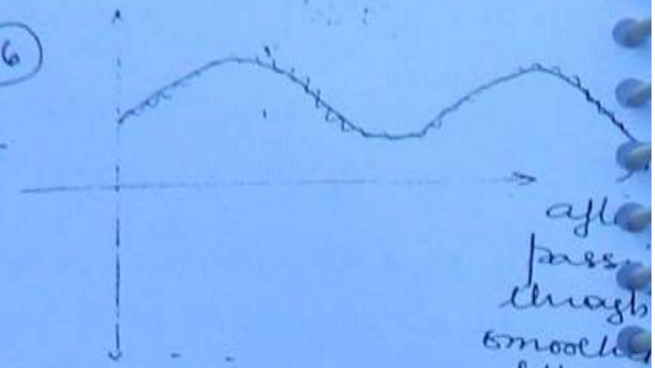
$S_{AM}(t) = A_c\{1 + K_a m(t)\} \cos 2\pi f_c t$

1.



less Amplitude Variations (more prone to Noise)





after passing through smoothing filter

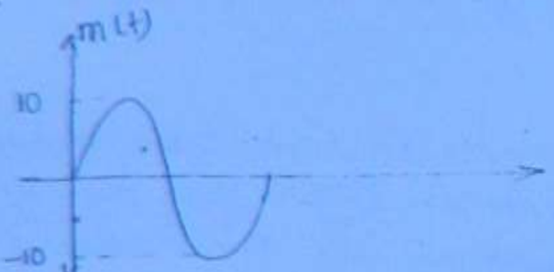
Note:

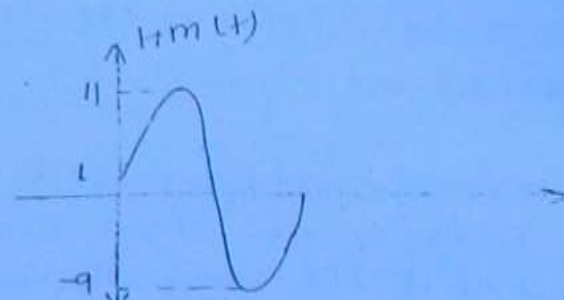
1. μ is high the effect of Noise is less on the reconstructed message signal

2. μ is less, the effect of Noise is dominant.

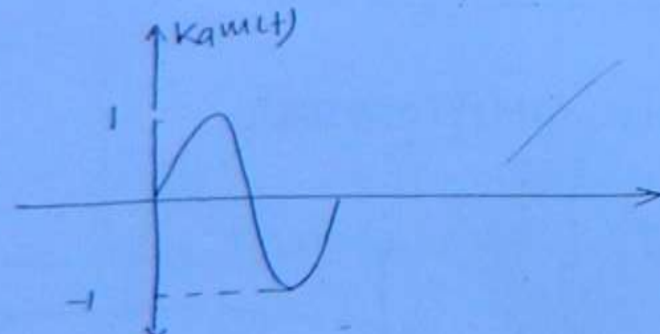
NOW, $S_{AM}(t) = A_c\{1+m(t)\}\cos 2\pi f_c t$

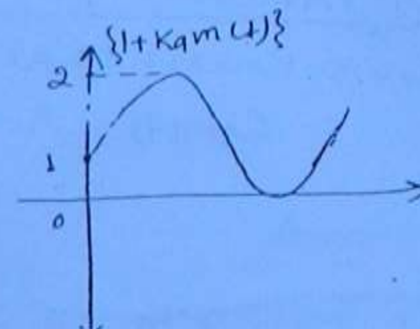
1.



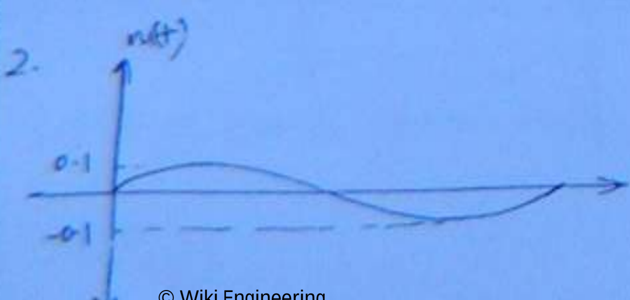


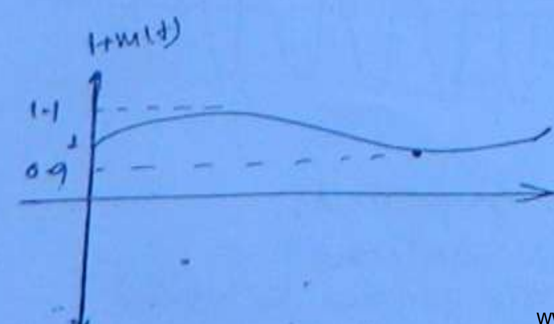
NOW, $K_a = 1/10 \Rightarrow K_a m(t) \rightarrow$ attenuating message signal





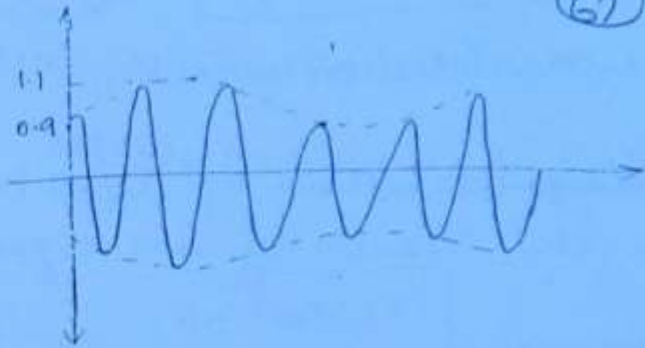
2.





The AM signal has little amp variations and noise effect of Noise is high.

67

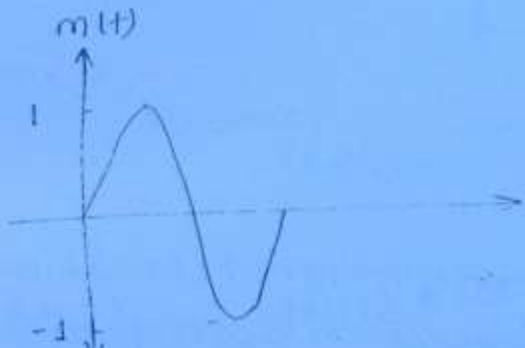


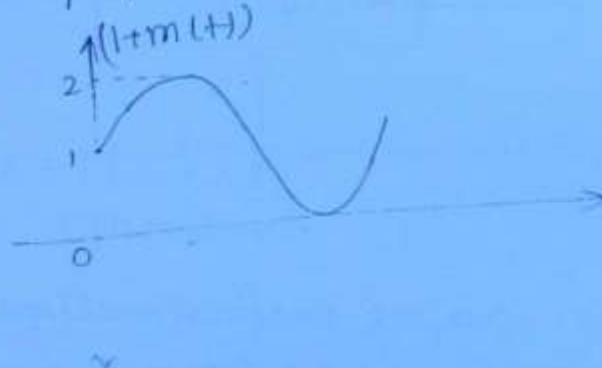
X ← As effect of Noise is high.

← If d.c value of 10 is also added, hence, $A_{max} = 10.1$; $A_{min} = 9.9$
Then also, amp variations are small and effect of Noise is present.

3. As effect of Noise was high, since for small amp. variations. So, the message needs to be amplified.

So, $K_a = 10 \Rightarrow K_a m(t) =$ amplified.





Note:-

1. Ka specifies, normalisation of message signal for proper envelope detection and according to SIMON-HAYKINS,

$$\boxed{\mu = K_a \cdot A_m}$$ ← For Square law & Switching modulator

where,

$K_a = \dfrac{2a_2}{a_1}$ {Square law modulator}

$K_a = \dfrac{4}{\pi A_c}$ {Switching modulator}

Note:- originally, $\mu = \dfrac{A_m}{A_c}$, corresponds to the value of μ when $K_a = 1$.
Now, If the signal needs to amplified or attenuated by K_a. Then $\mu = K_a \cdot A_m$ ← value of A_c is included in K_a or taken as 1

2. For envelope detection,

$$\boxed{\begin{gathered} \max\{K_a m(t)\} \le 1 \\ \text{or} \quad \mu \le 1 \end{gathered}}$$

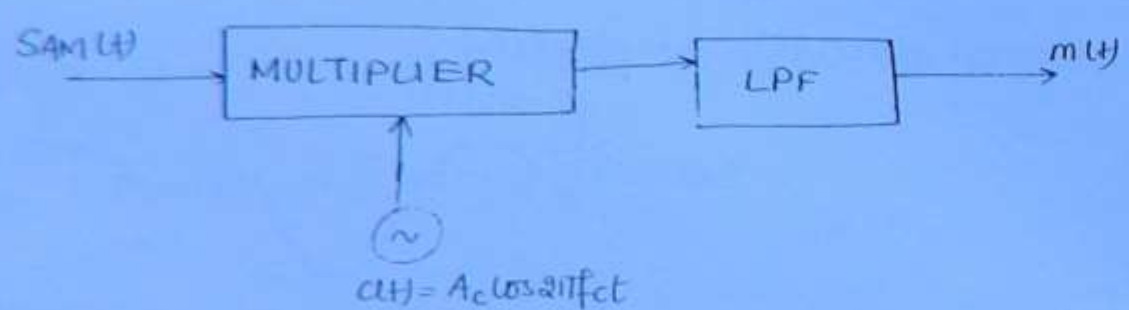
3. For: $\boxed{\max\{K_a m(t)\} > 1 \rightarrow \text{over modulation}}$ (68)

the O/P of the diode detection will be $|m(t)|$.

4. It specifies the voltage variations of superimposed message signal in the Resulting AM signal.

* SYNCHRONOUS DETECTION:

Block-diagram:



* For perfect Reconstruction of message signal, Local oscillator O/P should be perfectly synchronised in both frequency and phase.
* Frequency synchronisation can be easily achieved but achieving phase synchronisation is very complex & difficult.
* To achieve phase synchronisation additional complex cktry has to be used which make the synchronous detector very complex.

Analysis:

Case 1:

1. Assume L.O(O/P) $= A_c \cos 2\pi f_c t$ {perfect synchronisation}.

$\& \; S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$

$= A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$

The multiplier O/P is given as:

$S_{AM}(t) \times c(t)$.

So

$(Mul)_{o/p} = S_{AM}(t) \times c(t)$

$= \{A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t\} \cdot A_c \cos 2\pi f_c t$

$= A_c^2 \cos^2 2\pi f_c t + A_c^2 K_a m(t) \cos^2 2\pi f_c t$ (69)

$= \frac{A_c^2}{2}(1+\cos 4\pi f_c t) + \frac{A_c^2 K_a m(t)}{2}(1+\cos 4\pi f_c t)$

So $\boxed{(LPF)_{O/P} = \frac{A_c^2}{2} \cdot K_a m(t)}$

→ As the AM signal is transmitted for long distances, its ampli gets reduced.

→ So, for its Reconstruction, it is passed through Ampl^r

Case 2:-

Assume,

$(L\cdot O)_{O/P} = \cos\{2\pi f_c t + \Phi\}$ ← {No phase synchronisation}

So,

$(Mul)_{O/P} = S_{AM}(t) \times \cos\{2\pi f_c t + \Phi\}$

$= \{A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t\} \cos\{2\pi f_c t + \Phi\}$

✓ (But blocked by block cap. of Ampl^r)

$= \frac{A_c^2}{2}\cos(4\pi f_c t + \Phi) + \frac{A_c^2}{2}\cos\Phi + \frac{A_c^2 K_a m(t)}{2}\cos(4\pi f_c t + \Phi)$

$+ \frac{A_c^2 K_a m(t)}{2}\cos\Phi.$

So $\boxed{(LPF)_{O/P} = \frac{A_c^2 \cdot K_a \cdot m(t)}{2} \cos\Phi.}$

Note:-

1. The message signal can be Reconstruction back, if the value of $\cos\Phi$ ie Φ remains const.

2. But if the value of Φ changes continuously with time, we have to implement an Ampl^r whose gain changes with Φ to maintain m(t) at const value, & practically it is impossible to construct.

× Also if $\boxed{\phi = 90°}$

so $\cos\phi = \cos 90° = 0$

So the O/P at the demodulator is 0, ie no msg can be extracted. This is called as "Quadrature NULL EFFECT" (QNE).

Note:

1. For perfect Reconstruction of message signal $\boxed{\phi = const}$

2. To maintain, $\phi = const$ additional complex cktry has to be used which makes SD very complex.

3. The demodulation of AM by SD is affected by "QNE"

* ADVANTAGES OF AM:

1. Primary Advantage is that the demodulation is simpler

2. used for long distance commn

* DRAWBACKS OF AM:

1. Transmitter power is wasted. (maxm $\eta = 33\%$ of $P_t = P_{SB}$)

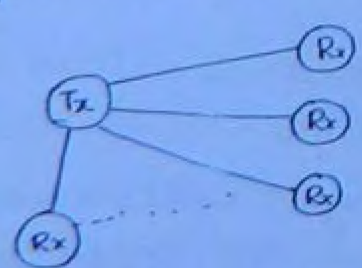
2. It needs high transmission Bandwidth

3. Highly affected by Noise

4. QNE.

* APPLICATIONS OF AM:

1. It is preferred to be used in Broadcasting. (Point to Multi-point)



Since AM Reciever is simple and cheaper, it is higher preferred in Broadcasting...

× DOUBLE SIDE BAND-SUPPRESSED CARRIER (DSB-SC):

Assume, message signal = $m(t)$

Carrier signal = $c(t) = A_c \cos 2\pi f_c t$ 7

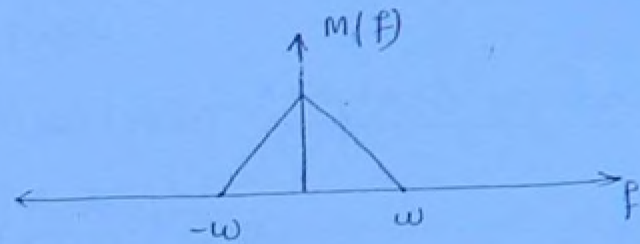
× General exp. of DSB-SC signal:

$$S_{DSB}(t) = m(t)\, c(t)$$

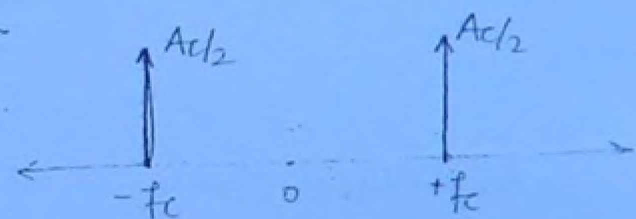
$$\boxed{S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t}$$

× Frequency spectrum:

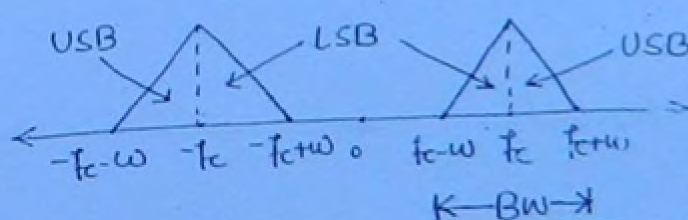
$m(t) \longleftarrow$



$c(t) = A_c \cos 2\pi f_c t \longleftarrow$



$S_{DSB}(t) \longleftrightarrow$



$\boxed{BW = 2W}$

$$\boxed{\text{Bandwidth} = 2 \times \text{message signal B.W}}$$

* SINGLE TONE DSB:

Assume,

$$m(t) = A_m \cos 2\pi f_m t$$

So,

$$S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t$$

$$= A_c A_m \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

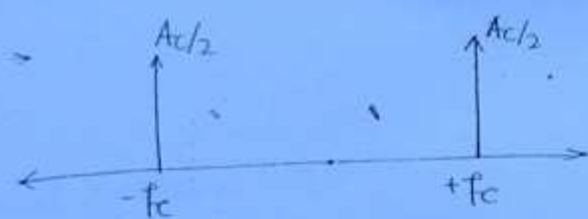
$$\boxed{S_{DSB}(t) = \frac{A_c \cdot A_m}{2} \cos 2\pi (f_c + f_m)t + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m)t}$$

'USB' 'LSB'

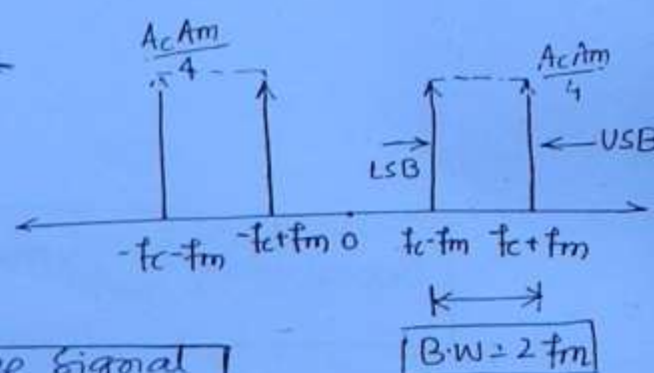
Frequency spectrum Analysis:-

$m(t) \longleftrightarrow$

$= A_c \cos 2\pi f_c t \longleftrightarrow$



$S_{DSB}(t) \longleftrightarrow$



$$\boxed{\text{Band width} = 2 \times \text{message signal frequency}}$$

Note:

let $m(t) = 5\cos \pi \times 10^3 t$

$c(t) = 10\cos 2\pi \times 10^6 t$

(73)

Then Peak amp. of the DSB signal $= \frac{A_c A_m}{2}$

same for AM also

Peak amp. of the USB of DSB in spectrum $= \frac{A_c A_m}{4}$

× Total Power of DSB-SC Signal:-

The total power is given as:

$$P_t = P_{SB}$$

$$= P_{USB} + P_{LSB}$$

$$P_{USB} = \left(\frac{A_c A_m}{2}\right)^2 / 2R$$

$$P_{USB} = \frac{A_c^2 A_m^2}{8R}$$

So, $P_t = 2 \cdot \frac{A_c^2 A_m^2}{8R}$

$$\boxed{P_t = \frac{A_c^2 A_m^2}{4R}}$$

× Modulation efficiency (η):-

As, $\eta = \frac{P_{SB}}{P_t}$

$$\eta = \frac{P_t}{P_t}$$

$$\boxed{\eta = 1 = 100\%}$$

Q. A Carrier of $20\sqrt{2}\cos\pi\times10^{5}t$ is DSB modulated by a msg signal of $2\sqrt{2}\cos\pi\times10^{3}t$. Find all the parameters and plot the spectrum?

(74)

Soln: Given:

$$m(t) = 2\sqrt{2}\cos\pi\times10^{3}t$$

$$A_m = 2\sqrt{2} \quad ; \quad f_m = \frac{10\times10^{9}}{2} = 0.5\ KHz$$

$$c(t) = 20\sqrt{2}\cos\pi\times10^{5}t$$

$$A_c = 20\sqrt{2} \ ; \quad f_c = \frac{10\times10^{4}}{2} = 50KHz$$

Now,

$$P_t = \frac{A_c^2 A_m^2}{4R} = \frac{800\times8^{2}}{4\times1}$$

$\boxed{P_t = 1600\ W}$ {Normalised power, $R = 1\Omega$} Ans

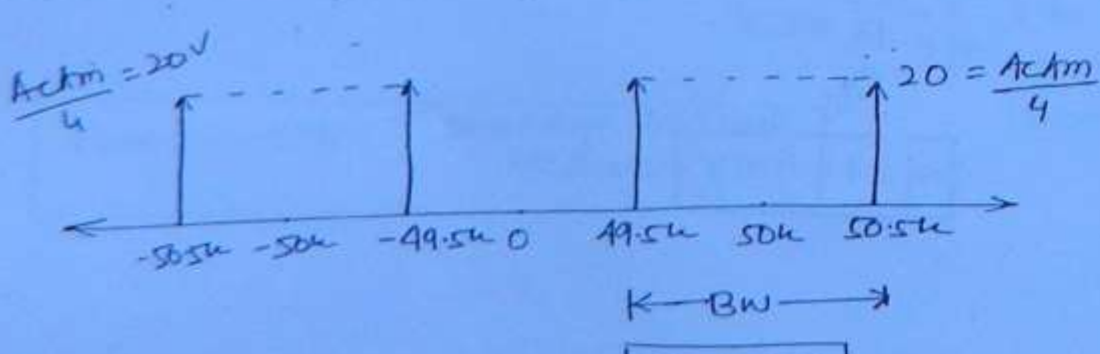
Band width $= 2f_m = 2\times0.5K$

$\boxed{B.W = 1KHz}$ Ans

2. $\boxed{P_{USB} = P_{LSB} = \frac{P_t}{2} = 800\ W}$ Ans

3. $\boxed{\eta = 100\%}$ Ans

Spectrum:



$\boxed{B.W = 1K}$ Ans

* MULTI-TONE DSB:

Assume,

$$m(t) = A_{m1}\cos 2\pi f_{m1} t + A_{m2}\cos 2\pi f_{m2} t$$ 75

$$S_{DSB}(t) = A_c m(t)\cos 2\pi f_c t$$

$$= A_c\{A_{m1}\cos 2\pi f_{m1} t + A_{m2}\cos 2\pi f_{m2} t\}\cos 2\pi f_c t$$

$$= A_c A_{m1}\cos 2\pi f_{m1} t\cos 2\pi f_c t + A_c A_{m2}\cos 2\pi f_{m2} t\cos 2\pi f_c t$$

$$S_{DSB}(t) = \frac{A_c A_{m1}}{2}\cos 2\pi(\underbrace{f_c + f_{m1}}_{USB1})t + \frac{A_c A_{m1}}{2}\cos 2\pi(\underbrace{f_c - f_{m1}}_{LSB1})t$$

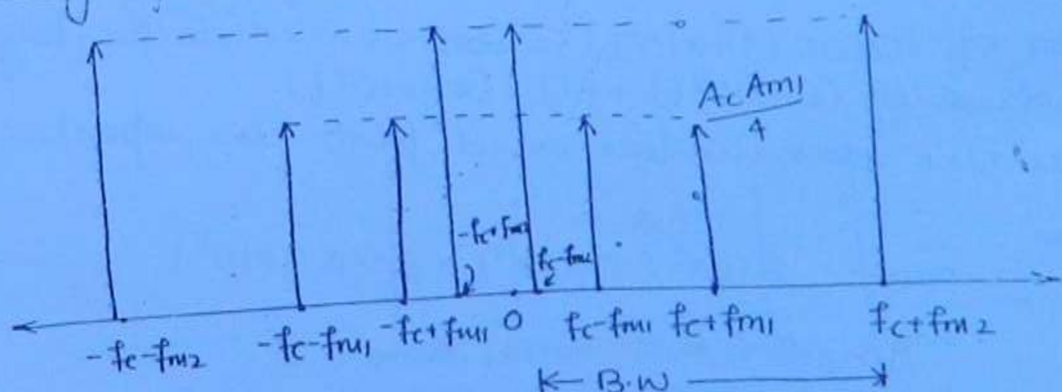
$$+ \frac{A_c A_{m2}}{2}\cos 2\pi(\underbrace{f_c + f_{m2}}_{USB2})t + \frac{A_c A_{m2}}{2}\cos 2\pi(\underbrace{f_c - f_{m2}}_{LSB2})t.$$

* For plotting of spectrum,

let $f_{m2} > f_{m1}$

$A_{m2} > A_{m1}$.

* Frequency Spectrum:



B.W = 2 × Max^m frequency of msg. signal.

$$\boxed{B.W = 2 \times f_{max2}}$$

* Total Power of Multi tone DSB:

As

$$P_t = P_{SB}$$

where,

$$P_{SB} = P_{USB(total)} + P_{LSB(total)}$$

$$= P_{USB1} + P_{USB2} + P_{LSB1} + P_{LSB2}$$

Now,

$$P_{USB1} = \left(\frac{A_c A_{m1}}{2}\right)^2 \Big/ 2R = \frac{A_c^2 A_{m1}^2}{8R} = P_{LSB1}$$

$$P_{USB2} = \left(\frac{A_c A_{m2}}{2}\right)^2 \Big/ 2R = \frac{A_c^2 A_{m2}^2}{8R} = P_{LSB2}$$

So,

$$P_t = \frac{A_c^2 A_{m1}^2}{4R} + \frac{A_c^2 A_{m2}^2}{4R}$$

$$\boxed{P_t = \frac{A_c^2 \left[A_{m1}^2 + A_{m2}^2\right]}{4R}}$$

Q1. A carrier of $10\cos(4\pi \times 10^6 t)$ is DSB modulated by a msg signal of $6\cos(6\pi \times 10^4 t) + 8\cos(\pi \times 10^5 t)$. Find all the parameters and plot the spectrum.

Soln: Given:

$$m(t) = 6\cos 6\pi \times 10^4 t + 8\cos \pi \times 10^5 t$$

$A_{m1} = 6$; $A_{m2} = 8$

$f_{m1} = 3 \times 10^4 \text{ Hz} = 30K$; $f_{m2} = \frac{10 \times 10^4}{2} = 50K$

$$c(t) = 10\cos(4\pi \times 10^6 t)$$

$A_c = 10$; $f_c = 2\text{MHz} = 2000K$

So, $\boxed{B.W = 2 \cdot f_{m(max)} = 2 \times 50K = 100K}$

2. $P_t = \frac{A_c^2 A_{m1}^2}{4R} + \frac{A_c^2 A_{m2}^2}{4R}$ (72)

$$P_t = \frac{A_c^2}{4R}\left\{A_{m1}^2 + A_{m2}^2\right\}$$

$$= \frac{100}{4}\left\{36+64\right\}$$

$$\boxed{P_t = 2500\,W}$$

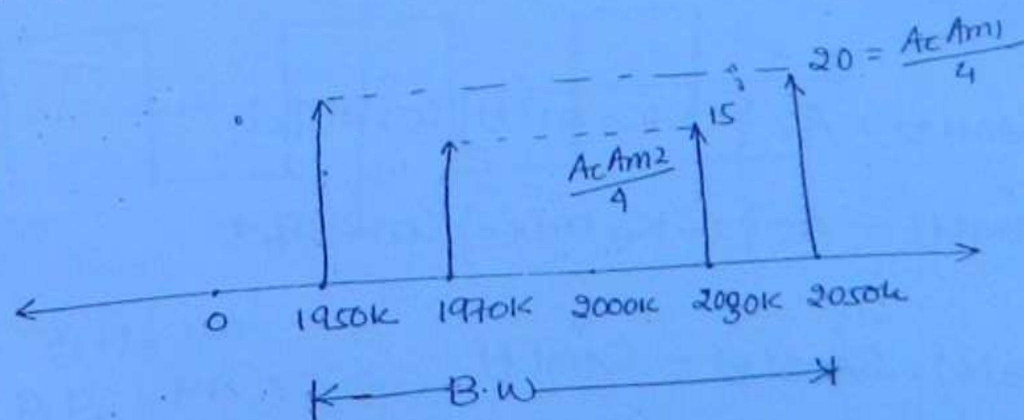
3. $\underset{(total)}{P_{USB}} = \underset{(total)}{P_{LSB}} = 1250\,W$

Now, $P_{USB1} = \frac{A_c^2 A_{m1}^2}{8R} = P_{LSB1}$

$$\boxed{P_{USB1} = P_{LSB1} = \frac{\cancel{100}^{25} \times \cancel{36}^{18}}{\cancel{8}} = 450\,W}$$

$$\boxed{P_{USB2} = 800 = P_{LSB2}}$$

Frequency spectrum:



$\boxed{B.W = 100K}$

* GENERATION OF DSB-SC SIGNAL:

The Generation methods are:

1) Balanced Modulator.

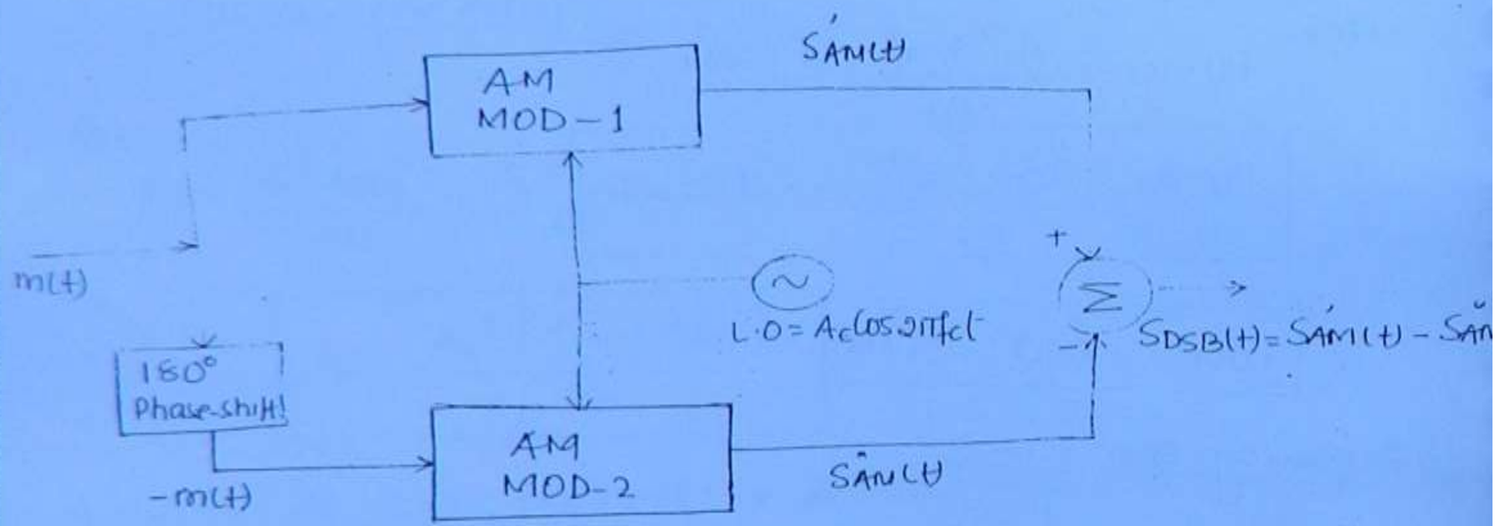
*** 2) Ring modulator.

(78)

* BALANCED MODULATOR:

1. In this two AM modulators are connected in Balanced, to generate DSB signal.

Block diagram:



Now,

$$S'_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$$

&

$$S''_{AM}(t) = A_c\{1 - K_a m(t)\}\cos 2\pi f_c t$$

So,

$$S_{DSB}(t) = S'_{AM}(t) - S''_{AM}(t)$$

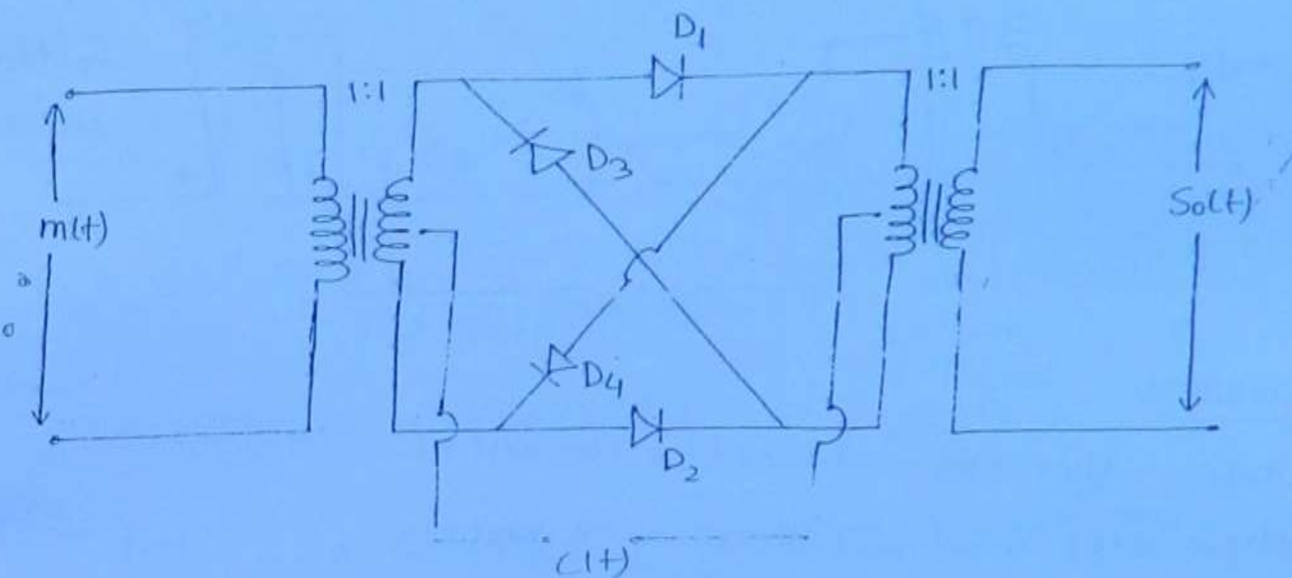
$$= 2A_c K_a m(t) \cos 2\pi f_c t$$

$$\boxed{S_{DSB}(t) = A_c' m(t) \cos 2\pi f_c t}$$

where $\boxed{A_c' = 2A_c K_a.}$

* In this 4 diodes are connected in the form of RING to generate DSB signal

* Block Diagram:

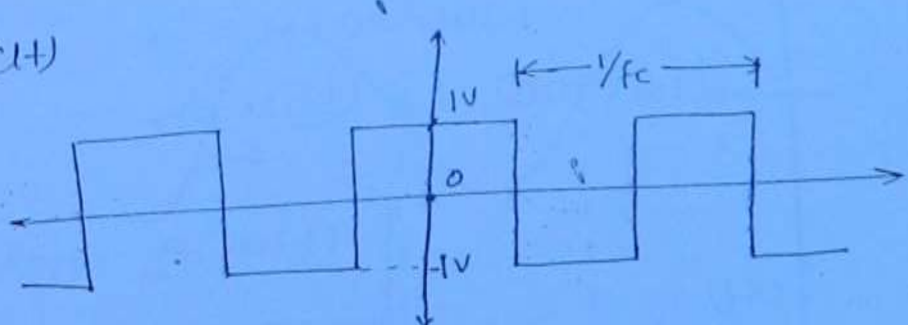


Note:

→ Assume the diode are ideal.

→ x^{mers} are centre - Tapped and of 1:1 type.

let, c(t)

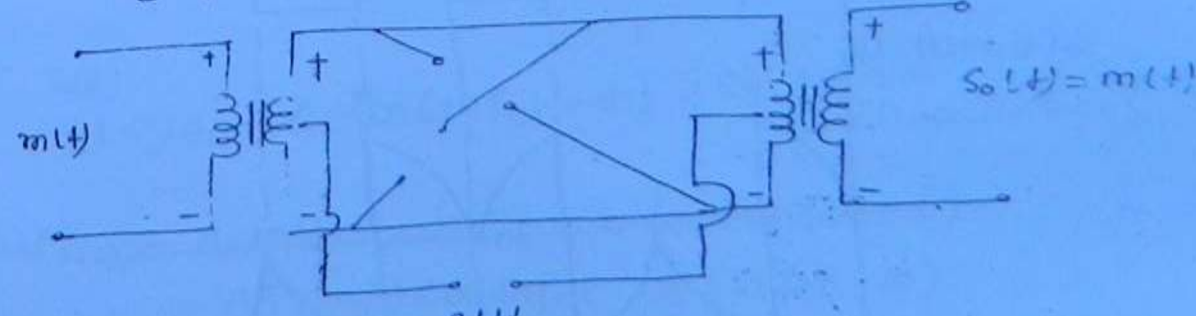


Op^n:

1. when $c(t) = +ve$

$D_1 D_2 = FB = S \cdot C$

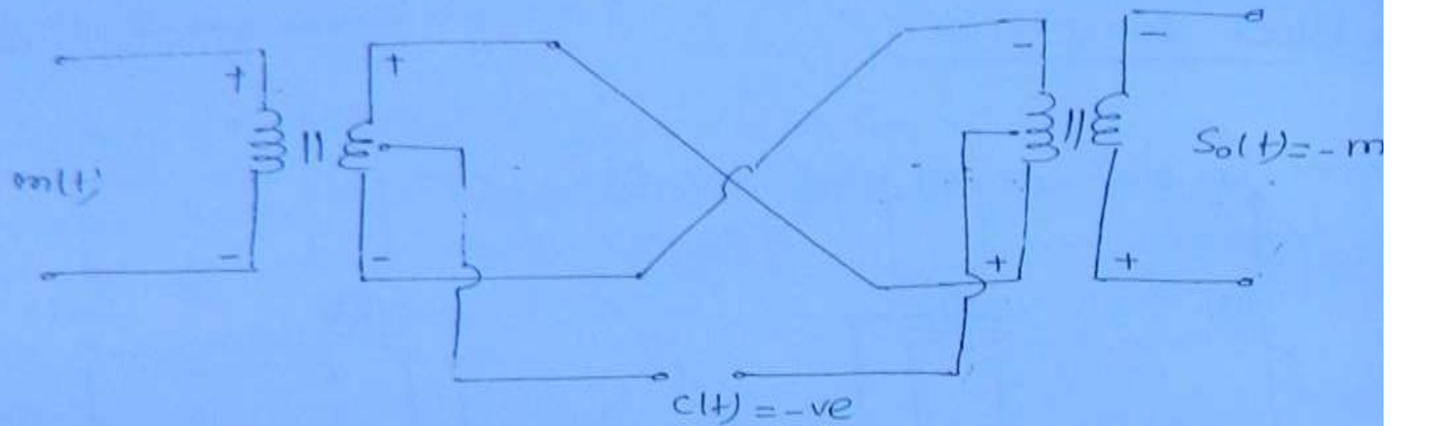
$D_3 D_4 = R \cdot B = O \cdot C$



2. c(t) = -ve : then

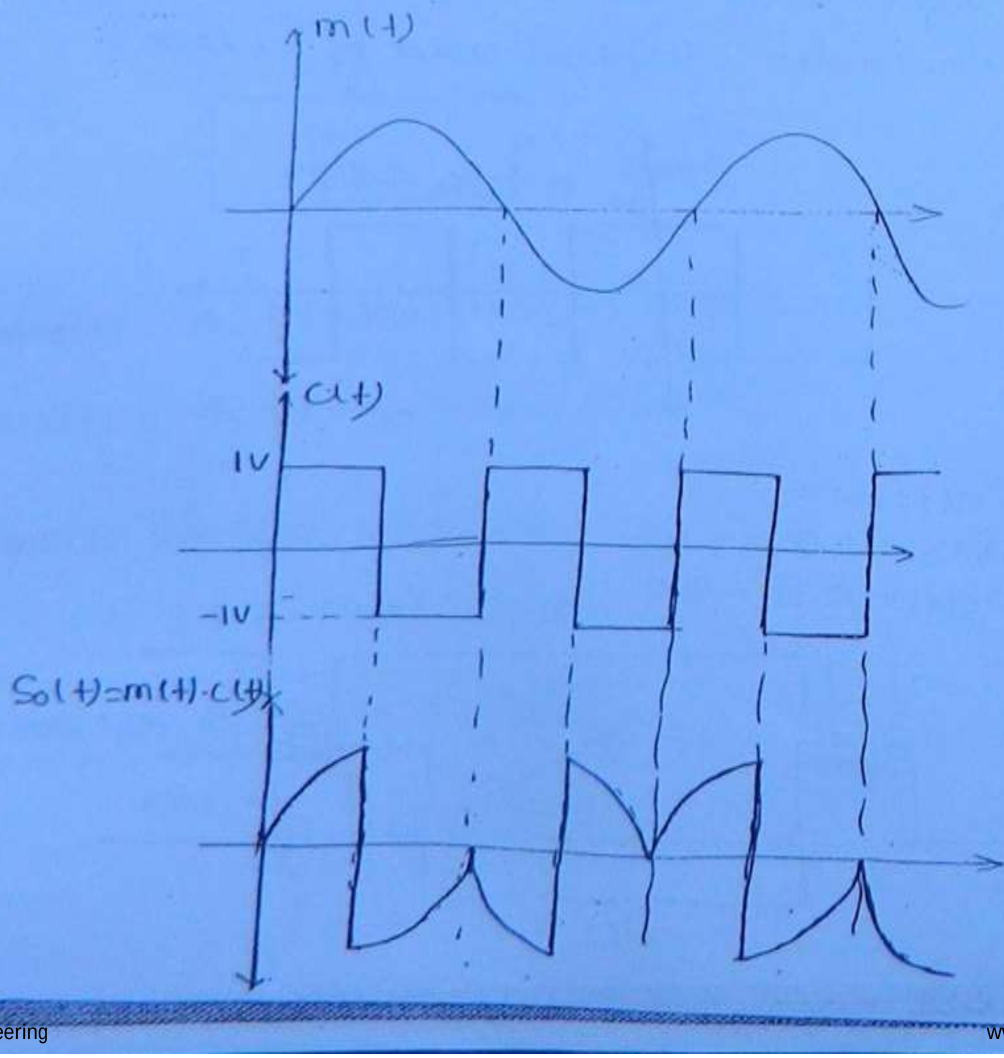
$D_3 D_4$ = FB = S C

$D_1 D_2$ = RB = O/C CKT



Conclusion:

1. when c(t) = +ve $\Rightarrow$ $S_0(t)$ = +ve m(t).
- when c(t) = -ve $\Rightarrow$ $S_0(t)$ = -ve. m(t).



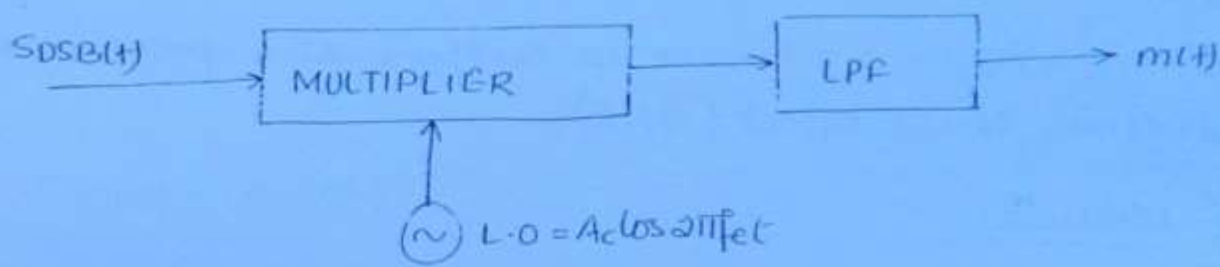
* DEMODULATION OF DSB-SIGNAL :-

* The demodulation of DSB is done by Synchronous detector

* SYNCHRONOUS - DETECTOR :

(81)

B·Diag^m



Now,

$$S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t$$

Case 1: let $L \cdot O = A_c \cos 2\pi f_c t$ (perfect synchronisation)

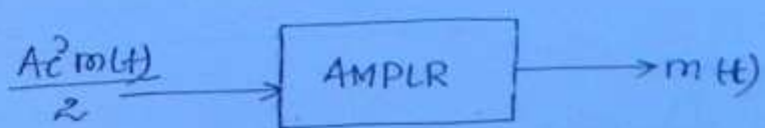
$$(Mul)\, o/p = S_{DSB}(t) \times (L \cdot O)\, o/p$$

$$= A_c\, m(t) \cos 2\pi f_c t \times A_c \cos 2\pi f_c t$$

$$= A_c^2\, m(t) \cos^2 2\pi f_c t$$

$$= \frac{A_c^2\, m(t)}{2} \{1 + \cos 4\pi f_c t\}$$

$$\boxed{(L \cdot P \cdot F)\, o/p = \frac{A_c^2\, m(t)}{2}}$$



Case 2: let,

$$(L \cdot O)\, o/p = A_c \cos(2\pi f_c t + \Phi) \quad \{\text{No phase synchronisation}\}$$

$$(Mul)\, o/p = A_c\, m(t) \cos 2\pi f_c t \times A_c \cos(2\pi f_c t + \Phi)$$

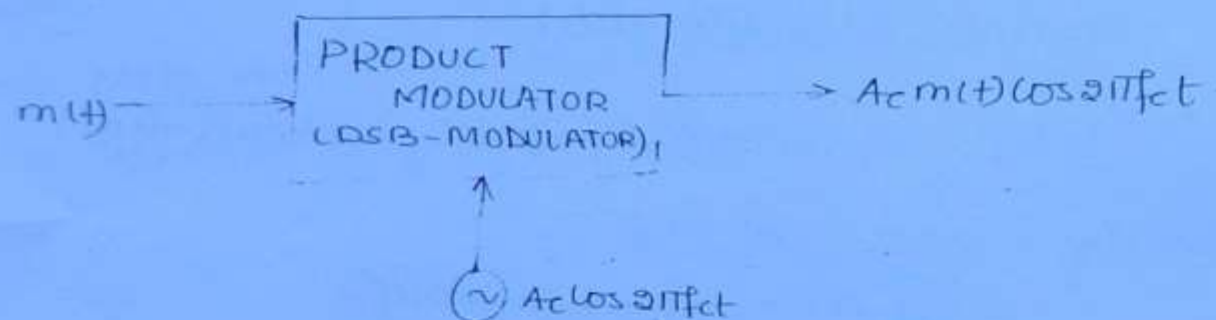
$$= A_c^2\, m(t) \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \Phi).$$

$$(Mul)_{o/p} = \frac{A_c^2 m(t)}{2}\cos(4\pi f_c t + \Phi) + \frac{A_c^2 m(t)}{2}\cos\Phi$$

* $\boxed{(LPF)_{o/p} = \frac{A_c^2 m(t)}{2}\cos\Phi}$

* To maintain the Φ const additional ckt$_{ry}$ has to be utilised.
* when $\Phi = 90^\circ$, o/p = 0, hence it suffers the problem of Quadrature Null effect (QNE).

* Product Modulator:-



Note:-
* Product modulator is the alternate name given to a DSB modulator.

* ADVANTAGES OF DSB:-

1. Transmitter power will be saved ($\eta\% = 100\%$).
2. used for long distance comm^n.

* DRAWBACKS OF DSB:-

1. Demodulation is complex.
2. It needs high xmission Bandwidth.
3. affected by QNE.

* APPLICATION:-

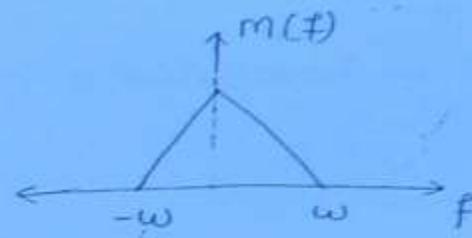
1. It is used in Quadrature carrier Multiplexing

* SINGLE SIDE BAND - SUPPRESSED CARRIER (SSB):-

* The advantage of SSB over AM and DSB is both the xmitter power and B.W will be saved.

let

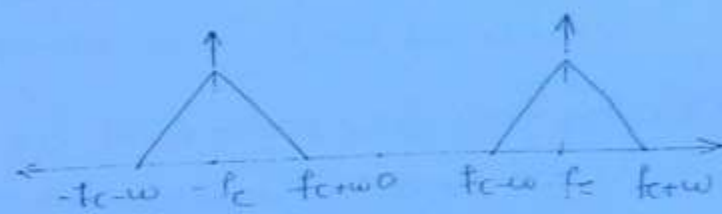
$m(t) \longleftrightarrow$



$c(t) = A_c \cos 2\pi f_c t$

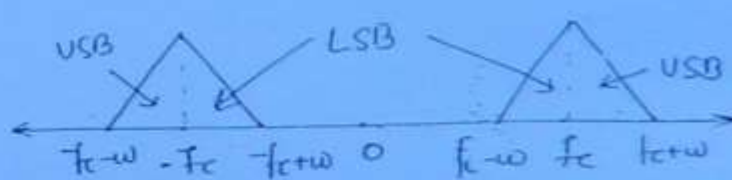
$S_{AM}(t) \longleftrightarrow$

$\boxed{AM\ B.w = 2w}$



$S_{DSB}(t) \longleftrightarrow$

$\boxed{DSB\ B.w = 2w}$



$S_{SSB}(t) \longleftrightarrow$

only USB

$\boxed{SSB\ Bw = w}$

message signal BW

OR

only LSB.

84

Note:

1. The demodulation of SSB is also done by the SD. The SD gives the message signal as o/p whenever the 2 sideBands (as in DSB) are given as i/p. It also gives the m(t) whenever only 1 sideBand (as in SSB) is given. Hence SSB is preferred over DSB.

(88)

2. By doing this:
   1) Power is saved (for xmitting 2 sideBand as in DSB)
   2) Band width is saved

Note:

1. In SSB, compared to DSB 50% of the Transmitter power and 50% of Transmission B.W will be saved.

2. The % of power saved in DSB and SSB compared to AM depends on the modulation Index (μ).

Eg.

* SINGLE TONE SSB:

let
$m(t) = A_m \cos 2\pi f_m t$

$S_{AM}(t) = A_c \{1 + \mu \cos 2\pi f_m t\} \cos 2\pi f_c t$

$S_{DSB}(t) = \frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c A_m}{2} \cos 2\pi (f_c - f_m) t$

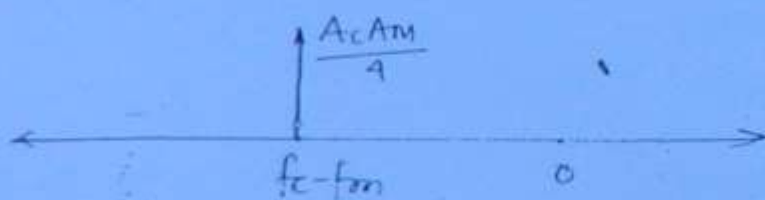
Now,
the corresponding SSB expression is given as:-

$$\boxed{S_{SSB}(t) = \frac{A_c A_m}{2} \cos 2\pi [f_c \pm f_m] t}$$

+ → USB
− → LSB

spectrum:-

or



Bandwidth of single tone SSB:-

$$\boxed{B.W = 0}$$

* Total Power:-

$P_t = P_{SB}$

$= P_{USB}$ or P_{LSB}

Now,

$P_{USB} = \left(\frac{A_c A_m}{2}\right)^2 / 2R$

$P_{USB} = \frac{A_c^2 A_m^2}{8R}$

$$\boxed{P_t = \frac{A_c^2 A_m^2}{8R}}$$

× Modulation efficiency (η):

(87)

$$\%\eta = \frac{P_{SB}}{P_t} = \frac{P_{SB}}{P_{SB}} \times 100$$

$$\boxed{\eta = 100\%}$$

× General expression of SSB:-

The General expression of AM is:

$$S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$$

& for DSB is:

$$S_{DSB}(t) = A_c m(t)\cos 2\pi f_c t$$

The General expression of SSB is given as:

$$\boxed{S_{SSB}(t) = \frac{A_c A_m}{2}\cos 2\pi(f_c \pm f_m)t}$$

+ → USB
− → LSB

$$\boxed{S_{SSB}(t) = \frac{A_c A_m}{2}\cos 2\pi f_c t \cos 2\pi f_m t \mp \frac{A_c A_m}{2}\sin 2\pi f_c t \sin 2\pi f_m t}$$

− → USB.
+ → LSB.

we have;

$$m(t) = A_m \cos 2\pi f_m t$$

and 90° phase shift of m(t) we get { $\hat{m}(t)$ = 90° phase shift of m(t) }

$$\hat{m}(t) = A_m \sin 2\pi f_m t$$

where, $\boxed{\hat{m}(t) = \text{HILBERT xmation of } m(t)}$

So, substituting in the above equation we get:

$$\boxed{S_{SSB}(t) = \frac{A_c m(t)}{2}\cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t}$$

− → USB.
+ → LSB.

single tone SSB signal

% of power saved in DSB and SSB V/s AM

let,

$\mu = 0.707$

Then, after modulation, for AM

$P_t = 100W$

$P_c = 80W$ $P_{SB} = 20W$

88

For AM:

P_t	P_c	P_{USB}	P_{LSB}
100W	80W	10W	10W

For DSB:

20W	–	10W	10W

For SSB:

10W	–	10W or 10W

Note:

1. In SSB as compared to AM, 90% of transmitter power is saved.
2. In DSB as compared to AM, 80% of transmitter power is saved.
3. In SSB as compared to DSB: 50% of xmitter power is saved.

Note:

1. The amount of power saved in DSB & SSB compared to AM depends on μ.
2. whatever be the value of 'μ' the amount of power saved in SSB compared to DSB is always 50%.

$$P_t = P_c \left\{1 + \frac{\mu^2}{2}\right\}$$

$$P_{SB} = \frac{P_c \mu^2}{2} \quad ; \quad P_{USB} = P_{LSB} = \frac{P_c \mu^2}{4}$$

Now;

% of power saved in DSB compared to AM $= \frac{\text{Power saved}}{\text{Total power}}$

(89)

$$= \frac{P_c}{P_t} = \frac{P_c}{\left(1+\frac{\mu^2}{2}\right) \cdot P_c}$$

**

$$\boxed{\text{\% of power saved in DSB V/s AM} = \frac{2}{2+\mu^2} = 1-\eta = 1-\frac{P_{SB}}{P_t}}$$

% of power saved in SSB compared to AM $= \frac{\text{Power saved}}{\text{total power}}$

$$= \frac{P_c + \frac{P_c \mu^2}{4}}{P_c \left\{1+\frac{\mu^2}{2}\right\}}$$

**

$$\boxed{\text{\% of power saved in SSB V/S AM} = \frac{4+\mu^2}{4+2\mu^2}}$$

Q1. An AM transmitter power is given by 500W. Find the amount of power saved if carrier and one of the SB is suppressed with i) $\mu = 0.5$?
ii) $\mu = 0.8$

Soln: Given:

$P_t = 500W$

for $\mu = 0$; $P_t = P_c = 500W$

So, $P_t = P_c\left\{1+\frac{\mu^2}{2}\right\}$

$= 500\left\{1+\frac{0.25}{2}\right\}$

$= 501.125$

So, % of power saved in SSB $= \frac{4+\mu^2}{4+2\mu^2}$

i) $\mu = 0.5$

% of power saved $= 94.5\%$

Amount of power saved $= 94.5\%$ of 500

$= \boxed{472.5W}$ Ans

ii) $\mu = 0.8$

% of power saved $= 0.878 \times 100$

Amount of power saved $= 87.8\%$ of 500

$= \boxed{439W}$ Ans

* Generation of SSB:

The Generation methods of SSB are:

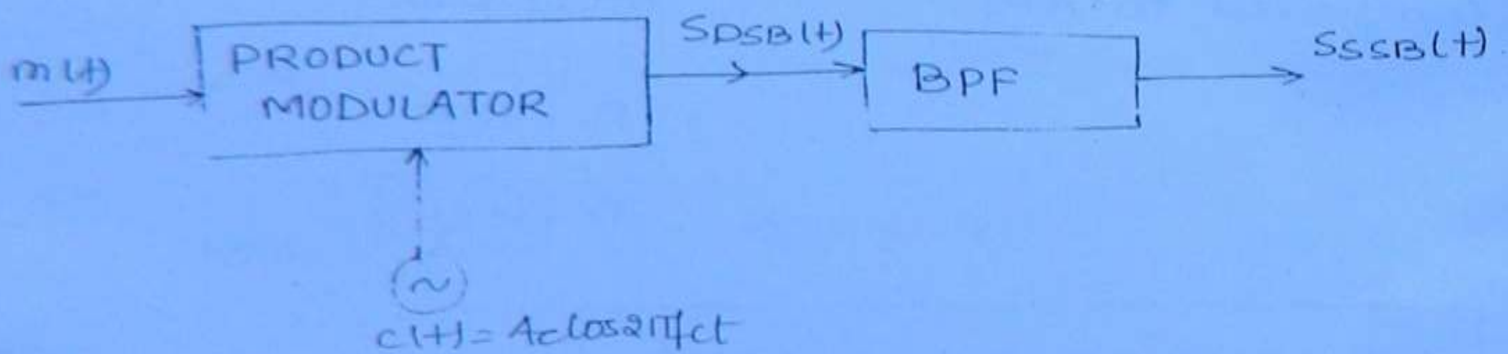
i) Frequency discrimination method

ii) phase discrimination method

* FREQUENCY DISCRIMINATION METHOD:

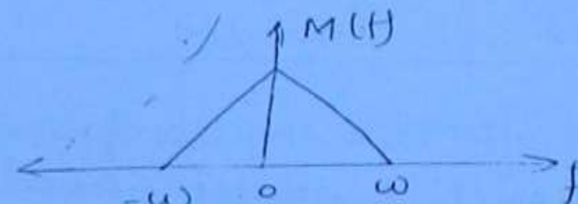
* In this DSB signal is transmitted through proper Band pass filter to generate SSB.

* Block Diagram:



Let

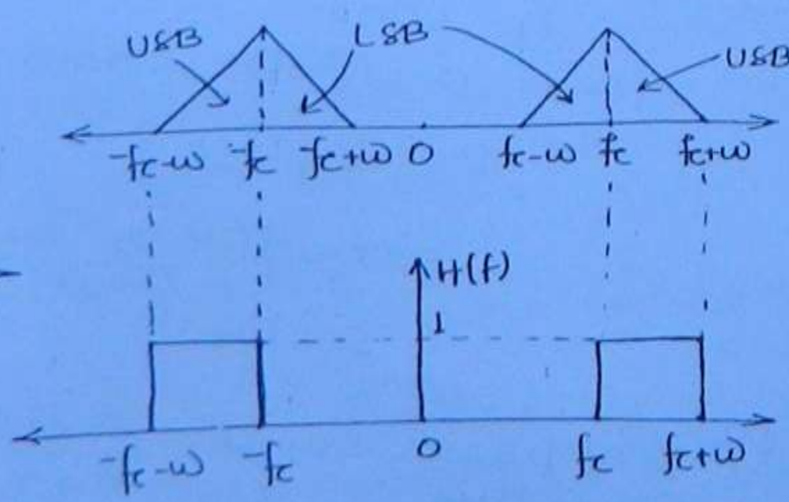
m(t) ⟷



$c(t) = A_c \cos 2\pi f_c t$

$S_{DSB}(t)$ ⟷

$(BPF)_{o/p}$ ⟷

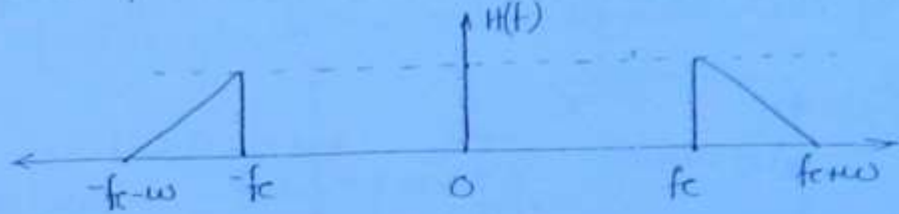


So the BPF; o/p is given as

$$Y(f) = H(f)\,X(f)$$

(91)

$(BPF)_{o/p}$ = SSB spectrum

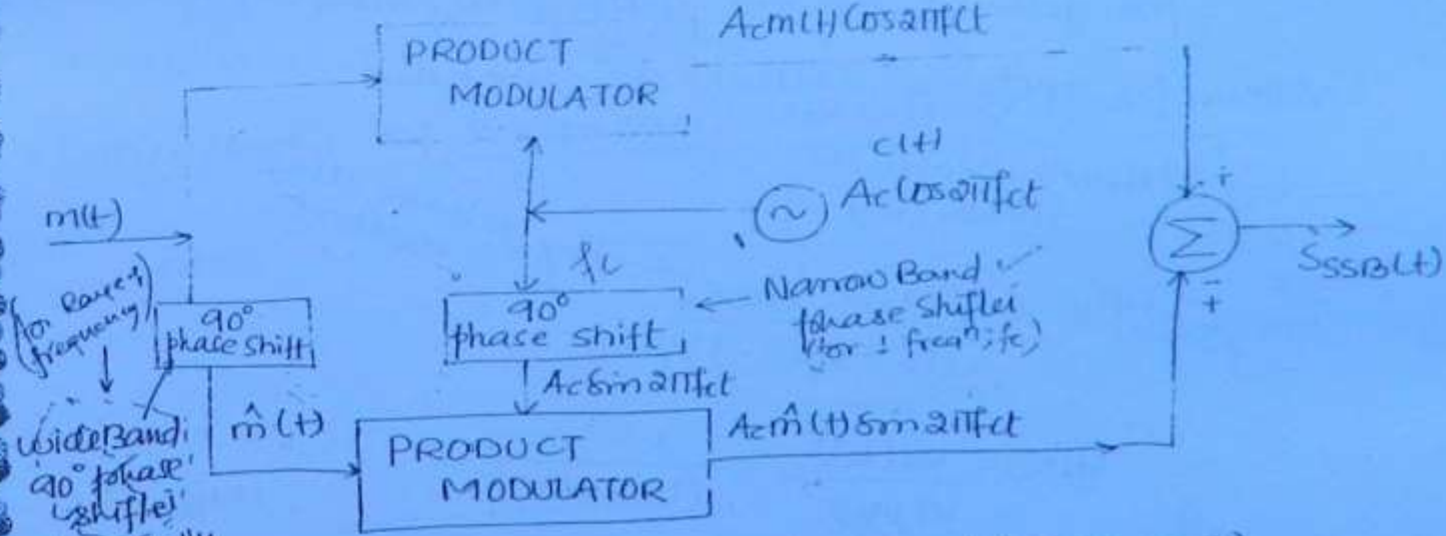


× PHASE DISCRIMINATION METHOD:

The General expression of SSB signal is given as:

$$S_{SSB}(t) = \underbrace{\frac{A_c m(t)}{2}\cos 2\pi f_c t}_{DSB} \mp \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t.$$

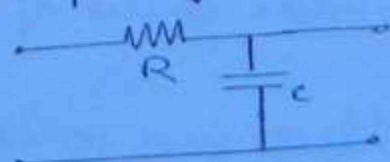
Block diagram:



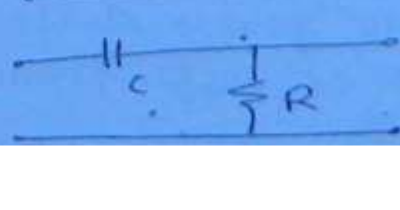
SSB Generation (for only multi tone SSB)

Note:

1. Phase shift n/w is nothing but Integrator or Differentiator
2. Integrator ⟶ low pass filter with high RC



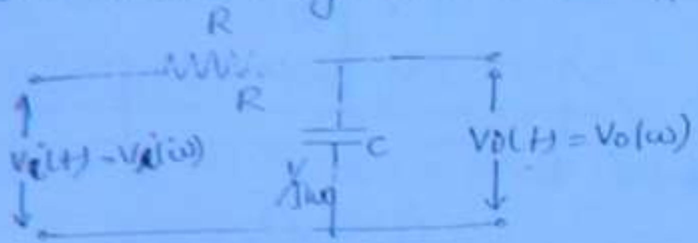
3. Differentiator ⟶ → high pass filter with low value of RC



1) Generally for Realisation of 90° phase shifter differentiator or Integrator will be used

92

2. The following LPF with high RC works as Integrator.



So, $V_0(\omega) = V_i(\omega) \cdot H(\omega)$

$|V_0(\omega)| = |V_i(\omega)|\,|H(\omega)|$ ← Magnitude

$\angle V_0(\omega) = \angle V_i(\omega) + \angle H(\omega)$ ← Phase

For getting phase shifted by 90°, $\angle H(\omega) = 90°$

Now, for getting $\angle H(\omega) = 90°$, we have:

$$H(\omega) = \frac{V_0(\omega)}{V_i(\omega)}$$

← By voltage division method

$$H(\omega) = \frac{V_i(\omega) \cdot 1/j\omega C}{(R + 1/j\omega C)\,V_i(\omega)}$$

$$H(\omega) = \frac{V_0(\omega)}{V_i(\omega)} = \frac{1}{(1 + j\omega RC)}$$

So, $\angle H(\omega) = 0 - \tan^{-1}\left(\frac{\omega RC}{1}\right)$

$$\boxed{\angle H(\omega) = -\tan^{-1}(\omega RC)}$$

→ To obtain $\angle H(\omega)$ the value of $\omega RC \simeq \infty$, but practically it is not possible.

→ let the m(t) be multitone ie contain range of frequency. Hence to obtain 90° phase shift for Range of frequency we need to vary R & C with frequency, which can't be implemented.

Note:

* The practical significance of single tone SSB generation ckt is not there; as for transmission, multi tone SSB takes place. Hence there is no practical significance of single tone SSB / Phase discrimination method.

93

Note:-

$$\angle H(i\omega) = -\tan^{-1}(\omega RC)$$

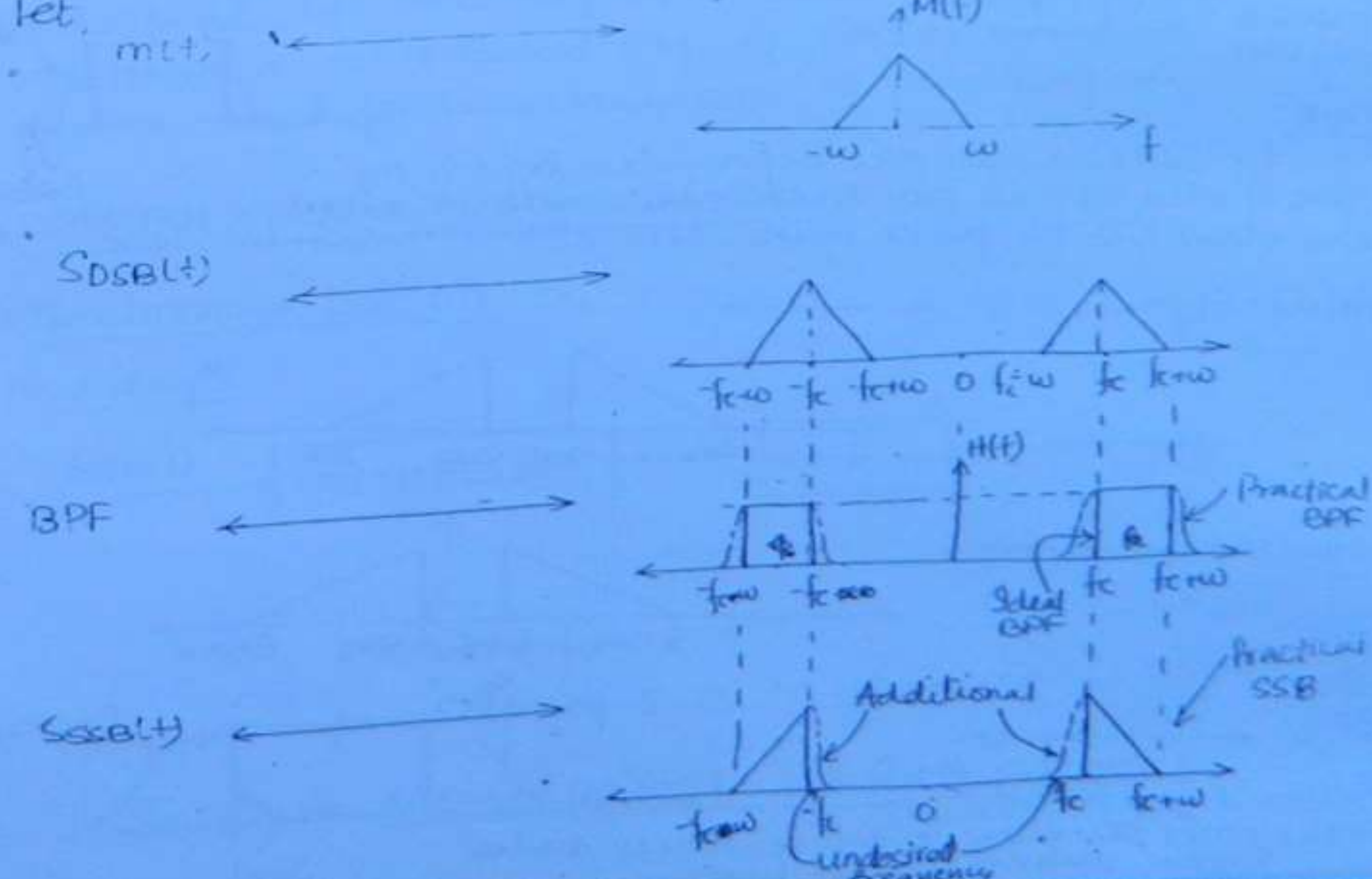
from above: diff. frequency components of input are experiencing diff. phase shifts. So, it is practically not possible to construct wide Band 90° phase shifter. So the above method is failed for the generation of Multi tone SSB signal.

Note:

For Generation of Multitone SSB frequency discrimination method will be used.

* Drawbacks of Frequency discrimination method:

let,



Note:

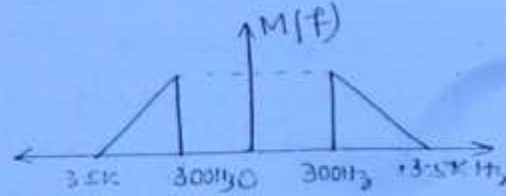
1. Since Ideal BPF Cannot be constructed, the Resulting SSB signal contains undesired frequencies in addition to actual side Bands.

94

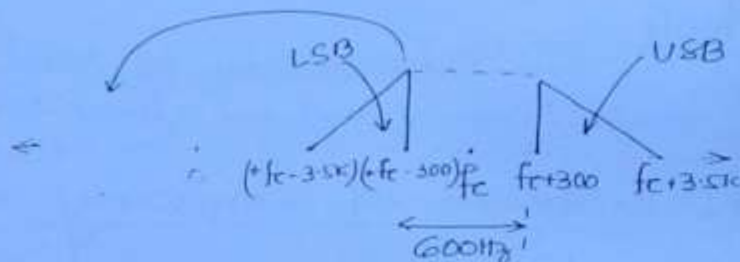
2. Because of above drawback SSB is limited only for voice signal transmission.

Analysis:

1. Voice Signal: m(t)



$S_{DSB}(t)$

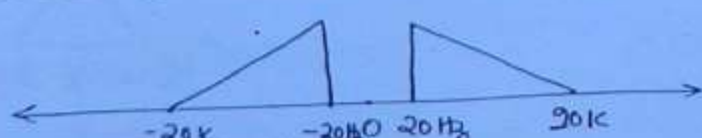


(BPF) passing only USB

Note:

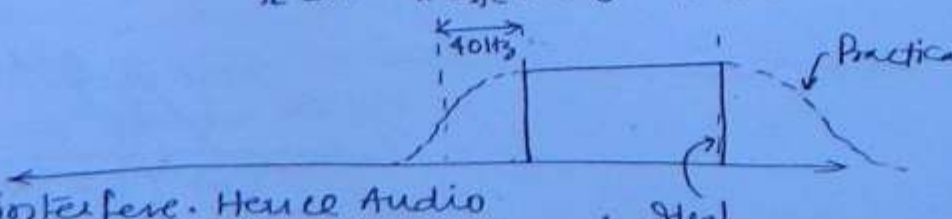
The SideBands are separated by 600Hz. Also, if the BPF is not Ideal, it will not allow the other SB to pass. Hence the xmission can be done

2. Audio Signal; m(t)



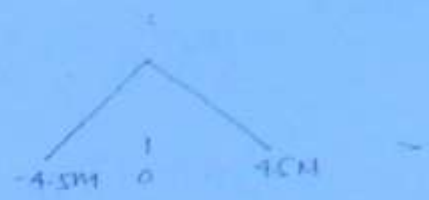
$S_{DSB}(t)$

(BPF)

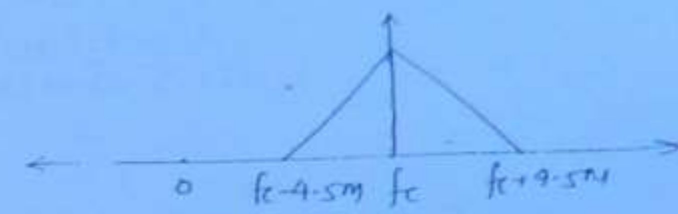


∴ Band gap is 40Hz

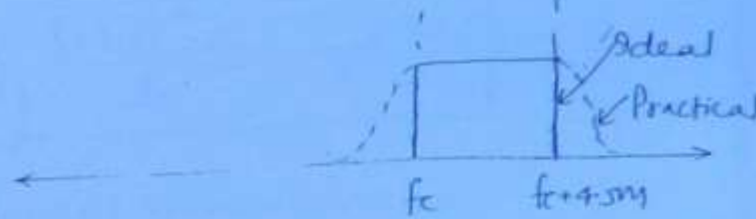
So, the other SB may interfere. Hence Audio signal xmission is not preferred.



SDSB(t)



(BPF)



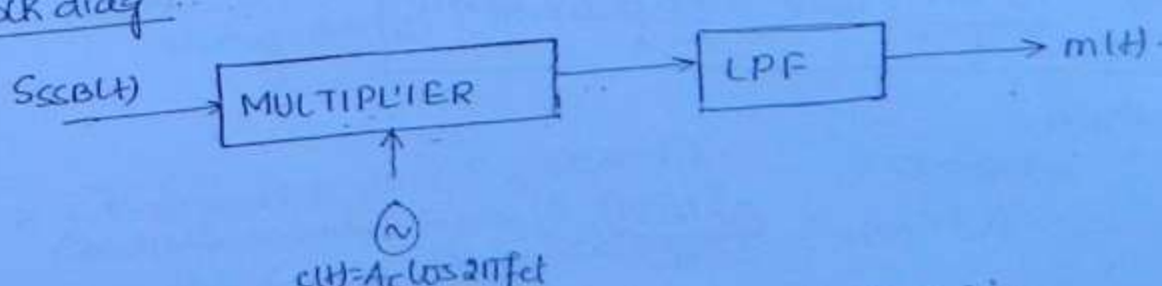
∴ Band gap = 0
Hence the video signal can't be xmitted through SSB scheme as other SB may interfere.

Conclusion:
1. For transmission of a message signal in the Respective DSB spectrum, a wide Band gap should be existing b/w the corresponding DSB side Bands; For voice signal in the corresponding DSB side Bands; For voice signal of 600Hz exist so SSB spectrum a wide Band gap of 600Hz exist so SSB spectrum can be used for transmission of voice signals.

*DEMODULATION OF SSB SIGNAL:-

*Synchronous Detector:

Block diag^m:



General expression of SSB is given as:

$$S_{SSB}(t) = \frac{A_c m(t)}{2}\cos 2\pi f_c t - \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t$$

Case 1:

i) $(LO)_{O/P} = A_c \cos 2\pi f_c t$ {Perfect synchronisation}

(96)

ii) $(Mul)_{O/P} = S_{SSB}(t) \times (LO)_{O/P}$

$$= \frac{A_c^2 m(t)}{2}\cos^2 2\pi f_c t \mp \frac{A_c^2 \hat{m}(t)}{2}\frac{\sin 4\pi f_c t}{2}$$

$\{\because 2\sin\theta\cos\theta = \sin 2\theta$

iii) $\boxed{(LPF)_{O/P} = \frac{A_c^2 m(t)}{4}}$ $\left\{\because \cos^2 2\pi f_c t = \frac{1-\cos 4\pi f_c t}{2}\right\}$.

Case 2:

let,

i) $(LO)_{O/P} = A_c \cos(2\pi f_c t + \Phi)$ {No phase synchronisation}

ii) $(Mul)_{O/P} = S_{SSB}(t) \cdot (LO)_{O/P}$

$$= \left\{\frac{A_c^2 m(t)}{2}\cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t\right\}\{A_c \cos(2\pi f_c t + \Phi)$$

$$= \frac{A_c^2 m(t)}{4}\cos(4\pi f_c t + \Phi) + \frac{A_c^2 m(t)}{4}\cos\Phi.$$

$$\mp \frac{A_c^2 \hat{m}(t)}{4}\sin(4\pi f_c t + \Phi) \pm \frac{A_c^2 \hat{m}(t)}{4}\sin\Phi.$$

iii) $\boxed{(LPF)_{O/P} = \frac{A_c^2 m(t)}{4}\cos\Phi \pm \frac{A_c^2 \hat{m}(t)}{4}\sin\Phi.}$

when,

a) $\Phi = 0$

$(LPF)_{O/P} = \frac{A_c^2 m(t)}{4}$

b) $\Phi = 90^\circ$

$(LPF)_{O/P} = \pm\frac{A_c^2 \hat{m}(t)}{4}$ ← {∵ O/P ≠ 0; hence SSB is not affected by QNE}.

No QNE

Note

× Demodulation of SSB is not affected by QNE

× Advantages of SSB:

(97)

1) Transmitter power is saved { % of $\eta = 100\%$ }.

2) Transmission Bandwidth is saved.

3) NO affect of QNE.

× Drawbacks of SSB:

1) Demodulation is complex.

2) Limited only for Voice signal transmission.

× APPLICATION:

1) Preferred in Voice signal transmission.

Note:

× Earlier SSB was used for Voice signal xmission in local telephone $comm^n$.

× VESTIGIAL SIDEBAND MOD^N (VSB):-

Video Signal = 0 – 4.5 MHz

B.W = 4.5 MHz

Hence using AM or DSB modulation = 2 × B.W = 9 MHz

× T.V Signal = 10 MHz (each)
(including Audio Signal)

× co-axial cable B.W = 600 MHz
(Broadly used)

→ 60 TV channels can be xmitted.

let, SSB modulation scheme is used.

Hence S.S.B, B.W = 4.5 MHz

Total B.W TV signal ≃ 5 MHz (Including audio signal)

⇒ 120 TV channel

∴ SSB not used for Video Signal

Hence it fails. though it has advantage over AM or DSB (In B.W & power)

Note :-

1. VSB provides almost of same Bandwidth as SSB and can be used for the xmission of Video signals

B.W = slightly greater than SSB

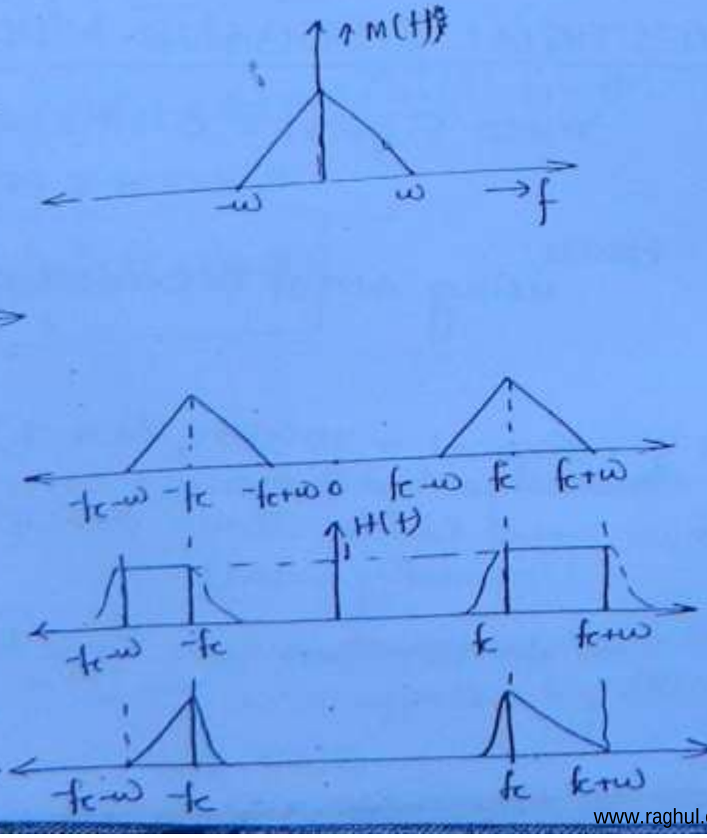
let, SSB

m(t)

SDSB(t)

BPF

SSSB(t)



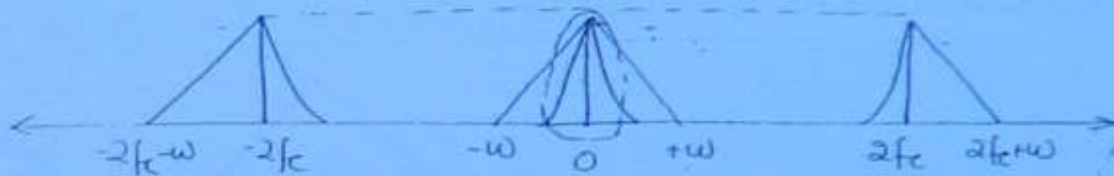
In the Recr of (synchronous detector)
(Mul) o/p spectrum is given as

99

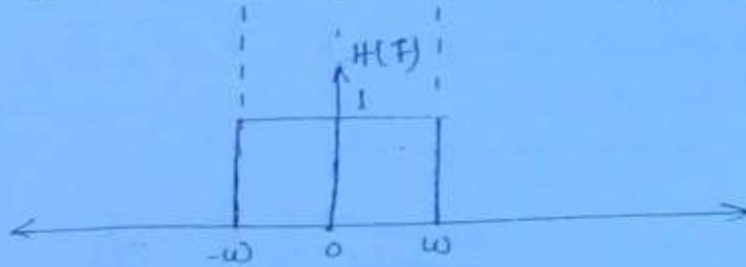
$$s(t)\cos 2\pi f_c t = \frac{S(f+f_c) + S(f-f_c)}{2}$$

(SSB(t) shifted left & Right by f_c)

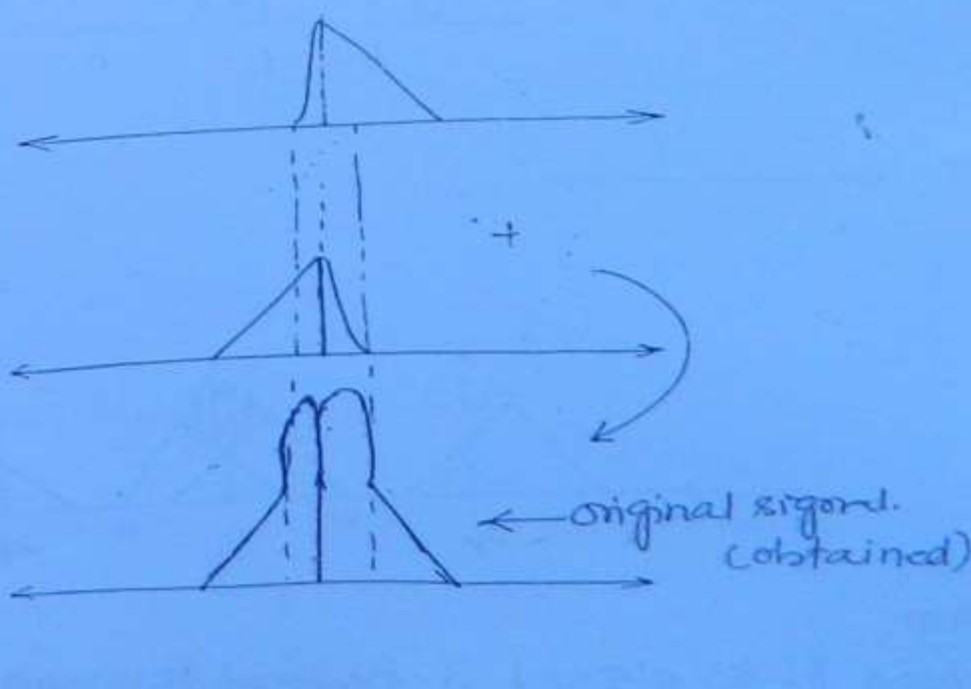
(1st) o/p



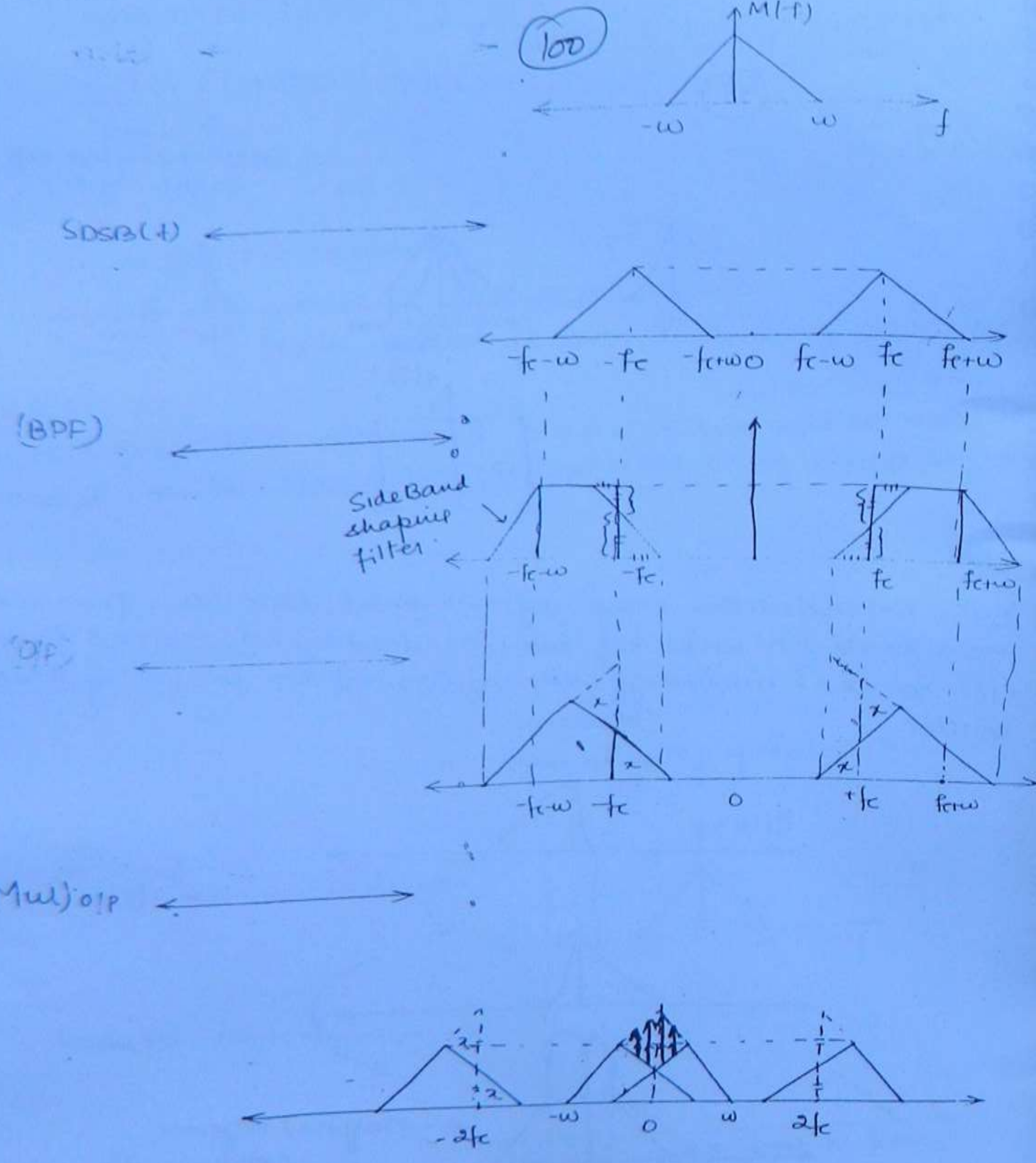
(LPF)



Note:
* In the Reconstructed message signal, the low frequencies near to origin will be interferenced by undesired frequencies so that message signal cannot be perfectly Reconstructed..



100



Conclusion:

1. The ~~above~~ DSB signal is passed through SideBand shaping filter to generate VSB.

2 H(f) of Side Band shaping filter should be symmetrical about fc.

101

3 For demodulation of VSB, synchronous Detector will be used.

Analysis of BW & Power with other Modulation schemes:

Bandwidth:-

$\boxed{AM \ \& \ DSB > VSB > SSB}$ ← Comparision of Bandwidth.

Power:-

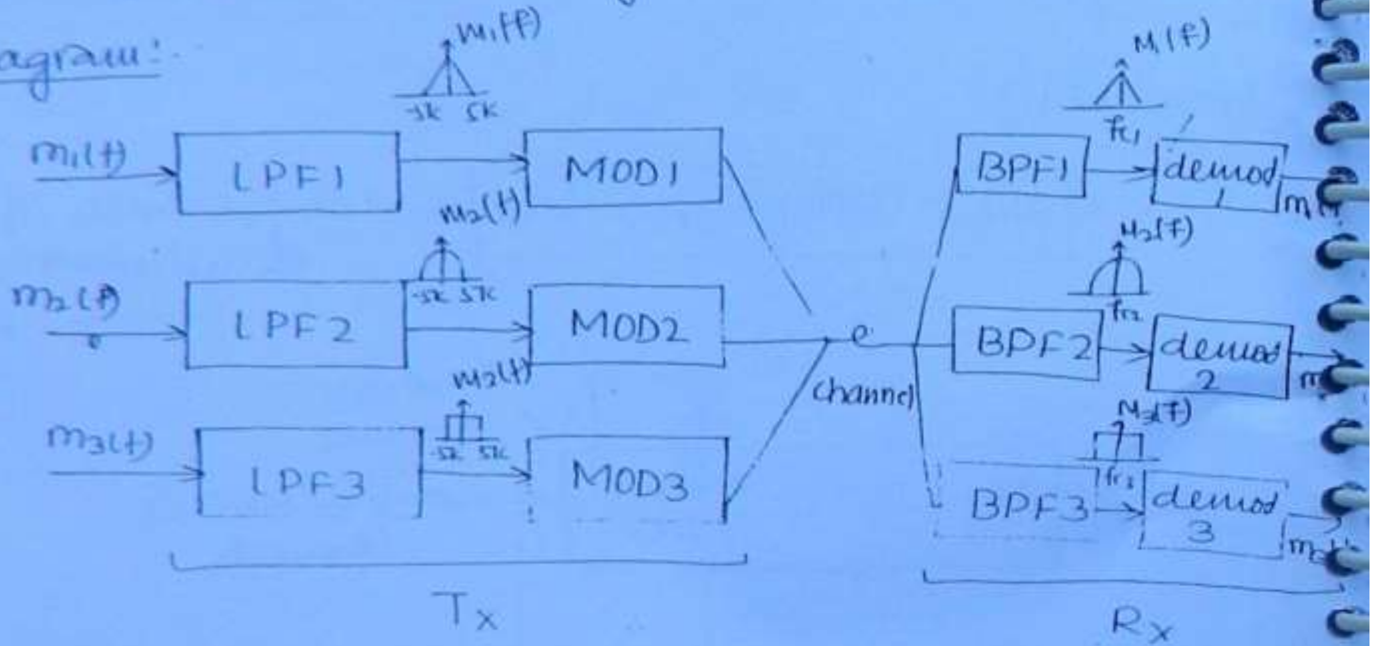
$\boxed{AM > DSB > VSB \ \& \ SSB}$ ← comparision of power

*FREQUENCY DIVISION MULTIPLEXING (FDM): (102)

→ used to multiplex continuous signal

*FDM is used for multiplexing continuous signals.

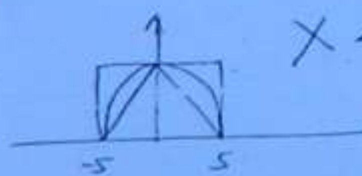
Block diagram:



*LPF are used to Band limit the signals

Case 1 (no modulators):-

∵ All the messages are occupying same Band of frequencies so, they all will get interefered with each other.

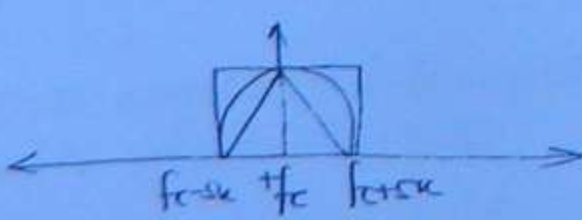


X ← Interference occurs.

Case 2 (Same Carrier frequency):-

let same carrier frequency, $f_c = A_c \cos 2\pi f_c t$

So,



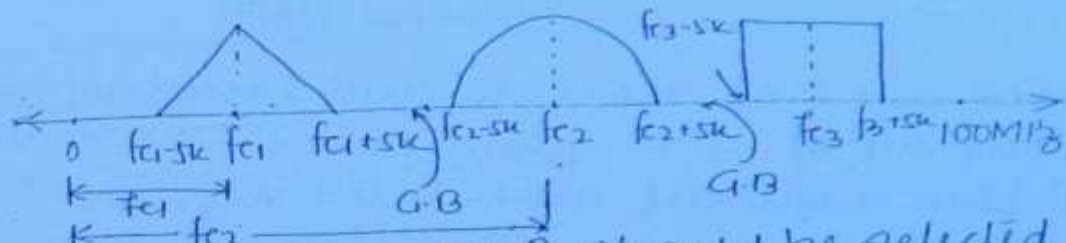
← Here also the signal occupy same Band of frequency Hence Interference is said to occur

Case 3 (diff. Carrier frequency) :-

let the channel B.W be 100MHz (0 to 100MHz)

1. The limitation is that the carrier frequency should be less than 100MHz

103

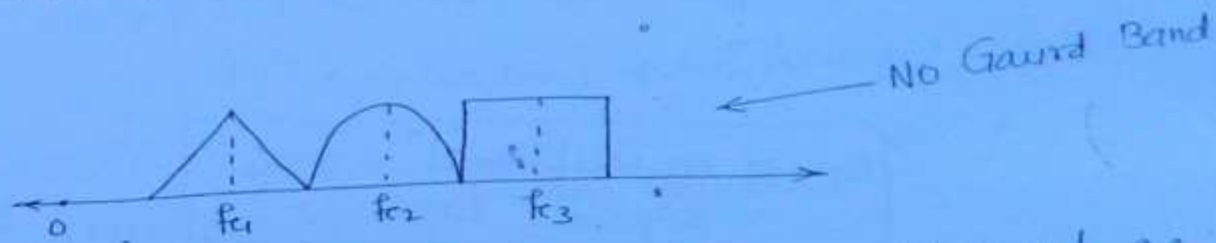


2. Also the carrier freq^n should be selected such that there exist some spacing b/w spectrums. This spacing is called as 'GAURD BAND'.

So, to avoid interference,

$$\boxed{\begin{array}{l} f_{c2} >> f_{c1}+10K \\ f_{c3} >> f_{c2}+10K \end{array}}$$

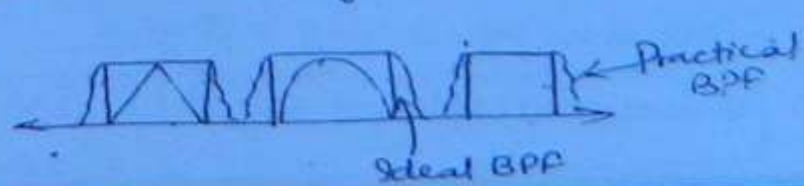
Case 4: $f_{c2} = f_{c1}+10K$; $f_{c3}+f_{c2}+10K$



→ B.W is efficiently used, but practically not used as no Gaurd Band exists. Hence in the Rx section the part of the other msg signal will be extracted

Note:-

1. When the modulated signal as in case 3 is given to the BPF (practical) the message signal can be extracted efficiently



2. whereas in case 4, when no G.B exists then.

Note :-

Since BPF are not ideal, to avoid undesired frequencies at the O/P of BPF; Gaurd Band has to be maintained b/w adjacent modulated signals.

Q1) Three msg signals each Bandlimited to 5KHz are multiplexed using FDM. Gaurd Band is 1 KHz. Find multiplexed signal BW, if the modulation schemes used are AM, DSB & SSB respt.

Soln: For 1st msg signal: 2nd msg signal 3rd

So, $\boxed{\text{B.W of multiplexed signal} = 27k}$

Q2. 10 msg signal, each Band limited to 10K are multiplexed using FDM. Gaurd Band is 0.5K. Find multiplexed signal B.W if modulation used is

a) AM b) DSB c) SSB

Soln: For AM

$BW = 2W = 20K$

So, B.W of 10 msg = 200K

So, $\boxed{\begin{aligned}\text{B.W of multiplexed signal} &= 200K + 9\times 0.5K \\ &= 204.5K \text{ (Same for DSB)}\end{aligned}}$

For SSB:

B.W = 10K

So, B.W of 10 msg = 100K (105)

So, $\boxed{\text{B.W of multiplexed signal} = 100 + 9 \times 0.5 = 104.5K}$ Ans

Previous exam Questions:

Q1. An AM signal is given by

$s(t) = \{1 + m(t)\} \cos \omega_c t$

where, $m(t) = \frac{1}{2} \cos \omega_{m_1} t + \frac{1}{2} \sin \omega_{m_2} t$

Find modulation efficiency?

Soln: Given,

$s(t) = \{1 + m(t)\} \cos \omega_c t$

$= \{1 + \frac{1}{2} \cos \omega_{m_1} t + \frac{1}{2} \sin \omega_{m_2} t\} \cos \omega_c t$

comparing with standard multitone AM:

$s(t) = A_c \{1 + \mu_1 \cos \omega_{m_1} t + \mu_2 \cos \omega_{m_2} t\} \cos \omega_c t$

for $A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t = m(t)$.

So, for if $m(t) = A_{m_1} \cos \omega_{m_1} t + A_{m_2} \sin \omega_{m_1} t$

then

$s(t) = A_c \{1 + \mu_1 \cos \omega_{m_1} t + \mu_2 \sin \omega_{m_2} t\}$

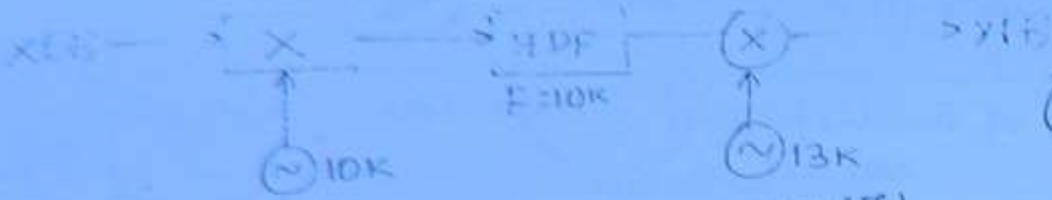
So, $\mu_1 = \frac{1}{2}$; $\mu_2 = 0.5$

$\mu_t = \sqrt{0.5^2 + 0.5^2} = \frac{1}{\sqrt{2}}$

So, $\eta = \frac{\mu_t^2}{2 + \mu_t^2} = \frac{\frac{1}{2}}{1 + \frac{1}{2}}$

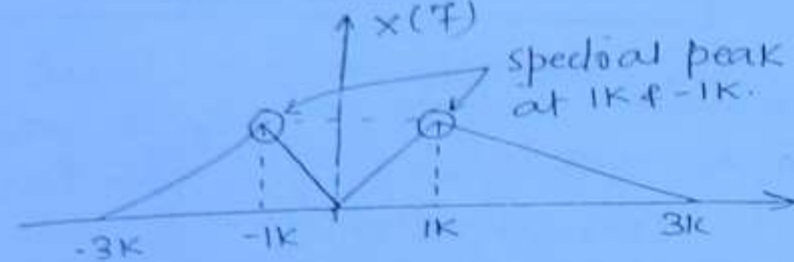
$\boxed{\eta = 20\%}$ Ans

Q) For the following ckt, find the frequencies for which spectral peaks will be observed in Y(f)

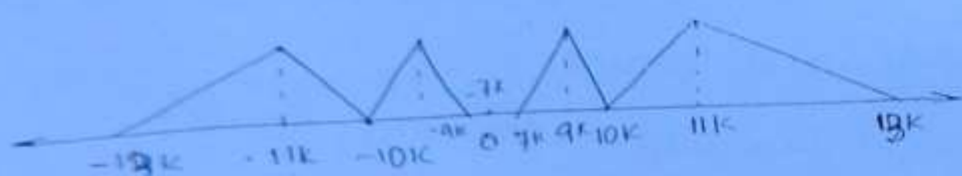


(1d)

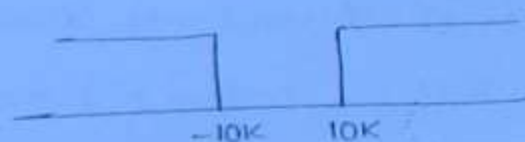
a) 2K, 24K
b) 1K, 22K
c) 1K, 2K, 24K
d) None



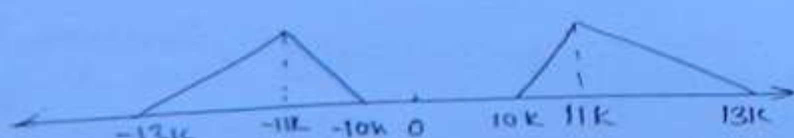
Soln: At O/P of multiplier 1st



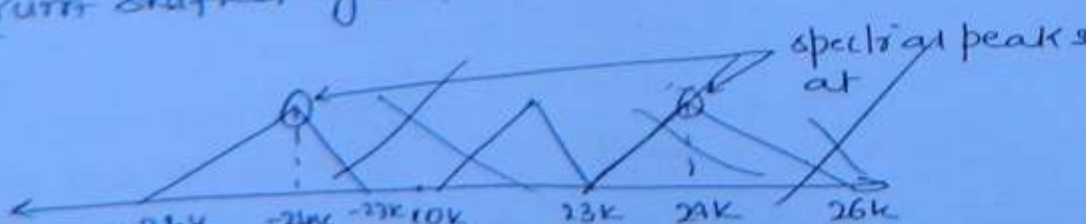
Spectrum of HPF:



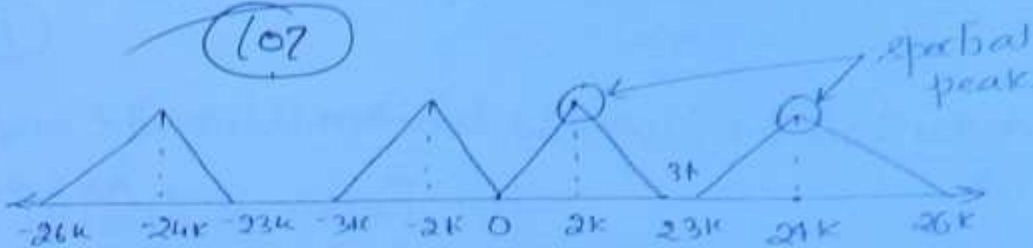
So. O/P of HPF



At O/P of multiplier 2nd:
spectrum shifted by 13K



Multiplier 2nd O/P → spectrum shifted by fc



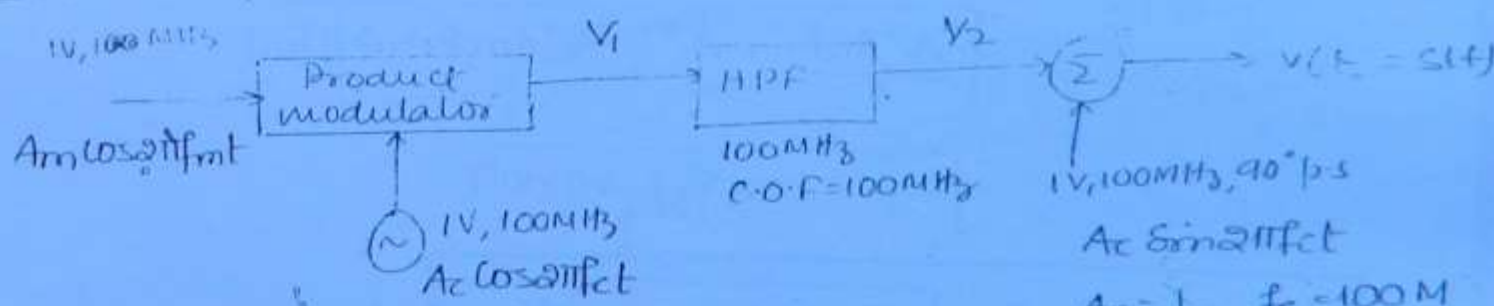
Q3. A sinusoidal carrier of 1V, 100MHz; is product modulated by a sinusoidal msg signal of 1V, 1MHz. The resulting signal is passed through HPF of fcutoff = 100MHz. Filter o/p is added with sinusoidal signal of 1V, 100MHz, 90° phase shift. Find the envelope of o/p?

a) $\sqrt{\frac{5}{4} - \sin 2\pi \times 10^6 t}$ b) $\sqrt{1+2\cos 2\pi \times 10^6 t}$

Soln:

c) $\sqrt{2 - \sin 2\pi \times 10^6 t}$ d) Const

Block diagram



$A_c \sin 2\pi f_c t$

$A_c = 1$, $f_c = 100M$

So, $A_m = 1$; $f_m = 1MHz$
$A_c = 1$; $f_c = 100MHz$

So, $V_1 = A_m \cos 2\pi f_m t \cdot A_c \cos 2\pi f_c t$

Blocked by HPF

$= \frac{A_c A_m}{2}\left[\cos 2\pi(f_c+f_m)t + \cos 2\pi(f_c-f_m)t\right]$

101M 99M

So, (HPF) o/p $V_2 = \frac{A_c A_m}{2} \cos 2\pi(f_c+f_m)t$

So, summer O/P $= s(t) = \frac{A_c A_m}{2} \cos 2\pi(f_c+f_m)t + \sin 2\pi f_c t$

Now as

$$A\cos 2\pi f_0 t + B\sin 2\pi f_0 t \xrightarrow{\text{envelope}} \sqrt{A^2+B^2}$$

(108)

Now

$$s(t) = \frac{A_c A_m}{2}\cos 2\pi f_c t \cos 2\pi f_m t - \frac{A_c A_m}{2}\sin 2\pi f_c t \sin 2\pi f_m t + A_c \sin 2\pi f_c t$$

$$= \underbrace{\frac{A_c A_m}{2}\cos 2\pi f_m t}_{A}\cos 2\pi f_c t + \underbrace{\left(A_c - \frac{A_c A_m}{2}\sin 2\pi f_m t\right)}_{B}\sin 2\pi$$

So, envelope $= \sqrt{A^2+B^2}$

$$= \sqrt{\frac{A_c^2 A_m^2}{4}\cos^2 2\pi f_m t + A_c^2 + \frac{A_c^2 A_m^2}{4}\sin^2 2\pi f_m t - 2\frac{A_c^2 A_m}{2}\sin 2\pi f_m t}$$

$$= \sqrt{\frac{A_c^2 A_m^2}{4} + A_c^2 - A_c^2 A_m \sin 2\pi f_m t}$$

$$= \sqrt{\frac{1}{4} + 1 - \sin 2\pi\times 10^6 t}$$

$$\boxed{\text{envelope} = \sqrt{\frac{5}{4} - \sin 2\pi\times 10^6 t}}$$ Ans

Q4. A non-linear device is characterised by

$$V_0 = aV_i + bV_i^3$$

where $V_i = m(t) + \cos 2\pi f_1 t$

by considering only DSB terms. find f_1 such that resulting DSB signal carrier frequency is 1MHz

Soln: As, $V_0 = a\{m(t) + \cos 2\pi f_1 t\} + b\{m(t) + \cos 2\pi f_1 t\}^3$

$$V_0 = a\,m(t) + a\cos 2\pi f_1 t + b\{m^3(t) + \cos^3 2\pi f_1 t + 3m^2(t)\cos 2\pi f_1 t + 3m(t)\cos^2 2\pi f_1 t\}$$

$(1+\cos 4\pi f_1 t/2\}$

So, by considering only USB terms we get

$$V_0 = b \cdot \frac{3m(t)}{2} \cos 4\pi f_1 t$$

(109)

$$V_0 = \frac{3b}{2} m(t) \cos 4\pi f_1 t$$

So, by standard DSB term

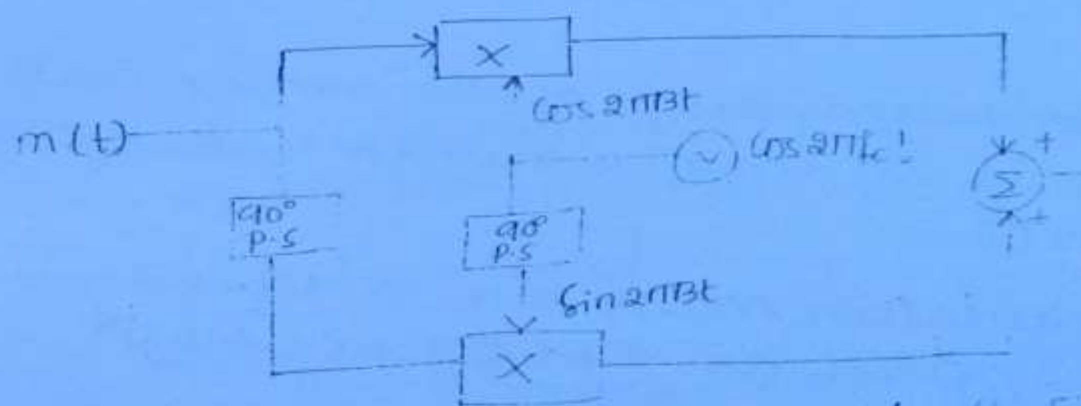
$$V_0 = A_c' m(t) \cos 2\pi f_c t$$

So, $f_c = 2f_1$

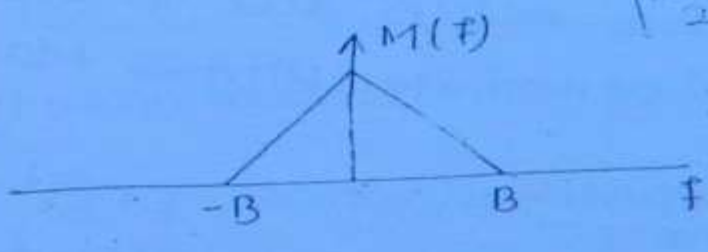
$f_1 = f_c/2$

$\boxed{f_1 = 0.5\ MHz}$ Ans.

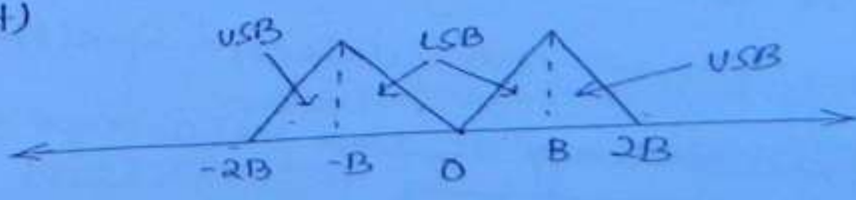
Q5



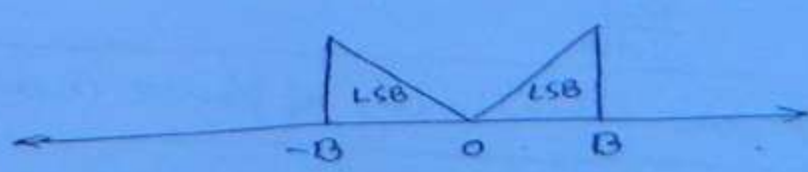
Spectrum of S(f) = ?



Sol^n: $S_{DSB}(t)$



So, spectrum of $s(t) \longleftrightarrow S(f)$



* ANGLE - MODULATION.

Classified Broadly as:

(110)

1) Frequency Modulation.

2) Phase modulation.

Now,

let,

carrier signal; $c(t) = A_c \cos\{\underbrace{2\pi f_c t}_{\text{radians}} + \underbrace{\phi}_{\text{radians}}\}$

Now, let

$2\pi f_c t + \phi = \theta(t)$

$A_c \cos\{\theta(t)\}$ total Angle

Note:-

1. In Angle modulation, angle of the carrier will be varied linearly in accordance with message signal voltage variations.

2. If Angle modulation, occurs due to dependence of f_c on, $m(t)$, then is called as FREQUENCY MOD^N

3. If Angle modulation occurs, due to dependence of ϕ on $m(t)$; then is called as PHASE MODULATION.

* PHASE MODULATION:

Carrier before phase modulation = $c(t) = A_c \cos\{2\pi f_c t\}$

carrier after phase modulation = $S_{PM}(t) = A_c \cos\{2\pi f_c t + \phi\}$

* ϕ is varied according to the message signal amp.

so, $\boxed{\phi = K_p m(t)}$

ϕ → rad; K_p → $\frac{\text{rad}}{\text{volts}}$; $m(t)$ → volts

$\boxed{K_p = \text{phase sensitivity of phase modulator (rad/volt)}}$

Note:

1. Kp, specifies the amount of phase change in the carrier for 1 volt change in the message signal

if $m(t) = 0 \Rightarrow$ no modulation $\Rightarrow \phi = 0$

* FREQUENCY MODULATION:

Assume,

frequency of carrier before modulation = f_c

frequency of carrier after frequency modulation = f_i

Frequency of frequency modulated signal = Instantaneous frequency

* f_i is varied in accordance to variations in $m(t)$

so, mathematically

$$\boxed{f_i = f_c + K_f\, m(t)}$$

where, K_f = frequency sensitivity of frequency modulation (Hz/volt)

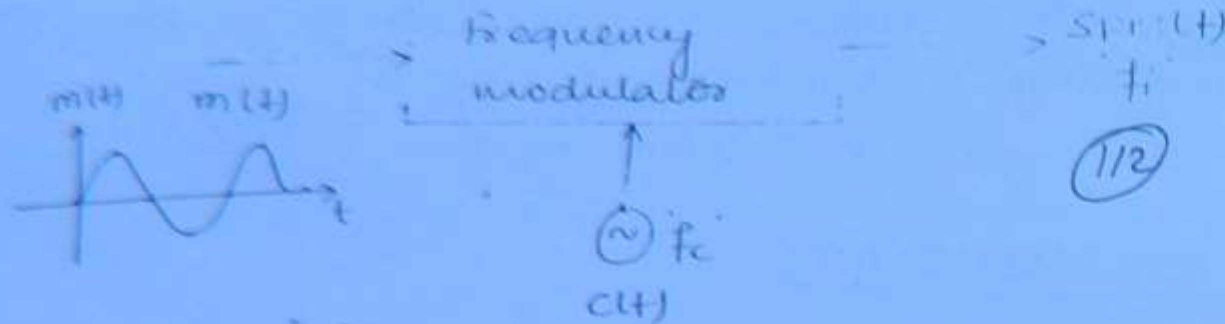
If $m(t) = 0 \Rightarrow$ no modulation $\Rightarrow f_i = f_c$

also, let, $K_f = 25$ KHz/volt ← hence for 1 volt change frequency changes by 25 KHz

Note:

K_f; specifies the amount of frequency change in the carrier for 1 volt change in the message signal.

$K_f = 25KHz/Volt$



Now,

as $f_i = f_c + K_f m(t)$

So,

i) $m(t) = 0 \Rightarrow f_i = f_c + 0$

2) $m(t) = +5V \Rightarrow f_i = f_c + 125K$

3) $m(t) = -5V \Rightarrow f_i = f_c - 125K$.

Conclusion :

when,

1) $m(t) = 0 \Rightarrow f_i = f_c$.

2) $m(t) = +ve \Rightarrow f_i > f_c$.

3) $m(t) = -ve \Rightarrow f_i < f_c$.

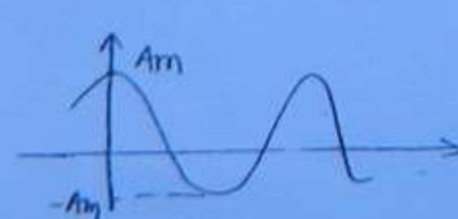
Note :

1. By changing the amp. of message signal (voltage), variations in frequency is obtained. Hence F.M is also called as "Voltage to Frequency conversion".

2. In FM, message signal voltage variation are converted as carrier signal frequency variation, so is also called as "VOLTAGE TO FREQUENCY CONVERSION"

*Analysis:

Assume, $m(t) = A_m \cos 2\pi f_m t$



$f_i = f_c + \boxed{K_f m(t)}$

frequency

So,

$\max^m$ frequency of FM signal $\Rightarrow$ $f_{max} = f_c + K_f A_m$

$\min^m$ frequency of FM signal $\Rightarrow$ $f_{min} = f_c - K_f A_m$

So, $\boxed{\max^m \text{ frequency deviation} = \text{Max}\{K_f m(t)\}}$ (113)

So, $\boxed{\Delta f = K_f A_m}$

So, putting value of Δf in above eq^n we get:

$$\boxed{\begin{aligned} f_{max} &= f_c + \Delta f \\ f_{min} &= f_c - \Delta f \end{aligned}}$$

So, $\boxed{\begin{aligned} \text{total frequency swing of FM signal} &= F_{max} - F_{min} \\ &= 2\Delta f \end{aligned}}$

Q1. An unmodulated carrier frequency is given by 1MHz. After frequency modulation, $\max^m$ frequency is given by 1.4 MHz. Find Δf and f_{min}?

Sol^n: Given:

$$f_{max} = 1.4\,MHz$$
$$= f_c + \Delta f$$
$$= 1 + \Delta f = 1.4\,MHz$$

$$\boxed{\Delta f = 0.4\,MHz} \text{ Ans}$$

And, $f_{min} = f_c - \Delta f$

$$= 1 - 0.4$$

$$\boxed{f_{min} = 0.6\,MHz} \text{ Ans}$$

Q2. For an FM signal fmax is given by 1.5 MHz. Total frequency swing is given by 900KHz. Find fc, Δf & fmin?

(114)

Solⁿ: Given:

$2\Delta f$ = Total frequency swing = 900KHz

$$\boxed{\Delta f = 450\ KHz}\ \text{Ans}$$

Now, $f_{max} = 1.5\ MHz$

$= f_c + \Delta f$

$f_c = f_{max} - \Delta f$

$= 1.5 \times 1000 - 450\ K$

$= 1050\ K$

$$\boxed{f_c = 1.05\ MHz}\ \text{Ans}$$

And, $f_{min} = f_c - \Delta f$

$= 1.05 \times 1000 - 450$

$$\boxed{f_{min} = 600\ KHz}\ \text{Ans}$$

Q3. A Sinusoidal carrier of 20V, 2MHz is frequency modulated by a mesg signal of $10 \sin 4\pi \times 10^3 t$. K_f is given by 50 KHz/volt. Find Δf; fmax & fmin?

Solⁿ: Given, $A_c \cos 2\pi f_c t = c(t)$

So, $A_c = 20V$; $f_c = 2\ MHz$

$A_m \sin 4\pi \times 10^3 t = m(t)$

$A_m = 10$; $f_m = 2\ KHz$

$K_f = 50\ KHz/volts$

So, $f_{max} = f_c + K_f m(t)$

$= 2000 + 50 \times 10$

$$\boxed{f_{max} = 2500\ KHz}\ \text{Ans}$$

$f_{min} = f_c - K_f m(t)$

$= 2000 - 50 \times 10$

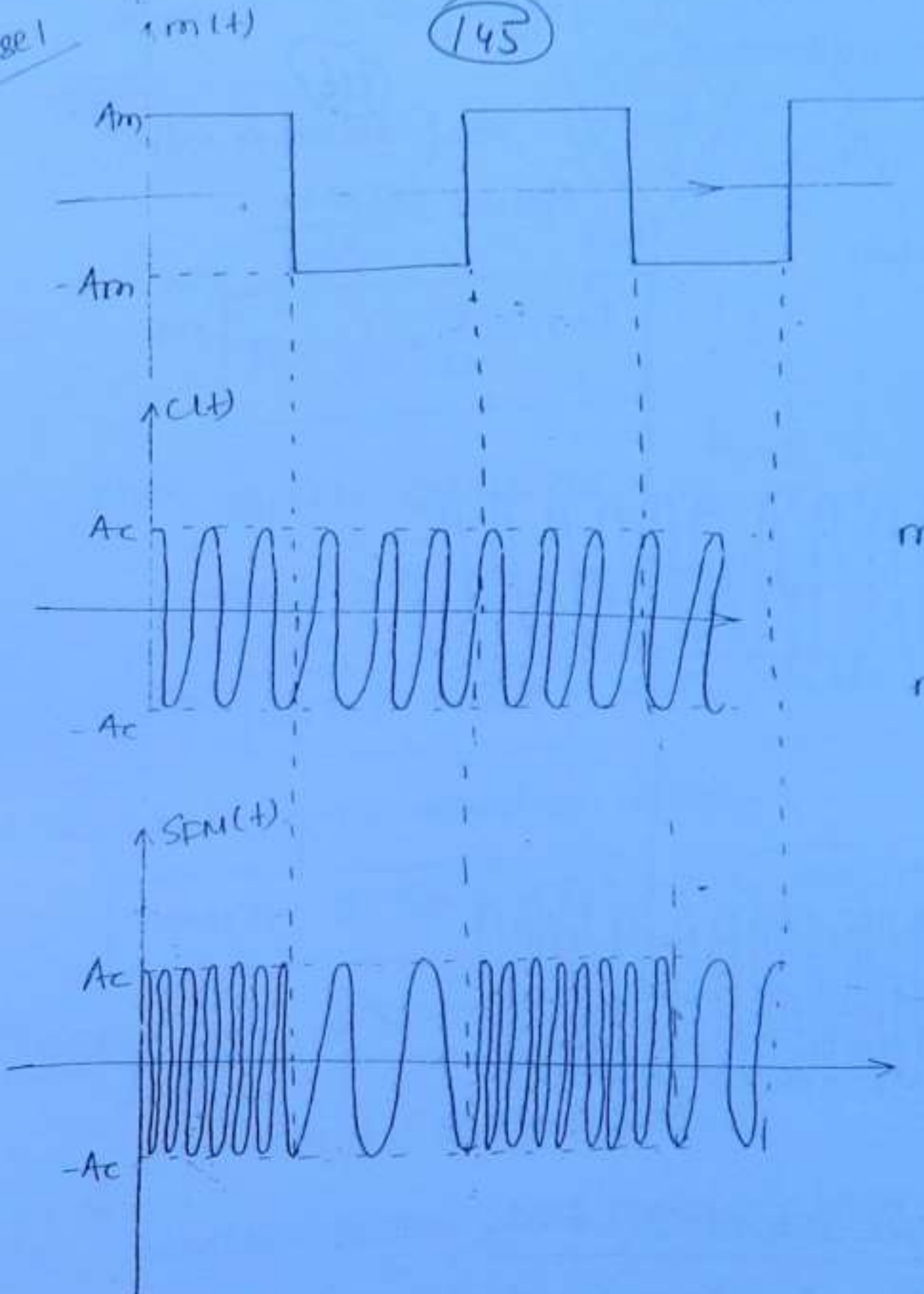
$$\boxed{f_{min} = 1500\ KHz}\ \text{Ans}$$

$\Delta f = f_{max} - f_{min}$

$$\boxed{\Delta f = \frac{1000\ KHz}{2}}$$

* Analysis:

Case 1



$f_i = f_c + K_f m(t)$

$m(t) = A_m \Rightarrow f_i = f_c + K_f A_m = f_1$

$f_1 > f_c$

$m(t) = -A_m \Rightarrow f_i = f_c - K_f A_m = f_2$

$f_2 < f_c$

As, $f_i = f_c + K_f m(t)$

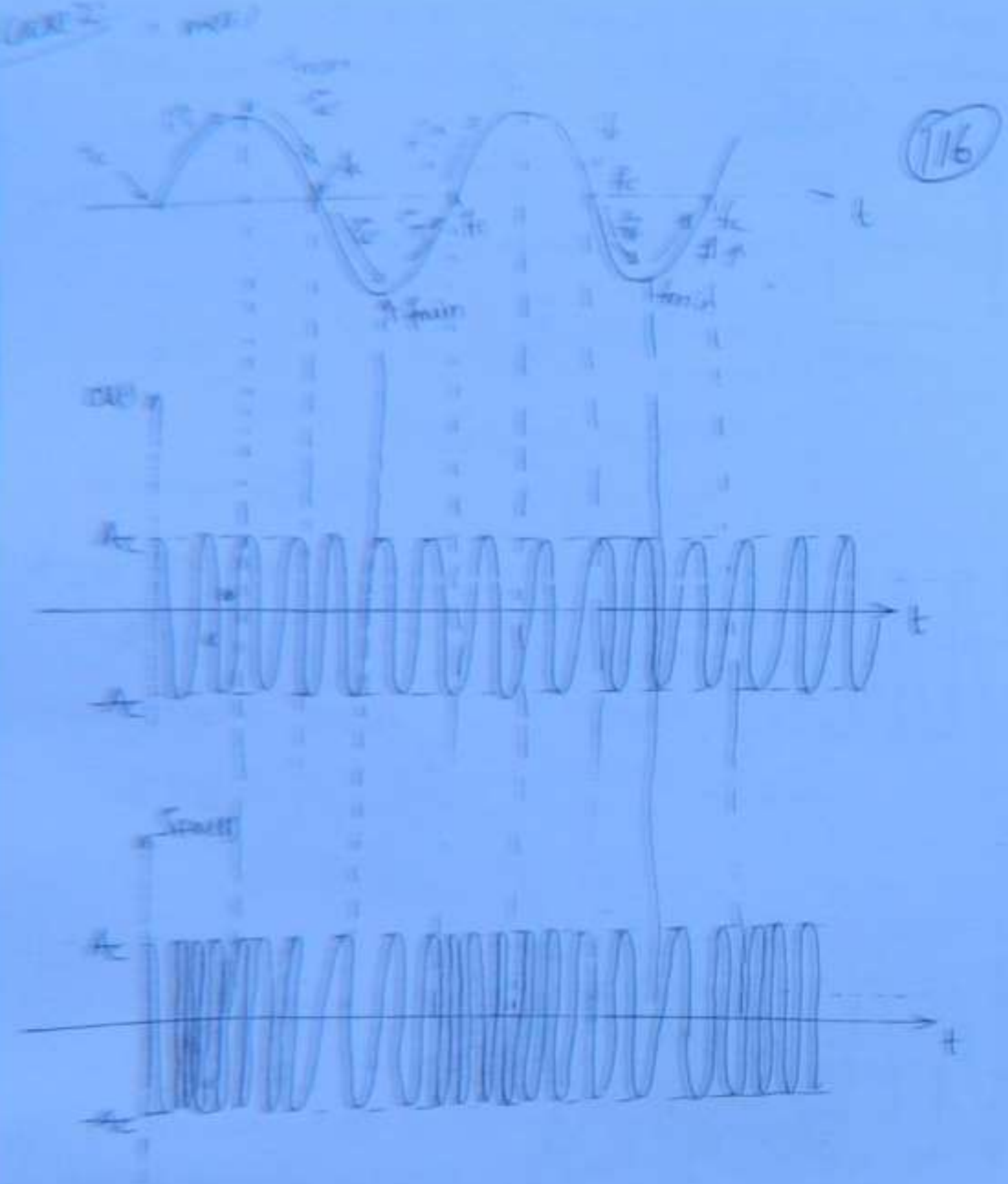
NOW,

$m(t) = A_m \Rightarrow f_i = f_c + K_f A_m = f_1$

$\boxed{f_1 > f_c}$

$m(t) = -A_m \Rightarrow f_i = f_c - K_f A_m = f_2.$

$\boxed{f_2 < f_c}$



GENERAL EXPRESSION OF FM:

let, Carrier Signal $= c(t) = A_c \cos\{2\pi f_c t + \phi\}$

$= A_c \cos\{\theta(t)\}$

where,

$\theta(t) = 2\pi f_c t + \phi$ ← no significance of ϕ.

To make ϕ insignificant;

$$\boxed{\frac{d\theta(t)}{dt} = 2\pi f_c}$$

before modulation

⇒ General expression of FM signal:

$$S_{FM}(t) = A_c \cos\{\theta_i(t)\} \quad (1)$$

← Instantaneous Angle

(117)

So, after modulation

$$\frac{d\,\theta_i(t)}{dt} = 2\pi f_i$$

***

So, $$\boxed{f_i = \frac{1}{2\pi} \times \frac{d}{dt}\theta_i(t)}$$

Now, $$\theta_i(t) = 2\pi \int f_i\,dt$$

$$\theta_i(t) = 2\pi \int \{f_c + K_f\, m(t)\}\,dt$$

So; $$\boxed{\theta_i(t) = 2\pi f_c t + 2\pi K_f \int m(t)\,dt}$$

Substituting in equation (1) we get:

$$\boxed{S_{FM}(t) = A_c \cos\left\{2\pi f_c t + 2\pi K_f \int m(t)\,dt\right\}}$$

⁎ SINGLE TONE F.M:-

Assume,

$$m(t) = A_m \cos 2\pi f_m t$$

So, $$S_{FM}(t) = A_c \cos\left\{2\pi f_c t + 2\pi K_f \int A_m \cos 2\pi f_m t\,dt\right\}$$

So, $$S_{FM}(t) = A_c \cos\left\{2\pi f_c t + 2\pi K_f \cdot A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right\}$$

***

Now, let $$\boxed{\frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m} = \beta}$$ ← modulation index of FM

Also,

$$\boxed{\text{modulation index} = \beta = \frac{\text{max}^m \text{ frequency deviation}}{\text{message signal freq}^n}}$$

So,
$$\boxed{S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}} \leftarrow \text{Single tone FM.}$$

* Depending upon the value of 'β', the FM is classified as:

118

FM

- NBFM ($\beta \leq 1$)
- WBFM ($\beta > 1$)

* Narrow Band FM (NBFM):-

The single tone FM is given as:

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

$$S_{FM}(t) = A_c\{\cos(2\pi f_c t)\cos(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) - \sin(2\pi f_c t)\sin(\underbrace{\beta \sin 2\pi f_m t}_{\theta})\}$$

≈ 0 ($\because \beta$ small)

Now, for NBFM ; $\boxed{\beta \leq 1}$ (Small value)

So, for small value of θ

$$\cos\theta \simeq 1$$
$$\sin\theta \simeq \theta$$

So,

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t \cdot 1 - A_c \sin 2\pi f_c t \times \beta \sin 2\pi f_m t$$

So,

$$S_{NBFM}(t) \cong A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \sin 2\pi f_m t$$

$$S_{NBFM}(t) = A_c \cos 2\pi f_c t - \frac{A_c \beta}{2}\{\cos 2\pi(f_c - f_m)t\} + \frac{A_c \beta}{2}\{\cos 2\pi(f_c + f_m)t\}$$

Now, Similarity b/w NBFM & AM: comparable ; both ≤1

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\cos 2\pi(f_c - f_m)t + \frac{A_c \mu}{2}\cos 2\pi(f_c + f_m)t$$

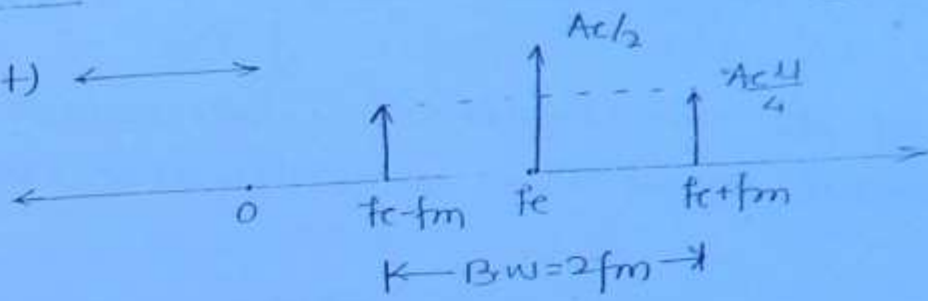
Note:

* General expression of AM & NBFM are same except 180° phase shift at LSB frequency component.

119

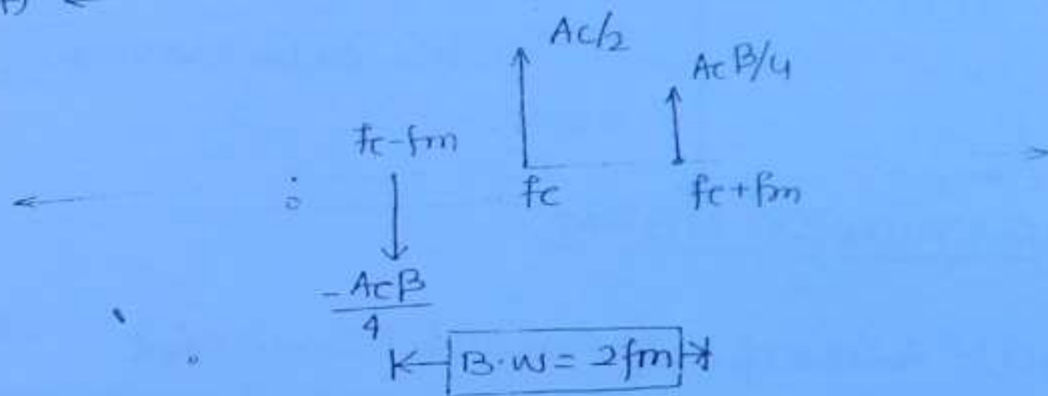
Spectrum of AM:

$S_{AM}(t) \longleftrightarrow$



Spectrum of NBFM:

$S_{NBFM}(t) \longleftrightarrow$



Note:

* Magnitude spectrum of AM & NBFM are same.

* **Power of NBFM:**

As, $P_t = P_c + P_{USB} + P_{LSB}$

where, $P_c = \frac{A_c^2}{2R}$; $P_{USB} = \left(\frac{A_c \beta}{2}\right)^2 / 2R = \frac{A_c^2 \beta^2}{8R} = P_{LSB}$

So, $P_t = \frac{A_c^2}{2R} + 2 \times \frac{A_c^2 \beta^2}{8R} = \frac{A_c^2}{2R} + \frac{A_c^2 \beta}{4R}$

$$P_t = \frac{A_c^2}{2R}\left\{1 + \frac{\beta^2}{2}\right\} \Rightarrow \boxed{P_t = P_c\left\{1 + \frac{\beta^2}{2}\right\}}$$

CONCLUSION:

* NBFM has much similarity with AM, hence practical significance of NBFM is negligible. (120)
* Bandwidth and power requirements of NBFM would be same as AM.
* Because of its much similarities with AM, NBFM is given least practical significance compared to WBFM.

Q1. AM and NBFM are having same modulation index were added. The resulting signal will be.

a) DSB

b) SSB

✓ c) SSB with carrier (USB with carrier).

d) None

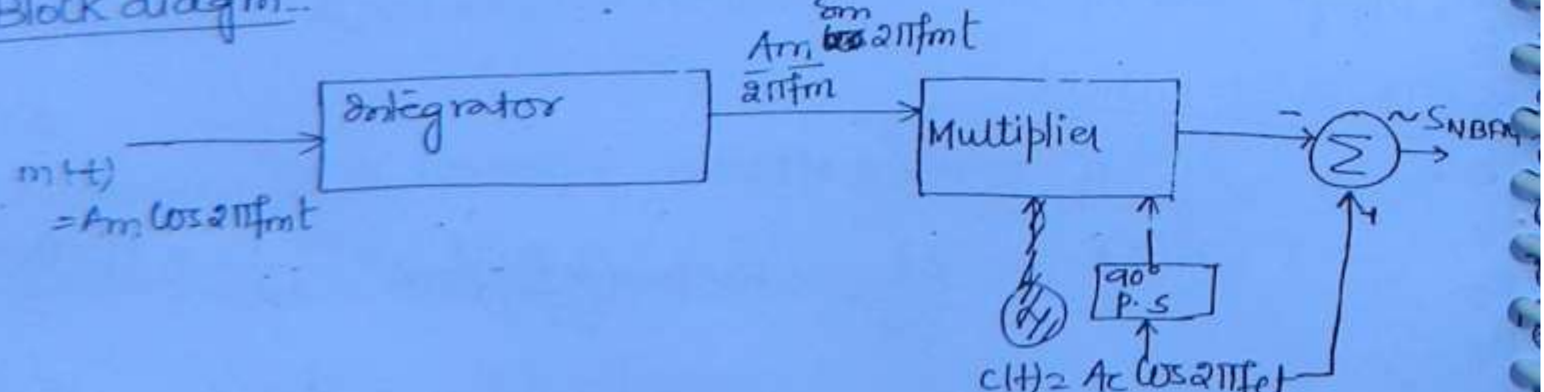
* GENERATION OF NBFM:

As

$$S_{NBFM}(t) \simeq A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

$$\simeq A_c \cos 2\pi f_c t - \frac{A_c K_f A_m}{f_m} \sin 2\pi f_c t \cdot \sin 2\pi f_m t$$

Block diagm:



* WIDE BAND FM (WBFM)

* BESSEL FUNCTION:

(21)

Standard defn is given as:

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i(x\sin\theta - n\theta)}\, d\theta \quad ; \theta = \text{dummy variable}$$

* Property of $J_n(x)$:-

i) $J_n(x)\downarrow$ decreases as $n\uparrow$ increases

So, $\boxed{J_0(x) > J_1(x) > J_2(x) > J_3(x) \ldots}$

ii) $J_n(-x) = (-1)^n J_n(x)$

So,
$$\boxed{\begin{aligned} J_{-n}(x) &= -J_n(x) \;;\; n = \text{odd} \\ &= J_n(x) \;;\; n = \text{even} \end{aligned}}$$

iii) $\boxed{\sum_{n=-\infty}^{\infty} J_n^2(x) = 1}$

iv) $J_n(x)$ is a Real quantity.

* General expression of WBFM:

General exp. is given as (of single tone):

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

Now, $\cos\theta = \text{Real}\{e^{j\theta}\}$

$$\text{So, } S_{FM}(t) = A_c \text{Real}\{e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\}$$

$$= A_c \text{Re}\{e^{j2\pi f_c t} \cdot \underbrace{e^{j\beta \sin 2\pi f_m t}}_{\text{Fourier Series}}\} \quad \text{---(1)}$$

Now, $e^{j\beta\sin 2\pi f_m t}$ is a continuous periodic signal with $\boxed{T = 1/f_m}$

Above signal is periodic. So, $x(t) = x(t+1)$ (22)

$$e^{j\beta \sin 2\pi f_m t} = e^{j\beta \sin 2\pi f_m (t + 1/f_m)} \quad \{\because T = 1/f_m\}$$

* Fourier series is used to find the frequency Analysis of continuous periodic signal. So, the exp Fourier series is given as:-

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad ; \omega_0 = \frac{2\pi}{T}$$

where,

$$C_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t} dt$$

So,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn 2\pi f_m t} \quad \text{---- (2)}$$

Now,

$$C_n = \frac{1}{(1/f_m)} \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin 2\pi f_m t} \cdot e^{-jn 2\pi f_m t} dt$$

$$= f_m \int_{-1/2f_m}^{+1/2f_m} e^{j(\beta \sin 2\pi f_m t - n 2\pi f_m t)} dt$$

Now, as

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(x \sin\theta - n\theta)} dt$$

So, $\theta = 2\pi f_m t$

$2\pi f_m dt = d\theta$.

Now, when $t = -1/2f_m \Rightarrow \theta = -2\pi$.

$t = 1/2f_m \Rightarrow \theta = +\pi$.

So,

$$C_n = f_m \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \frac{d\theta}{2\pi f_m}$$ (123)

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} d\theta = J_n(\beta) \quad \{\because x = \beta\}.$$

So,

$$\boxed{C_n = J_n(\beta).}$$

putting this value in eq^n (2) we get:

$$\boxed{e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot e^{jn2\pi f_m t}}$$

substituting this value in eq (1) we get:

$$S_{FM}(t) = S_{WBFM}(t) = A_c Re\left\{ e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t} \right\}.$$

$$S_{WBFM}(t) = A_c Re\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi(f_c + nf_m)t} \right\}.$$

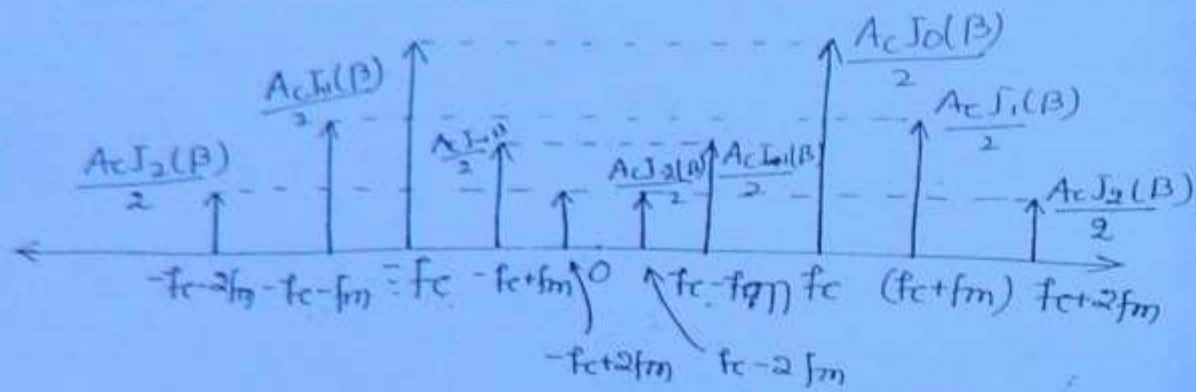
So, the General exp. of WBFM is given as:-

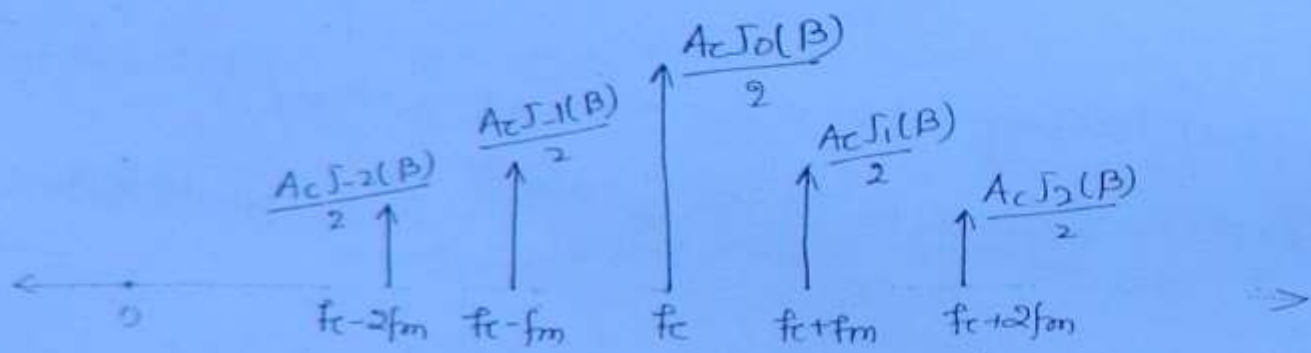
***
$$\boxed{S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + nf_m)t}$$

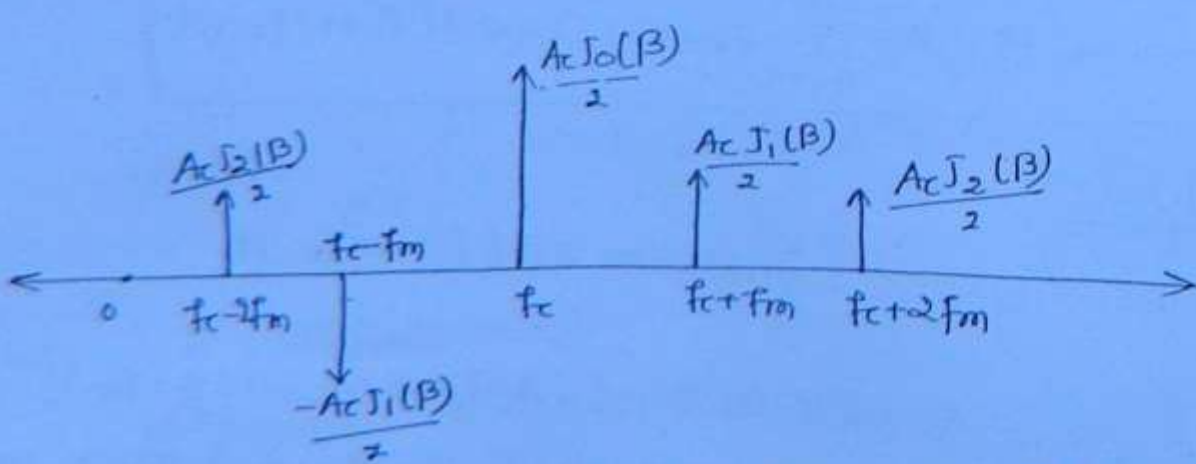
Analysis:-

$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi(f_c + nf_m)t$$

$$= A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos 2\pi(f_c + f_m)t + A_c J_{-1}(\beta) \cos 2\pi(f_c - f_m)t + A_c J_2(\beta) \cos 2\pi(f_c + 2f_m)t + A_c J_{-2}(\beta) \cos 2\pi(f_c - 2f_m)t + \cdots$$







$J_0(x) > J_1(x) > J_2(x) > J_3(x) \ldots$

Conclusion:-

1. WBFM consists of carrier frequency component, ∞ no of USB's and ∞ no. of LSB's. (125)

2. The Actual Bandwidth of WBFM is ∞.

3. For WBFM, strength of higher order side Bands go on decreasing and finally becomes zero.

4. For WBFM, lower order sideBands are said to be significant sidebands and higher order SideBands are insignificant.

* Power of WBFM:-

As,

$$P_t = P_c + (P_{USB1} + P_{USB2} + \cdots) + (P_{LSB1} + P_{LSB2} + \cdots)$$

*** where,

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R} \; : \; P_{USB1} = \frac{A_c^2 J_1^2(\beta)}{2R} \; ; \; P_{LSB1} = \frac{A_c^2 J_{-1}^2(\beta)}{2R}$$

$$P_{USB2} = \frac{A_c^2 J_2^2(\beta)}{2R} \; ; \; P_{LSB2} = \frac{A_c^2 J_{-2}^2(\beta)}{2R}$$

So,

$$P_t = \cdots + \frac{A_c^2 J_{-2}^2(\beta)}{2R} + \frac{A_c^2 J_{-1}^2(\beta)}{2R} + \frac{A_c^2 J_0^2(\beta)}{2R} + \frac{A_c^2 J_1^2(\beta)}{2R} + \frac{A_c^2 J_2^2(\beta)}{2R} + \cdots$$

So,

$$P_t = \frac{A_c^2}{2R}\left\{\cdots + J_{-2}^2(\beta) + J_{-1}^2(\beta) + J_0^2(\beta) + J_1^2(\beta) + J_2^2(\beta) + \cdots\right\}$$

$$P_t = \frac{A_c^2}{2R}\left\{\sum_{n=-\infty}^{\infty} J_n^2(\beta)\right\}.$$

**

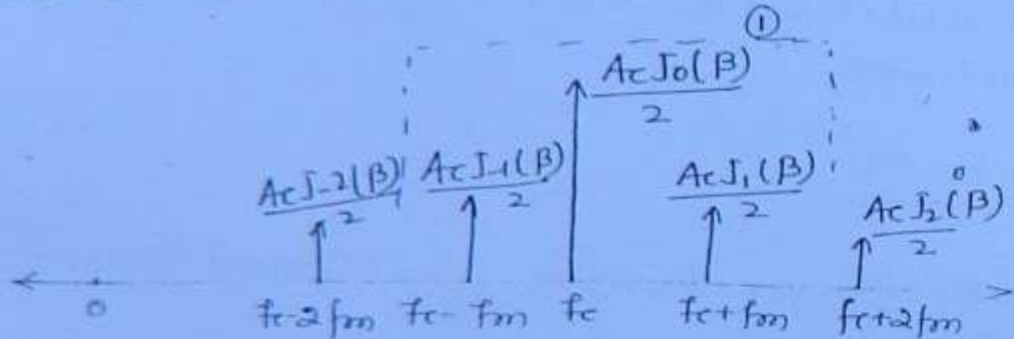
$$\boxed{P_t = \frac{A_c^2}{2R}} \quad \left[\because \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1\right]$$

Note:

1. For WBFM, the power of carrier before modulation same as after modulation. (126)

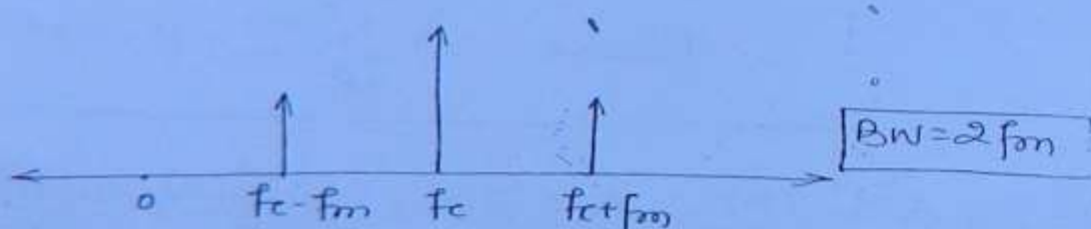
*** Practical Bandwidth of WBFM: {CARSON's RULE}**

* The Actual B.W of WBFM is '∞'. For transmission of signal, it should be band limited by retaining only significant sidebands and eliminating insignificant side bands



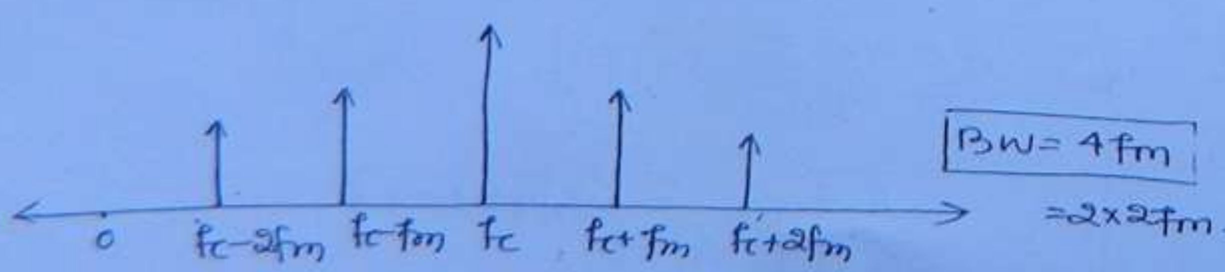
Case 1 (WBFM consist of significant SB's upto 1st order):-

After passing through Band limiting we get



Case 2 (WBFM consist of significant SB's upto 2nd order):-

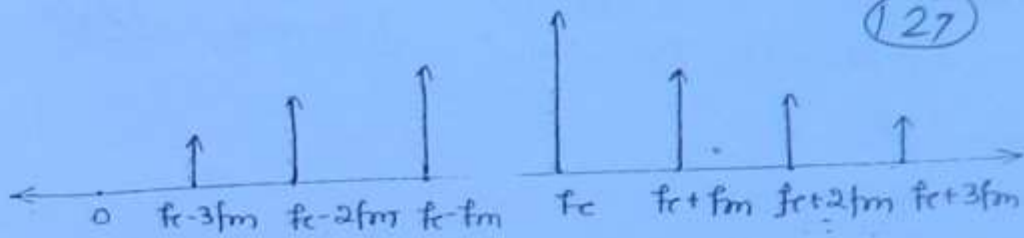
After Band limiting the signal we get:-



Case 3 : upto 3rd order

After passing through 1 and limiting filter we get

(127)



$$\boxed{\begin{aligned} B.W &= 6f_m \\ &= 3\times 2f_m \end{aligned}}$$

CARSON'S RULE:

According to carson, WBFM consist of the significant sideband upto order "$\beta+1$" ; when the modulation index is β.

So,

$$\boxed{B.W = (\beta+1)\times 2f_m}$$

Also

$$B.W = (\beta+1)\times 2f_m$$

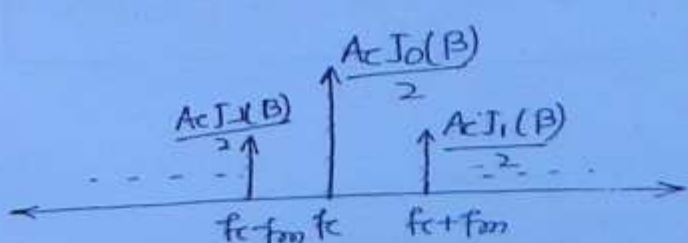
$$= \left(\frac{\Delta f}{f_m}+1\right)\times 2f_m$$

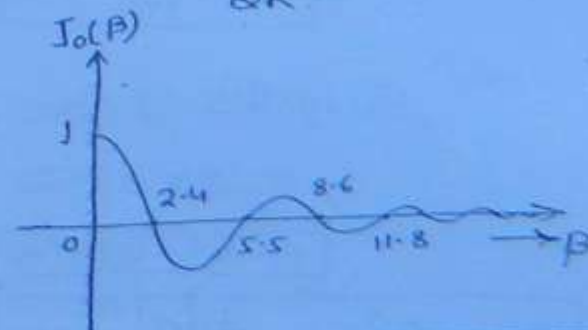
$$\boxed{B.W = 2(\Delta f + f_m)}$$

* MODULATION EFFICIENCY (η):

NOW,

$$P_c = \frac{A_c^2 J_0^2(\beta)}{2R}$$





Standard values. $\rightarrow$ $\boxed{J_0(\beta) = 0;\ \beta = 2.4, 5.5, 8.6, 11.8, \dots}$

for $\beta = 2.4, 5.5, 8.6, 11.8$, hence $P_c = 0$, are called as eigen values of β.

NOTE

* For above values of β, power taken by carrier freqⁿ component will be zero so, that modulation η will become 100%.

$$J_n(\beta) = 0 ;\ \beta = 2.4; 5.5; 8.6; 11.8;$$

128

$$P_c = 0$$

$$\eta = 100\%$$

Q1. A Sinusoidal carrier of 20V, 2MHz is frequency modulated by a sinusoidal msg signal of 10V, 50KHz.

$K_f = \frac{25KHz}{volts}$.

a) find Δf; β; Bandwidth & power.

b) repeat above if msg signal amplitude is doubled.

Solⁿ: Given:

$$A_c = 20V ;\ f_c = 2000\ KHz$$

$$A_m = 10V;\ f_m = 50\ KHz$$

$$K_f = \frac{25KHz}{volts}$$

So, $f_1 = \{f_c + K_f m(t)\}$; $f_2 = f_c + K_f m(t)$

$= (2000 + 25\times 10)$; $= 2000 - 25\times 10$

$= 2250K$; $= 1750\ KHz$

NOW, $\boxed{\beta = \frac{\Delta f}{f_m} = \frac{K_f . A_m}{f_m} = \frac{25\times 10}{50K} = 5\ (WBFM)}$

So, $B.W = 2(\beta+1) f_m$

$= 2\times 6\times 50$

$\boxed{B.W = 600K}$

$\boxed{P_t = \frac{A_c^2}{2R} = \frac{400}{2} = 200W}$

2. $A_m = 2\times10 = 20V$

So, $\uparrow \Delta f = K_f A_m \uparrow$

A_m is doubled Δf is doubled.

$$\boxed{\Delta f = 25K \times 20 = 500K}$$

$$\uparrow \beta = \frac{K_f A_m \uparrow}{f_m}$$

$$\boxed{\beta = \frac{25K\times20}{50} = 10}$$

* $B\cdot W = (\beta+1)\times 2f_m$

$= 11\times2\times50$

$$\boxed{B\cdot W = 1100K}$$

* $$\boxed{P_t = \frac{A_c^2}{2R} = 200W}$$

Conclusion:

* when mesg signal amp. is doubled, both Δf and β will also be doubled.

Q2. An FM signal is given by:

$$S(t) = 10\cos\{2\pi\times10^6 t + 8\sin 4\pi\times10^3 t\}$$

i) Find β; Δf; B.W & Power.

ii) Repeat above if mesg. signal frequency is doubled.

Soln: Given, $S(t) = 10\cos\{2\pi\times10^6 t + 8\sin 4\pi\times10^3 t\}$

comparing with:

$$S(t) = A_c\cos\{2\pi f_c t + \beta\sin 2\pi f_m t\}.$$

So, $A_c = 10V$; $f_c = 1000K$; $\boxed{\beta = 8}$ $f_m = 2K$.

Now, $\boxed{\Delta f = \beta\times f_m = 16K}$

$* \; B.W = 2(\beta+1) f_m$

$= 2 \times 9 \times 2K$

$\boxed{B.W = 36K}$

130

$* \; \boxed{P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50W}$

2. Frequency of msg signal doubled so, new f_m

$f_m = 4KHz$

Now,

$\Delta f = K_f \cdot A_m$

$= 16 KHz$ (no change)

Also,

$\Delta f = \beta \downarrow 2 \times f_m \uparrow 2$

Now, $\beta \downarrow 2 = \frac{K_f A_m}{f_m \uparrow 2} = 4$ (halfed)

So,

$* \; B.W = 2(\beta+1) f_m$

$= 2 \times 5 \times 4K$

$\boxed{B.W = 40K}$

$* \; \boxed{P_t = 50W}$

Conclusion:

when message signal frequency doubles; "Δf" will not be changed and "β" will halfed.

Q3. A carrier is frequency modulated with max^m frequency deviation of 16 KHz. Msg signal freq^n is given by 4KHz.

a) Find β & B.W?

b) Repeat above if message signal Amp. is doubled and frequency is reduced to 1KHz?

Sol^n:- Given:-

$\Delta f = 16 KHz$

$f_m = 4KHz$

Now,
$$\Delta f = \beta f_m \qquad ; \quad \Delta f = \frac{K_f A_m}{f_m}$$
$$\boxed{\beta = 4}$$

(T3)

Now, $B.W = 2(\beta+1) f_m$
$= 2 \times 5 \times 4$
$$\boxed{BW = 40K}$$

b) Given:
$\Delta f = 16 KHz$ (previous case)
$f_m = 1 KHz$.

So, $2\uparrow \Delta f = \frac{K_f A_m \uparrow 2}{f_m}$

So, $\boxed{\Delta f = 32K.}$

Now, $\boxed{\beta = \frac{\Delta f}{f_m} = \frac{32}{1} = 32}$

So, $B.W = 2(\beta+1) f_m$
$= 2 \times 33 \times f_m$
$$\boxed{B.W = 66K}$$

Q4. A carrier is frequency modulated with max^m frequency deviation of 100K. Find β & B.W if msg signal frequency is a) 10 K b) 1 MHz

Soln: Given:
a) $\Delta f = 100K$
$f_m = 10K$

Now, $\Delta f = \beta f_m$
$\beta = \frac{\Delta f}{f_m} = 10$ (WBFM)

So, $B.W = 2 \times (\beta + 1) f_m$
$= 2 \times 11 \times 10K$
$$\boxed{BW = 220K}$$

b) $\Delta f = 100K$ {NBFM}
$f_m = 1000K$
$\Delta f = \beta f_m$
$\beta = \frac{\Delta f}{f_m} = \frac{100}{1000}$
$$\boxed{\beta = 0.1}$$

$B.W = 2 f_m$
$= 2 \times 1 M$
$$\boxed{B.W = 2M}$$

Q3. Given:

$c(t) = 5\cos 2\pi \times 10^6 t$.

$m(t) = \cos 4\pi \times 10^3 t$

(132)

a) $c(t)$ & $m(t)$ are used to generate AM with $\mu = 0.707$. Find B.W & power?

b) $c(t)$ & $m(t)$ are used to generate FM; with max^m freq^n deviation is 3 times B.W of AM. Find the coeff of $\cos 2\pi \times (1016 \times 10^3)t$ in F.M expression.

a) $5J_6(8)$ c) $5J_3(6)$.

b) $5J_8(6)$ d) $5J_3(8)$.

Sol^n: Given,

$m(t) = \cos 4\pi \times 10^3 t$

$A_m = 1$; $f_m = 2\,KHz$

$c(t) = 5\cos 2\pi \times 10^6 t$

$A_c = 5$; $f_c = 1000K$.

Now,

$B.W = 2f_m = 2 \times 2K$

$$\boxed{B.W = 4K}$$

$$Power = P_t = P_c\left\{1 + \frac{\mu^2}{2}\right\}$$

$$P_c = \frac{A_c^2}{2R} = \frac{25}{2}$$

$$P_t = \frac{25}{2}\left\{1 + \frac{1}{4}\right\} = \frac{25}{2} \times \frac{5}{4}$$

$$\boxed{P_t = \frac{125}{8}\,W}$$

b) $\Delta f = 3 \times B.W$ of AM

$= 3 \times 4K$

$$\boxed{\Delta f = 12K}$$

Now, $\beta = \frac{\Delta f}{f_m}$ (133)

$\boxed{\beta = \frac{12}{2} = 6}$ (WBFM) $\beta = 3 \times 2 f_m$

Now,

$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} (J_n(\beta)\cdot)$

$= A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + n f_m).$

Now, $\cos 2\pi (2016 \times 10^3 t)$.

So, $f_c + n f_m = 1016$

$n f_m = 1016 - 1000$

$n = \frac{1016 - 1000}{2}$

$\boxed{n = 8}$ Ans

So $\boxed{\text{Coeff.} \Rightarrow A_c J_n(\beta) = 5 J_8(6)}$

Q6 A Sinusoidal carrier of frequency f_c is used for both AM & FM xmitter; msg signal freq^n is given by 5KHz. Max freq^n deviation = 2 × B.W of AM. Find mod^n index of both AM & FM such that strength of freq^n component $f_c + 5K$ is same in both AM & FM spectrum.

$J_1(0) = 1$; $J_1(2) = 0.57$; $J_1(4) = 0.37$; $J_1(8) = 0.08$

Sol^n:- B.W of AM = $2 f_m$; $\mu = ?$

$= 10 \text{KHz}$; $\beta = ?$

$\Delta f = 2 \times 10K = 20K.$; $\frac{\Delta f}{f_m} = \boxed{\beta \Rightarrow \beta = \frac{20}{5} = 4}$

Now, $\frac{A_c \mu}{2} = \frac{A_c J_1(\beta)}{2} \Rightarrow \mu = J_1(4)$

Strength of $f_c + 5K$ is same in both AM & FM

$\boxed{\mu = 0.37 \times 2} \Rightarrow \boxed{\mu = 0.74}$

Q1 A Sinusoidal carrier is frequency modulated by a sinusoidal msg signal of frequency m and amp Am. Conducting and experiment with $f_m = 1\,KHz$ and increasing Am from 0V, it is observed that the strength of the carrier frequency component in the spectrum becomes 0 for the 1st time with $A_m = 2V$

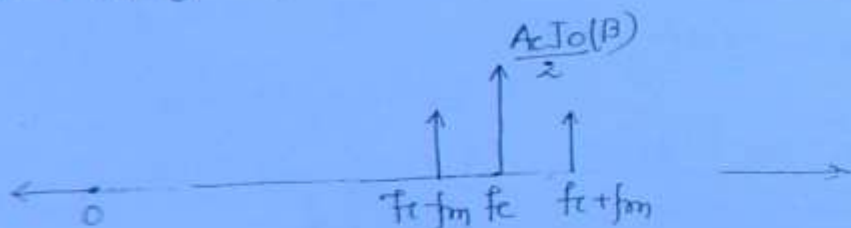
(134)

Find i) K_f

ii) Find msg signal amplitude for which strength of carrier frequency component becomes zero for the 2nd time?

Soln: Strength of frequency component $= \frac{A_c J_0(\beta)}{2}$

$f_m = 1\,KHz$; $A_m = \overset{2V}{0V} \uparrow$



NOW. $J_0(\beta) = 0$; $\beta = 2.4, 5.5, 8.6; 11.8$

NOW. $\beta_1 = \frac{K_f A_m}{f_m}$

NOW; $2.4 = \frac{K_f \times 2}{1K}$

$$\boxed{K_f = 1.2\,KHz/Volt}$$ Ans

NOW, $\beta_2 = \frac{K_f A_m}{f_m}$

$5.5 = \frac{1.2 \times 10^3 \times A_m}{1 \times 10^3}$

$$\boxed{A_m = 4.58V}$$ Ans

Q8 An unmodulated FM xter power is given by 100W; with modulation; it is observed that at ngth of 1st order side Band is 0. Find

i) the power of carrier frequency component

ii) Total ~~second~~ side Band power (135)

iii) total Second order side Band power.

$J_0(2.4) = 0$; $J_1(2.4) = 0.3$; $J_2(2.4) = -0.28$

$J_0(3.8) = 0.4$; $J_1(3.8) = 0$; $J_2(3.8) = 0.2$

$J_0(5.1) = -0.36$; $J_1(5.1) = -0.23$; $J_2(5.1) = 0$

Soln: Given:- $P_c = 100W - A_c^2/2R = A_c = \sqrt{200}$

1st order SideBand $= \frac{A_c J_1(\beta)}{2} = 0.$

$J_1(\beta) = 0 = J_1(3.8)$

$$\boxed{\beta = 3.8}$$

Now, $\frac{A_c J_0(\beta)}{2}$ = Carrier frequency component

$$\text{Power} = \frac{A_c^2 J_0^2(\beta)}{2}$$

$$= \frac{200 \times J_0^2(3.8)}{2} = \frac{\cancel{200}^{100} \times 0.16}{\cancel{2}}$$

$\boxed{\text{Power} = 16\ W}$ Ans.

Now,

ii) Total power $= \frac{A_c^2}{2R} = 100W = P_t$

So, $\boxed{\text{Total side Band power} = 100 - 16 = 84W}$ Ans

iii) second order power $= \frac{A_c^2 J_2^2(\beta)}{2R} + \frac{A_c^2 J_{-2}^2(\beta)}{2R}$

$$= \frac{A_c^2}{2}\{J_2^2(\beta) + J_{-2}^2(\beta)\}$$

$$= A_c^2\{J_2^2(\beta)\} = 200 \times J_2^2(3.8)$$

$P_{2nd\,order} = 2000 \times \frac{0.04}{100}$

$\boxed{P_{2nd\,order} = 8W}$ Ans

* GENERATION OF FM:

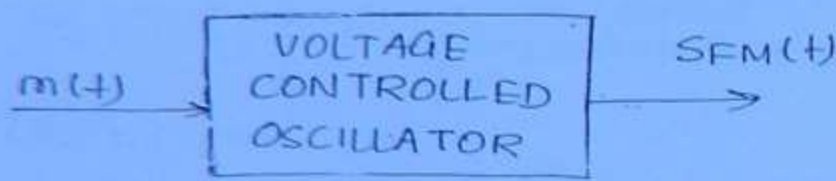
Generation of FM done by:-

1) Direct Method.

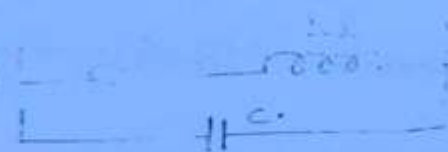
***2) Indirect Method or Amstrong Method.

* DIRECT METHOD:

FM ⟷ VCO



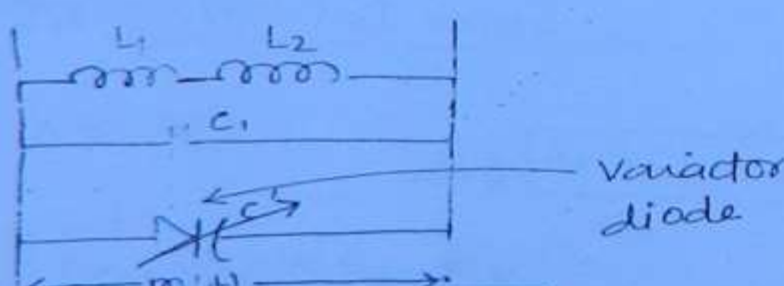
Hartley osc



$\Rightarrow \boxed{\frac{1}{2\pi\sqrt{(L_1+L_2)\cdot C}}}$

tank ckt of VCO :-

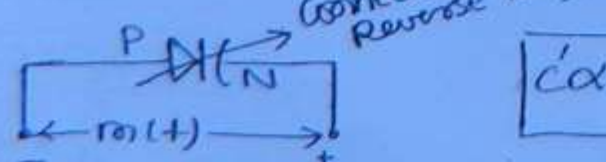
VCO



$$\boxed{f_i = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_1+C')}}}$$

As, $f_i = f_c + K_f m(t)$

case 1 :- $m(t) = +ve$

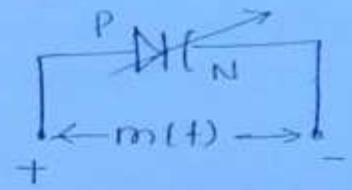


$\boxed{C' \alpha 1/W}$

when connected in the Reverse mode

hence $\boxed{\omega \uparrow \rightarrow C \downarrow \longrightarrow f_i \uparrow}$

Case 2 $m(t) = -ve$



when connected in the Forward mode

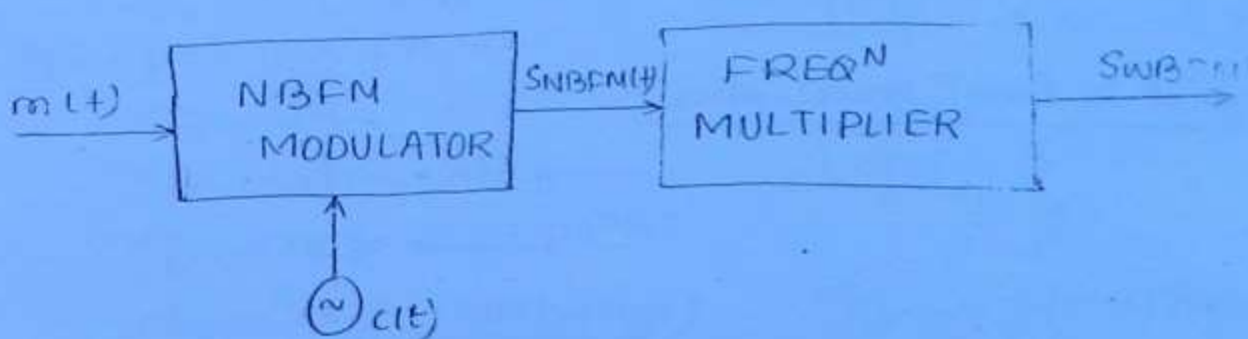
$\boxed{\omega \downarrow \rightarrow C \uparrow \longrightarrow f_i \downarrow}$

Hence, the frequency is varied in accordance to the msg signal voltage variations.

* INDIRECT METHOD (ARMSTRONG METHOD):-

* In this method WBFM is generated from NBFM.

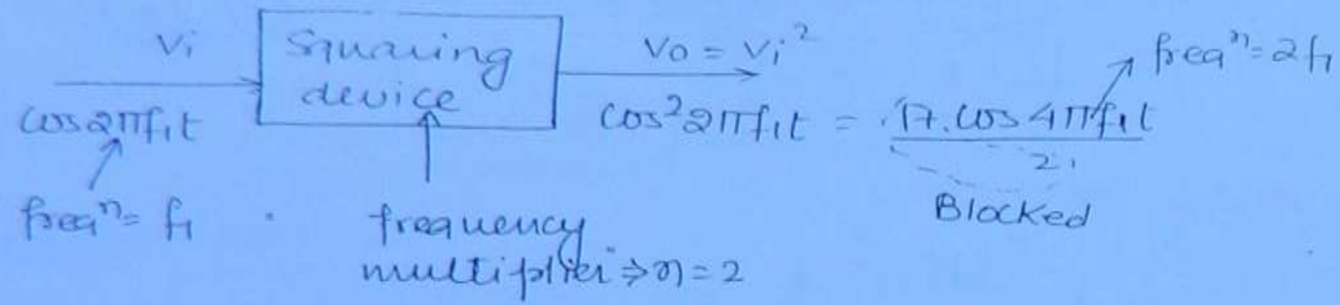
Block diagram:



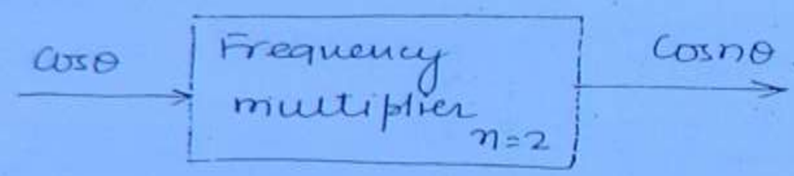
* Frequency multiplier is nothing but square law device followed by proper Band pass filter

$V_i \rightarrow \boxed{SLD} \rightarrow V_2 = a_1V_i + a_2V_i^2 + a_3V_i^3 + \dots$

* Pass Band of BPF is such that it allows the V_i^2 of the Square law device.



Note:-



* The General exp of single tone FM is

$$S_{NBFM}(t) = A_c \cos\{\underbrace{2\pi f_c t + \beta \sin 2\pi f_m t}_{\theta}\} \; ; \; \beta \le 1$$

$$(\text{Freq}^n\ \text{mul})_{O/P} = S_{WBFM} = A_c \cos\{2n\pi f_c t + n\beta \sin 2\pi f_m t\}$$

n should such that

$$\boxed{n\beta > 1}$$

Input of Freq^n multiplier	O/P
f_c	$\rightarrow nf_c$
β	$\rightarrow n\beta$
f_m	$\rightarrow f_m$

$$\boxed{\overset{n}{\uparrow}\Delta f = \overset{n}{\uparrow}\beta f_m}$$

So, when β is increased by n

Hence, Δf is increased by n

So $\boxed{\text{new } \Delta f = n\Delta f}$

Q1 An FM is given by:-

$$s(t) = 10\cos\{2\pi\times10^6 t + 0.2\sin 2\pi\times2\times10^3 t\}$$

It is passed through cascaded frequency multiplier of having multiplying const of 4 and 5 Respt Find all the parameters of FM signal at the O/P of each of the multiplier?

Solⁿ: Given:

$$s(t) = 10\cos\{2\pi\times10^6 t + 0.2\sin 4\pi\times10^3 t\}.$$

comparing with standard eqⁿ:

$$s(t) = A_c\cos\{\cos 2\pi f_c t + \beta\sin 2\pi f_m t\}.$$

$$s(t) \rightarrow \boxed{\text{Frequ}^n \text{ multiplier } n=4} \xrightarrow{s'(t)} \boxed{\text{Freq}^n \text{ multiplier } n=5} \xrightarrow{s''(t)}$$

$A_c = 10V$; $\beta = 0.2$

$f_c = 1MHz$; $f_m = 2KHz$

$$\boxed{\begin{aligned}\Delta f &= \beta f_m \\ &= 0.4\,KHz\end{aligned}}$$

After passing: $(n=4)$

✓ $A_c = 10V$

✓ $f_c = 4\times1 = 4MHz$

✓ $\beta = 4\times0.2 = 0.8$ (NBFM)

✓ $f_m = 2KHz$ (nochange)

✓ $\Delta f = 4\times0.4$ $(\beta\times f_m)$

$= 1.6K$ ✓

✓ $B.W = 2f_m$

$= 4K$

✓ $P_t = \frac{A_c^2}{2R}\left\{1+\frac{\beta^2}{2}\right\}$

$= \frac{100}{2}\left\{1+\frac{0.64}{2}\right\}$

$P_t = 66W$ ✓.

After Passing $(n=5)$:

✓ $A_c = 10V$

✓ $f_c = 5\times4M = 20MHz$

✓ $\beta = 0.8\times5 = 4$ (WBFM)

✓ $f_m = 2K$

✓ $\Delta f = 5\times1.6K$

$= 8K$

✓ $B.W = 2(\beta+1)f_m$

$= 2\times5\times2K$

$= 20K$

✓ $P_t = \frac{A_c^2}{2R} = 50W$

* Demodulation of FM:

i) slope detector.
ii) Balanced slope detector.

* Phase discrimination method:

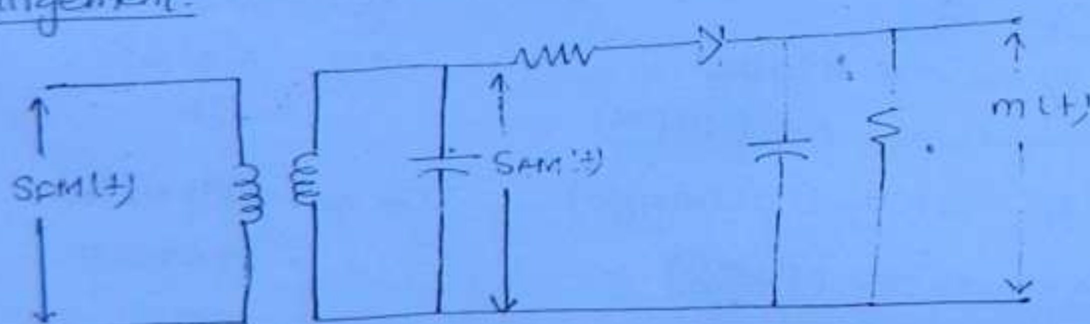
i) Foster Seley detector.
ii) Ratio method.
iii) PLL- method. (mostly used). (IES < subj. / obj.)

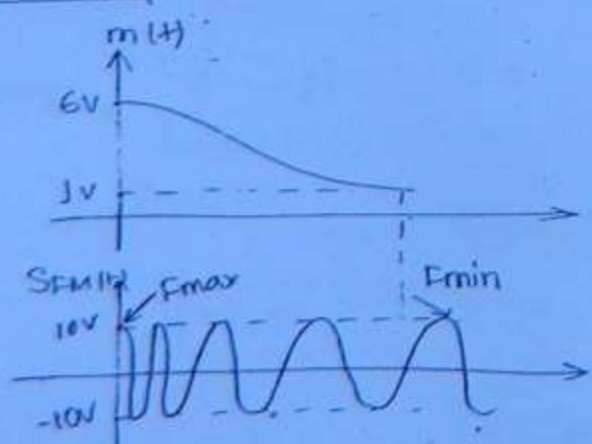
* Every other demodulator except PLL, needs x^{mer} for their constn, so for FM demodulation; generally PLL method will be used.

i) Slope detector:

Block diagm:

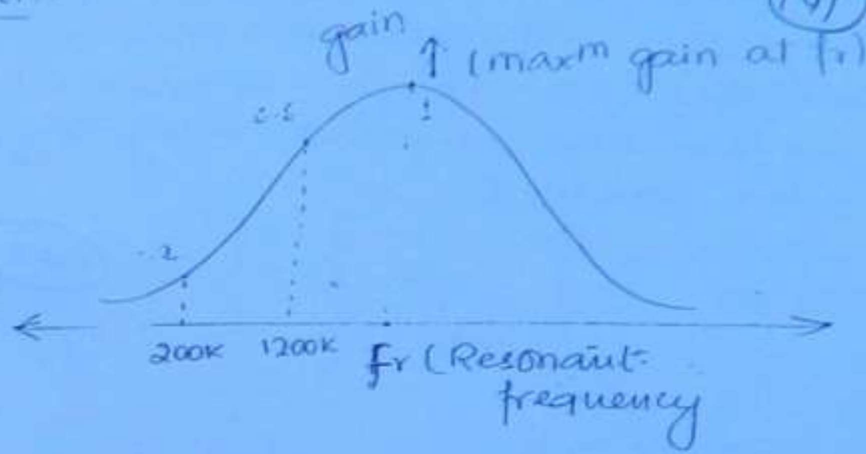
ckt arrangement:





Message signal Amp. is decreased by 6 times so, frequency is also decreased by 6 times.
Fmax = 1200K
Fmin = 200K

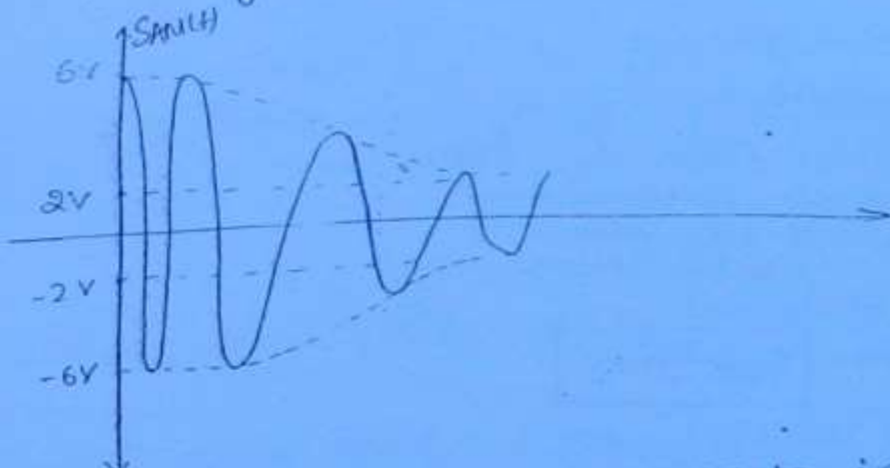
Tuned Amp Char:



* The fr should be such that $\boxed{f_r > f_{max}}$ (of Tuned Ampl^r)

* The gain offered by Tuned ampl^r at 1200K is 0.6; so the resulting signal peak amp. is decreased from 10V to 6V and for fmin = 200K; the peak amp. is 2V

So, o/p is given as:

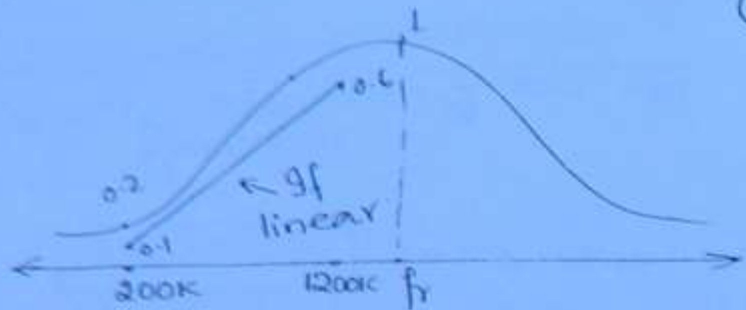


So, the O/P of envelope detector is

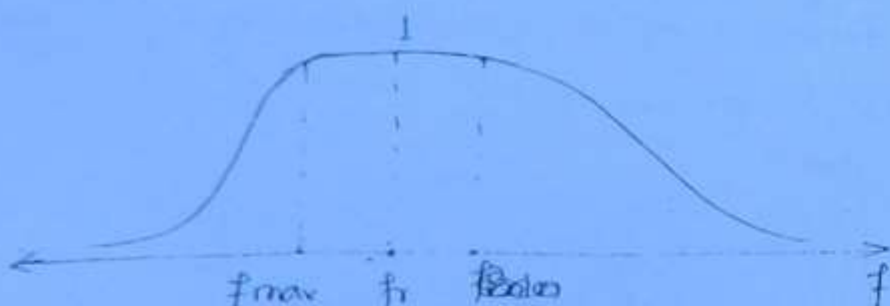
The above doesn't resemble to m(t) but has some deviation and error is obtained.

× The gain freq^n characteristics was linear ie the same amount of gain is obtained as the frequency decreases the the msg. signal O/P of FD would be same as the original msg signal and no error was present

(142)



× To avoid above a practical tuned Ampl^r is constructed of gain frequency char as following:



(taken corresponding where the gain starts decreasing)

So, $\boxed{f_r > f_{max}}$

Note :-

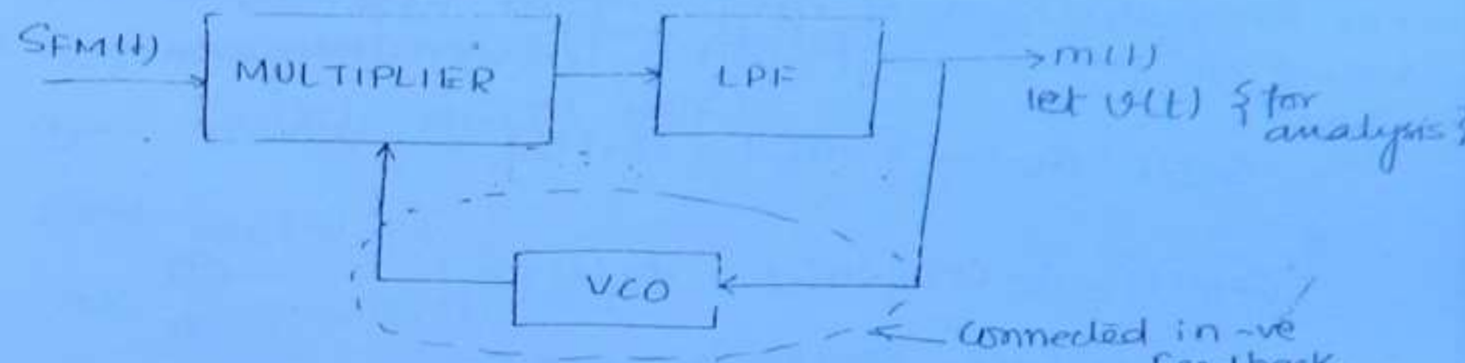
1. Since gain frequency char. of Tuned Ampl^r is nonlinear in nature, some amount of non-linearity will be introduced in frequency to voltage conversion.
So the Reconstructed msg. signal is not perfectly corresponds to transmitted msg. signal and is called as SLOPE ERROR.

2. In Balanced slope detector, 2 slope detectors are connected in Balanced to decrease the slope Error, to the min^m possible extent.

* PLL METHOD:

Block diagm:

(143)



Note:

1. The frequency of FM signal changes continuously w.r.t msg. signal voltage variations. so to maintain freqⁿ synchronisation L.O of Synchronous detector is replaced by VCO
2. For the VCO, the msg. signal is taken as input, so that VCO O/P frequency changes continuously w.r.t m(t) voltage variations, frequency synchronisation can be achieved.

* As,

$$S_{FM}(t) = A_c \cos\{2\pi f_1 t + \underbrace{2\pi K_f \int m(t)\,dt}_{\Phi_1}\}$$

$$\&\ \underset{S_v(t)}{(VCO)\,O/P} = A_v \cos\{2\pi f_2 t + \underbrace{2\pi K_v \int v(t)\,dt}_{\Phi_2}\}$$

For perfect Reconstruction of msg. signal,

i) f_1 should be made equal to f_2 ie

$$\boxed{f_1 = f_2 = f_c}$$

Then PLL is said to be working in the LOCK MODE.

ii) $\Phi_1(t)$ should be made equal to $\Phi_2(t)$

$$\boxed{\Phi_1(t) = \Phi_2(t)}$$

Then PLL said to be working in the CAPTURE MODE

Now
1 Because of ve feedback within no time f_1 will be made equals to f_2 and PLL will go to LOCK MODE.

144

* For Reconstruction of msg signal VCO output should have 90° phase shift w.r.t transmitter carrier; so that

$$S_V(t) = A_V \sin\{2\pi f_c t + \Phi_2(t)\} \leftarrow \text{VCO O/P} \quad -①$$

&

$$S_{FM}(t) = A_C \cos\{2\pi f_c t + \Phi_1(t)\} \quad -②$$

So the multiplier o/p is given as:

$$S_V(t) \times S_{FM}(t) = \frac{A_C A_V}{2}\left\{ \sin(4\pi f_c t + \Phi_1(t) + \Phi_2(t)) + \sin\{\Phi_2(t) - \Phi_1(t)\}\right\}$$

$$= \frac{A_C A_V}{2}\sin(4\pi f_c t + \Phi_1(t) + \Phi_2(t)) - \frac{A_C A_V}{2}\sin(\Phi_1(t) - \Phi_2(t))$$

let $\Phi_1(t) - \Phi_2(t) \simeq \Phi_e(t) \leftarrow$ phase error

For PLL, the care will be taken to make $\Phi_1(t)$ very close to $\Phi_2(t)$ so that $\Phi_e(t)$ will be very small.

So,

$$\underset{(\text{Mul})\text{O/P}}{S_V(t) \cdot S_{FM}(t)} = \frac{A_C A_V}{2}\underbrace{\cancel{\sin\{4\pi f_c t + \Phi_1(t) + \Phi_2(t)\}}}_{\text{no effect as blocked by LPF}} - \frac{A_C A_V}{2}\sin\{\Phi_e(t)\}$$

The 1st term in the above exp corresponds to very high frequencies, so it is not allowed by the LPF and since $\Phi_e(t)$ is very small:

$$\sin\{\Phi_e(t)\} \simeq \Phi_e(t)$$

So

$$(\text{Mul})_{O/P} \simeq -\frac{A_C A_V}{2}\Phi_e(t)$$

$$(\text{Mul})_{O/P} \simeq -\Phi_e(t).$$

This is given to the LPF.

So, $v(t) = \Phi_e(t) * h(t)$

So, taking Fourier xform on both side we get:

$\therefore \quad V(F) = \Phi_e(F) \cdot H(F) \quad \text{- - - - - (A)}$

Now, $\Phi_E(F) = ?$

as, $\Phi_e(t) = \Phi_1(t) - \Phi_2(t)$

$= \Phi_1(t) - 2\pi K_v \int v(t)\,dt$.

where, $v(t) \overset{\circ}{=} \Phi_e(t) * h(t)$.

$$\frac{d\Phi_e(t)}{dt} = \frac{d\Phi_1(t)}{dt} - 2\pi K_v \cdot v(t)$$

$$\frac{d}{dt}\Phi_e(t) = \frac{d}{dt}\Phi_1(t) - 2\pi K_v \left\{\Phi_e(t) * h(t)\right\}.$$

taking F.T on both sides

$$J \cdot 2\pi f\, \Phi_e(f) = j2\pi f\, \Phi_1(f) - 2\pi K_v \left\{\Phi_e(f) \cdot h(f)\right\}$$

$$\left\{\because \frac{d}{dt}\Phi(t) \longleftrightarrow j\omega \Phi\right\}$$

$$\Phi_e(f)\left\{Jf + K_v H(f)\right\} = jf\,\Phi_1(f)$$

$$\boxed{\Phi_e(f) = \frac{jf\,\Phi_1(f)}{\left\{jf + K_v H(f)\right\}}}$$

Also,

$$\Phi_e(f) = \frac{\Phi_1(f)}{1 + \frac{K_v}{jf} H(f)}$$

Now when $\boxed{\text{pass Band gain of LPF} = \infty}$

so, $\boxed{\Phi_e(f) = 0 \Rightarrow \Phi_e(t) \Rightarrow \Phi_1(t) = \Phi_2(t)}$

But practically it is not possible, so the pass Band gain of LPF is made high.

Note:

1. If the pass band gain of LPF is ∞ ie

$$\boxed{H(f) = \infty}$$

146

then, $\phi_e(t) = 0$

but for a practical LPF; H(f) will be finite

2. By making H(f) = very large; $\phi_e(t)$ will be made very small quantity.

Now, substituting value of $\Phi_e(f)$ in eqn (A) we get

So,

$$V(f) = \frac{\Phi_1(f)}{1 + \frac{K_v}{jf} H(f)} \cdot H(f)$$

∵ H(f) is very large

So,

$$1 + \frac{K_v}{jf} H(f) \simeq \frac{K_v}{jf} H(f)$$

So,

$$V(f) = \frac{\Phi_1(f)}{\frac{K_v}{jf} \cancel{H(f)}} \cdot \cancel{H(f)}$$

$$\boxed{V(f) = \frac{jf}{K_v} \Phi_1(f)}$$

$$V(f) = \frac{jf\,\Phi_1(f)}{K_v} \times \frac{2\pi}{2\pi}$$

By Inverse F.T we get

$$v(t) = \frac{1}{2\pi K_v} \cdot \frac{d}{dt} \Phi_1(t)$$

$$= \frac{1}{2\pi K_v} \cdot \frac{d}{dt} \times \left\{ 2\pi K_f \int m(t)\,dt \right\}$$

So,

$$\boxed{v(t) = \frac{K_f}{K_v} m(t)}$$

147

Now.

Note :

1 If $S_v(t) = A_v \cos\{2\pi f_c t + \Phi_2(t)\}$

$S_{FM}(t) = A_c \cos\{2\pi f_c t + \Phi_1(t)\}$

So,

$$(Mul)\ O/P = \frac{A_c A_v}{2} \cos\{4\pi f_c t + \Phi_1(t) + \Phi_2(t)\} + \frac{A_c A_v}{2} \cos\{\Phi_1 - \Phi_2(t)\}.$$

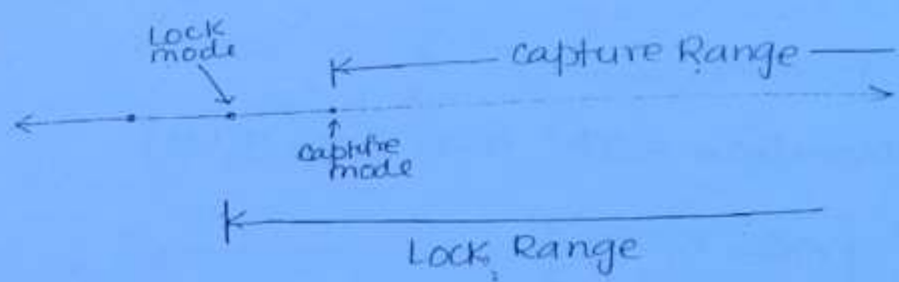
$\Phi_e(t) = \Phi_1(t) - \Phi_2(t) \simeq 0$

So, $\cos^0\{\Phi_e(t)\} \simeq 1$

So, Input of LPF = 1

Hence msg signal couldn't be obtained.

2.



So,

$$\boxed{L \cdot R \geq C \cdot R}$$

Note :-

1. For PLL; -ve feedback is Responsible for LOCK MODE.
2. LPF is responsible for CAPTURE MODE of PLL.
3. The frequencies produced by VCO corresponds to LOCK MODE of PLL.
4. For PLL : $\boxed{L.R \geq C.R}$

* PLL demodulation is based on the differentiation process

* PHASE MODULATION :

* Carrier before phase modulation $= A_c \cos\{2\pi f_c t\}$.
* Carrier after phase modulation $= S_{PM}(t) = A_c \cos\{2\pi f_c t + \underbrace{\phi}_{\text{Phase deviation}}\}$
(phase modulated signal)

where, $\phi(t) = K_p m(t)$

K_p = Phase sensitivity of phase modulator (rad/volt).

So,

$$\boxed{S_{PM}(t) = A_c \cos\{2\pi f_c t + \underbrace{K_p m(t)}_{\text{Phase deviation}}\}}$$ ← General exp. of phase modulation

let, $m(t) = A_m \cos\{2\pi f_m t\}$

So,
$\max^m$ phase deviation $= \Delta\phi = \max\{K_p m(t)\}$

So, $\boxed{\Delta\phi = K_p A_m}$

So,
$S_{PM}(t) = A_c \cos\{2\pi f_c t + K_p \cdot A_m \cos 2\pi f_m t\}$.

let, $K_p \cdot A_m = \beta$ = modulation index of P.M.

Now,
Corresponding to F.M we have,

$$\beta = \frac{K_f A_m}{f_m} = \frac{\Delta f}{f_m}$$

$$\boxed{M.I = \frac{\text{Max}^m \text{ freq}^n \text{ deviation}}{\text{msg. signal freq}^n}}$$

And corresponding to PM we have:

$\beta = K_p A_m = \Delta\phi$

$\boxed{M.I = \text{max}^m \text{ phase deviation}}$

Note:

For phase modulation, β & $\Delta\phi$ are independent of msg. signal frequency variations

Now, let $\beta = K_p A_m$

So,

$\boxed{S_{PM}(t) = A_c \cos\{2\pi f_c t + \beta \cos 2\pi f_m t\}}$

& $\boxed{S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}}$

Note:

1. General expressions of FM & PM are same except 90° phase shift at msg frequency component.
2. The magnitude spectrum of PM will be same as FM so that B.W and power requirements of PM & FM will be the same.

* B.W of PM (WBFM):

$\boxed{B.W = 2(\beta+1) f_m} \Rightarrow \boxed{2(\Delta\phi+1) f_m = B.W}$

* Power of PM:

$\boxed{P_t = \frac{A_c^2}{2R}}$

Q1 A phase modulated signal is given by:

$$S_{PM}(t) = 10\cos\{2\pi\times10^6 t + 6\sin 6\pi\times10^3 t\}$$

(156)

i) Find all parameters of PM.

ii) Repeat above if msg signal freq^n is doubled

Sol^n:

$$S_{PM}(t) = 10\cos\{2\pi\times10^6 t + 6\sin 6\pi\times10^3 t\}$$

$$S_{PM}(t) = A_c\cos\{2\pi f_c t + \beta\sin 2\pi f_m t\}$$

i)

$A_c = 10$

$f_c = 1M = 1000K$

$f_m = 3K$

$\beta = 6\text{ rad} = \Delta\phi$.

So,

$B.W = 2(\beta+1) f_m$; $\boxed{\beta = \Delta\phi = 6\text{ rad}}$ Ans.

$= 2\times7\times3$

$\boxed{B.W = 42K}$ Ans.

$\ast\ \boxed{P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50W}$ Ans.

Now,

ii) let $f_m = 6K$; $\boxed{\beta = \Delta\phi = K_p A_m = 6\text{ rad}}$ Ans.

So, $\uparrow B.W = 2(\beta+1) f_m \uparrow$

$= 2\times7\times6$

$\boxed{B.W = 84K}$ Ans.

$\ast\ \boxed{P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50W}$ Ans.

Note:- For PM; as the msg signal frequency doubles the corresponding B.W will also be doubled.

* Generation of FM from PM & PM from FM

As,

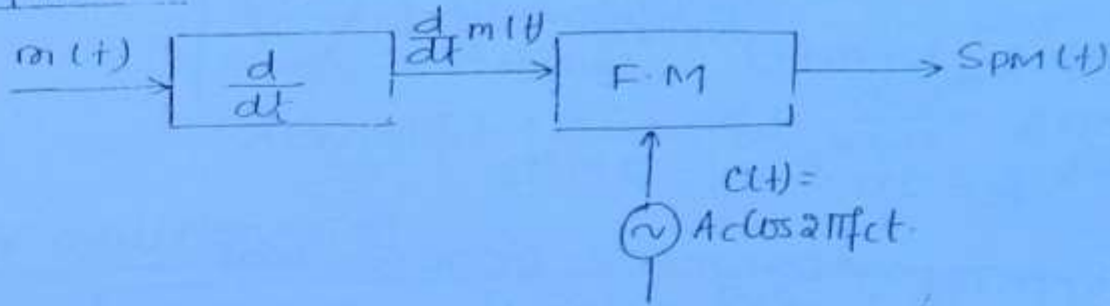
$$S_{PM}(t) = A_c \cos\{2\pi f_c t + K_p m(t)\}$$

151

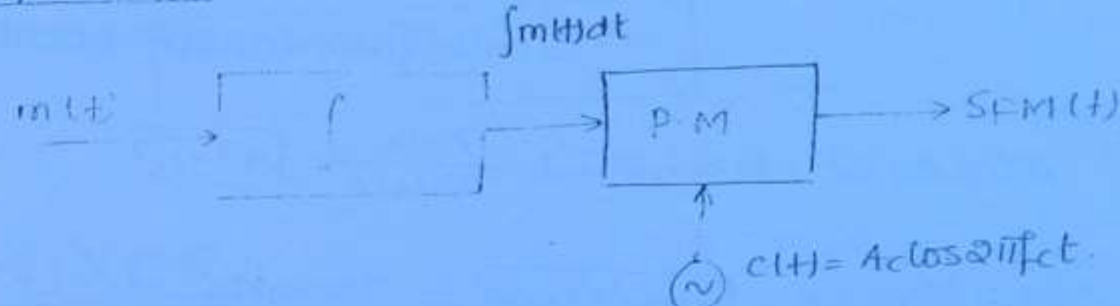
&

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + 2\pi K_f \int m(t)\,dt\}$$

* PM from FM:



* FM from PM:



Note:-

1. Phase modulation of m(t) is nothing but frequency modulation of $d/dt\; m(t)$.

Q1. An Angle modulated Signal is given by

$$s(t) = \cos\{2\pi(10^6 \times 2t + 30\sin 150t + 40\cos 150t\}.$$

Find max^m freq^n deviation & max^m phase deviation?

Sol^n: Given:-

$$s(t) = \cos \underbrace{2\pi\{2\times10^6 t + 30\sin 150t + 40\cos 150t\}}_{\theta_i(t)}.$$

$\because$ Angle modulated Signal, So let

$$\theta_i(t) = 2\pi\{2\times10^6 t + 30\sin 150t + 40\cos 150t\}$$

For FM:- So, $f_i = \dfrac{1}{2\pi}\cdot\dfrac{d}{dt}\theta_i(t)$

$$f_i = \frac{1}{2\pi}\cdot\frac{d}{dt}\left\{2\pi(2\times10^6 t + 30\sin 150t + 40\cos 150t)\right\}$$

152

$$= \frac{1}{2\pi}\left\{2\pi(2\times10^6 + 4500\cos 150t - 6000\sin 150t)\right\}$$

Now,

$f_i = f_c + K_f m(t)$ ← frequency deviation

So, $f_c = 2\times10^6$ Hz $= 2$ MHz

& $K_f m(t) = 4500\cos 150t - 6000\sin 150t$

So max^m freq^n deviation, $\Delta f = \max\{K_f m(t)\}$.

$= \max\{4500\cos 150t - 6000\sin 150t\}$

as. $A\cos 2\pi f_0 t + B\sin 2\pi f_0 t \xrightarrow[\text{value}]{\text{max}} \sqrt{A^2+B^2}$

So, $\Delta f = \sqrt{4500^2 + 6000^2} = 7500$ Hz

$\boxed{\Delta f = 7.5\text{ KHz}}$ Ans.

For PM:

As,

$S_{pm}(t) = A_c\cos\{2\pi f_c t + K_p m(t)\}$. ← phase deviation

Now,

$s(t) = A_c\cos 2\pi\{2\times10^6 t + 30\sin 150t + 40\cos 150t\}$

$= A\cos\{4\pi\times10^6 t + 60\pi\sin 150t + 80\pi\cos 150t\}$

So, $2\pi f_c = 4\pi\times10^6$

$K_p m(t) = 60\pi\sin 150t + 80\pi\cos 150t$

$\Delta\phi = \max\{K_p m t\} = \sqrt{(60\pi)^2 + (80\pi)^2}$

$\boxed{\Delta\phi = 100\pi\text{ rad.}}$ Ans.

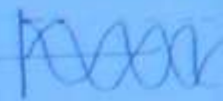
WSKO...
Pg: 16/8/21

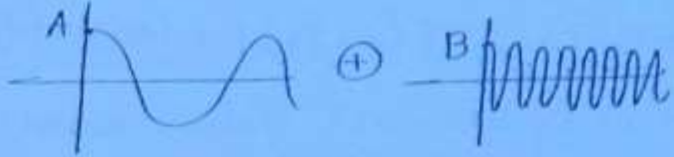
$f_{max} = 1000$

$s(t) \quad 10\cos\{\omega_c t + 5\sin 3000t + 10\sin 2000\pi t\}$ $\quad A\cos 2\pi f_a t + B\sin 2\pi f_b t$

153

Now, $A\cos 2\pi f_1 t + B\cos 2\pi f_2 t \xrightarrow[\text{value}]{\text{max}^m} (A+B)$





→ when added max^m value $(A+B)$.

$A\sin 2\pi f_1 t + B\sin 2\pi f_2 t \xrightarrow[\text{value}]{\text{max}^m} A+B)$

* For multitone FM:

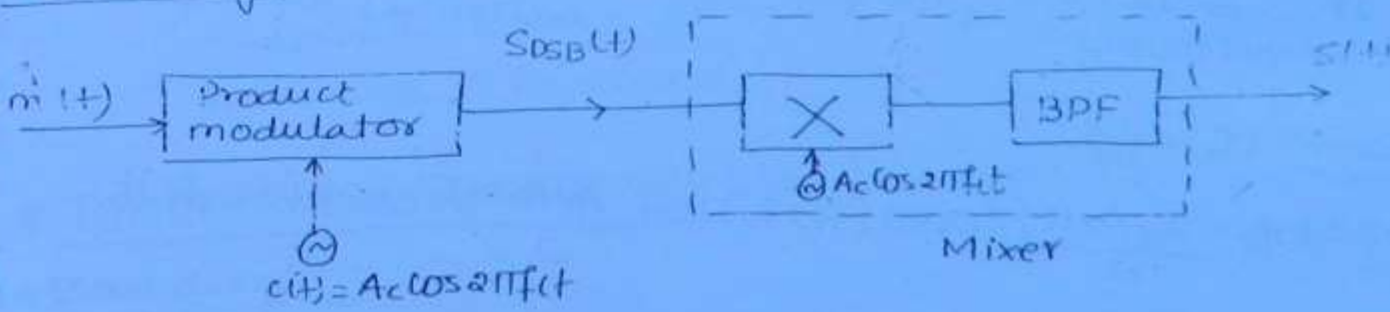
$$\boxed{\text{Mod}^n \text{ Index} = \text{deviation Ratio} = \frac{\Delta f}{f_{max}}}$$

$$\boxed{B.W = 2(\beta+1) f_{max}}$$

* MIXER :-

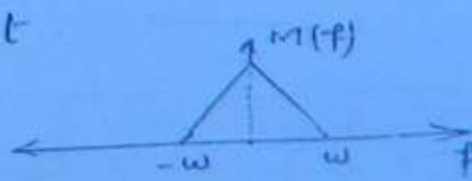
Mixer is used for frequency xlation of Modulated signal.

Block diag^m:

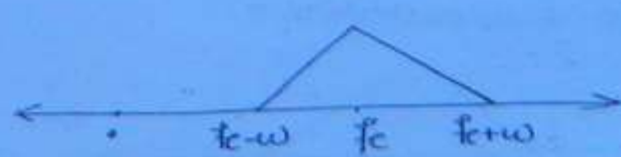


$S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t$

let $m(t) \longleftrightarrow M(f)$



$S_{DSB}(t) \longleftrightarrow$



(154)

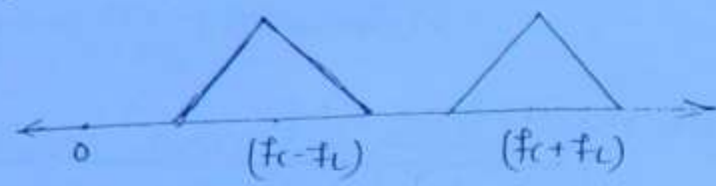
(Mul) o/p = SDSB(t) · (LO) o/p

$= A_c m(t) \cos 2\pi f_c t \times \cos 2\pi f_L t$

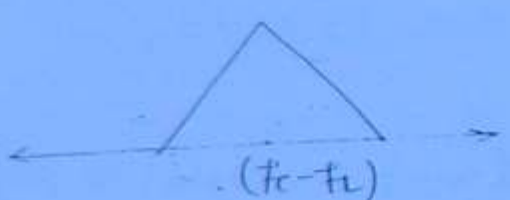
Case i $(f_c > f_L)$

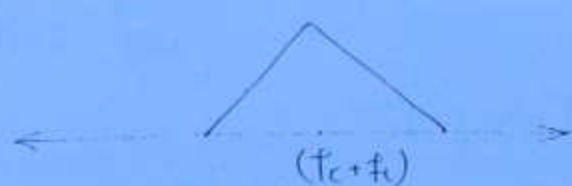
$$(\text{Mul})\,o/p = \frac{A_c m(t)}{2} \left\{ \cos 2\pi (f_c + f_L)t + \cos 2\pi (f_c - f_L)t \right\}.$$

(mul) o/p $\longleftrightarrow$



So, (Mixer) o/p (depending upon Pass Band of filter)





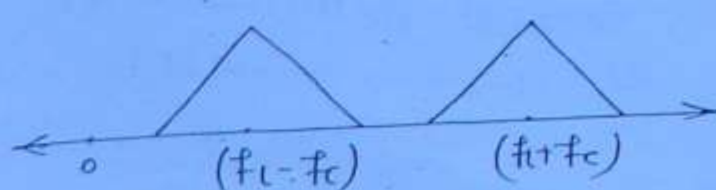
when B.P.F o/p is this, it is called as DOWN-CONVERSION

when B.P.F allows the $(f_c + f_L)$ component, it is called as UP-CONVERSION

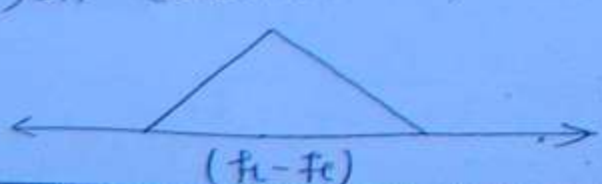
Case ii : $(f_c < f_L)$.

$$(\text{Mul})\,o/p = \frac{A_c m(t)}{2} \cos 2\pi (f_L + f_c)t + \frac{A_c m(t)}{2} \cos 2\pi (f_L - f_c)t$$

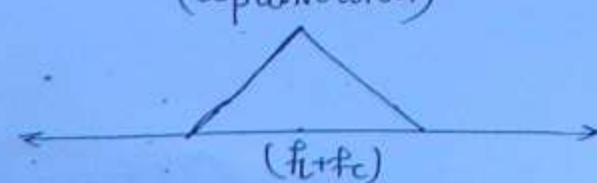
(mul) o/p $\longleftrightarrow$



(Mixer) o/p (down conversion) (up conversion)



or



Note:

1. Down conversion will give substraction of the frequencies and upconversion gives sum of the frequencies

Date: 11.11.11 *PSU's)

* RECIEVERS:-

→ Tuned Radio Frequency (TRF).

→ Super heterodyne (SHD).

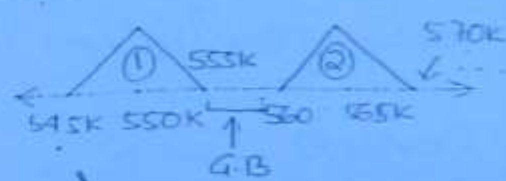
* Depending on modulation scheme:
  1) AM Reciever.
  2) FM Reciever.

AM standards:-

According to federation commitee of commn (FCC):-

Carrier freq^n = 550 K to 1650K.

A.M. B.W = 10K.



FM standards:

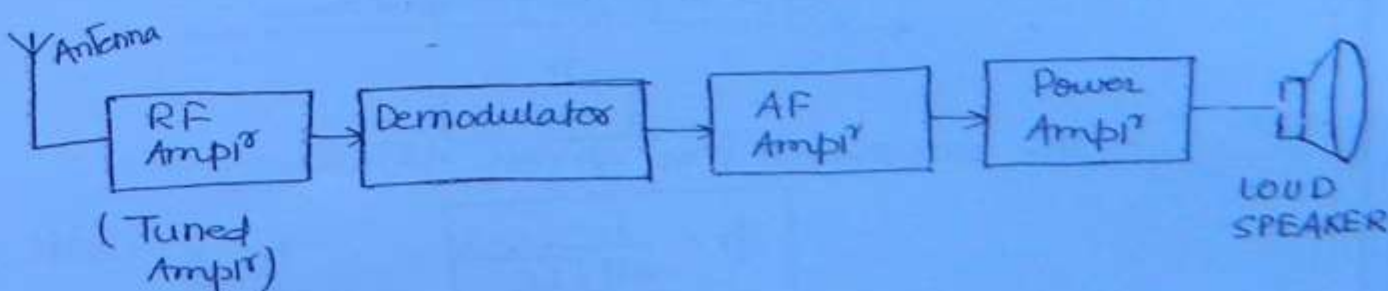
According to FCC:

Carrier Freq^n = 88 to 108 MHz.

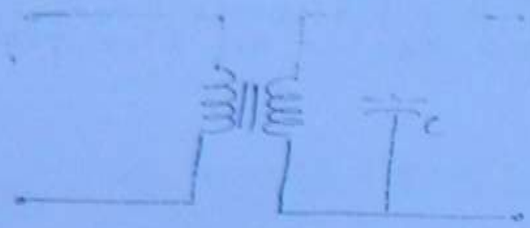
FM ~~FB~~ BW = 200K.

* TUNED RADIO FREQUENCY (TRF) Reciever:

* Block diag^m:



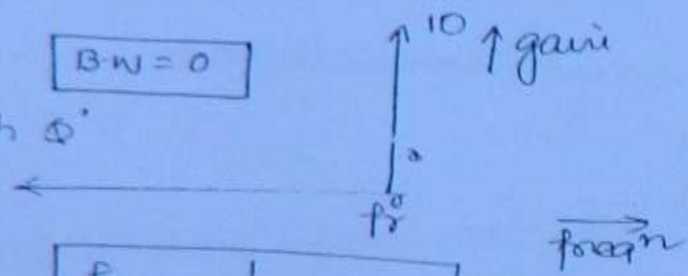
Tuned ckt (of Ampl^r):



× Gain Frequency char. of Tuned Ampl^r:

Ideal Tuned Ampl^r → "high Q"

$\boxed{B.W = 0}$

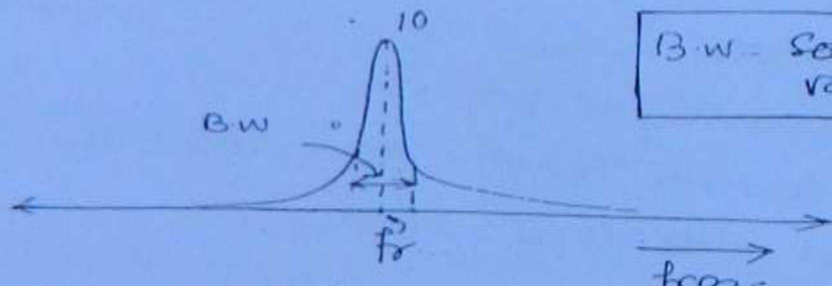


← allows only single frequency ie fr and rejects other frequency

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

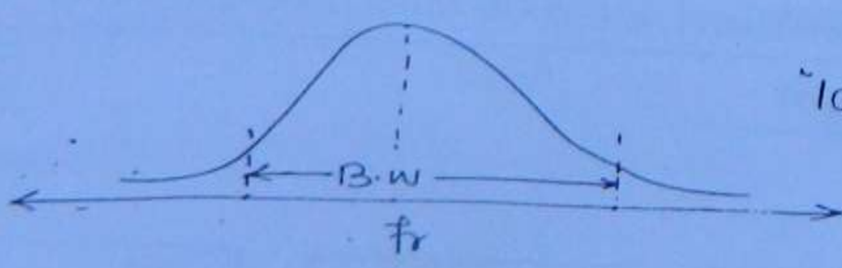
× The Range of frequencies which the Tuned ampl^r passes w/o any Attenuation is called as BW of Tuned Ampl^r.

Practical Tuned Ampl^r → (highly Qualitative)



B.W - Some finite value

Practical Tuned Ampl^r (General case)



"low Q"

The Resonant freq^n given as:

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

Quality factor given as:

$$\boxed{Q = \frac{1}{2\pi}\sqrt{\frac{L}{C}}}$$

Note:

1. Quality factor is also said as the sharpness of the GAIN FREQ^n CHAR.

Sharpness high = Q high.

Sharpness low = Q low.

(157)

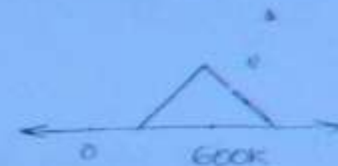
2. Also, B.W is inversely ~~proo~~proportional to Q.

$$\boxed{\begin{aligned} B.W &\propto 1/Q \\ B.W &= f_r/Q \end{aligned}}$$

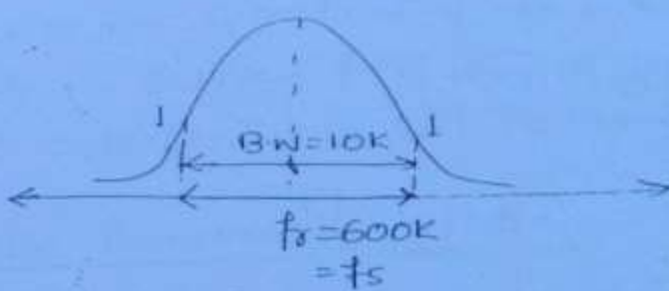
*Op^n of TRF Rx:

Case 1:

1) Assume Rx is tuned to 600K station i.e $f_c = 600K$



2) Assume Tuned Ampl^r should be



3) f_r should be equal to 600K ; $f_r = 600K$

4) To get B.W of 10K ; Q should be

$$B.W = \frac{f_r}{Q}$$

$$Q = f_r/B.W = \frac{600K}{10K}$$

$$\boxed{Q = 60}$$

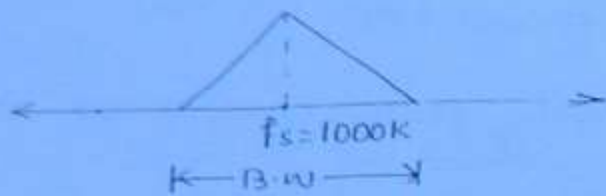
5) Now, $f_r = \frac{1}{2\pi\sqrt{LC}} = 600K$.

$$Q = \frac{1}{2\pi} \times \sqrt{\frac{L}{C}} = 60.$$

Case 2

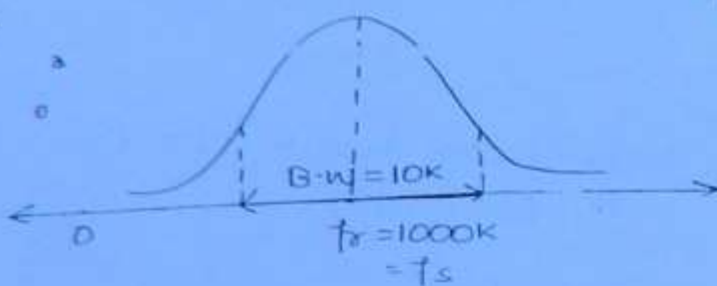
1. Assume the Rx is tuned to 1000K station
   ie $f_s = 1000K$

158



2. Tuned Ampl should be as:-

$f_r = 1000K$
$B \cdot W = 10K$



3. To get B.W of 10K; Q should be

$$\boxed{Q = 100}$$

4. As. $f_r = \frac{1}{2\pi\sqrt{LC}}$; $Q = \frac{1}{2\pi}\sqrt{\frac{L}{C}}$

For changing the frequency. capacitance is varied.
Such that $f_r = 1000K$
but correspondly due to diff formulae of f_r & Q,
the value of Q is not exactly 100.
So, let for $f_r = 1000K \Rightarrow Q = 120$

So, $B \cdot W = \frac{f_r}{Q} = \frac{1000}{120}$

$$\boxed{B \cdot W = 8 \cdot 3K}$$

Hence, some B.W of Tuned Ampl is lost and the msg signal correspondly can't be Reconstructed

Conclusion:

1. If B.W of T.Ampl^r = 10K ⇒ m(t) can be perfectly Reconstructed

2. If B.W of Tuned Ampl^r < 10K ⇒ m(t) can't be Reconstructed perfectly & freq^n attenuated

3. If B.W of T.Ampl^r > 10K ⇒ m(t) can't be Reconstructed perfectly & some unwanted freq^n are allowed.

* Due to the above discussion the selectivity of the TRF Rx is poor.

Note:

1. For TRF Rx, as the Tuning changes B.W of the Tuned Ampl^r will be changed accordingly.

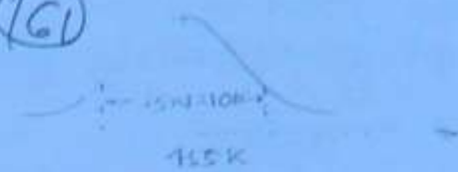
2. For excellence in selectivity, to what may be the station, Rx is Tuned for B.W of Tuned Ampl^r should be 10K only. So selectivity of TRF Rx is worst.

* Characteristic Parameter of Rx:

1. Selectivity: It is the ability of the Rx to allow only the desired freq^n components and Rejecting undesired frequency components.
It mainly depends on characteristics of Tuned Ampl^r.

2. Sensitivity: It specifies the min^m strength of the signal to be maintained at the Rx input; to produce faithful output.

× IF Ampl^r = Tuned Ampl^r
always tuned to 455 K
and B.W = 10K



so, $\boxed{Q = \frac{f_r}{B.W} = 45.5}$

× Mixer is designed to work as a down convertor and f_L is preferred to take as greater than f_s ($f_L > f_s$); so that Mixer O/P is equal to :

$$\boxed{f_L - f_s = 455\,K = IF = \text{Intermediate freq}^n}$$

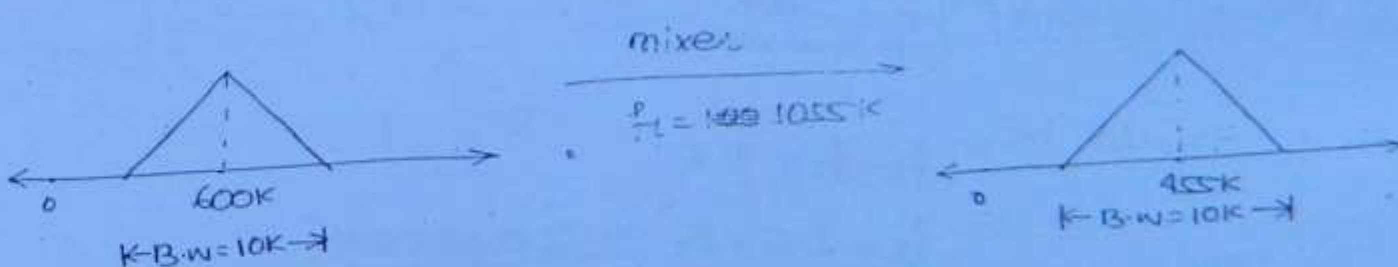
Op^n:

case 1:

1. Assume R_x is tuned to 600 KHz station.

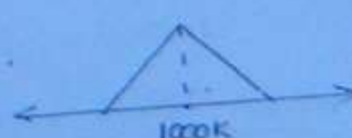
2. I.F ampl^r tuning →

3.

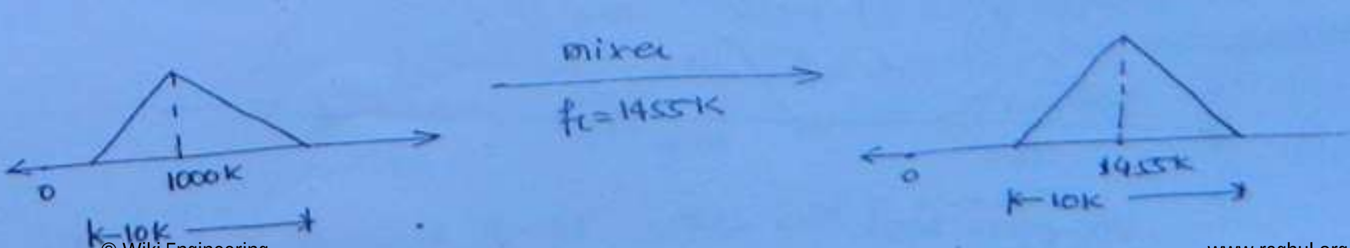


case 2:

1. Assume R_x is tuned to 1000 KHz station.



2.



Note:

1. In TRF Rx, tuning is achieved by changing 'fr' of the tuned Ampr.

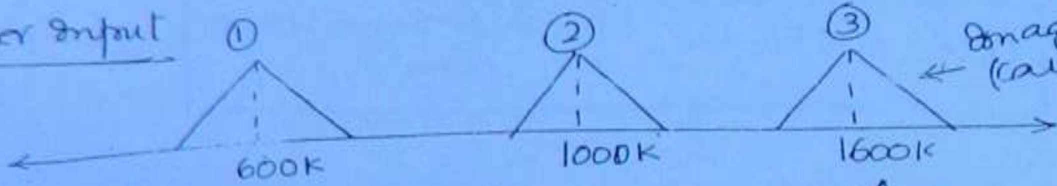
2. In SHD Rx; tuning is achieved by changing "fl" of the local oscillator.

(162)

*** Image Frequency:

let the messages spectrum be:

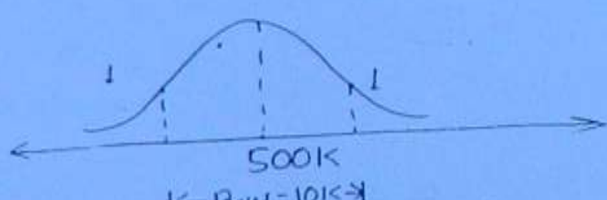
mixer Input



2. Assume Rx is tuned to 600K

$f_s = 600K$

3. let Intermediate frequency, IF = 500K



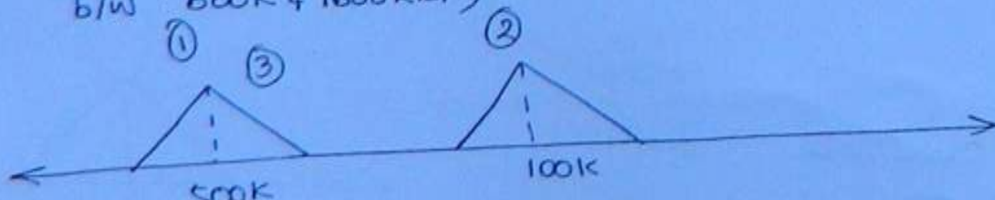
4. f_l should be, $f_l - f_s = I.F$

$f_l = I.F + f_s = 500K + 600K$

$\boxed{f_l = 1100K}$

Mixer O/P:

(Interference occurs b/w 600K & 1600K stn)



1100 - 600 = 500K

1600 - 1100 = 500K

× 1600K station is causing interference to 600K station so 1600K is said to be Image frequency of 600K

Now,

$\boxed{\text{Image freq}^n.\ f_{si} = f_s + 2\,I.F}$

let,

$I.F = 500K$

$f_s = 600K$

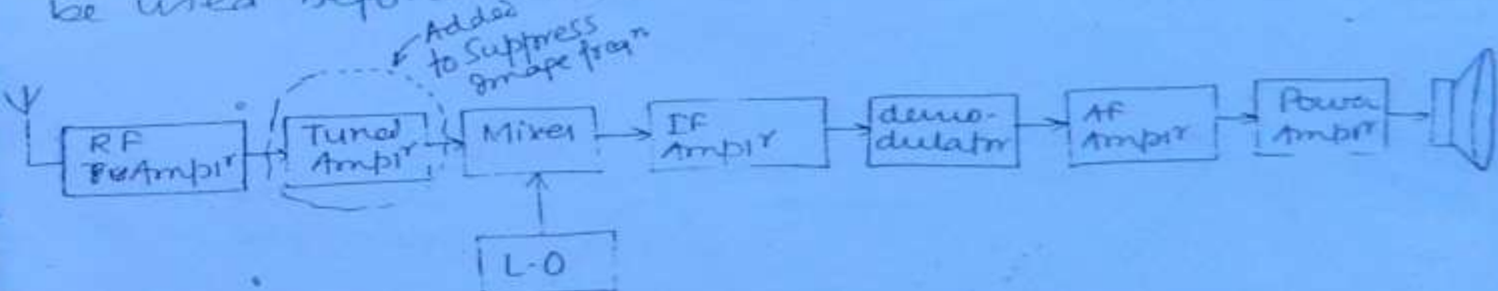
$f_{si} = 600 + 1000$

$\boxed{f_{si} = 1600K}$

* Suppression of Image frequency:-

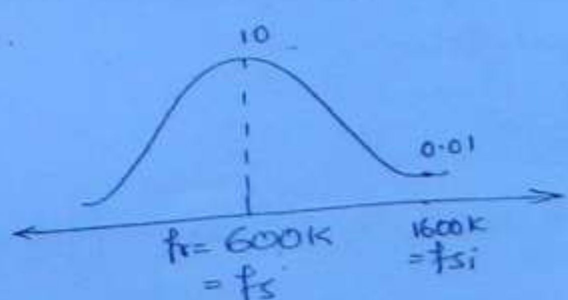
1. Since, desired station and Image station are occupying same frequency Band at Mixer output, so for comfortable Reconstruction of desired signal Image frequency should be suppressed.

2. To suppress Image frequency, a tuned Amplr will be used before mixer.



"Practical SHD"

× Gain-Freq^n char. of Tuned Amplr:-



$\uparrow 10V \qquad \uparrow 10V$

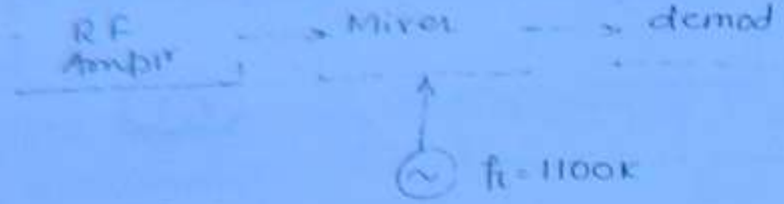
$f_s = 600K\ ;\ f_{si} = 1600K$

the Resonant freq^n of this Amplr is not fixed as that of IF Amplr. But it can be varied in accordance to the desired st^n.

So, o/p of Tuned Amplr ; $f_s = 600K \rightarrow 100V$

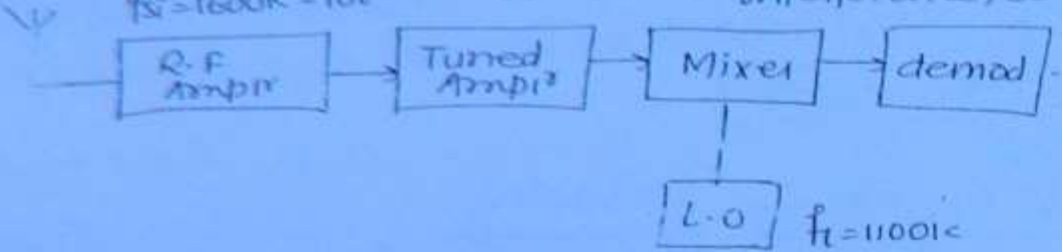
$f_{si} = 1600K \rightarrow 0.1V$

fr = 600K – 10V   500K – 10V } same strength hence interference
fsi = 1600K – 10V   500K – 10V } is high & m(t) is distorted

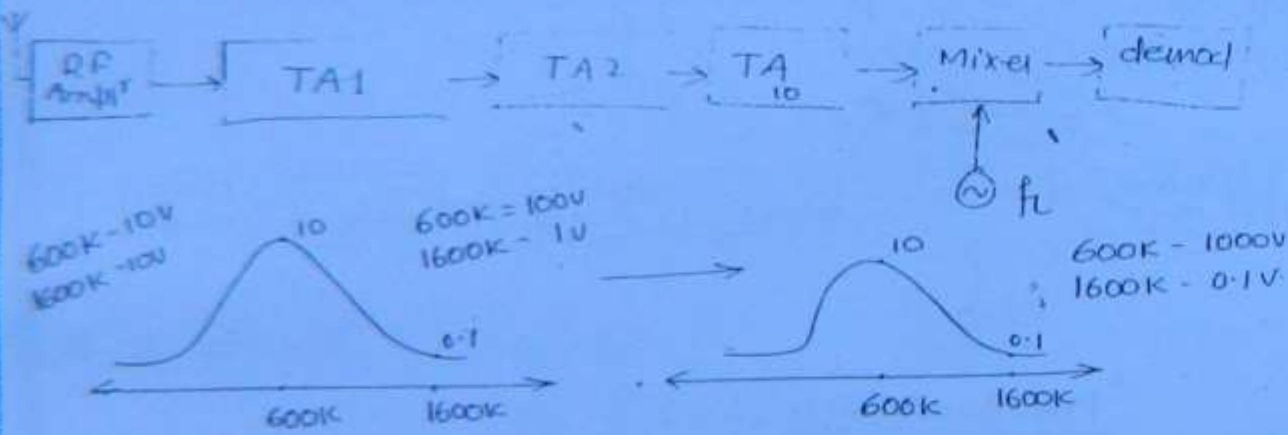


164

fr = 600K – 10V   500K – 100V { diff strength; hence less
fsi = 1600K – 10V   500K – 0.1V. { Interference; so m(t) easily Reconstructed



* For much separation of Image freqn; Cascaded Tuned Ampl will be used at the Input of Mixer.
So,



Note:
* For getting optimum suppression, Cascaded Tuned Ampl should have same characteristics.

* Image Rejection Ratio (IRR):
It specifies, effectiveness of tuned Ampl in suppressing Image frequency.
OR
It specifies how many times, Image frequency is attenuated w.r.t desired frequency.

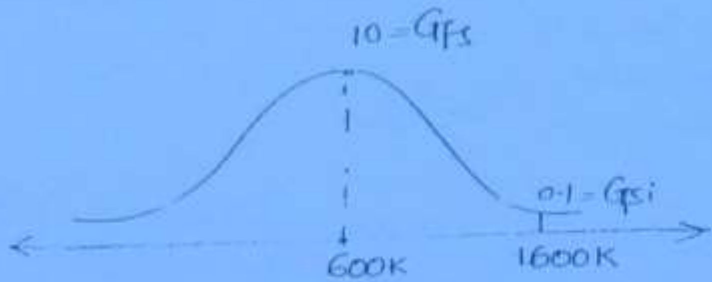
Mathematically.

$$\boxed{IRR = \frac{\text{Gain offered by TA to } f_s}{\text{Gain offered by TA to } f_{si}}}$$

165

or $$\boxed{\alpha = IRR = \frac{G_{fs}}{G_{fsi}}}$$

So,



$$\alpha = \frac{G_{fs}}{G_{fsi}} = 10/0.1$$

$$\boxed{\alpha = 100}$$

If Tuned Ampl^r are connected in cascade then,

$$\boxed{\text{effective } IRR = \alpha_1 \cdot \alpha_2 \cdot \alpha_3}$$

So from above discussion

$\alpha_1 = 100$; $\alpha_2 = 100$

$\therefore$ effective $\alpha = 100 \times 100$
$= 10000$

× If G_{fs} & G_{fsi} are not known then ;

$$\boxed{\alpha = \sqrt{1 + P^2 Q^2}}$$

where, Q = Quality factor of TA

$$\boxed{P = \frac{f_{si}}{f_s} - \frac{f_i}{f_{si}}}$$

× If 2 tuned Ampl^r of having diff. char are connected in CASCADE, then

$$\boxed{\alpha = \sqrt{P^2 Q_1^2 + 1} \times \sqrt{1 + P^2 Q_2^2}}$$

× If tuned Ampl^r having same char. then:

$$\boxed{\begin{aligned}\alpha &= \sqrt{1+P^2Q^2} \times \sqrt{1+P^2Q^2} \\ &= 1+P^2Q^2\end{aligned}}$$

188

Q1. A Rx is tuned to 600KHz, I.F = 450KHz
Find f_L & f_{si}?

Soln: Given: $f_s = 600K$
$f_i = 450K$

So, $f_i = f_L + f_s$

$f_L = f_i + f_s = 600 + 450$

$\boxed{f_L = 1050K}$ Ans

And, $f_{si} = f_s + 2 I_f$
$= 600K + 900K$

$\boxed{f_{si} = 1500K}$ Ans

Q2. A Rx is tuned to 500K; local osc. freq^n is given b
1050K. Find i) I.F and f_{si}
ii) Find IRR, if $Q = 50$.

Soln: Given:
$f_s = 500K$; $f_i = f_L - f_s$
$f_L = 1050K$ $= 1050 - 500$
$\boxed{f_i = 550K}$

So, $f_{si} = f_s + 2 I_f$

$= 500 + 1100K$

$\boxed{f_{si} = 1600K}$

Now, $P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1600}{500} - \frac{500}{1600}$

$P = 2.8$

So, $\alpha = \sqrt{1+P^2Q^2}$

$= \sqrt{1+(2.8)^2 \times 100}$

(157)

$\boxed{\alpha = 144.3}$ Ans

Q3. A Rx is tuned to 750K; Corresponding image freq^n is given by 1750K. Find i) f_L & I.F
ii) Find IRR if 2 tuned ampl^r of having Q = 50 & 70 are connected in cascade

Sol^n: Given: $f_s = 750K$

$f_{si} = 1750K$

So, $f_{si} = f_s + 2I_f$

$I_f = \frac{1750-750}{2} = 500K$

Now, $f_i = f_L - f_s$

$500 = f_s - 750K$

$f_s = 1250K$

Now, $P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1750}{750} - \frac{750}{1750} = 1.9$

Now, $\alpha = \sqrt{1+P^2Q_1^2} \times \sqrt{1+P^2Q_2^2}$

$= \sqrt{1+1.9^2 \times 2500} \times \sqrt{1+1.9^2 \times 4900}$

$\boxed{\alpha = 12636.05}$

Q4. A Rx is tuned to 1MHz, I.F = 455 KHz and Q = 100.
i) Find IRR
2) Find IRR if the Rx is tuned to 25MHz

Sol^n: Given, $f_s = 1000K$

I.F = 455K

$f_{si} = f_s + 2 I_f$

$= 1000 + 910\,K$

(168)

$f_{si} = 1910\,K$

So, $P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{1910K}{1000} - \frac{1000}{1910}$

$P = 1.386$

So, $\alpha = \sqrt{1 + P^2 Q^2}$

$= \sqrt{1 + 1.386^2 \times 10000}$

$\boxed{\alpha = 138.6}$

Now,

$f_s = 25M = 25000\,K$

So, $f_{si} = f_s + 2 I_f$

$= 25000 + 910$

$= 25910\,K$

$P = \frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = \frac{25910}{25000} - \frac{25000}{25910}$

$P = 0.072$

So, $\alpha = \sqrt{1 + P^2 Q^2}$

$= \sqrt{1 + 0.072^2 \times 10000}$

$\boxed{\alpha = 7.26}$

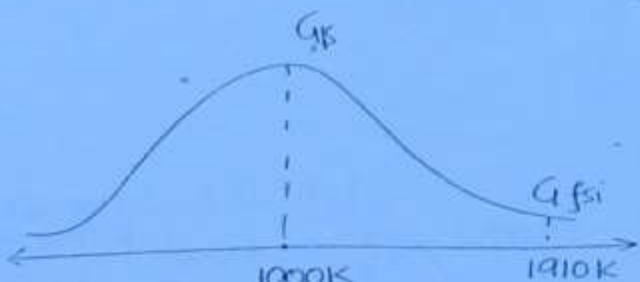
Note:

1. In the 2nd case, when $f_s = 25\,MHz$; α is very small ie 7.
2. For comfortable reconstrn of signal α has to be increased.

So, f_{si}

Case 1 (when $f_s = 1M$):

Case 2 (when $f_s = 25M$)



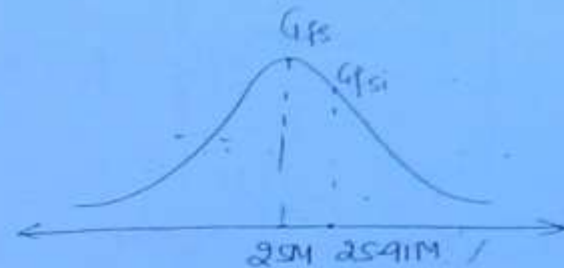
$\leftarrow 2f_i = 910K \rightarrow$

$f_r = 1M$

$f_r = 1000K$

$Q = 100$

$\boxed{B.W = 10K}$



$f_r = 25M$

$f_r = 25000K$

$Q = 100$

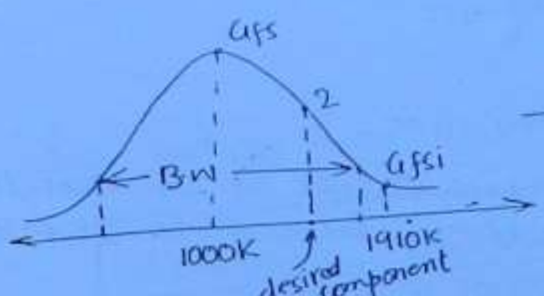
$\boxed{BW = 250K}$

Interview

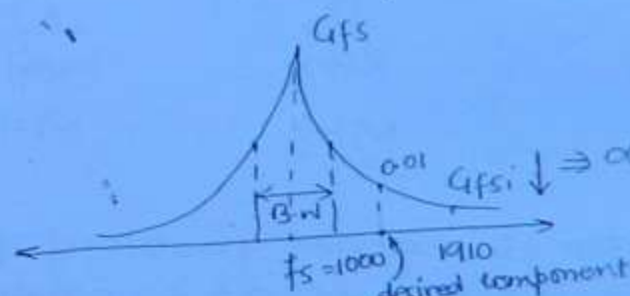
* Measures to Increase α:

As $\alpha = \sqrt{1 + P^2 Q^2}$

i) by increasing Q:



$Q\uparrow \rightarrow$



$G_{fsi} \downarrow \Rightarrow \alpha \uparrow$

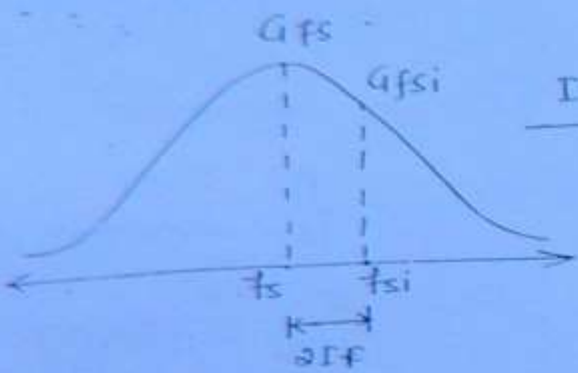
Practically it is not preferred, as the B.W decreases, & hence the selectivity of the Rx is affected.

Hence, it may be concluded that some of the desired frequency components will be attenuated. Hence the desired msg. signal (audio signal) cannot be perfectly Reconstructed.

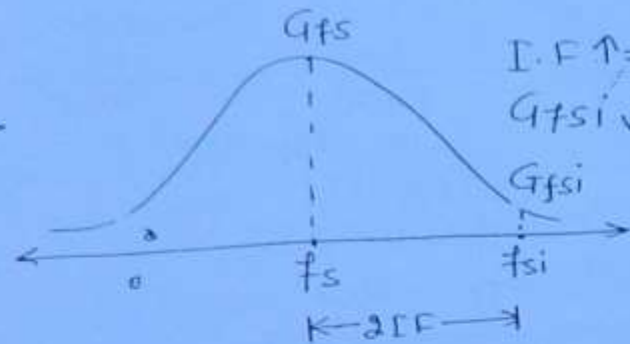
Note :- The above is not suggested because B.W of the Tuned Ampl^r will be very much decreased and it affects the selectivity of the Rx.

As $\uparrow P = \frac{f_{si}\uparrow}{f_s} - \frac{f_s}{f_{si}}$

Hence, $\uparrow f_{si} = f_s + 2IF \uparrow$



I.F $\uparrow$



I.F $\uparrow \Rightarrow \alpha \uparrow$
$G_{fsi} \downarrow \Rightarrow \alpha \uparrow$

Conclusion:

To get high value of IRR; both Fc and IF should be in the same order.

f_s	IF
KHz	KHz
MHz	MHz

Q. For the above problem, find new value of IF required to get IRR of 138.6? when the Rx is tuned to 25 MHz?

Soln: Given, $\alpha = 138.6$

$f_s = 25000\,K$

So, $\alpha = \sqrt{1 + P^2 Q^2} \Rightarrow 138.6$

$1 + P^2 Q^2 = (138.6)^2$

$PQ = 138.6$

$P = 1.385$

Now, $\frac{f_{si}}{f_s} - \frac{f_s}{f_{si}} = 1.385$

$\frac{f_{si}^2 - f_s^2}{f_s \times f_{si}} = 1.385$

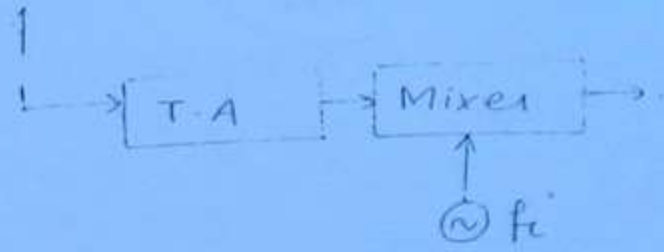
$\Rightarrow \frac{f_{si}^2 - 625}{f_{si}} = 34.65$

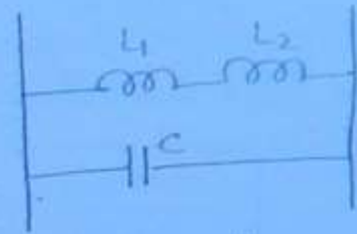
$f_{si}^2 - 34.65 f_{si} - 625 = 0$

$f_{si} = 47.74\,M$

$\Rightarrow f_s + 2I_f = 47.74$

$\boxed{I_f = 11.37\,M}$ Ans





$$f = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot C}}$$

Case 1: ($f_L > f_s$)

Assume $IF = 500K$

$IF = f_L - f_s = 500K$

NOW, For AM, Range is $f_s \to 550K$ to $1650K$

1. when $f_s = 550K \Rightarrow f_L = 1050K$

So, $f_r = \frac{1}{2\pi\sqrt{(L_1+L_2)C_1}}$ $\Rightarrow 1050 = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot C(max)}}$ — (1)

2. when, $f_s = 1650K$; $\Rightarrow f_L = 2150K$

So, $2150 = \frac{1}{2\pi\sqrt{(L_1+L_2)C(min)}}$ — (2)

So, $eq^n(2) \div eq^n(1)$ we get:

$$\boxed{\frac{C_{max}}{C_{min}} \simeq \left(\frac{2150}{1050}\right)^2 \simeq 4}$$

Case 2: ($f_L < f_s$)

let $IF = 500K$

$IF = f_s - f_L = 500K$

1. $f_s = 550K \Rightarrow f_L = 50K$

So, ~~$50K = \frac{1}{2\pi\sqrt{(L_1+L_2)C_{min}}}$~~ --- (1)

$50K = \frac{1}{2\pi\sqrt{(L_1+L_2)C_{max}}}$ --- (1)

∴ when $f_s = 1650K \Rightarrow f_l = 1150K$

So, $1150K = \dfrac{1}{2\pi\sqrt{(L_1+L_2)C_{min}}}$ — (2)

$eq^n(2) \div eq^n(1)$ we get

$$\boxed{\frac{C_{max}}{C_{min}} \approx \left(\frac{1150}{50}\right)^2 \approx 500}$$

Conclusion:-

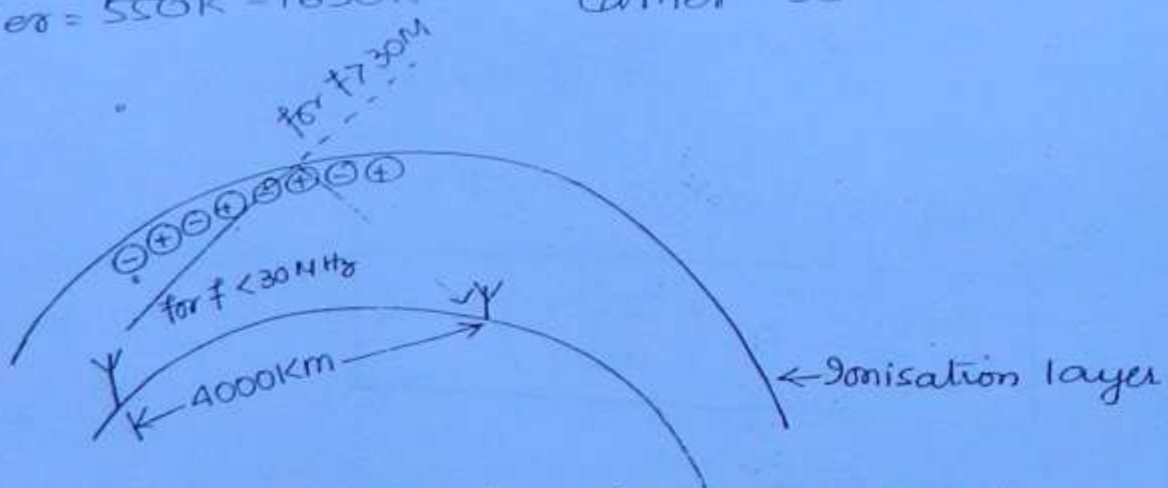
Tuning of capacitor will be easy for $f_l > f_s$. So, it is preferred.

* A.M — For long distance
  F.M — For short distance

Discussion :-

As, for AM:-
Carrier = 550K - 1650K

For FM
Carrier - 88M to 108M.

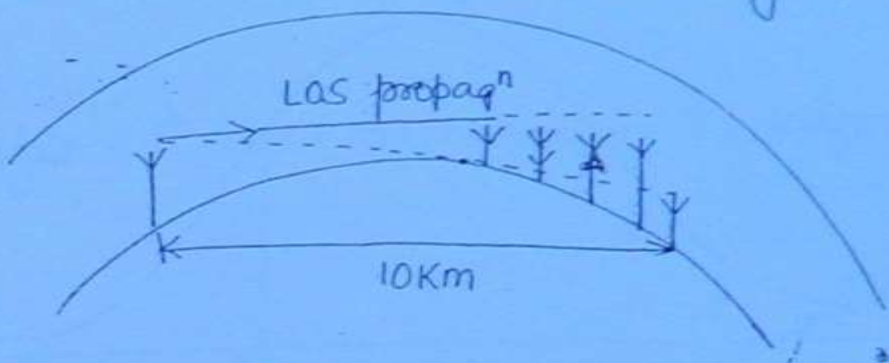


* Due to the property of Ionisation layer, if the freq^n < 30M, it will reflect back the signal.
* But if f > 30M, escapes the layer and never comes back.
* If the distance b/w xmitter and Rx is 4000 Km (upto) it is called as Single HOP propagation.

* If for distance >4000Km, transceivers are used to regenerate the signal, and is called as Ionospheric layer propagation.

173

For FM: For FM, line of Sight propagation used. The max^m distance that may be covered by FM is 10Km.



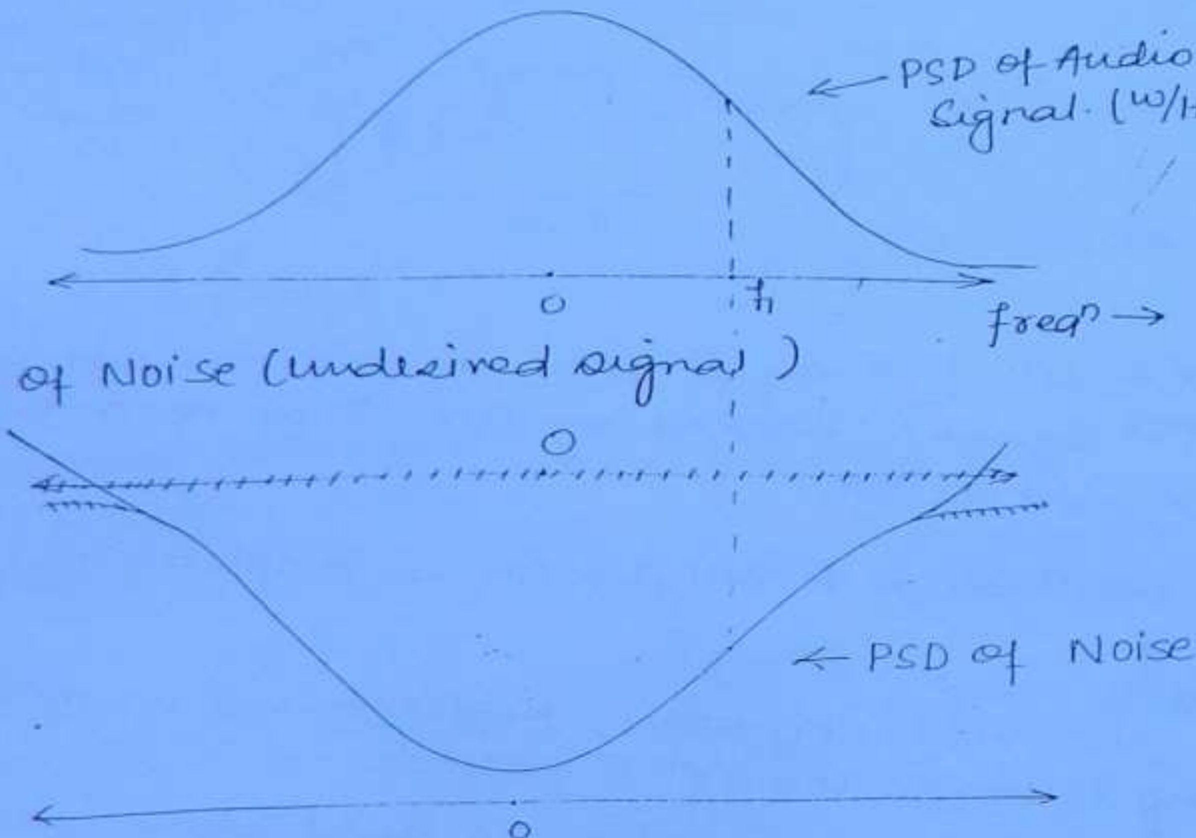
* If LOS propagation is not maintained, then the FM signal is blocked by curvature of Earth and signal is lost.

* Freq^n Reuse Technique can be Implemented.

Note:
1. For AM; Sky wave propagation is used where long distance comm^n is possible.
2. In FM; LOS propagation is used, where short dist. comm^n is possible.
3. Freq^n Reuse concept can be effectively implemented in FM xmission.

* PRE-EMPHASIS & DE-EMPHASIS: (174)

As we know that the PSD of Audio Signal is given by:



Analysis:

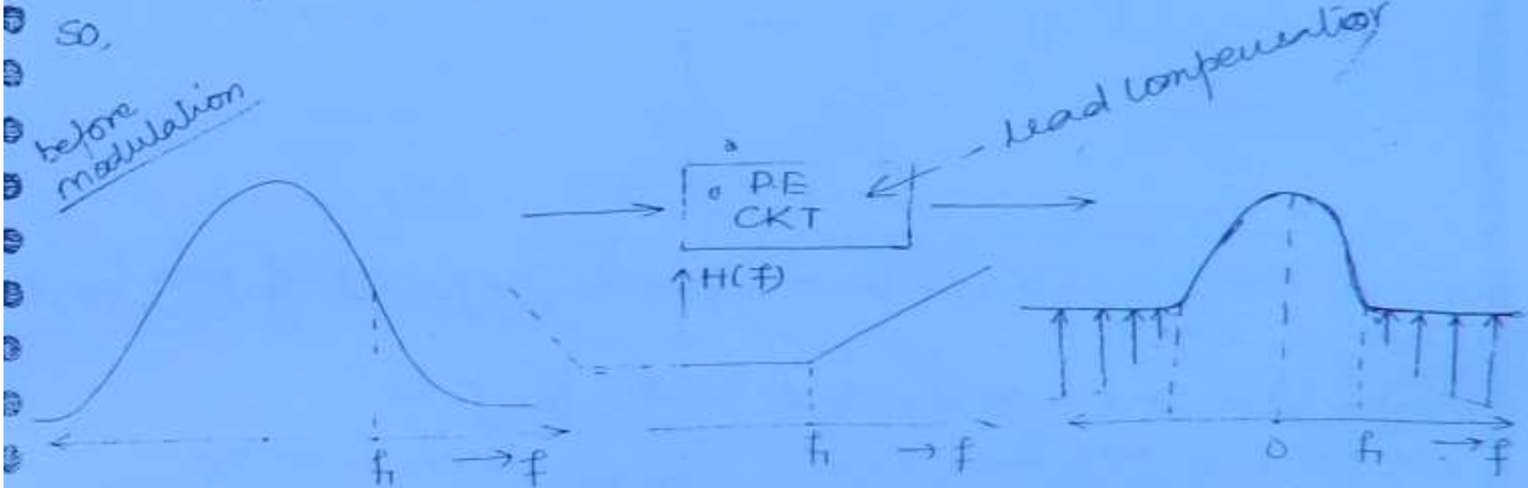
* upto f_1; $\frac{S}{N} > 1$; these freq^n components can be Reproduced at the Rx output
* Above f_1; $S/N < 1$; these high freq^n components cannot be Reproduced at the Rx output
* Hence, for high freq^n, :.
  i) either S power has to be increased
  ii) decreasing N power (but it is Indeterministic).
* So, artificially increasing the signal strength at high freq^n is called as PRE-EMPHASIS.

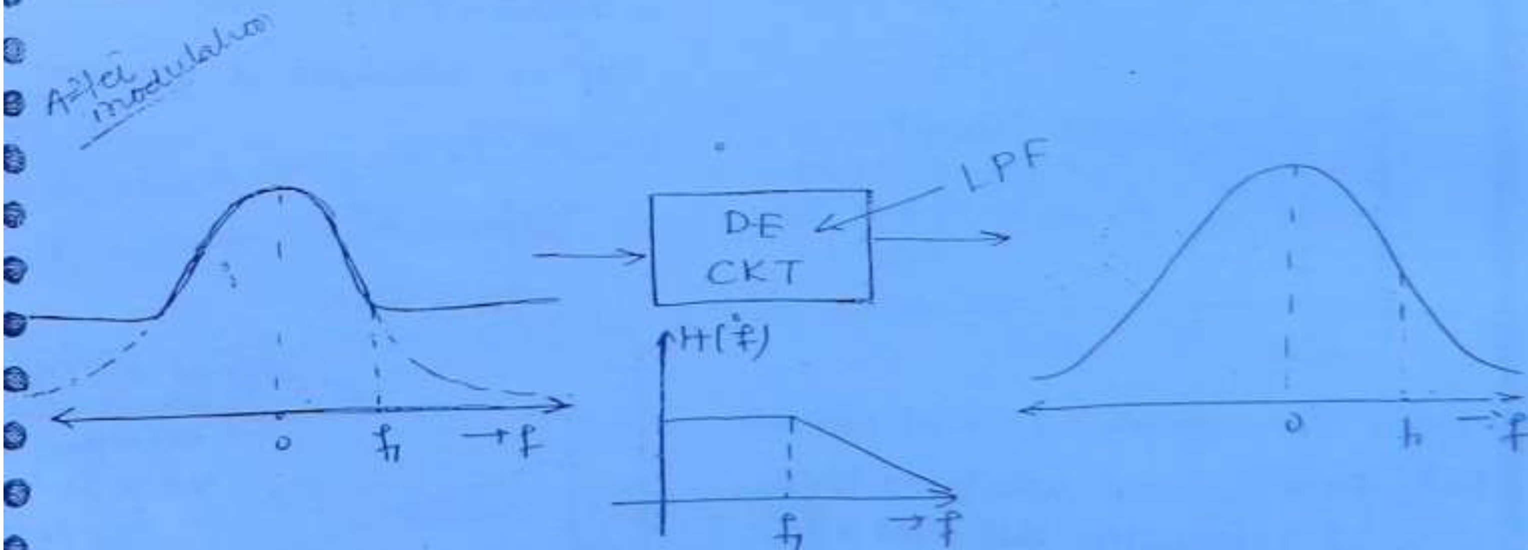
* PRE-EMPHASIS:-

* To improve fidelity S/N of high freqn components of the Audio signal has to be increased. (175)

* The process of increasing the strength of high freqn components of Audio signal is called as PRE-EMPHASIS

* Pre-Emphasis will be done in Tx before modulation so,





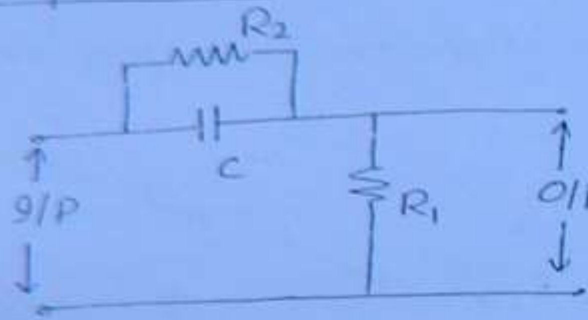
DE-EMPHASIS:-

* It is the process of decreasing the strength of High freqn components of Audio signal.
* De-emphasis will be done in the Rx after demodulation.

× LPF:

176

× Lead compensator:

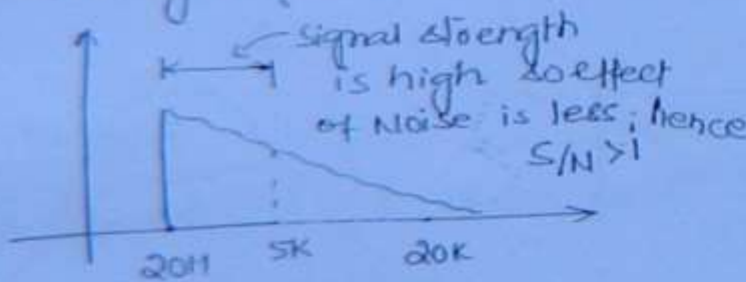


Q. Why Pre-emphasis & De-emphasis needed in FM but not in AM?

As for AM the standard AM B.W is 10 KHz

AM B.W = 10 KHz

So, Mesg B.W = 5 KHz



Hence, Pre-emphasis not needed.

But, freq^n above 5K has less signal strength. so S/N < 1.

Now for voice signal Range is 300Hz to 3.5K, hence the S/N > 1. So, it can be xmitted very Easily w/o any effect of Noise

As, for F.M

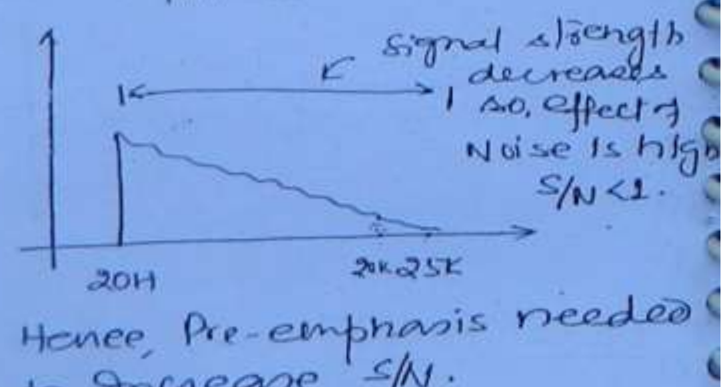
B.W = 200 KHz

$2(\Delta f + f_m) = 200K$

Δf ie standard for FM is 75K Hz.

So $2f_m = 50K$

$f_m = 25K$



Hence, Pre-emphasis needed to increase S/N.

Note:
In AM Transmission, low freq^n of Audio signal is only considered for Transmission. So, PE & DE are not Read.

* In FM transmission the high freq of Audio signal also will be considered for xmission; and for high freqn S/N < 1 so PE & DE are Required.

(177)

* F.M RECIEVERS:

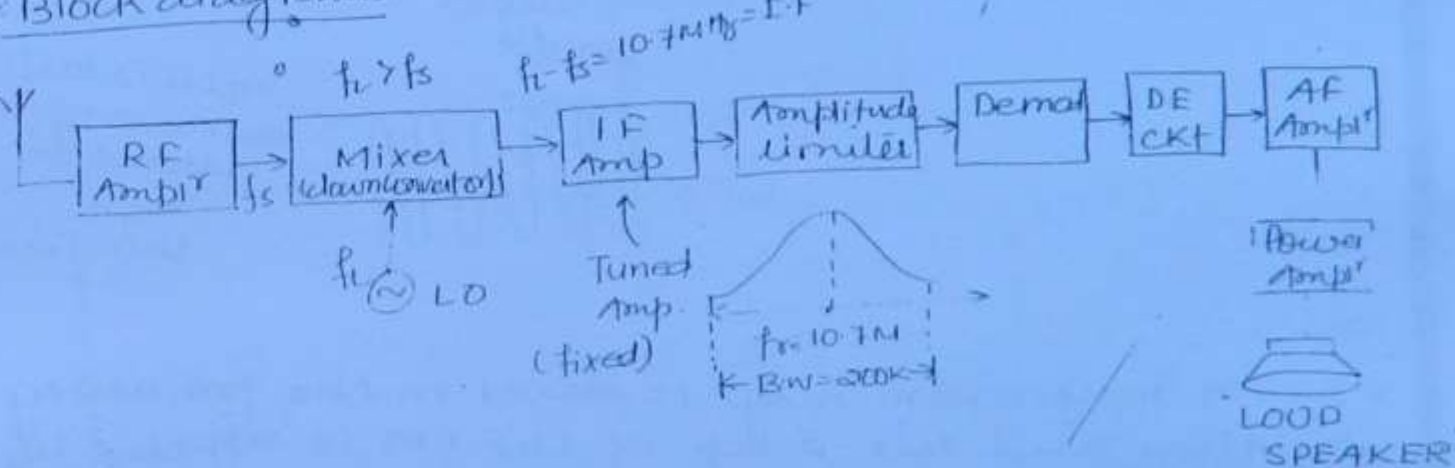
For FM, the standards are:

Carrier freqn = 88 M to 108 MHz.

F.M BW = 200 kHz.

I.F = 10.7 MHz.

* Block diagram:

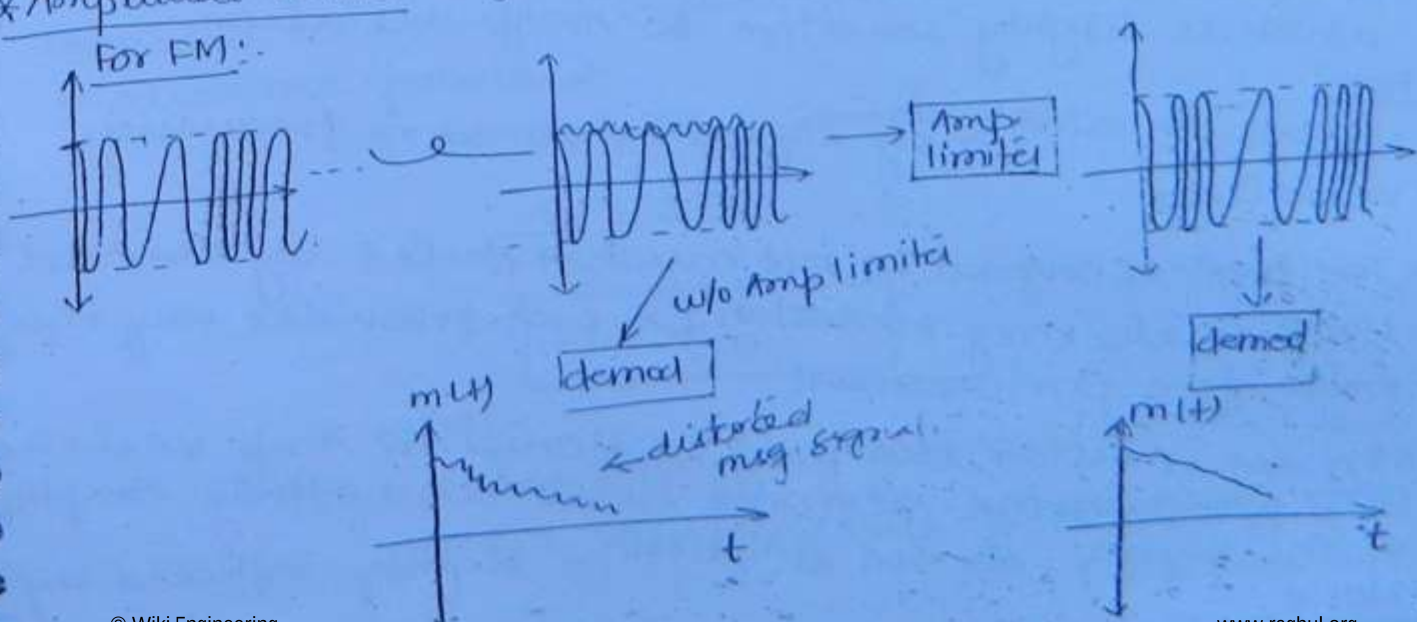


Note:

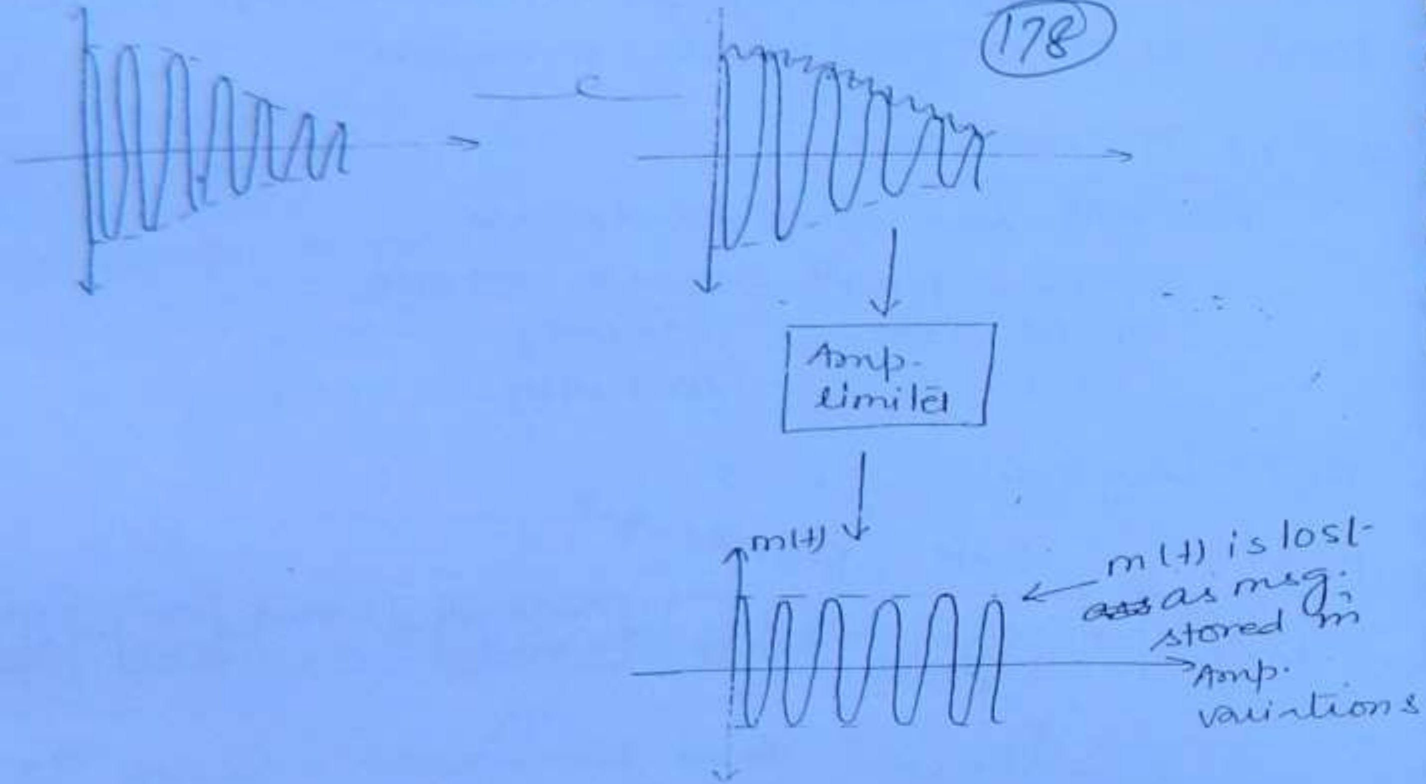
As the tuning of the Rx (ie f_s) changes, correspondingly f_L has to be changed such that; $\boxed{f_L - f_s = I.F = 10.7\,MHz}$

* Amplitude limiter: ← gives a const amplitude

For FM:



* If Amp. limiter is used in AM, then



Note:

* In FM transmission msg is stored in the frequency variation and the amp. of the FM is affected by Noise during channel xmission.
* The msg can be Reconstructed, as msg stored in freqn variation, but the amp. limiter is used to Remove the variation of Amp. in channel xmission. This is done to overcome the drawback of demodulation which is highly sensitive to Amp variation.

Note:

* In FM xmission, msg signal is stored in frequency variations.
* The freqn of signal is not much affected by channel Noise. So, the msg xmission in FM provides very much Noise free environment
* In AM xmission, the msg is stored in Amp. variation and most of the channel Noise also affects Amplitude of the signal, so AM xmission is highly affected by Noise

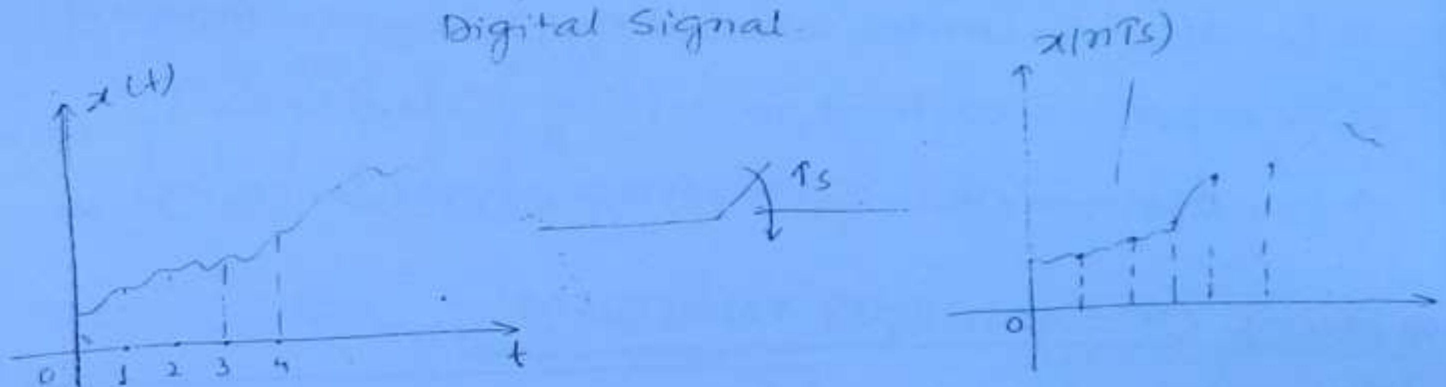
— DIGITAL COMMUNICATION —

Why Digital comm"?

→ It provides highly Noise free environment, hence widely used.

* Process to convert continuous to Digital Signal:

Continuous Signal
↓
Sampling (discrete / Sampled Signal)
↓
Quantisation
↓
coding
↓
Digital Signal



Sampling Theorem:

A continuous signal Bandlimited to fm Hz can be converted to sampled signal eqvt. without any information loss provided

$$f_s \geq 2f_m \text{ samples/sec}$$

$$\frac{1}{T_s} \geq 2f_m$$

$$\boxed{T_s \leq 1/2f_m \text{ sec}}$$

where,

T_s = Sampling interval (sec)

f_s = Sampling Rate or sampling freq^n (samples/sec).

e.g. if f_m = 1KHz

then $\boxed{f_s \geq 2000 \text{ samples/sec}}$ (188)

or $\boxed{T_s \leq 0.5 \text{ ms}}$

Note: A continuous signal perfectly reconstructed from its sampled equivalent provided;

$f_s \geq 2f_m$ sample/sec

or $T_s \leq 1/2f_m$ sec

* Depending on sampling rate; sampling is divided as:-

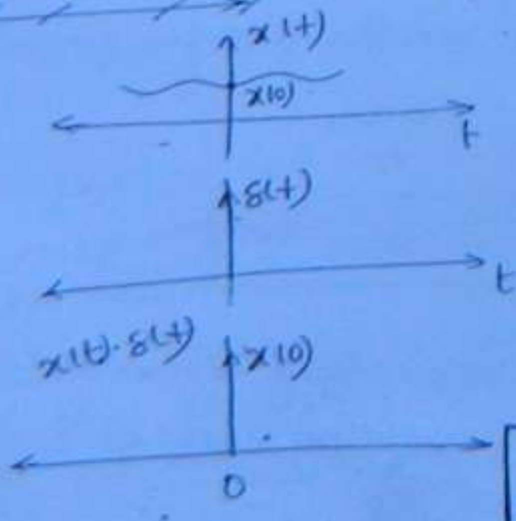
1) $f_s < 2f_m$ → under sampling. × (causes Alaising)
2) $f_s = 2f_m$ → critical sampling (Ideal cases)
3) $f_s > 2f_m$ → over sampling (Practical cases)

* PROOF OF SAMPLING THEOREM:-

let,

$$\boxed{\text{As, } \sum_{n=-\infty}^{\infty} \delta(t-nT_s) \xleftrightarrow{F.T} f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)}$$

Note



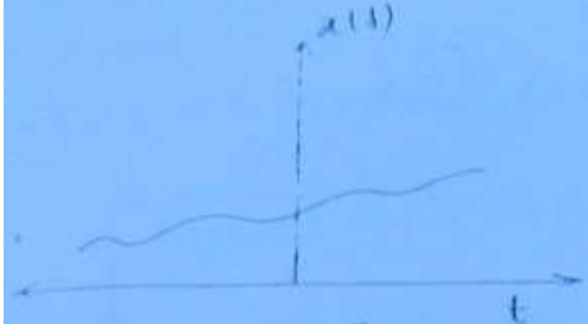
$x(t)\cdot\delta(t) = \infty;\ t = 0$
$= 0;\ t \neq 0$

$x(t)\cdot\delta(t) = x(0)\cdot\delta(t)$

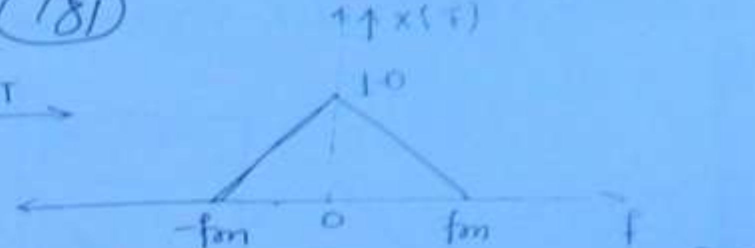
$x(t)\cdot\delta(t-t_0) = x(t_0)\cdot\delta(t-t_0)$

Impulse train;

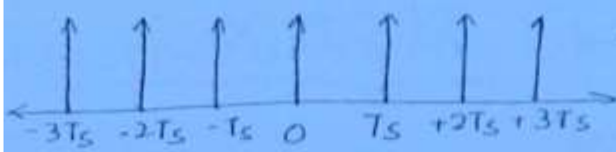
$$\boxed{\sum_{n=\infty}^{\infty} \delta(t-nT_s) = \cdots + \delta(t+T_s) + \delta(t) + \delta(t-T_s) + \cdots}$$



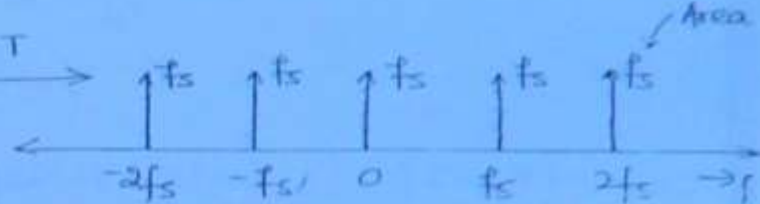
$\xleftrightarrow{F.T}$



$$\sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$

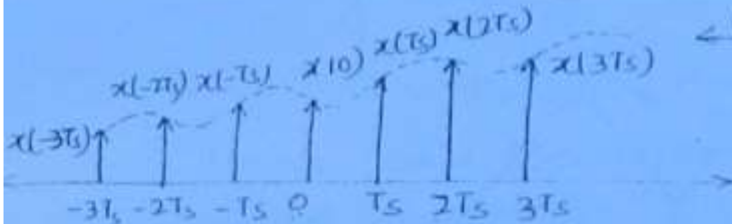


$\xleftrightarrow{F.T}$

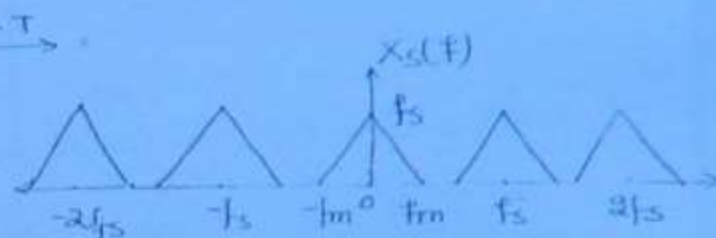


$$\boxed{\sum_{n=-\infty}^{\infty} \delta(t-nT_s) \longleftrightarrow f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s)}$$

$$x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$$



$\xleftrightarrow{F.T}$



NOW, $x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$

$$X_s(f) = X(f) * f_s \sum_{n=-\infty}^{\infty} \delta(f-f_s)$$

$$\boxed{X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)}$$

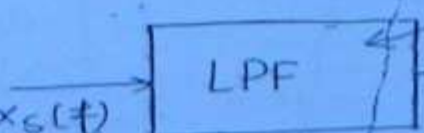
Time domain multiplication ; freq^n domain convolution.

$x(t) * \delta(t) = x(t)$

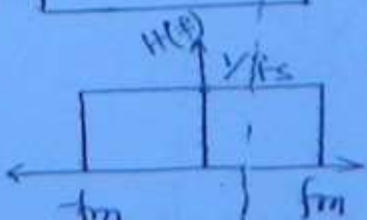
$x(t) * \delta(t-t_0) = x(t-t_0)$

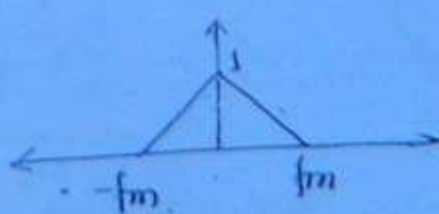
$$X_s(f) = \dots + X(f+f_s) + X(f) + X(f+f_s).$$

Reconst^n filter



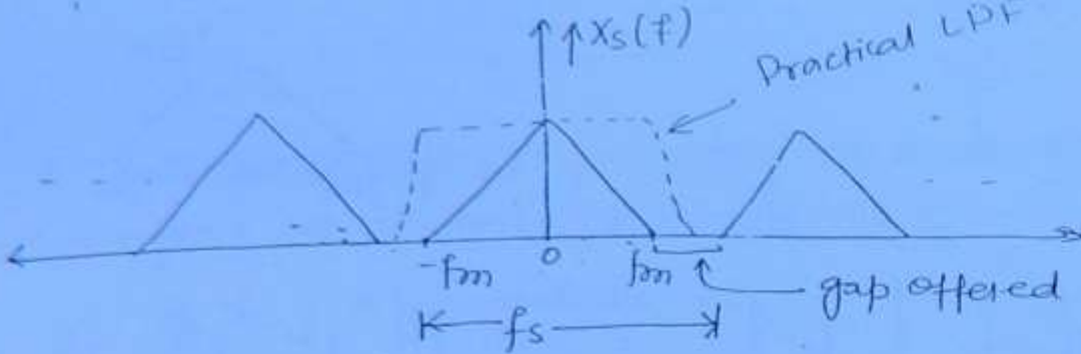
$X_s(f) \cdot H(f) = X(f)$



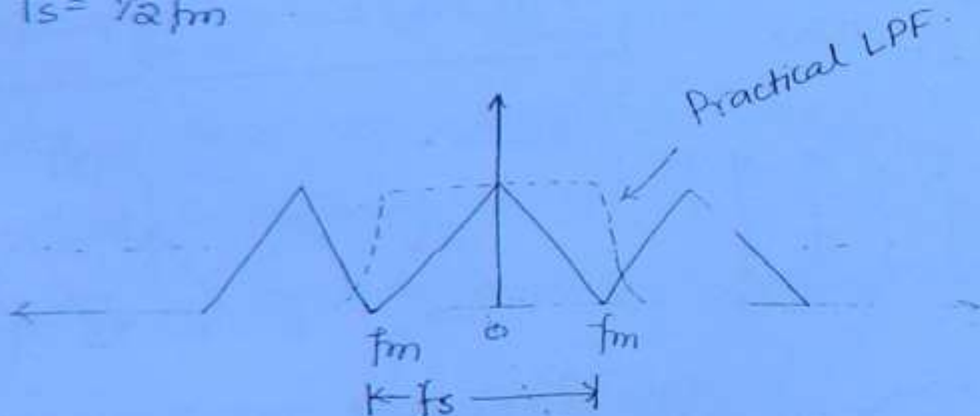


Case1 ($f_s > 2f_m$): over sampling

182



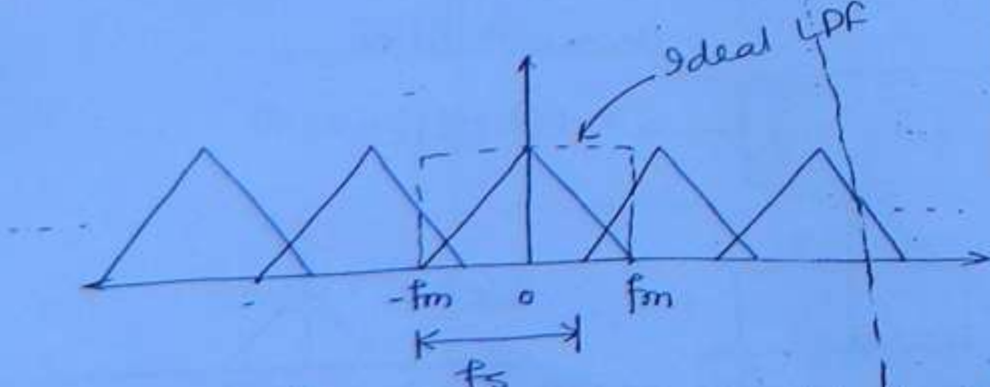
Case2 ($f_s = 2f_m$): Critical sampling
$T_s = 1/2f_m$



Since Practical LPF is available so, the o/p obtained is as shown above.
Hence the critical sampling is preferred for Ideal Sampling

Case3: ($f_s < 2f_m$); under sampling → Causes ALAISING.

$f_s < 2f_m$
or
$T_s > 1/2f_m$ sec

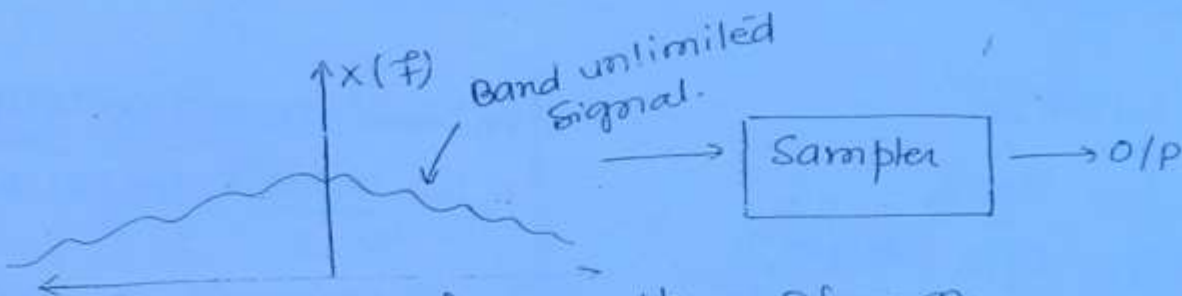


using the Ideal LPF, we cannot obtain the original msg and undesired frequencies are obtained. Hence not preferred

* Hence min^m^ sampling rate and max^m^ sampling interval to be maintained to avoid ALIASING are called as "NYQUIST RATE" & "NYQUIST INTERVAL" Resp.

$$\boxed{\begin{array}{l}\text{Nyquist Rate} = 2f_m \text{ samples/sec.} \\ \text{Nyquist interval} = \frac{1}{2f_m} \text{ sec}\end{array}}$$

* ANTI-ALIASING FILTER:



$f_m = \infty$; then $2f_m = \infty$

Hence sampler should take ∞ samples/sec to avoid aliasing

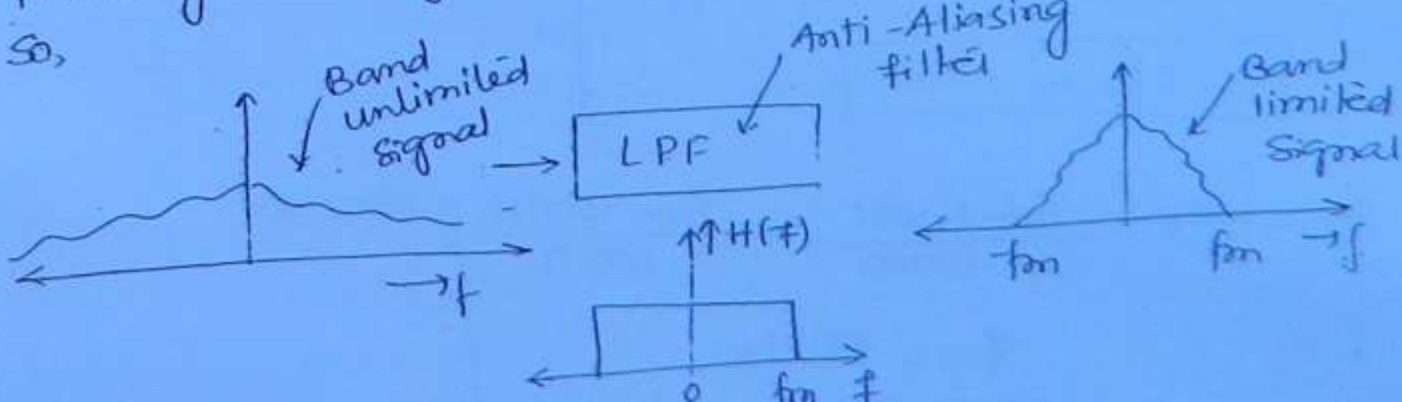
But for practical sampler

f_s = finite value

So, $\boxed{f_s < 2f_m}$

To avoid this we have to Band-limit the signal, ie passing through a filter (Called Anti Aliasing filter)

So,



Q) Find Nyquist Rate of the following

i) $10\sin 8\pi\times10^3 t$

ii) $6\sin 4\pi\times10^3 t + 8\cos 12\pi\times10^3 t$

iii) $\sin 4\pi\times10^3 t \cdot \cos 12\pi\times10^3 t$

iv) $\sin^2 4\pi\times10^3 t \cdot \cos 12\pi\times10^3 t$

v) $\mathrm{Sinc}\,100t$

vi) $\mathrm{Sinc}^2 100t$

vii) $\mathrm{Sinc}\,400t \cdot \mathrm{Sinc}\,600t$

viii) $\mathrm{Sinc}\,400t * \mathrm{Sinc}\,600t$

184

Soln:

i) $10\sin 8\pi\times10^3 t$

$f_m = 4\,KHz$

So, $\boxed{f_s = 2f_m = 8\,KHz}$

ii) $6\sin 4\pi\times10^3 t + 8\cos 12\pi\times10^3 t$

$f_{m1} = 2K \qquad f_{m2} = 6K$

$$\boxed{\begin{aligned} f_s &= 2f_{max} \\ &= 12K \end{aligned}}$$

iii) $\sin 4\pi\times10^3 t \cdot \cos 12\pi\times10^3 t$

$= \frac{1}{2}\{\sin 16\pi\times10^3 t - \sin 8\pi\times10^3 t\}$

$f_{m1} = 8K \qquad f_{m2} = 4K$

$\boxed{f_s = 2f_{max} = 16K}$

iv) $\sin^2 4\pi\times10^3 t \cdot \cos 12\pi\times10^3 t$

$\frac{1-\cos 8\pi\times10^3 t}{2} \cdot \cos 12\pi\times10^3 t$

$= \frac{\cos 12\pi\times10^3 t}{2} - \frac{\cos 8\pi\times10^3 t \cdot \cos 12\pi\times10^3 t}{2}$

$= \frac{\cos 12\pi\times10^3 t}{2} - \frac{1}{4}\{\cos 20\pi\times10^3 t + \cos 4\pi\times10^3 t\}$

$$\boxed{\begin{aligned} f_s &= 2f_{max} \\ &= 2\times10K = 20K \end{aligned}}$$

v) $\mathrm{Sinc}\,100t = \frac{\sin \pi 100t}{\pi\times100} = \frac{\sin 100\pi t}{100\pi}$

$f_m = 50\,Hz$

$\boxed{f_s = 100\,Hz}$

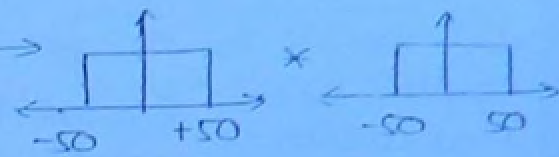
sinc 100t = A rect(f) ... property)

$\tau = 100$

$f_{max} = 50\,Hz$

So, $\boxed{f_s = 100\,Hz}$

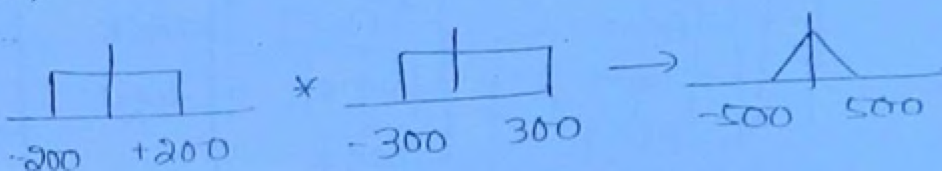
vi) $\text{sinc}^2\{100t\} = \text{sinc}\,100t \cdot \text{sinc}\,100t \longleftrightarrow$



$f_{max} = 100$

$\boxed{f_s = N \cdot R = 200\,Hz}$

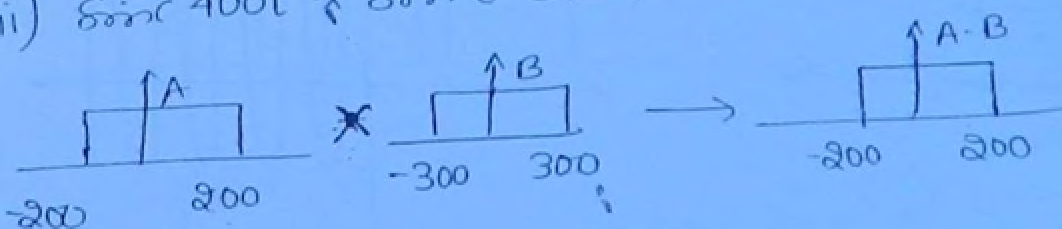
vii) $\text{sinc}\,400t \cdot \text{sinc}\,600t$



$f_{max} = 500\,Hz$

$\boxed{f_s = NR = 1000\,Hz}$

viii) $\text{sinc}\,400t * \text{sinc}\,600t \longrightarrow \text{sinc}$



$f_{max} = 200\,Hz$

$\boxed{f_s = 400\,Hz}$

Modulation

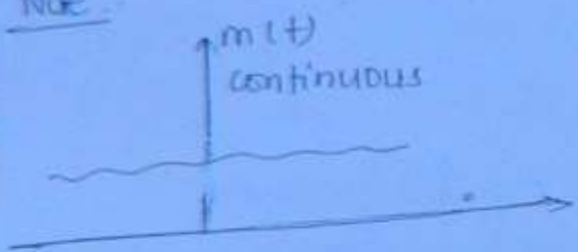
(186)

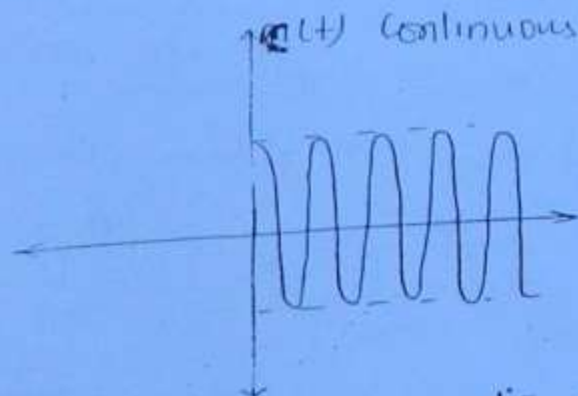
- Continuous
  - Amplitude modulation
    - AM
    - DSB
    - SSB
    - VSB
  - Angle modulation
    - FM
    - PM
- Digital
  - Base Band modulation
    - Pulse Analog Modulation
      - PAM
      - PWM
      - PPM
    - Pulse digital modⁿ
      - *** PCM
      - DPCM
      - *** DM & ADM
  - Band Pass modulation
    - Binary Signalling Modⁿ
      - ASK
      - PSK
      - FSK

      (Num)
    - No. of Symbols (M) M-Array Signalling
      - 4 Array PSK
      - 8 Array PSK

      Prob. of error IES (Sub)

Note:





In continuous modulation, one of the parameter of continuous signal will be varied continuously in accordance with msg signal voltage variations.

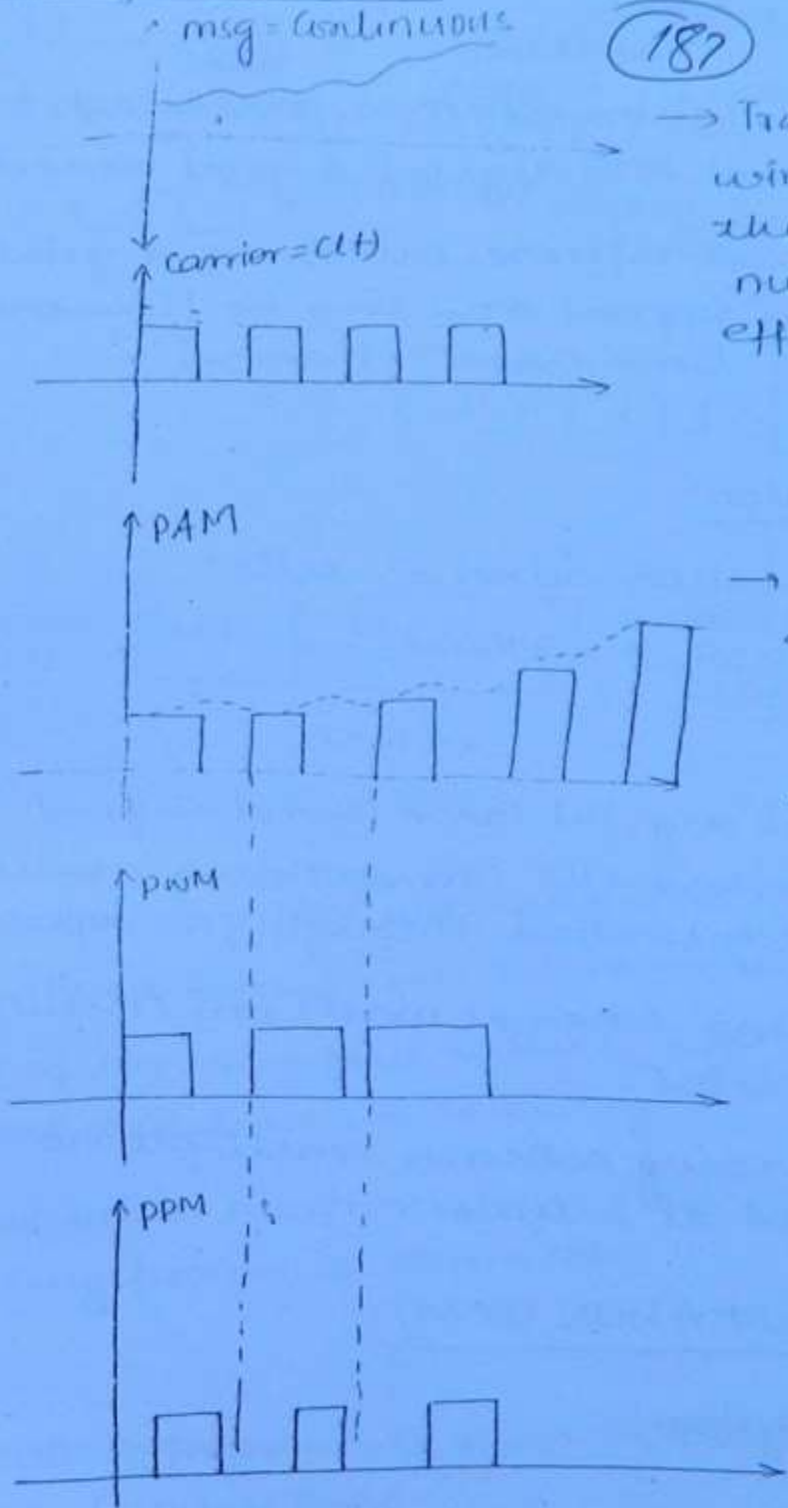
Note:

1. Base Band signals will be transmitted directly through wired channel. so, it is also called as BASE BAND channel or low pass channel.

2. Band Pass signal will be transmitted through free space. so free space is also called as Band Pass

Pulse Analog modulation:-

187



→ Transmitted through wired channel and as the amplitude is continuous and hence the effect of Noise is high.

→ PAM, PWM & PPM are also transmitted through wired n/w but the amp variation is not continuous. so effect of Noise is less

Note:-

In Pulse modulation, Pulse by pulse xmission is exploited where each of the pulse corresponds to Base Band signal. So these modulation schemes are called as BASE BAND Modulation.

Note: (Pulse digi l modulation).

1. Pulse digital modulation schemes are used to convert a continuous signal as, digital signal equivalent.

2. The electrical signal representation of digital correspo to a Base Band signal and can be transmitted directly through Base Band channel.

*Band Pass Modulation:

* Band Pass modulation schemes like:

i) ASK

ii) PSK

iii) FSK

are used to convert digital Base Band signal as and the corresponding modulated Band Pass signal and the corresponding modulated signal can be transmitted through free space.

* In Binary signalling scheme (modⁿ) one (1) bit is transmitted at a time.

* In M-ARRAY signalling schemes multiple no. of bits will be transmitted at a time.

* PULSE CODE MODULATION (PCM):

* PCM transmission system:

* Function of LPF:

i) Band limit the signal

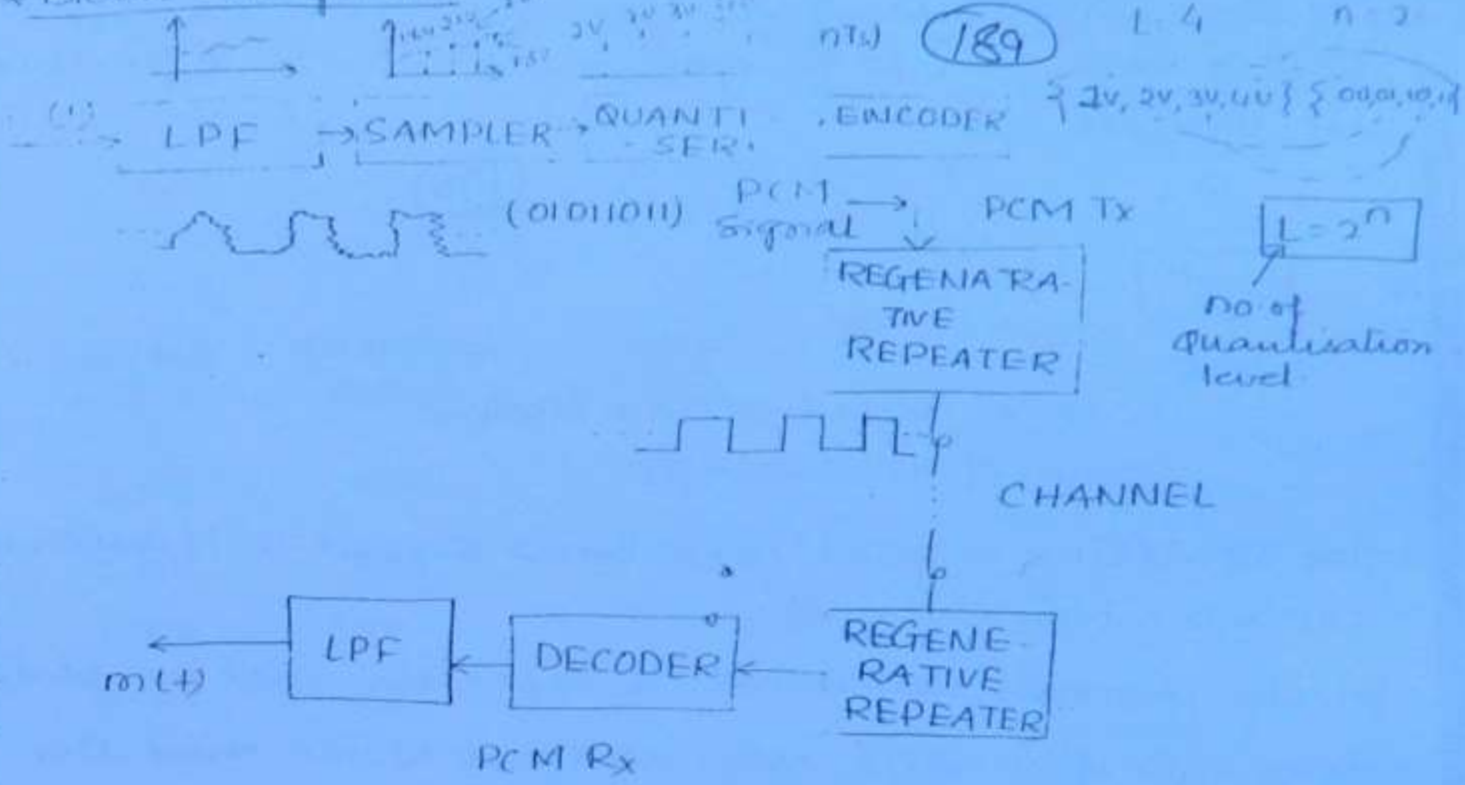
ii) avoids the "ALAISING."

* Function of RR (Regenerative Repeater)

i) to eliminite the chainel noise which distorts the amp. of the signal

ii) to regenerate the fresh copy of the original signal after removing the effect of ~~eff~~ Noise on its amplitude.

* Block diagram:



Note:

* LPF before sampler will work as "ANTI ALAISING" Filter
* The filtered Signal will be oversampled by the sampler
* Each of the sampled values will be Rounded off to nearest Quantization level by the "QUANTISER"
* Quantization mechanism is irreversible, but encoding & decoding process is Reversible.

Note:

* ENCODER represents each of the Quantized level by a unique Binary code.

* Importance of Quantiser:

lll, $f_m = 1 MHz$

$f_s = 2 \times 10^6$ samples

So, over a period of time a large no. of sample has to be taken and to encode such large no. of sample is not possible. Hence, Quantiser is needed.

Note

* If there is no Quantiser, numerous no of Unique Binary codes has to be generated which is practically not possible

(190)

* $\boxed{L=2^n}$

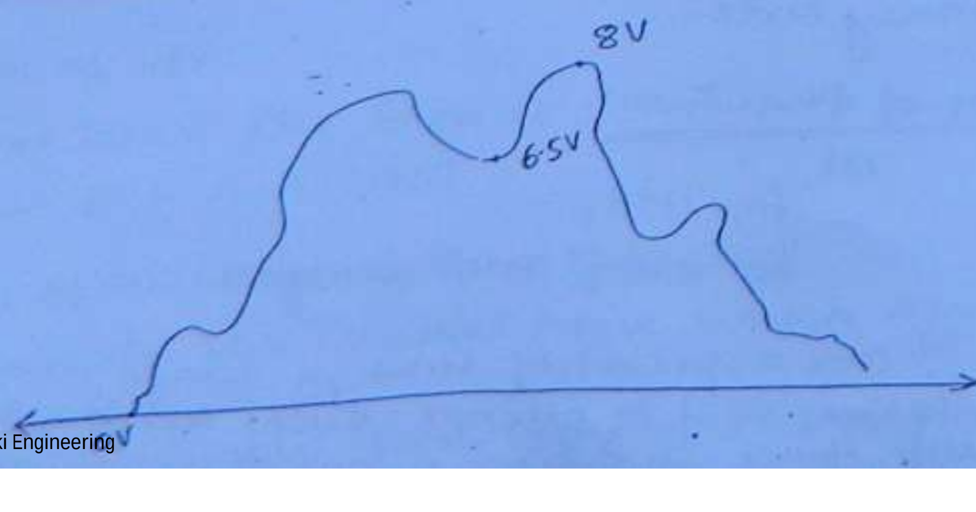
where,

L = no of Quantisation levels.

n = no. of bits/sample.

* The Resulting digital base band Signal is transmitted through wired channel.
* In the channel, Regenerative Repeaters will be placed
* These RR eliminates total channel Noise and the fresh copy of transmitted signal will be Regenerated.
* Decoder will do the Reverse of Encoder.
* The O/P of decoder will be the quantised signal & is given to Reconstruction filter (LPF)
* In the Reconstructed Signal, finite amount of the quantisation error will be permanently Retained.
* Quantisation process is Irreversible, and ENCODING process is Reversible.

*QUANTISATION PROCESS:



* 2 bit encoder

$n = 2$ bits/sample

$L = 4$

191

8V

Q 6V → 11

Q 4V → 10

$\boxed{\Delta = 2V}$ Q 2V → 01

Q 0V → 00

$\boxed{q.V = 0V, 2V, 4V, 6V}$

Sampled value	quantised value	Encoder O/P	$Qe = S.V - q.V$
0.5V	0V	00	0.5V
2.6V	2V	01	0.6V
6.8V	6V	11	0.8V
7.9V	6V	11	1.9V

$$[Qe]_{max} = \Delta$$

Note:

* The total dynamic range of the system is divided into 'L' equal no. of steps.
* The Bottom of Each step is assumed as Quantisation level
* The sampled value in each specific step will be Rounded off to Bottom of the step.
* In this Quantisation process; $[Qe]_{max} = \Delta$.
* In the following Quantisation process $[Qe]_{max}$ is decreased to $\pm \Delta/2$.

8V

Q = 7V → 11

6V

Q = 5V → 10

4V

Q = 3V → 01

2V

$\Delta = 2V$ Q = 1V → 00

0V

Quantisation voltages (values) = 1V; 3V; 5V; 7V

S.V	Q.V	Encoder O/P	Qe: S.V ~ q.V
0.8	1V	00	0.2
2.2V	3V	01	0.8
7.9V	7V	11	0.9
7V	7V	11	0

192

Note :

If
$S.V = 0V$; then $q.V = 1V \Rightarrow Q_e = 1V$
$S.V = 8V$; then $q.V = 7V \Rightarrow Q_e = 1V$

* The total dynamic Range of the signal is divided into L equal no. of steps.
* The middle of each step will be assumed as the quantisation level.
* The sampled value in a specific step will be Rounded off to middle of the step or to the nearest Quantisation level.
* In this process,

$$[Q_e]_{max} = \pm \Delta/2.$$

* BASIC FORMULA'S IN PCM SYSTEM:

let,
n = no. of bits per sample
L = No. of Quantisation level.

$$\boxed{L = 2^n}$$

$$\boxed{\Delta = \frac{V_{max} - V_{min}}{L}} \Rightarrow \boxed{\frac{V_{peak\ to\ peak}}{2}}$$

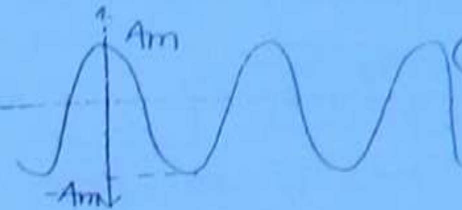
Vmax

L = 4.

Vmin

" $m(t) = A_m \cos 2\pi f_m t$ or $A_m \sin 2\pi f_m t$

So, $\boxed{\Delta = \frac{2A_m}{L} = \frac{2A_m}{2^n}}$



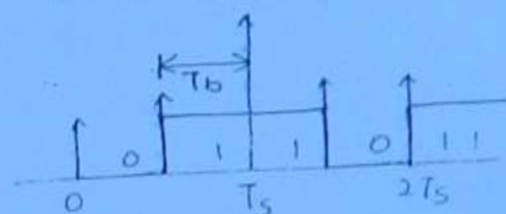
* $\boxed{\text{Quantised Error; } Q_e = \text{Sampled value} - \text{Quantised value}}$

So, $\boxed{[Q_e]_{max} = \pm \Delta/2}$

, let $n = 2$ bits/sam

* $\boxed{\text{Bit Duration, } T_b = T_s/2}$

$\boxed{T_s = 2T_b}$



So, for n no. of bits;

$\boxed{\text{Bit duration, } T_b = T_s/n}$

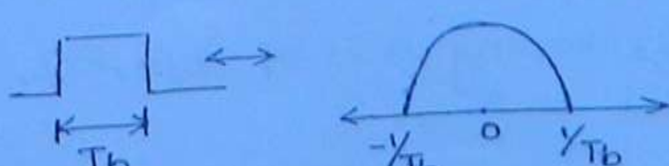
* Bit Rate ; $R_b = \text{bits/sec} = \frac{\text{bits}}{\text{sample}} \times \frac{\text{sample}}{\text{sec}}$

$\boxed{R_b = n f_s}$

or $\boxed{R_b = \frac{n}{T_s} = \frac{1}{T_b}}$

* <u>Max^m Transmitter B.W:-</u>

one bit is Transmitted at a time



$B.W = \frac{1}{T_b}$ - ①

$\boxed{B.W = \frac{1}{T_b} = R_b}$

* 2 bits are transmitted at a time

194

$$\boxed{B.W = \frac{1}{2T_b} = \frac{R_b}{2}}$$

* 3 bits are Transmitted at a time

$$\boxed{B.w = \frac{1}{3T_b} = \frac{R_b}{3}}$$

Note:

* As the time domain signal width increases, then the frequency domain signal width decreases.

So, $\boxed{[BW]_{max} = R_b}$

***

* Transmission B.W or Min Tx B.W:

* In practical cases, min^m 2 bits are transmitted in a Bunch, hence the channel B.W required is $\boxed{R_b/2}$.

Note:

$$\boxed{Min^m \; T_x \; B.W = R_b/2}$$

* In this, if the channel has B.W of $R_b/2$; then it will allow the signals having B.W of $R_b/3$, $R_b/4$ ... R_b/n Hence the min^m B.W Required is $R_b/2$.

Q: msg signal of 10cos8πx10³t is transmitted through it PCM system. Find all the parameters of the PCM system?

(195)

Soln: Given,

$m(t) = 10\cos 8\pi \times 10^3 t \Rightarrow A_m = 10V$

$f_m = 4KHz$

"bits read to xmit 1 sample" $\longrightarrow n = 4$ bits/sample.

So,

$L = 2^n = 16$ quantisation levels.

Now,

1) step size : $\Delta = \frac{V_{max} - V_{min}}{L}$

$$\boxed{\Delta = \frac{20}{16} V} \text{ Ans}$$

∵ Sampling Rate is not mentioned. so let it be equal to Nyquist Rate

So,

ii) $\boxed{f_s = N.R = 2f_m = 8K}$ Ans

iii) $\boxed{[Qe]_{max} = \pm \Delta/2 = \pm \frac{20}{32} \text{ volts}}$ Ans

iv) $R_b = nf_s$

$= 4 \times 8$

$\boxed{R_b = 32K}$ Ans

v) $\boxed{T_b = 1/R_b = \frac{1}{32K} = \frac{1}{32} \text{ msec}}$ Ans

vi) $\boxed{[BW]_{max} = R_b = 32K}$ Ans

vii) $\boxed{B.W = \frac{R_b}{2} = 16K}$ Ans

Q2. A sinusoidal msg signal of peak voltage 20V and having freqⁿ of 5KHz is transmitted through 256 level PCM system. Sampling Rate is 25% high than N.R. Find all the parameters?

(196)

Solⁿ: Given, $A_m = 20V$; $f_m = 5KHz$

$L = 256$ level ; $f_s = N.R + 25\%$ of $N.R$

So, $L = 2^n = 256$

$2^8 = 256 = 2^n$

$\boxed{n = 8}$

Now, $N.R = 2f_m = 10KHz$

So, $f_s = 10K + \frac{25}{100} \times 10000$

$\boxed{f_s = 12.5K}$

So,

i) $\boxed{\Delta = \frac{V_{P-P}}{L} = \frac{2 \times 20}{256} = \frac{40}{256} \text{ volts} = 0.15625 V}$

ii) $\boxed{[Q_e]_{max} = \pm \Delta/2 = \pm 20/256 \text{ volts}}$

iii) $\boxed{R_b = n f_s = 100 \text{Kbps}}$

iv) $\boxed{T_b = 1/R_b = 0.01ms}$

v) $\boxed{[BW]_{max} = R_b = 100K}$

vi) $\boxed{B.W = R_b/2 = 50K}$

Q3. A msg signal of $8\cos 2\pi \times 10^4 t$ is transmitted through a PCM system. Sampling Rate is 50% higher than NR. Maxᵐ Quantization error can be atmost of 0.1% of peak amp. of msg. signal. Find all parameters?

Sol: Given

$m(t) = 8\cos 2\pi \times 10^4 t$

$A_m = 8$; $f_m = 10\,KHz$

197

So, $[Q_e]_{max} \le \pm\frac{\Delta}{2} = \frac{0.1}{100} \times 8 = 0.008$ $\{\because \Delta = 0.1\%\ of\ A_m\}$

Now,

$\pm\,\Delta \ge 0.016$

$= \frac{V_{P-P}}{L} \ge 0.016$

So, $L \le \frac{16 \times 1000}{0.016}$ $\Rightarrow 2^n \le 1000$

$\boxed{n = 10}$ Ans

~~$L < 1000 \le 2^n \Rightarrow \boxed{n = 10}$~~

Now, $f_s = 1.5 \times N.R$

$= 1.5 \times 20K$

$\boxed{f_s = 30K}$ Ans

$[Q_e]_{max} = \pm\frac{\Delta}{2} = \frac{8}{1024 \times 2}$

$= \frac{16}{2048}$ volts

ii) $R_b = nf_s$

$\boxed{R_b = 300\,Kb/sec.}$ Ans

iii) $\boxed{T_b = 1/R_b = \frac{1}{300}\,ms}$ Ans

iv) $\boxed{[B.W]_{max} = R_b = 300K}$ Ans

$\boxed{B.W = R_b/2 = 150K}$ Ans

Q4.. A Sinusoidal msg signal is transmitted through PCM system where $[Q_e]_{max}$ can be atmost of 2% of Peak to peak amp. of msg signal. Find no. of bits per sample reqd?

Soln: $[Q_e]_{max} \le \Delta/2 = 2\%\ of\ 2A_m$

$\frac{1}{2}\left\{\frac{2A_m}{2^n}\right\} \le \frac{2}{100}A_m$ ~~$\frac{2A_m}{2^n \times 2} \le \frac{2}{100} \times 2A_m \Rightarrow$~~

$\Rightarrow 1/2^n \le 1/25 \Rightarrow 2^n \ge 25 \Rightarrow n = 5$

Q. A sinusoidal msg signal of $4 \sin 4\pi \times 10^3 t$ is transmitted through 8 level PCM system. Sampling Rate is 5 times N.R

(198)

i) Find all the parameters?

ii) Given sampled values $-3.2V, -2.8V, -0.1V, 1.5V, 3.9V$. Find corresponding Quantiser and Encoder O/P?

iii) Plot the Quantiser characteristics?

Soln: Given,

$m(t) = 4 \sin 4\pi \times 10^3 t$

$A_m = 4V \; ; \; f_m = 2KHz$

i)

$L = 8 \Rightarrow n = 3$ bits/sample

1) $f_s = 5 \times 2 f_m$

$\boxed{f_s = 20KHz}$

2) $\boxed{\Delta = \frac{V_{PP}}{L} = \frac{8}{8} \text{ volts} = 1 \text{ volts}}$

3) $\boxed{[Q_e]_{max} = \pm \Delta/2 = \frac{\pm 1}{2} \text{ volts} = \pm 0.5 \text{ volts}}$

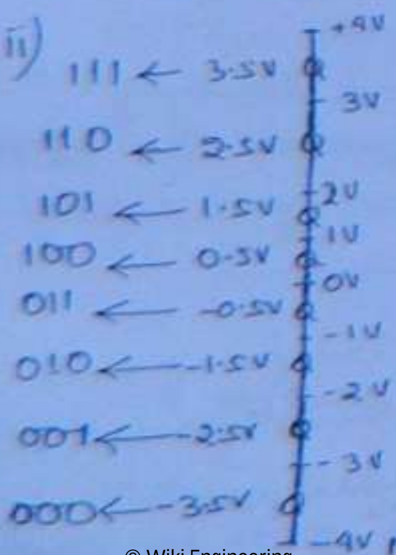
4) $\boxed{R_b = n f_s = 3 \times 20K = 60 Kbps}$

5) $\boxed{T_b = 1/R_b = \frac{1}{60} ms}$

6) $\boxed{[B.W]_{max} = R_b = 60K}$

$\boxed{B.W = R_b/2 = 30K}$

ii)



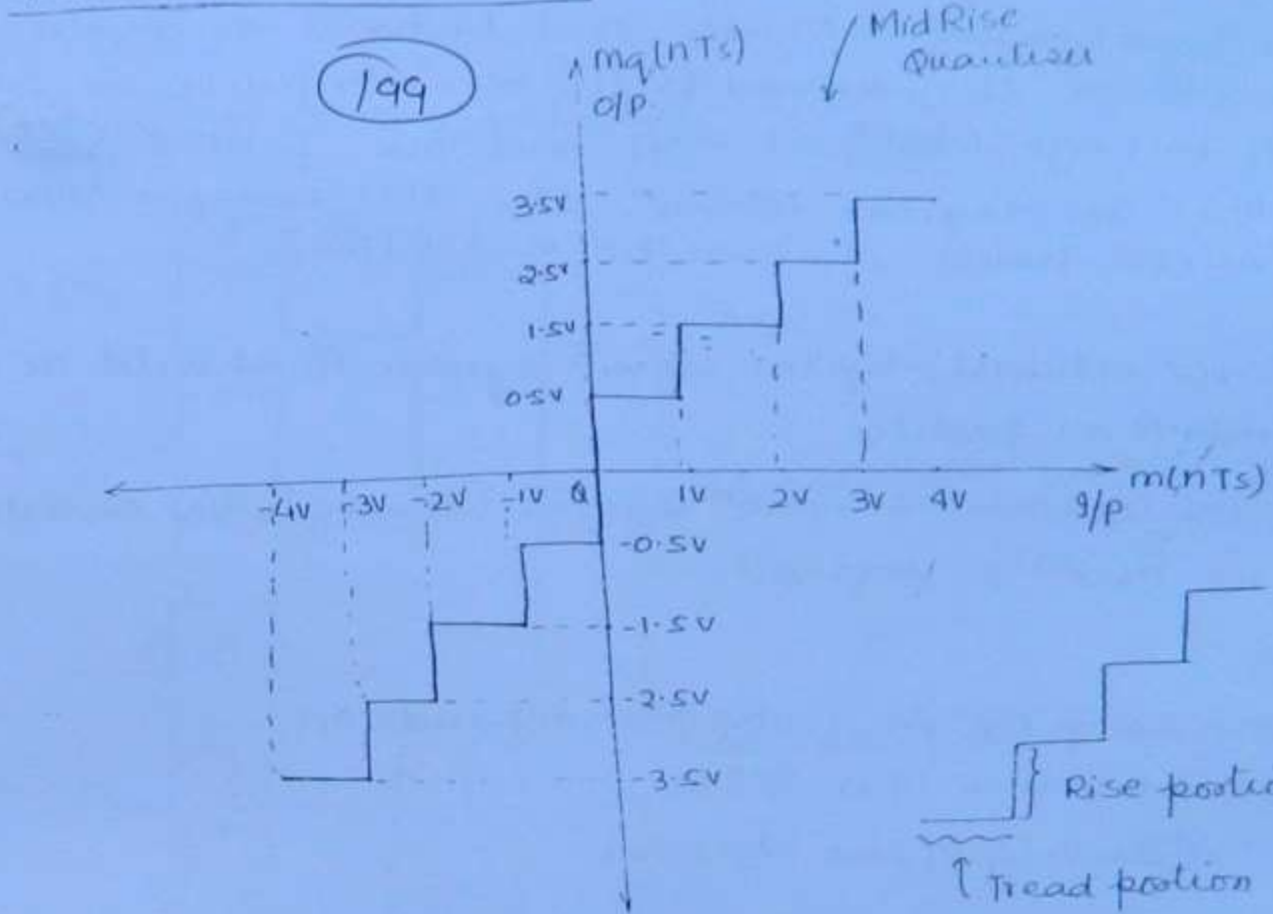
Quantiser O/P

Now, the sampled values are :-

S.V	Q.V	Encoder O/P	Qe = S.V.-
-3.2V	-3.5V	000	0.3 V
-2.8V	-2.5V	001	-0.3 V
-0.1V	-0.5V	011	0.4V
1.5V	1.5V	101	0V
3.9V	3.5V	111	0.4V

Encoder O/P

iii) Quantiser characteristics :-



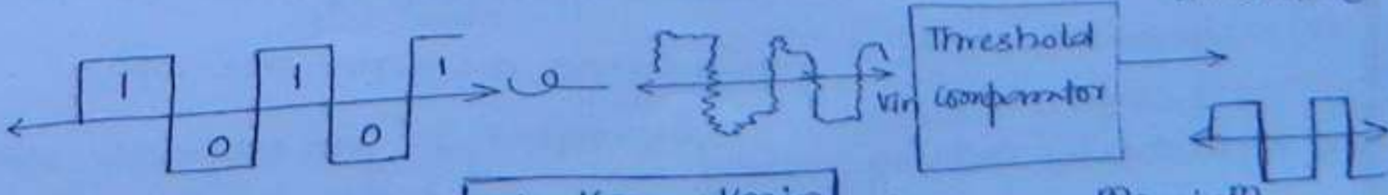
Note :

* If the origin lies in the Rise (middle) portion, then Quantiser called as Mid Rise Quantiser.
* If the origin lies in the middle of Tread portion, then Quantiser is called as Mid-Tread Quantiser (No importance)

* RE-GENERATIVE REPEATER (RR) : (no significance in obj ES - Subj)

* Digital Transmission provides very much Noise free environment due to Regenerative Repeater
* RR is nothing but, threshold comparator.

Vin > Vth = 1
Vin < Vth = 0



$$\boxed{V_{th} = \frac{V_{max} + V_{min}}{2}}$$; avg of max^m & min^m voltage

Note

* Regenerative Repeater can't be used in Analog comm as bcos the signal {S(t)} takes infinite amount of voltage levels at diff instants of time. But in digital commⁿ, the S(t) sweeps b/w +5V or -5V, hence it can be used (RR)

1. For efficient digital commⁿ system Pe should be as minᵐ as possible
2. For efficient continuous commⁿ system (Analog), S/N should be as maxᵐ as possible

Note:

1. The no. of RR in a channel depends on
   i) distance b/w Tx & Rx
   ii) Quality of the channel

* Electrical Representation of Binary Signals:

The electrical Representation schemes available are:

i) ON-OFF $0 \to 0V$, $1 \to +ve$

ii) NRZ $0 \to -ve$, $1 \to +ve$

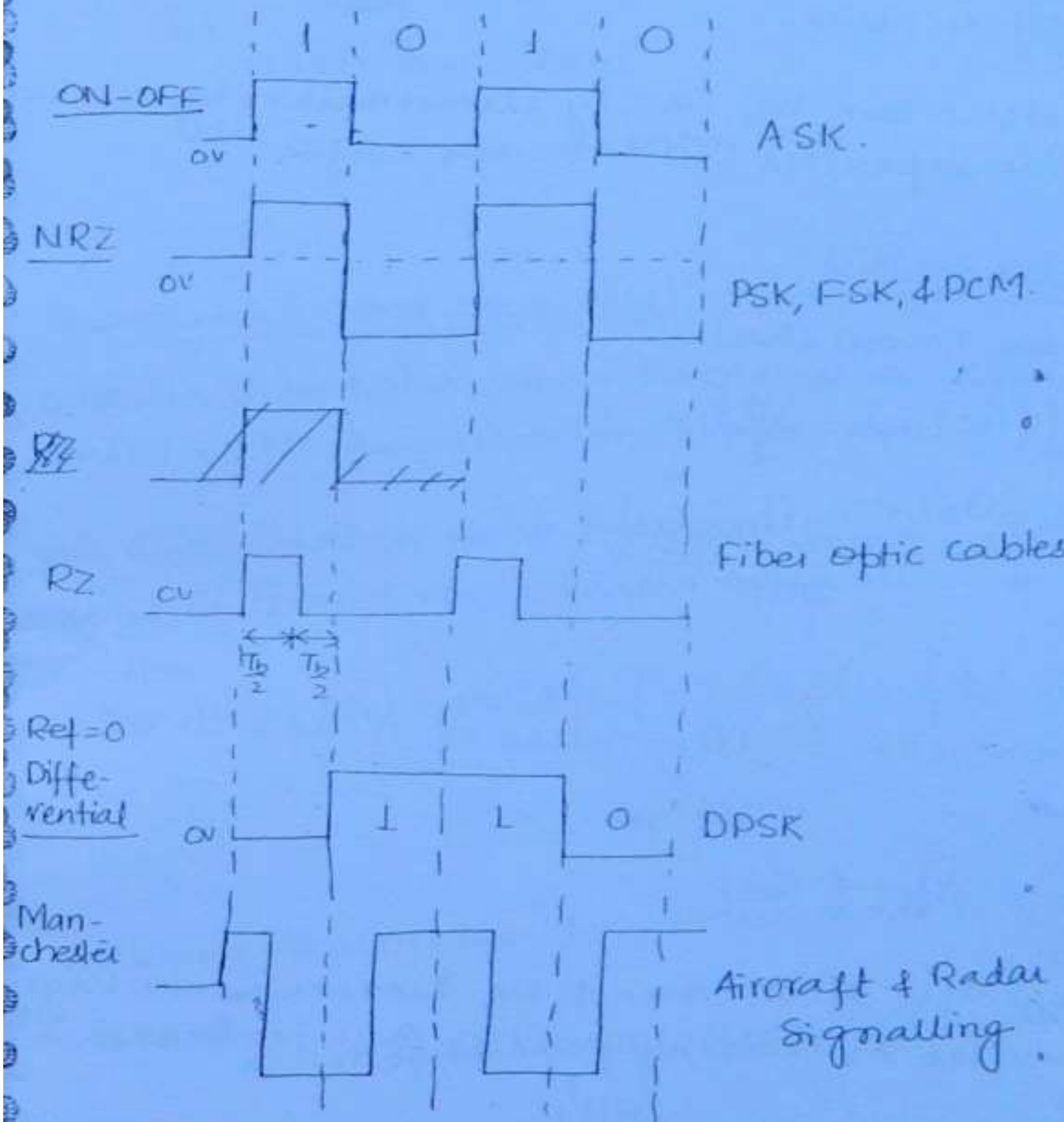
iii) RZ $0 \to 0V$

$1 \to$ first $T_b/2 \to +ve$, next $T_b/2 \to 0V$

iv) differential coding: $0 \to$ Complement of Previous O/P; $1 \to$ Same as previous O/P.

v) Manchester coding: $0 \to -ve, +ve$; $1 \to +ve, -ve$

* Analysis of Coding Techniques: (201)



Note:

d.E.S

For Generation of ~~RZ~~, X-NOR gate is used.

0 0 → 1
0 1 → 0
1 0 → 0
1 1 → 1

whenever the next bit is 0, the O/P corresponds to the complement of the other bit and for 1, the same bit will be the O/P.

* For Generation of differential encoded signal, X ... gate will be used.

* Noise in PCM System:

Noise in PCM system is classified as:

1) Channel Noise

2) Quantisation Noise.

RO2

* Channel Noise can be easily eliminated by using Regenerative Repeaters (RR).

* Quantisation Noise:

As, we know that

$$\downarrow [Qe]max = \frac{\Delta \downarrow}{2}$$

Now, $\downarrow \Delta = \frac{V_{max} - V_{min} \downarrow}{2^n \uparrow}$

Note:

1. To decrease the Δ, the value of n has to be increased.

ie $n\uparrow \longrightarrow \Delta\downarrow \longrightarrow Qe\downarrow$

2. But the value of 'n' cannot be increased to high values, because increasing 'n', the B.W is increased.

ie

$$n\uparrow \longrightarrow B.W\uparrow = \frac{nf_s}{2}$$

Hence, the value of 'n' should be such that:-

1) B.W is not high

2) Qe is min^m

* In PCM system as the Qe↓, correspondingly B.W requirements will be increased.

This is the drawback of PCM system.

* For a PCM system, it should be noted that ... both of Qe and B.W requirements will be satisfied

**ES

* Signal to Quantisation Noise power Ratio (SQNR):-

203

let,
$m(t) = A_m \cos 2\pi f_c t$

So, Signal power, $S = \frac{A_m^2}{2R}$.

Quantisation Noise power (Nq):-

Quantisation Noise power is called as the power associated with Quantisation Noise.

* As, Quantisation Noise is undeterministic in nature. So it is treated as Random Variable.

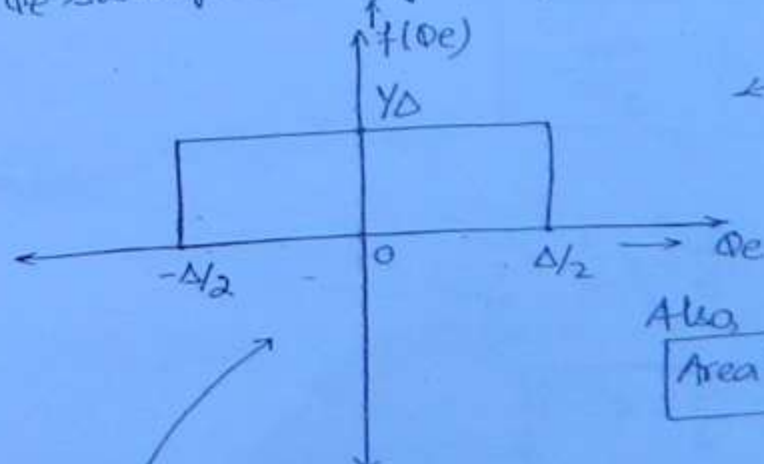
Now,
$$N_q = \text{Power}\{Q_e\} = \text{msq}\{Q_e\} = E\{Q_e^2\} = \int Q_e^2 \, f(Q_e) \, dQ_e$$

Expectation

probability density funcⁿ of Qe.

Following Assumption has to be made:-

1. Assume, Qe satisfies uniform probability density funcⁿ.



← Probability of Error is uniform.

Also
$\boxed{\text{Area}\{\text{Pdf}\} = \text{Probability}}$

And,
Area $\{F(Q_e)\} = 1$

So,

$$N_q = \int_{-\Delta/2}^{\Delta/2} q_e^2 \cdot \frac{1}{\Delta} \cdot dq_e$$

204

$$= \frac{1}{\Delta} \cdot \frac{q_e^3}{3} \Bigg|_{-\Delta/2}^{\Delta/2}$$

$$= \frac{1}{3\Delta} \left\{ \frac{\Delta^3}{8} + \frac{\Delta^3}{8} \right\}$$

$$= \frac{1}{24\Delta} \times 2\Delta^3$$

$$\boxed{N_q = \frac{\Delta^2}{12}}$$

For, $m(t) = A_m \cos 2\pi f_m t$

So, $\Delta = \frac{2A_m}{2^n}$

So, $N_q = \frac{1}{12} \left\{ \frac{2A_m}{2^n} \right\}^n$

$$= \frac{1}{12} \times \frac{4A_m^2}{2^{2n}}$$

$$\boxed{N_q = \frac{1}{3} \cdot \frac{A_m^2}{2^{2n}}}$$

Now,

$$SQNR = \frac{S}{N_q} = \frac{A_m^2}{2} \Big/ \frac{1}{3} \cdot \frac{A_m^2}{2^{2n}}$$

***

$$\boxed{\uparrow \frac{S}{N_q} = \frac{3}{2} \cdot 2^{2n} \uparrow}$$

Note:

As $n \uparrow \rightarrow L \uparrow \rightarrow \Delta \downarrow \rightarrow q_e \downarrow \rightarrow N_q \downarrow \rightarrow S/N_q \uparrow$

But $n \uparrow \rightarrow B.W \uparrow$.

1. For $n = 1 \Rightarrow (S/N_q)_1 = \frac{3}{2} \cdot 2^{2 \cdot 1}$

2. For $n = 2 \Rightarrow (S/N_q)_2 = \frac{3}{2} \cdot 2^{2 \cdot 2}$

3. For $n = 3 \Rightarrow (S/N_q)_3 = \frac{3}{2} \cdot 2^{2 \cdot 3}$

Now,

$$\boxed{\left(\frac{S}{N_q}\right)_2 = 2^{2 \cdot 1} \cdot (S/N_q)_1 \qquad \left(\frac{S}{N_q}\right)_3 = 2^{2 \cdot 1} \cdot (S/N_q)_2}$$

$$\boxed{\left(\frac{S}{N_q}\right)_3 = 2^{2 \cdot 2} \cdot (S/N_q)_1}$$

Conclusion:

As n changes to $(n+K)$ hence (S/N_q) increases by $2^{2 \cdot K}$ times

* As the no of bits/samples increases from n to $(n+K)$
$\left(\frac{S}{N_q}\right)$ is increased by 2^{2K} times

$$\boxed{n \rightarrow (n+K) \Rightarrow (S/N_q)_{n+K} \rightarrow 2^{2 \cdot K} (S/N_q)_n}$$

* $(S/N_q)_{dB} = ?$

$$\left(\frac{S}{N_q}\right)_{dB} = 10\log_{10}(S/N_q)$$

$$= 10\log_{10}\left\{\frac{3}{2} \cdot 2^{2n}\right\}$$

$$= 10\log 3/2 + 10\log_{10} 2^{2n}$$

$$\left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6.02n$$

$$\boxed{\left(\frac{S}{N_q}\right)_{dB} \approx (1.8 + 6n)\,dB}$$

Note

n	$(S/N_q)_{dB}$
1	(7.8) dB
	13.8 dB
2	
3	19.8 dB

6dB

6dB

12dB

206

As, the no. of bits / sample increased from n to (n+K) the SQNR is increased by 6K ~~times~~ dB.

$$\boxed{n \to n+K \Rightarrow \left(\frac{S}{N_q}\right)_{dB} = 6K \cdot \left(\frac{S}{N_q}\right)_{dB}}$$

Q1. A msg signal of $8\sin 8\pi \times 10^3 t$ is transmitted through PCM System. Sampling Rate is 50% higher than NR & min^m SQNR should be 22 dB. Find

i) transmission B.W

2) $(SQNR)_{dB}$ = ?

Sol^n: Given, $m(t) = 8\sin 8\pi \times 10^3 t$

$A_m = 8;\ f_m = 4K$

So, $N \cdot R = 2f_m = 8K$

So, $f_s = 1.5\,NR$
$= 1.5 \times 8K$

$$\boxed{f_s = 12K}$$

Now, $(SQNR) = \frac{3}{2} \cdot 2^{2n}$

For $(SQNR) = \min \Rightarrow 22 \le$ ~~$\frac{3}{2} 2^{2n}$~~ $1.8 + 6n$

~~$3n - \frac{49}{32}$~~

$6n \ge 20.2$

$$\boxed{n \ge 3.36 \approx 4 \text{ bits}}$$

Now,
$$B.W = \frac{nf_s}{2}$$
$$= \frac{4\times12\times10^3}{2}$$

(207)

$$\boxed{B.W = 24\,KHz}$$ Ans.

Now, $(SQNR) = 1.8 + 6n$ dB
$= 1.8 + 24$

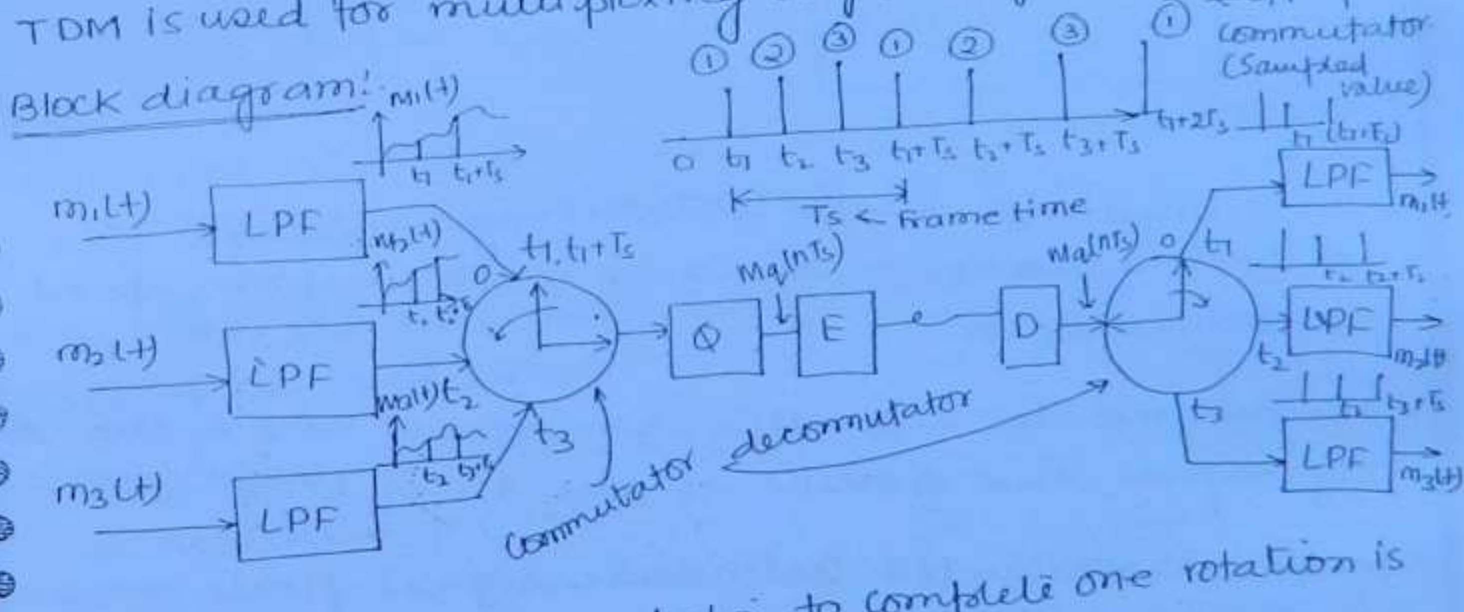
$$\boxed{(SQNR) = 25.8\,dB}$$ Ans.

* PSU's

* TIME DIVISION MULTIPLEXING:

TDM is used for multiplexing digital signals.

Block diagram:



* Time taken by commutator to complete one rotation is called as Frame time, Ts.

* LPF will work as Anti-Aliasing filters.

* Commutator is a Rotating switch which rotates in the Anticlockwise direction with uniform speed.

* The time taken by commutator to make one complete rotation is called as Frame time and it is specified as Ts (sampling interval).

* For proper opⁿ speed synchronisation has to be maintained b/w commutator & de commutator

* To ensure speed synchronisation, small 'a' amount of additional bits will be transmitted at the end of each frame time and these are called as the synchronisation bits

209

So,

$$(nN+a)T_b = T_s$$

$$\boxed{T_b = \frac{T_s}{(nN+a)}}$$

$$\boxed{R_b = (nN+a)f_s}$$

Note:

1. In FDM; carrier frequencies will be unique for the msg signals to be multiplexed.
2. In TDM; the time slots are unique for the msg signals to be multiplexed.
3. The time slot of $t_1 + KT_s$; $K = 0, 1, 2 \ldots$ is dedicated for $m_1(t)$
   $t_2 + KT_s$; $K = 0, 1, 2, \ldots = m_2(t)$
4. In FDM; sharing of channel B.W will be done whereas in TDM; no sharing of B.W.
5. FDM is complex than TDM.
6. The effect of Noise will be high in FDM as compared to TDM.

Q1 Five signals are multiplexed ~~~~ using TDM No of quantisation levels used are 256 Find xmission B.W of system

Soln: Given:

$N = 5$

$f_m = 5KHz$

$n = 8$

$L = 256$ levels.

236

$N \cdot R = 2f_m$

$$\boxed{f_s = N \cdot R = 10K}$$

$$B.W = \frac{R_b}{2} = \frac{nNf_s}{2} = \frac{8 \times 5 \times 10^4}{2}$$

$$\boxed{B.W = 200K}$$ Ans

Q2. 10 Signals each Band limited to 2K are multiplexed using TDM. Time taken by commutator to make 1 complete rotation is 125μs. Find Bit Rate of the Tx if 5 bit encoder is used.

Soln: Given.

$N = 10$

$f_m = 2K$

$T_s = 125\mu s = 1/f_s \Rightarrow f_s = 8K$

$n = 5$ bits/sample

$$\boxed{f_s = 2f_m = NR = 8K}$$

So, ~~Bit Rate~~ $R_b = nNf_s = 5 \times 10 \times 8K$
$= 400Kbps.$

$$\boxed{\text{Bit Rate}, R_b = 400Kbps}$$

Q3. 10 Sinusoidal msg Signal, each having freq^n of 10KHz are multiplexed using TDM. Sampling Rate is 25% high than NR. Max^m Qe can be atmost of .1% of Peak to Peak amp. of msg signal 5 no. of synchronisation bits are Transmitted at the end of each frame find Bit Rate of the Tx?

Sol^n: Given;

(281)

$N=10$

$f_m = 10K$

$f_s = 1.25\,NR$

$[Q_e]_{max} = 1\%$ of Peak to Peak Amplitude

$a = 5$

Now, $f_s = 1.25 \times 2 f_m$

$= 1.25 \times 20$

$f_s = 25K$

Now, $[Q_e]_{max} \leq 1\%$ of $2A_m$.

$$\frac{\Delta}{2} \leq 0.02 A_m$$

$$\frac{1}{2}\left\{\frac{2A_m}{2^n}\right\} \leq 0.02 A_m$$

$$2^n \geq 50 \Rightarrow \boxed{n=6}$$

Now,

$$R_b = (nN + a) f_s$$

$$= (10 \times 6 + 5)\,25K$$

$$\boxed{R_b = 1625\text{ Kbps}}$$ Ans

(ES subj. only)

* DIFFERENTIAL PULSE CODE MODULATION (DPCM): -

As, $\downarrow [Q_e]_{max} = \frac{\Delta \downarrow}{2}$

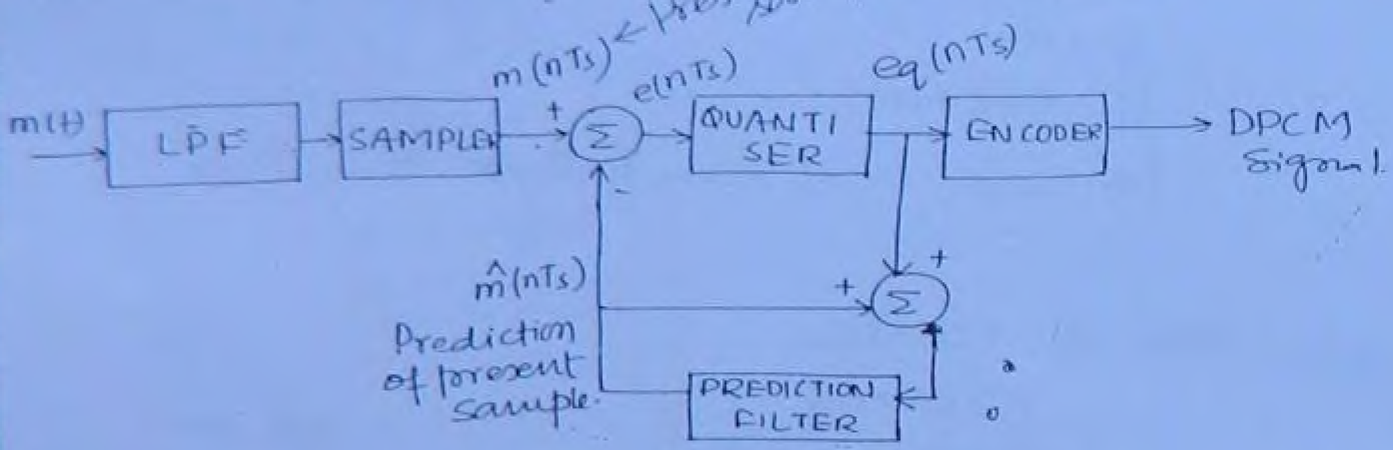
$$\downarrow \Delta = \frac{[V_{max} - V_{min}] \downarrow}{2^n}$$

* In DPCM, the dynamic Range of the signal is altered without any effect of B.W, then Q_e is Reduced.

* In DPCM the dynamic range of signal to be quantized will be decreased such that w/o affecting the B.W. requirement. Qe will be decreased

* DPCM Transmitter:

212



Prediction Filter:

(ie Past Behaviour of Signal)

* By Analysing to some finite instants, nearby future values will be predicted by the Filter

* It has numerous no. of delay elements

* It has huge storage capacity and has complex logical cktary

Op^n of DPCM Tx:

$$\boxed{(\text{Summer})_{O/P} = e(nT_s) = m(nT_s) - \hat{m}(nT_s)} \quad \text{---} (1)$$

where, $e(nT_s)$ = Prediction error

* Prediction filter will be very Precision ckt, so that dynamic Range of $e(nT_s)$ will be very small so that corresponding Qe will also be small

Now,

quantisation error → $q(nT_s) = e(nT_s) - e_q(nT_s)$ --- (2)

Prediction Filter I/P $= \hat{m}(nT_s) + e_q(nT_s)$.

from eq(2) we have $e_q(nT_s) = e(nT_s) - q(nT_s)$. so.

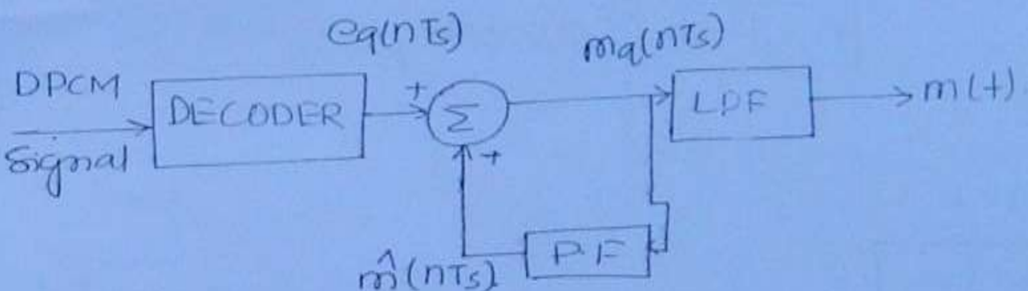
Filter I/P = $m(nT_s) + e(nT_s) - q(nT_s)$

$= m(nT_s) - q(nT_s)$ {from eq(1)}

quantised value of $m(nT_s)$ $= m_q(nT_s)$ {∵ S.V − q.V = q.e}

* All the quantised values will be stored in the Prediction filter
* By Analysing previous quantised values of the given signal, prediction filter predicts, present sample of the given signal.

* DPCM RECIEVER:



$(LPF)_{i/p} = e_q(nT_s) + \hat{m}(nT_s)$

$= m_q(nT_s)$

Note:
* For the Reconstructed msg signal, finite amount of quantisation error will be Retained, which is very small compared to PCM xmission.

* DPCM is complex than PCM.

* DELTA MODULATION: ← also called as 1 bit DPCM

* Delta Modulation needs very much min^m Tx B.W compared to PCM and DPCM. 214

As $\downarrow B.W = \frac{\downarrow n f_s}{2}$; for DM ⇒ n = 1 bit/sample.

To decrease B.W, f_s can't be decreased as to avoid the cause of undersampling $\{f_s > 2f_m\}$. ← oversample to be maintained.

Note :-

* In Delta Modulation Tx B.W Required will be decreased to ; min^m possible extent by selecting lowest possible value of 'n' ie 1 bit/sample.

$$\boxed{n = 1\ bit/sample}$$

As, $R_b = nf_s$

and in DM; n = 1

so, $\boxed{R_b = f_s}$

* In DM;

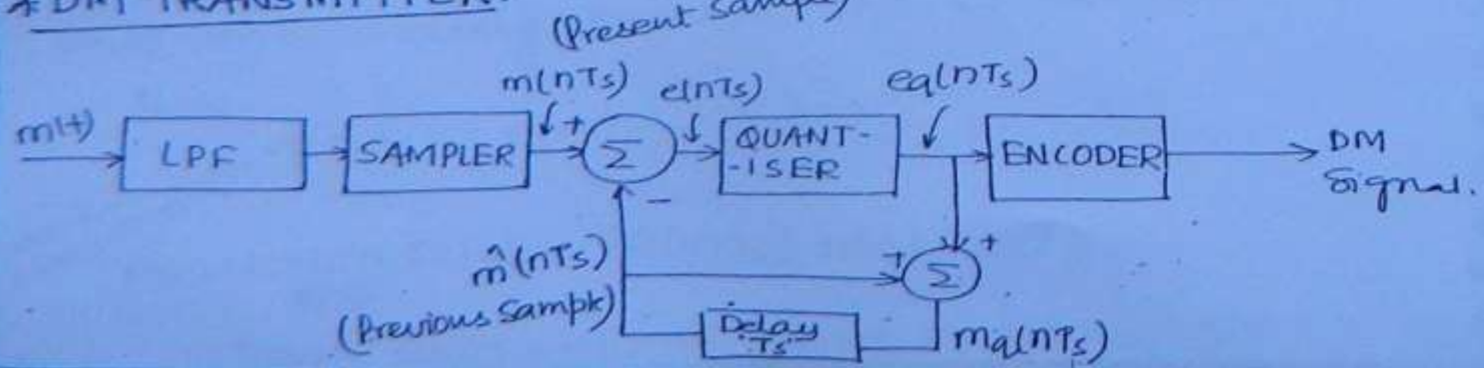
$$\boxed{\text{Sampling Rate} = \text{Bit Rate} = \text{Pulse Rate}}$$

* As; in DM ; n = 1

so, L = 2 → +Δ (step size) ≠ $V_{max} - V_{min}/L$ $\xrightarrow{\text{encoder O/P}}$ 1

→ −Δ (step size) ≠ $V_{max} - V_{min}/L$ $\xrightarrow{\text{encoder O/P}}$ 0

*DM TRANSMITTER:

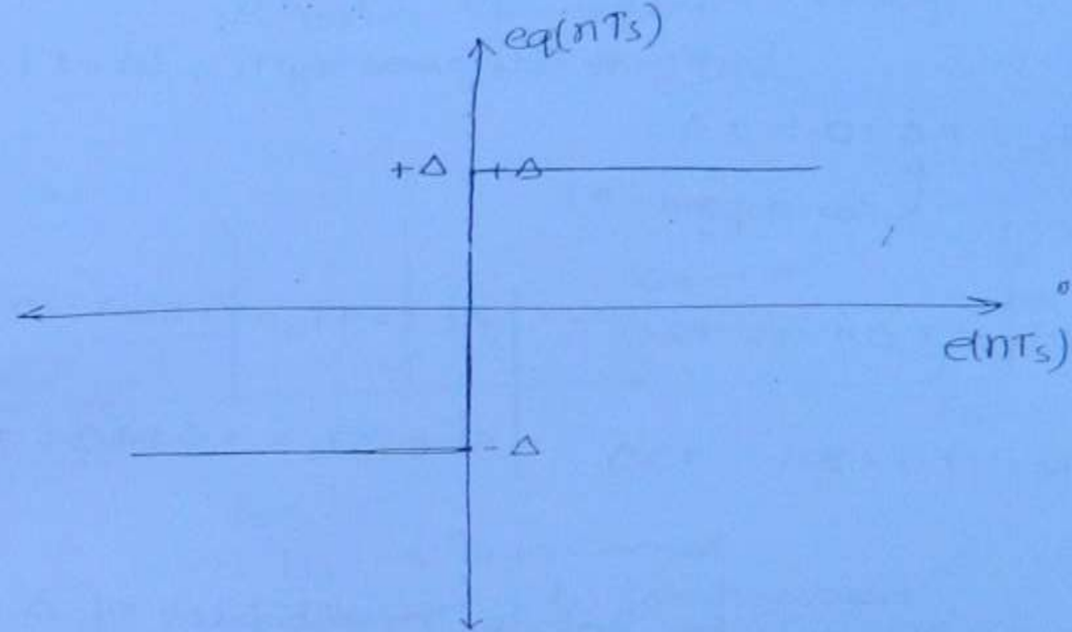


y/P of Quantiser

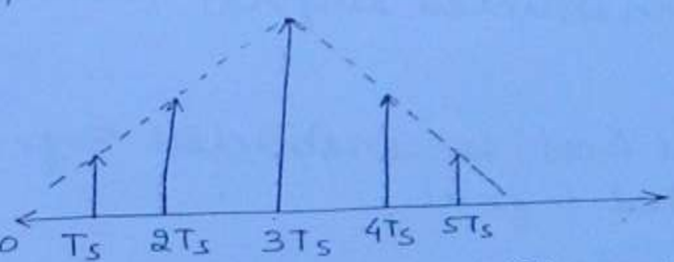
$$\boxed{e(nT_s) = \underbrace{m(nT_s)}_{\text{Present sample}} - \underbrace{\hat{m}(nT_s)}_{\text{Previous sample}}}$$

285

Quantiser characteristics:

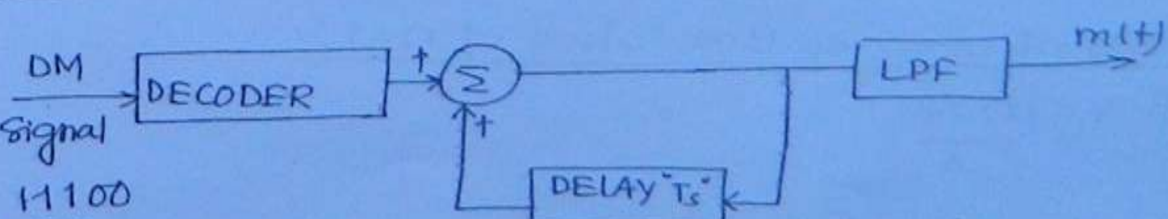


let,



$e(nT_s)$	+ve	+ve	+ve	-ve	-ve	{Present sample - Previous sample}.
$eq(nT_s)$	$+\Delta$	$+\Delta$	$+\Delta$	$-\Delta$	$-\Delta$	
DM Signal	1	1	1	0	0	

* DM RECIEVER:

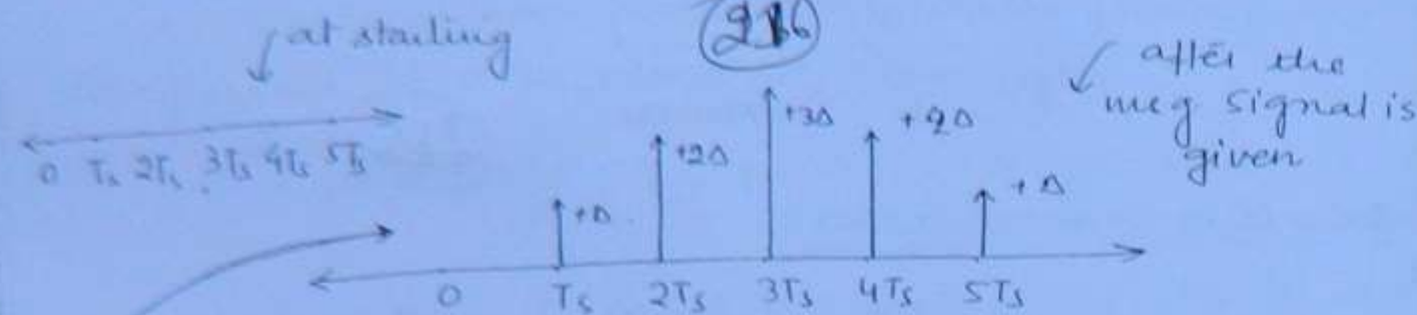


11100

①③③-

Summer O/P calculation

216



→ at $t = T_s$, delayed by T_s, so at $t = 0 \Rightarrow 0$.

At $t = T_s = +\Delta + 0 = +\Delta$

correspond = 1

At $t = 2T_s = +\Delta + \Delta = +2\Delta$

At $t = 3T_s = +\Delta + 2\Delta = +3\Delta$

At $t = 4T_s = -\Delta + 3\Delta = +2\Delta$

At $t = 5T_s = -\Delta + 2\Delta = +\Delta$

Note:

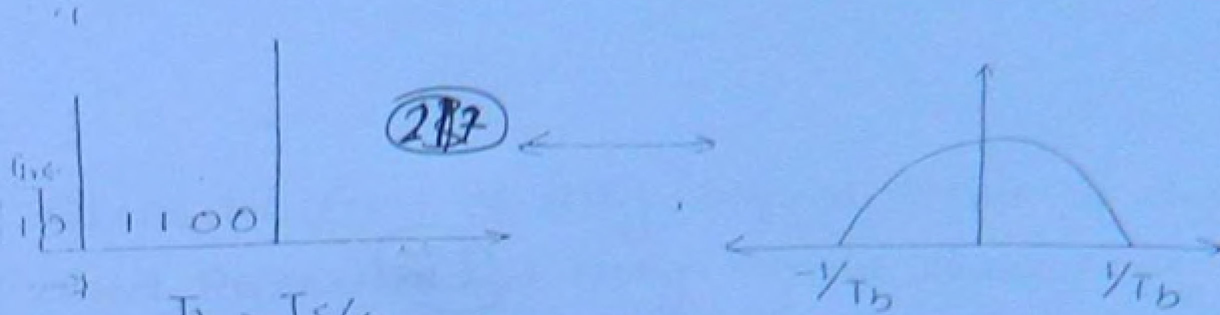
1. The I/P of LPF will be Integer multiples of Δ.
2. The pattern of the Reconstructed signal mainly depends on choice of Δ.
3. For optimum step size (Δopt) reconstructed signal will be same as Transmitted signal.
4. If $\Delta < \Delta opt$ → Slope Overload error occurs.

   $\Delta > \Delta opt$ → Granular error occurs.
5. To overcome SOE; step size has to be Increased.
6. To overcome GE; step size has to be decreased.

× Physical Significance of B.W is less if $n=1$:

* As $\downarrow B.W = \downarrow \frac{n f_s}{2}$

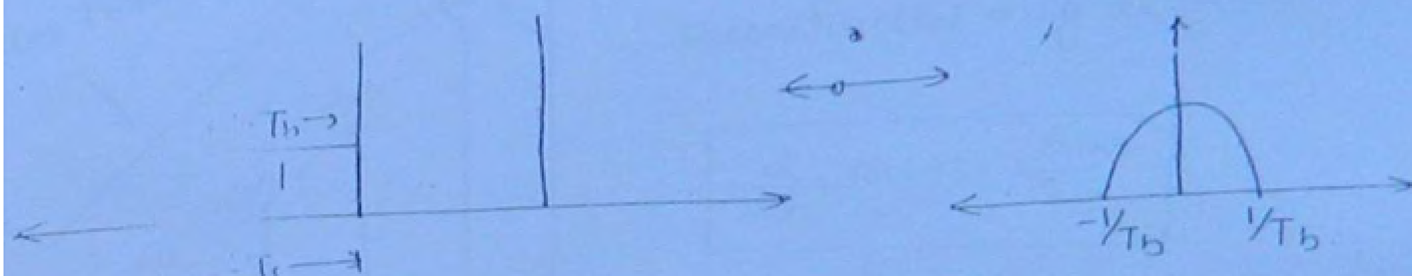
(217)



$T_b = T_s/4$

as T_b is small.
hence spectrum is large.
so large B.W reqd.

el :



$T_b = T_s$

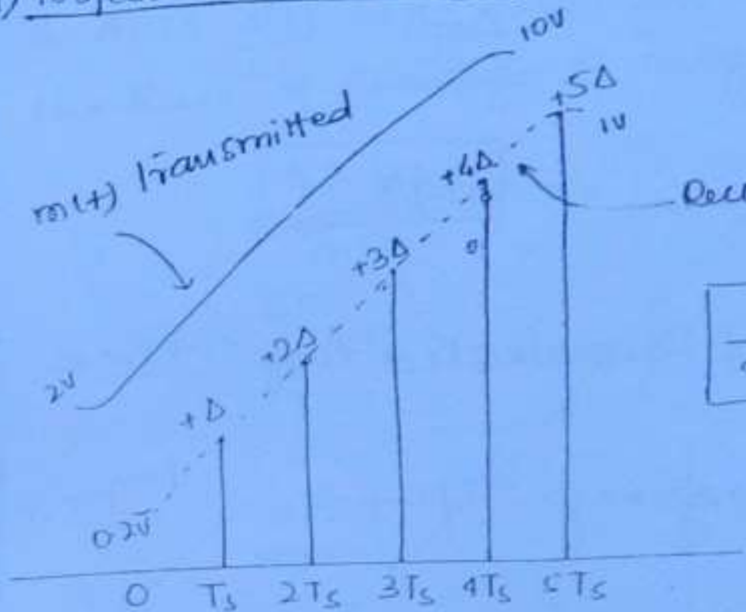
as, T_b is increased
spectrum gets compressed
hence B.W reqd is less.

Note:

* For proper Reconstruction of msg. signal slope of the transmitted and Reconstructed msg. signal should be same.

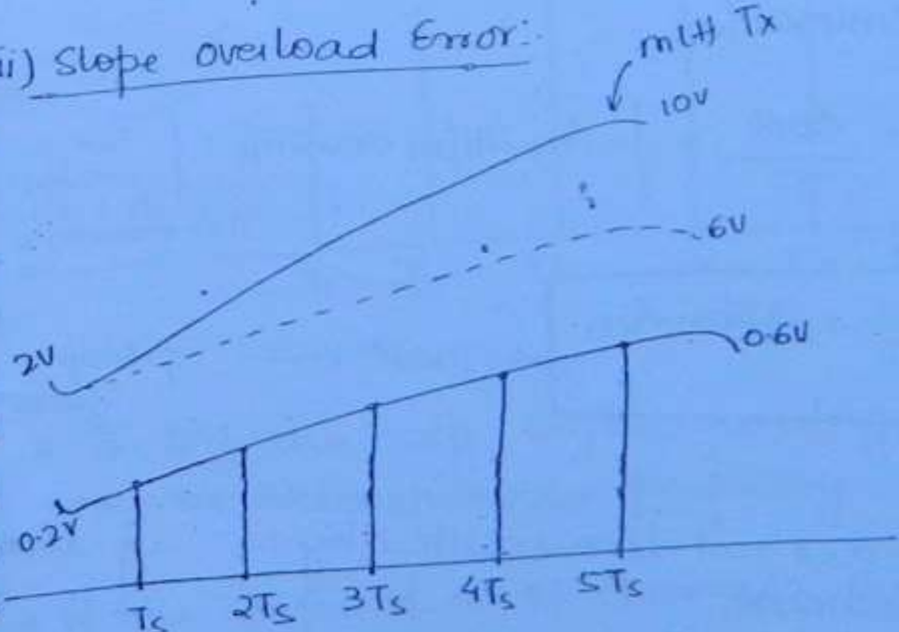
(279)

i) Perfect Reconstruction:



$$\boxed{\frac{d}{dt} m(t) = \frac{\Delta_{opt}}{T_s}}$$

ii) Slope overload Error:



$$\boxed{\frac{d}{dt} m(t) > \frac{\Delta}{T_s}}$$

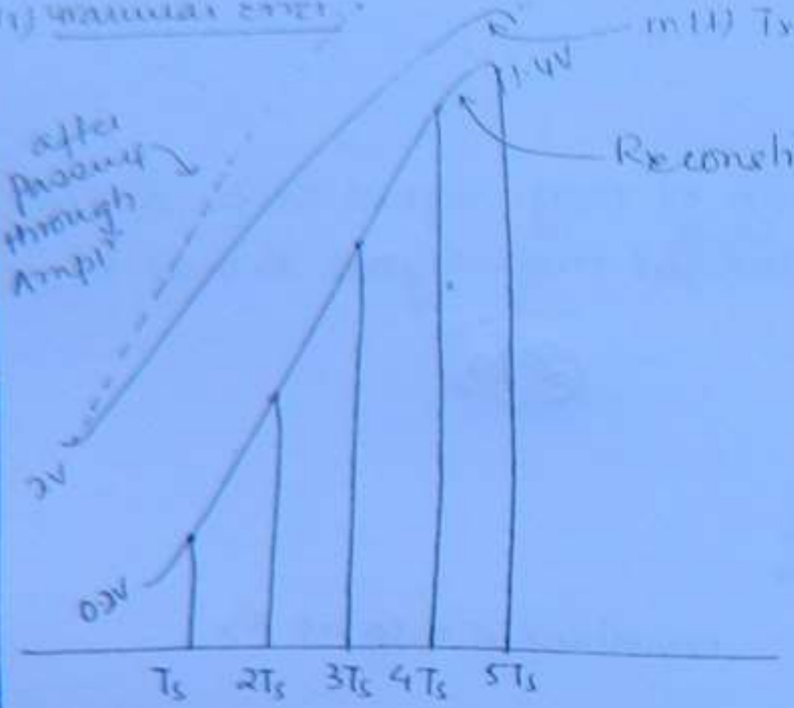
$$\frac{\Delta_{opt}}{T_s} > \frac{\Delta}{T_s}$$

$$\boxed{\Delta_{opt} > \Delta}$$

Slope of Tx signal > slope of Reconstructed signal then SOE takes place.

* Can be avoided by increasing step size.

granular error



$$\frac{d}{dt}m(t) < \Delta/T_s$$

$$\frac{\Delta opt}{T_s} < \Delta/T_s$$

$$\boxed{\Delta opt < \Delta}$$

* slope of msg Tx < slope of Reconstructed msg : then G.E takes place
* Can be avoided by decreasing step size.

Analysis:

Assume $m(t) = A_m \cos 2\pi f_m t$

$$\left|\frac{d}{dt}m(t)\right| = \frac{\Delta opt}{T_s} = \left|-A_m 2\pi f_m \sin 2\pi f_m t\right|$$

$$\boxed{\frac{\Delta opt}{T_s} = 2\pi f_m \cdot A_m}$$ ← max^m value of slope

Note:

* Sinusoidal signal are not transmitted by using the Delta Modulation scheme.
* For slope overload error to occur.

$$\frac{\Delta opt}{T_s} > \frac{\Delta}{T_s} \Rightarrow \boxed{\frac{\Delta}{T_s} < 2\pi f_m A_m}$$

* For [illegible]

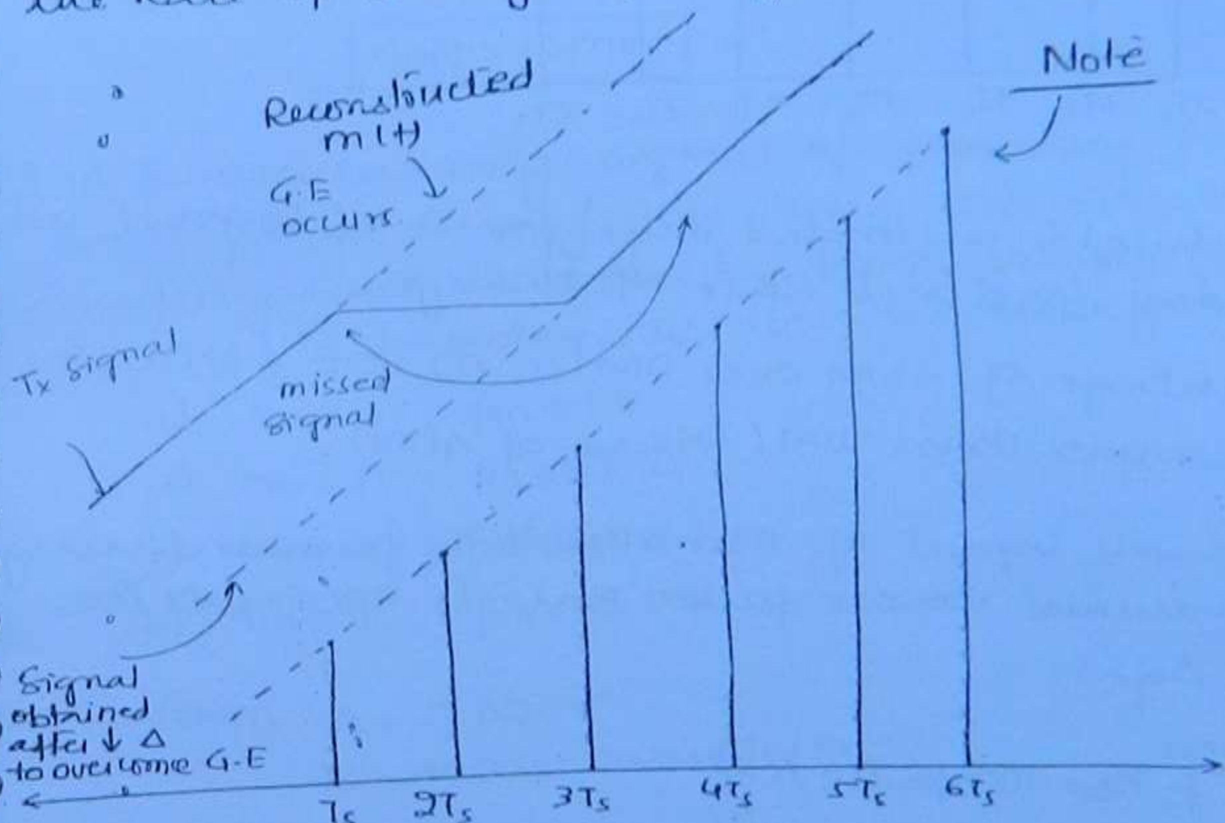
$$\frac{\Delta_{opt}}{T_s} < \Delta/T_s$$

$$\boxed{\frac{\Delta}{T_s} > 2\pi f_m A_m}$$

(129)

* ADAPTIVE DELTA MODULATION:

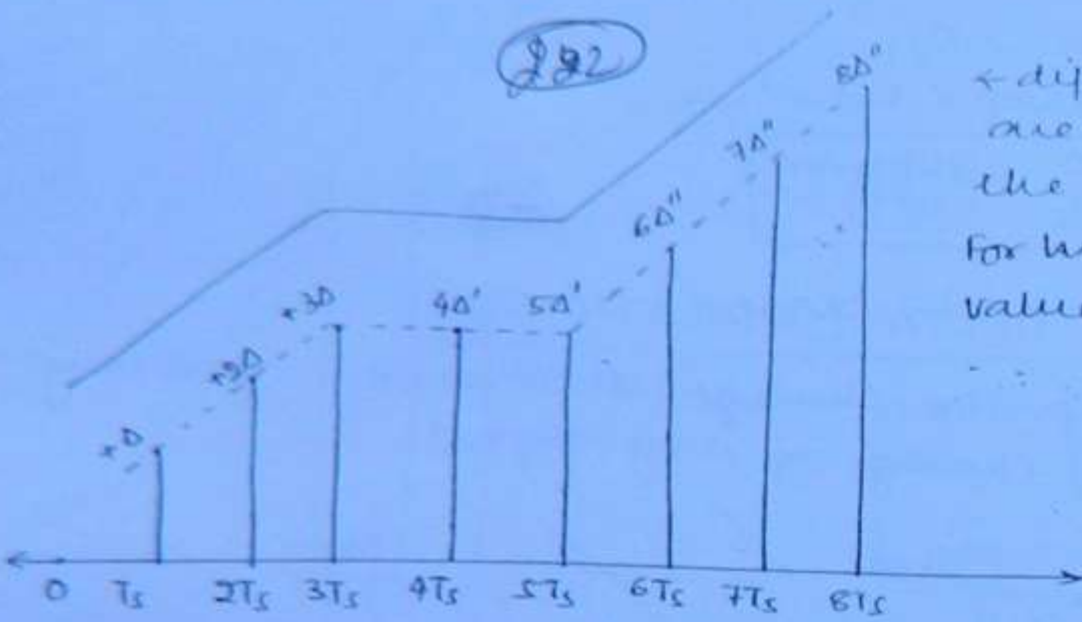
In ADM, step size changes continuously according to the Rate of change of msg. Signal.



Note:

* In DM, the msg signal having constant slope can only be Reconstructed perfectly
* If the slope of m(t) changes, then it can't be Reconstructed back as shown in above fig
* To overcome above, ADM is used.

But in ADM.



← different step sizes are used to track the Tx m(t).
For high slope, high value of Δ

Note:

* Delta Modulator is limited only for Tx of Signal which are having constant Rate of change.
* The Advantage of ADM over DM is no SOE & GE.
* ADM is complex than DM (Disad of ADM)

Q. A continuous Signal of $8\sin 8\pi\times10^3 t$ is passed through Delta Modulator whose pulse Rate is 4000pulses/sec. Find the Δ_{opt}?

Soln: Given

$f_s = 4000$ pulse/sec

$m(t) = 8\sin 8\pi\times10^3 t$

$A_m = 8V$; $f_m = 4KHz$

$$\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m$$

for DM, pulse Rate = Sampling Rate

$f_s = 1/T_s = 4000$ Samples/sec

$\Delta_{opt} \times 4000 = 2\pi \times 4K \times 8$

$\boxed{\Delta_{opt} = 16\pi}$ volts.

223

Q2 A msg signal [illegible] DM whose bit Rate is 1Kbps. find Δ_{opt}

Soln: Given, $m(t) = 10t$

$R_b = 1$ Kbps.

Now, Since given m(t) is not Sinusoidal so,

$$\frac{\Delta_{opt}}{T_s} = \cancel{2\pi f_m A_m}\ \frac{d}{dt} m(t) = \frac{\Delta_{opt}}{T_s}$$

$$\frac{d}{dt} 10t = \Delta_{opt} \times 1000 \quad \left\{ \because R_b = 1/T_s \right\}.$$

$$\boxed{\Delta_{opt} = 10mV}$$

Q3 A Sinusoidal msg signal of frequency, f_m, amplitude A_m is passed through DM whose step size is 0.628V. Sampling Rate is given by 40000 samples/sec. For which of the following, DM will be slope overloaded

a) $A_m = 3V$; $f_m = 1K$

b) $A_m = 2V$; $f_m = 1.5k$

c) $A_m = 2V$; $f_m = 2.5k$

d) $A_m = 1V$; $f_m = 2.5k$

Soln: Given, $\Delta = 0.628V$

$f_s = 40000$ sample/sec

Now,

$$\frac{\Delta}{T_s} = 0.628 \times 40000 = 25120$$

Now,

$$\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m$$

For slope overload error

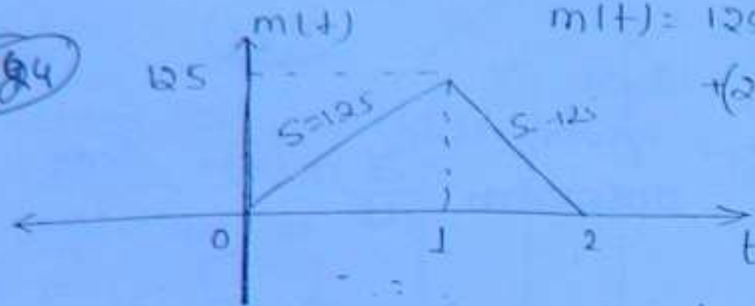
$$\frac{\Delta}{T_s} < \left(\frac{\Delta_{opt}}{T_s} = 2\pi f_m A_m \right)$$

So, for $A_m = 2V$; $f_m = 2.5K$, $2\pi f_m A_m = 31415.925$

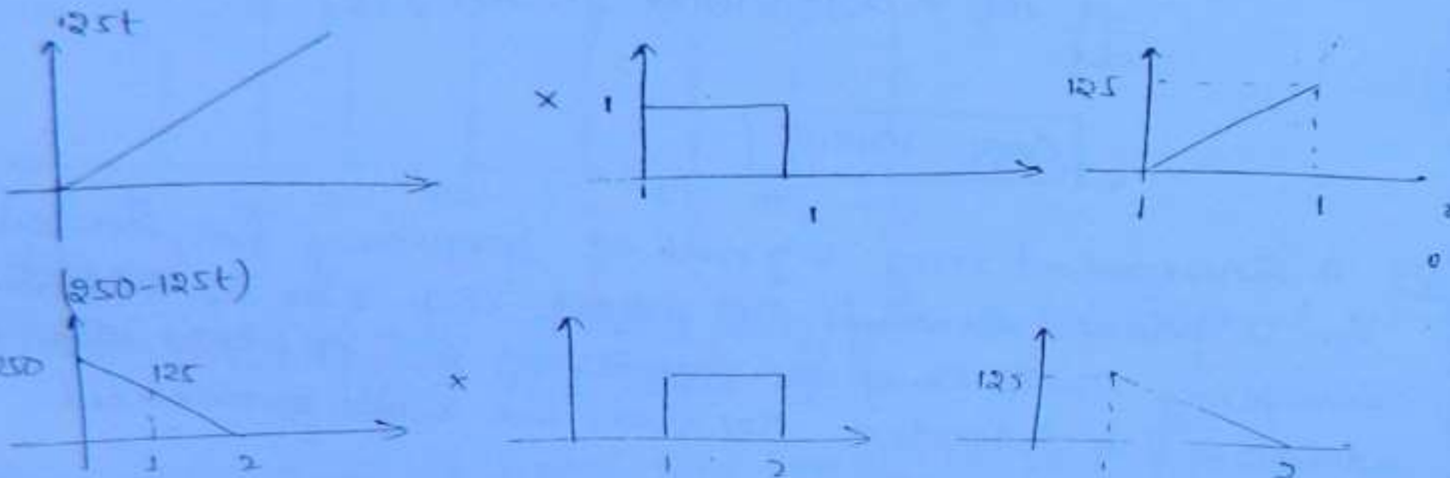
Hence SOE occurs.

Q4 For following msg signal find Δ_{opt} with f_s = 1000 samples/sec.

284



$m(t) = 125\{u(t) - u(t-1)\}$
$+ (250 - 125t)\{u(t-1) - u(t-2)\}$



Now, Since signal is not sinusoidal, so

$$\frac{d}{dt} m(t) = \frac{\Delta_{opt}}{T_s}$$

$$125 = \Delta_{opt} \times 1000$$

$$\boxed{\Delta_{opt} = 125\,mV}$$

Q5 A msg signal of Peak to Peak voltage 1.536 volts is passed through PCM system of having 128 quantization level. Find Quantization Noise power.

Soln: Given:

$V_{p\cdot p} = 1.536$ V

$L = 128 = 2^n \Rightarrow n = 7$

Now, $(N_q) = \frac{\Delta^2}{12} = \frac{1}{12}\left\{\frac{1.536}{128}\right\}^2$

$$\boxed{N_q = 12\,\mu W}$$

Q6 For a [illegible] to get min^m SQNR of 1000 a) 2 b) 3 c) 4 d) 5

Sol^n: Given,

$$SQNR \geq 1000$$

$$\frac{3}{2} \cdot 2^{2n} \geq 1000$$

$$2^{2n} \geq \frac{2000}{3}$$

$$2^{2n} \geq 666.66$$

$$n \geq 5$$

225

$$\boxed{n = 5}$$ Ans

Q7. A msg Signal Bandlimited to 4K is transmitted through 256 level PCM system. Find Tx B.W of the system

a) 16 K b) 32 K c) 64 K d) 128 K

Sol^n: Given: $f_m = 4K$

$$L = 256 \Rightarrow n = 8$$

Now, $f_s = 2f_m = 8K$

So, $B.W = \frac{nf_s}{2} = \frac{8 \times 8K}{2}$

$$\boxed{B.W = 32K}$$

Q8. A msg Signal sampled at 8K is transmitted through 512 level PCM system. $(SQNR)_{dB} = ?$

$R_b = ?$

Sol^n: Given, $f_s = 8K \Rightarrow f_m = 4K$

$L = 512 \Rightarrow n = 9$

So, $(SQNR)_{dB} = 1.8 + 6n$

$= 1.8 + 54$

$$\boxed{(SQNR)_{dB} = 55.8\,dB}$$

$R_b = \frac{nf_s}{2}$

$R_b = 9 \times 8K$

$$\boxed{R_b = 72\,Kbps}$$

Q9. A msg signal whose peak Amp is ... ing b/w -3V & 5V is transmitted through PCM system of having step size of 1V. Find $(SQNR)_{dB}$ =?

Solⁿ:- $(SQNR)_{dB} = 1.8 + 6n$ (286)

Given: $V_{max} = 5V$, $V_{min} = -3V$, $\Delta = 1V$

$$\Delta = \frac{V_{max} - V_{min}}{2^n} = 1V$$

$$\frac{5+3}{2^n} = 1 \Rightarrow 2^n = 8$$

$$n = 3$$

So, $SQNR = 1.8 + 18$

$$\boxed{(SQNR)_{dB} = 19.8 dB}$$

Q10. For a PCM system as the no. of Quantisation level increases from 2 to 8, then Tx B.W Reqd will be

a) Increased by 4 times

b) Tripled.

c) doubled.

d) No change.

Solⁿ: Quantisation level $L \rightarrow 2$ to 8

$n \rightarrow 1$ to 3

$$(B.W)_{n=1} = nf_s/2 = f_s/2$$

$$(B.W)_{n=3} = nf_s/2 = 3f/2$$

Hence Tripled

Q11. For a PCM system of having Bit Rate of 10^8 bits/sec No. of Quantisation levels are given by 256. Find the maxᵐ freqⁿ of the signal allowed by the PCM system?

Soln: Given:

$R_b = 10^8$ bits/sec

$L = 256 \Rightarrow n = 8$

Now,

$B.W = nf_s/2 = R_b/2$

227

$$\frac{10^8}{2} = \frac{8 \times 2f_m}{2}$$

$$f_m = \frac{10^8}{16}$$

$$\boxed{f_m = 6.25\,MHz}$$ Ans

Q12. A msg signal of $m(t) = 6\cos 2000\pi t + 2\cos 4000\pi t$ is passed through DM whose ~~step~~ pulse Rate is 5000 pulse/s. Find, min^m value of Δ required to overcome slope overload error?

Soln: $m(t) = 6\cos 2000\pi t + 2\cos 4000\pi t$

$A_{m_1}\cos 2\pi f_{m_1}t + A_{m_2}\cos 2\pi f_{m_2}t$

$A_{m_1} = 6$ $\qquad$ $A_{m_2} = 2$

$f_{m_1} = 1000\,Hz$ $\qquad$ $f_{m_2} = 2000\,Hz$

$$\frac{\Delta_1}{T_s} = 2\pi f_{m_1} A_{m_1} \qquad \frac{\Delta_2}{T_s} = 2\pi f_{m_2} A_{m_2}$$

$$\frac{\Delta_1}{T_s} = 2\pi \times 1000 \times 6 \qquad \Delta_2 \times 5000 = 2\pi \times 2000 \times 2$$

$$\Delta_1 \times 5000 = 2\pi \times 6000 \qquad \Delta_2 = \frac{8\pi}{5}$$

$$\Delta_1 = \frac{12\pi}{5} = 7.5\,V \qquad \Delta_2 = 5.03\,V$$

So, min^m step size Required to overcome SOE is given by max$^m\{\Delta_1, \Delta_2\}$

So, $\boxed{\Delta = 7.52\,V}$

min^m step size Reqd to overcome GE is $\boxed{\min\{\Delta_1, \Delta_2\} = 5.03\,V}$

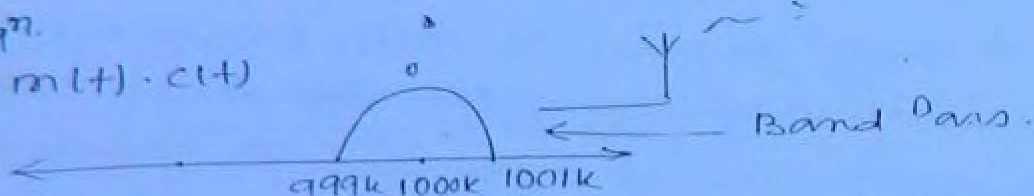
* Band Pass Data Transmission

1.

i) when, $T_b = 1\,msec \Rightarrow 1/T_b = 1K = f_{b1}$

then $f_c = 1M$ is used and the Resulting is Band Pass signal as it donot contain msg in low freq^n.

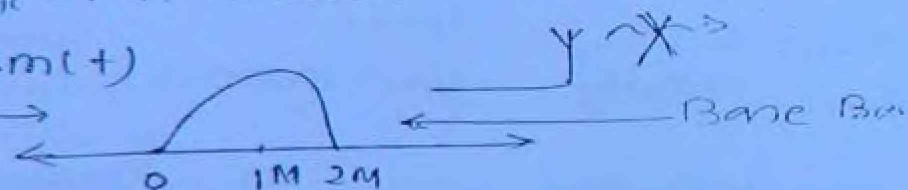
$s(t) = m(t) \cdot c(t)$



2) when, $T_b = 1\mu sec \Rightarrow 1/T_b = 1M \Rightarrow f_{b2}$

and $f_c = 1M$. is used, then the spectrum.

$s(t) = c(t) m(t)$



This is still Base Band Signal as it [illegible] in the significant low frequencies

Hence, f_c should be in order of 10M.

Note :-

* By using ASK, PSK & FSK, a digital Base band Signal is converted as Band Pass Signal and can be transmitted through free space.

* In these modulation schemes, one of the [illegible] of Carrier Signal switches b/w 2 specific [illegible] as the msg. Signal switching b/w 2 specific voltages. So these modulation schemes are [illegible] ed

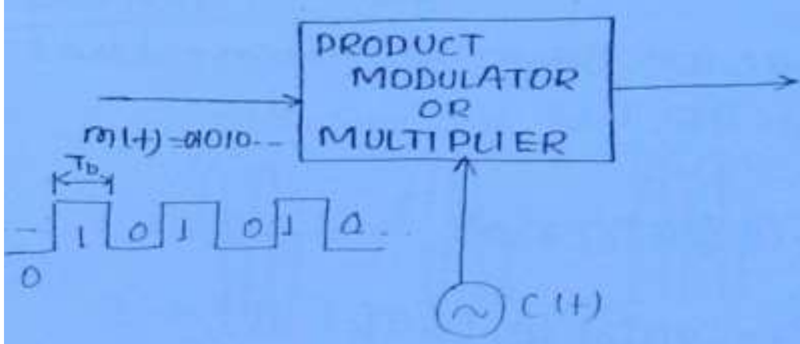
as signalling, ...

* Amplitude Shift Keying (ASK)

* In this modulation, binary 1 is represented by presence of carrier and binary 0 by absence of carrier

229

* ASK Transmitter



$1 \rightarrow S_1(t) = A_c \cos 2\pi f_c t$;

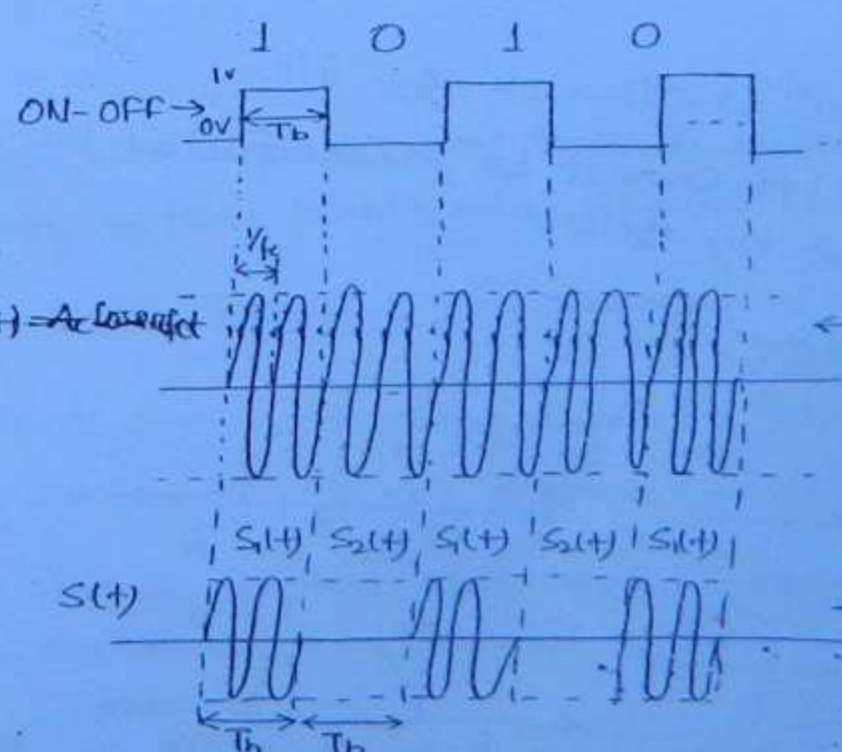
$0 \rightarrow S_2(t) = 0V$.

* electrical signal representation technique is ON-OFF coding. In which

$1 \rightarrow +ve$

$0 \rightarrow -ve$

* Graphical Representation



← Integer multiple of complete cycle in duration of T_b is taken.

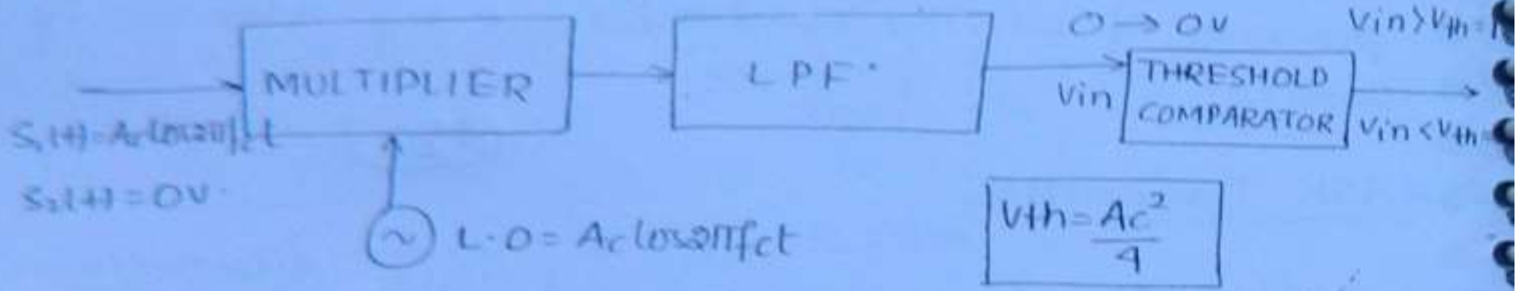
$\rightarrow$ $\boxed{T_b = 2/f_c}$

× ASK RECIEVER (250)

× for demodulation of ASK, SD will be used

$1 \to A_c^2 \cos^2 2\pi f_c t$ $\quad 1 \to \frac{A_c^2}{2}$

$0 \to 0V$ $\quad 0 \to 0V$



$$V_{th} = \frac{A_c^2}{4}$$

Since, SD is used hence, we have to observe that either ONE occurs or not

For, this let,

$$(L\cdot O)\,O/P = A_c \cos(2\pi f_c t + \phi)$$

$$\text{so } (Mul)\,O/P = A_c^2 \cos 2\pi f_c t \cos(2\pi f_c t + \phi) \leftarrow 1$$

$$0 \qquad \leftarrow 0$$

$$(LPF)\,O/P = \frac{A_c^2}{2}\cos\phi \leftarrow 1$$

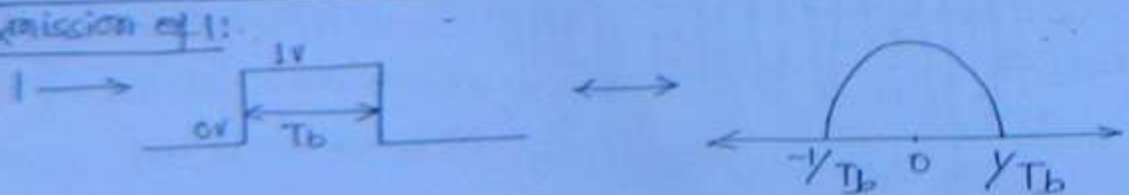
$$0 \qquad \leftarrow 0$$

Now, if $\phi = 90° \Rightarrow O/P = 0V$ for $I/P = 1$

Hence, the m(t) cannot be Reconstructed back and it is affected by ONE.

* XMISSION B.W OF ASK:

× xmission of 1:



* Transmission of 0

$S_0(t) = 0$

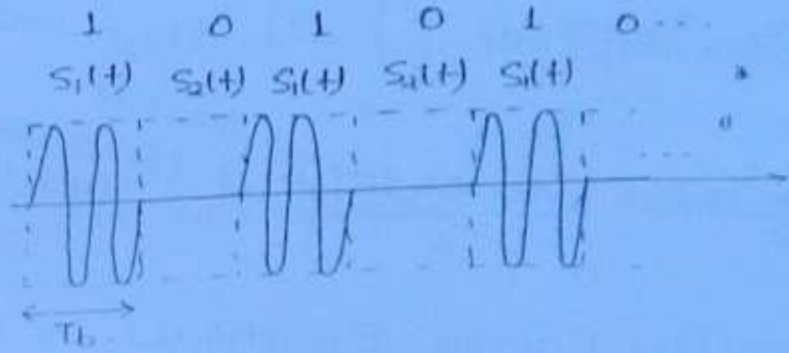
(237)

So, ASK B.W $= (f_c + 1/T_b) - (f_c - 1/T_b)$

$= 2/T_b$

$\therefore$ $\boxed{\text{ASK B.W} = 2R_b}$

* ENERGY PER BIT:



Now, as $E = \int_{-\infty}^{\infty} x^2(t)\,dt$

& $P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x^2(t)|\,dt = \frac{E}{T}\Big|_{T\to\infty}$ $\left\{\begin{matrix}\text{when} \\ T=\infty\end{matrix}\right\}$.

Note:

1. when the signal is of infinite duration, then the Energy of that particular signal is also ∞.
2. For finite signals, the energy is also finite.
3. $0 < E < \infty \longrightarrow$ Energy signal (Power = 0).

   $0 < P < \infty \longrightarrow$ Power signal (Energy = ∞).
4. An energy signal can never be power signal and also the vice-versa is correct.

Note
* All Periodic signals are Power Signals and its Energy

* Transmission of 1:

238

$$E_b = \int_0^{T_b} S_1^2(t)\,dt$$

$$= \int_0^{T_b} (A_c \cos 2\pi f_c t)^2 dt$$

$$= \int_0^{T_b} A_c^2 \cos^2 2\pi f_c t\,dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2}dt + \int_0^{T_b} \cancel{\frac{A_c^2 \cos 4\pi f_c t}{2}}\,dt$$

Area = 0 (since complete cycles)

To Save Transmitter Energy, Eb should be small.

So, $\boxed{E_b = \frac{A_c^2 T_b}{2}} \Rightarrow \boxed{A_c = \sqrt{\frac{2E_b}{T_b}}}$

* Transmission of 0:

$$E_b = \int_0^{T_b} S_2^2(t)\,dt$$

$$\boxed{E_b = 0}$$

***
* CONSTELLATION DIAGRAM:-

$$1 \rightarrow S_1(t) = A_c \cos 2\pi f_c t = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t.$$

$$0 \rightarrow S_2(t) = 0.$$

As, $$E_b = \int_0^{T_b} S_1^2(t)\,dt = \int_0^{T_b} \left\{\sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_c t\right\}^2 dt$$

$$\cancel{E_b} = \cancel{E_b} \int_0^{T_b} \left\{\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t\right\}^2 dt.$$

$$\int_0 \left\{ \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \right\}^2 dt = 1$$

Normalised funcⁿ

So,

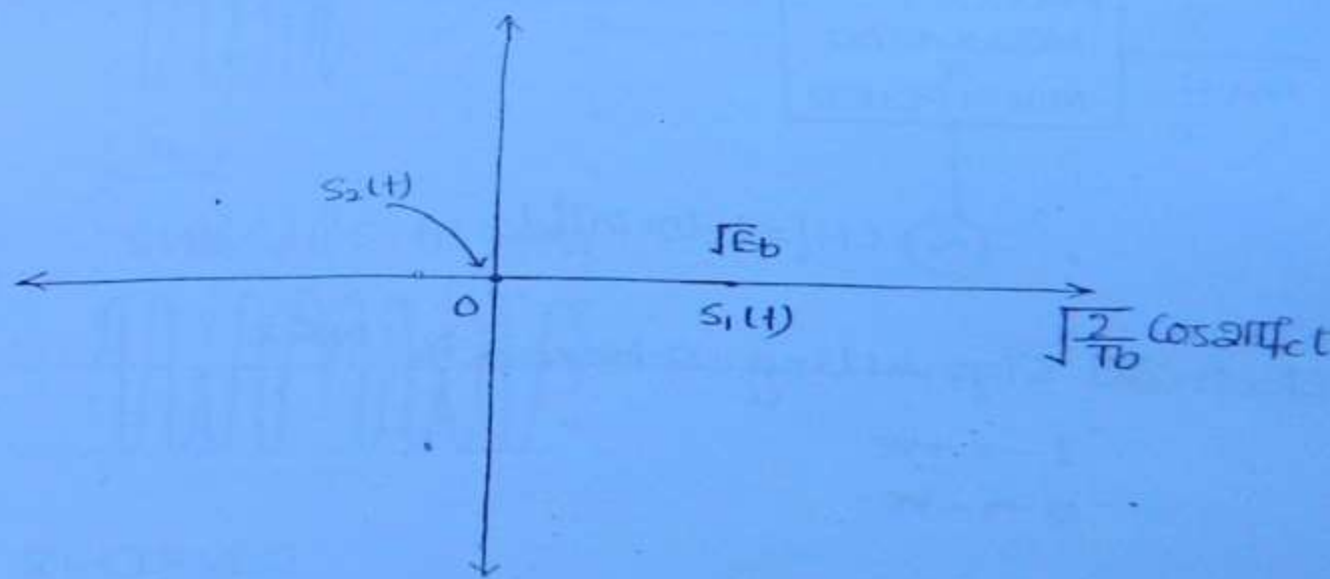
$$\boxed{\text{Energy} \left\{ \sqrt{\frac{2}{T_b}} \cos 2\pi f_c t \right\} = 1}$$

(239)

* In constellation diagram the funcⁿ whose energy is equal to 1 is said to be as Normalised function

Now, $S_1(t) = \sqrt{E_b} \cdot \underbrace{\sqrt{\frac{2}{T_b}} \cos 2\pi f_c t}_{f(t)}$; $S_2(t) = 0$.

* Each axis corresponds to a Normalised function



In constellation diagᵐ the reference axes corresponds to Normalised functions.

* Conclusion:

distance of $S_1(t)$ from the origin $= \sqrt{E_b}$

$$\boxed{\text{Energy}\{S_1(t)\} = (\sqrt{E_b})^2 = E_b.} \quad \{\text{Square of distance}\}.$$

* distance of $S_2(t)$ from origin = 0

$$\boxed{\text{Energy}\{S_2(t)\} = 0}$$

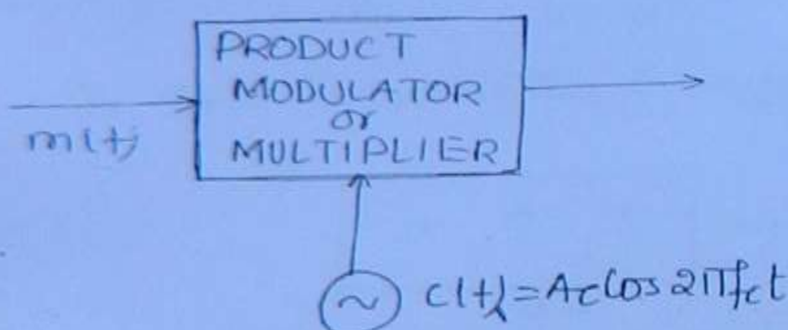
~~160~~ 234

* distance b/w signalling points $\boxed{d_{12} = \sqrt{E_b}}$

* PHASE SHIFT KEYING (PSK):

* In PSK, Binary 1 is Represented by Actual carrier and Binary 0 by 180° phase shift of carrier

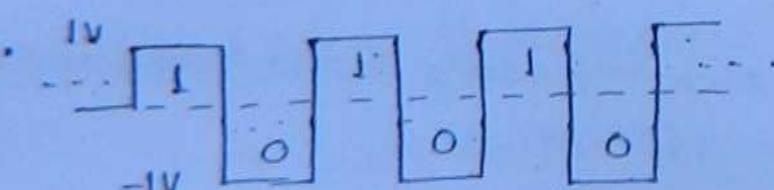
* PSK TRANSMITTER:



* Electrical signalling scheme is NRZ.

$1 \rightarrow +ve$

$0 \rightarrow -ve$

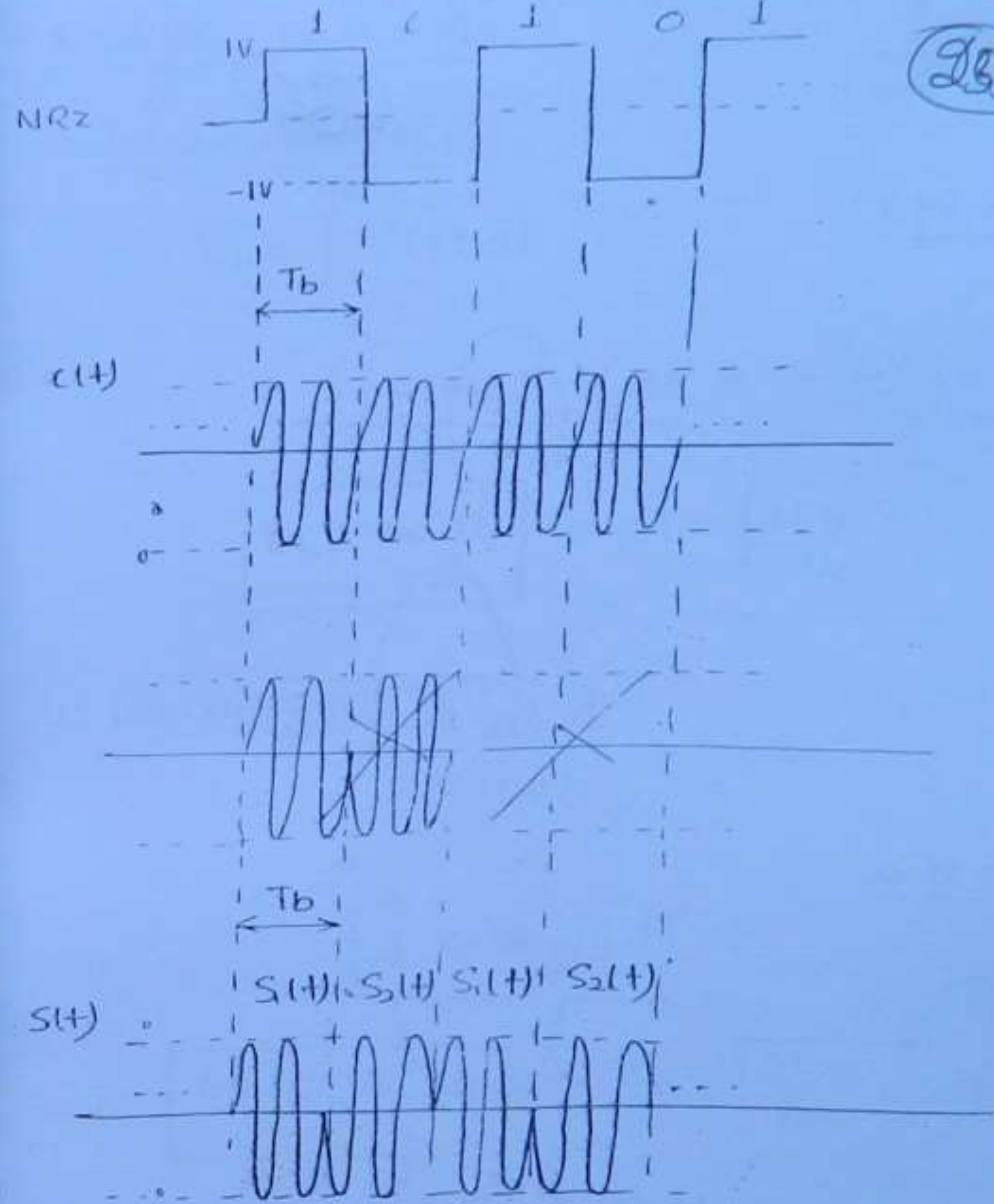


$1 \rightarrow S_1(t) = A_c \cos 2\pi f_c t$

$0 \rightarrow S_2(t) = -A_c \cos 2\pi f_c t = A_c \cos\{2\pi f_c t + 180^\circ\}.$

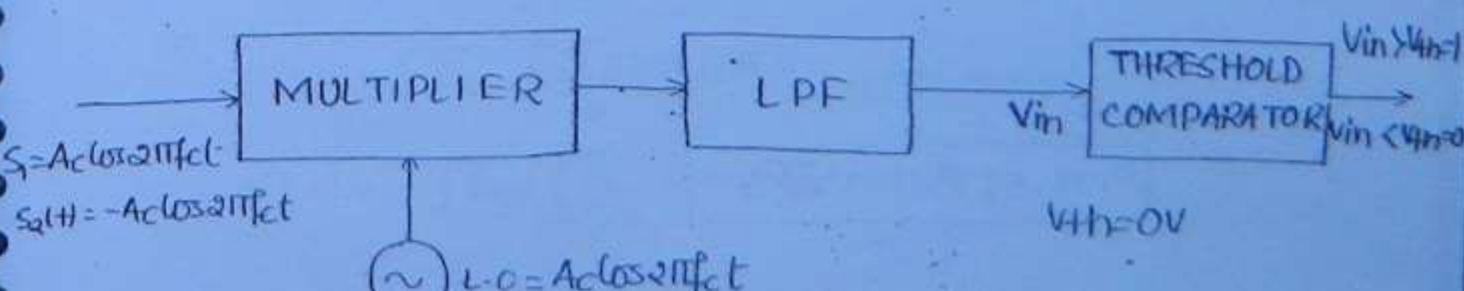
* Graphical representation

265



* PSK RECIEVER:

* For demodulation of PSK, SD will be used.



$1 \rightarrow A_c^2 \cos^2 2\pi f_c t$ $\quad 1 \rightarrow A_c^2/2$

$0 \rightarrow -A_c^2 \cos^2 2\pi f_c t$ $\quad 0 \rightarrow -A_c^2/2$

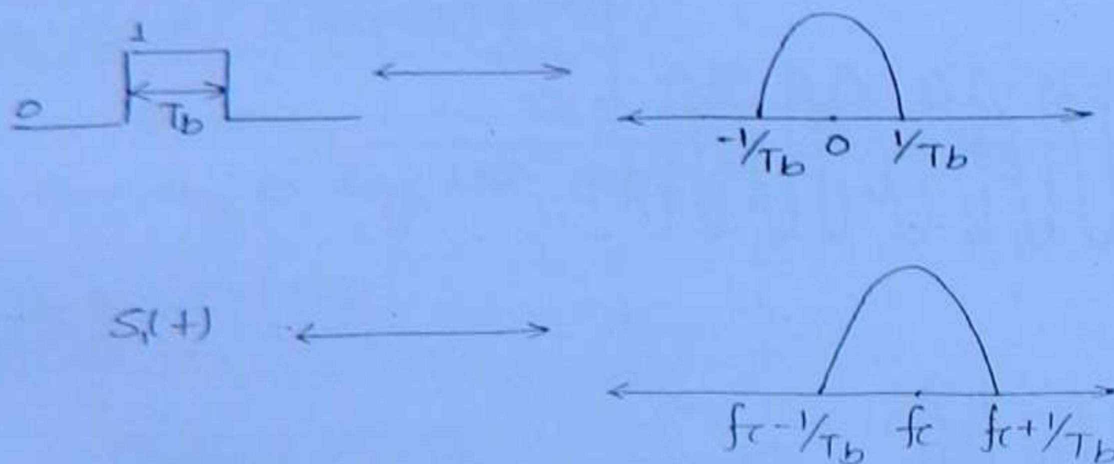
Note:

* Demodulation of PSK is affected by QNE.

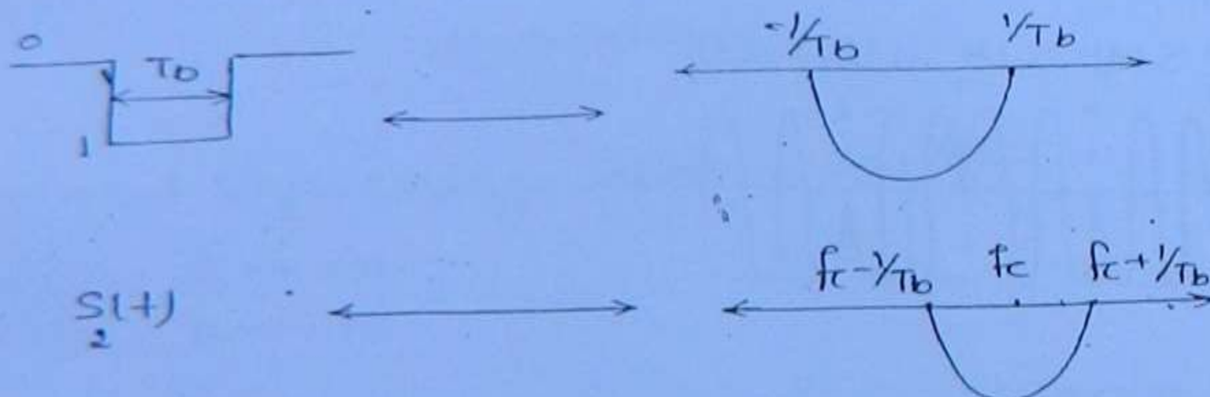
226

* Bandwidth of PSK:

a) Transmission of 1:



b) Transmission of 0:



So,

$$B.W \text{ of } PSK = \left(f_c + 1/T_b\right) - \left(f_c - 1/T_b\right)$$

$$= 2/T_b = 2R_b$$

$$\boxed{B.W \text{ of } PSK = 2R_b}$$

Note.
* channel B.W Requirements of ASK, PSK will be the same.

* ENERGY PER BIT:

~~169~~ 237

a) Transmission of 1:

$$E_b = \int_0^{T_b} s_1^2(t)\,dt$$

$$= \int_0^{T_b} \{A_c \cos 2\pi f_c t\}^2\,dt$$

$$\boxed{E_b = \frac{A_c^2 T_b}{2}} \Rightarrow A_c = \sqrt{\frac{2E_b}{T_b}}$$

b) Transmission of 0:

$$E_b = \int_0^{T_b} s_2^2(t)\,dt$$

$$= \int_0^{T_b} (-A_c \cos 2\pi f_c t)^2\,dt$$

$$\boxed{E_b = \frac{A_c^2 T_b}{2}} \Rightarrow A_c = \sqrt{\frac{2E_b}{T_b}}$$

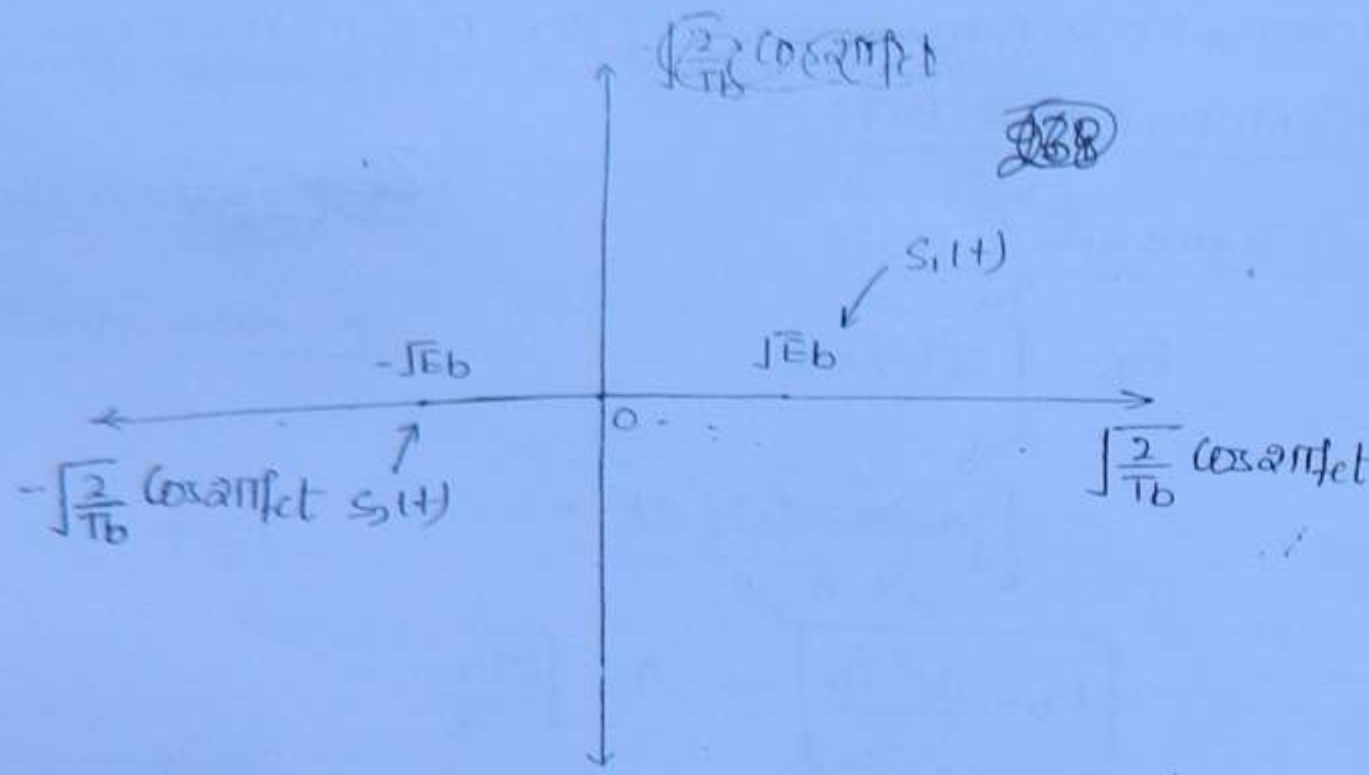
* CONSTELLATION DIAGRAM:

$$1 \rightarrow s_1(t) = A_c \cos 2\pi f_c t = \sqrt{2E_b/T_b}\cos 2\pi f_c t.$$

$$0 \rightarrow s_2(t) = -A_c \cos 2\pi f_c t = -\sqrt{\frac{2E_b}{T_b}}\cos 2\pi f_c t.$$

So,
$$s_1(t) = \sqrt{E_b}\cdot\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t.$$

$$s_2(t) = -\sqrt{E_b}\cdot\sqrt{\frac{2}{T_b}}\cos 2\pi f_c t.$$



So,

$$\boxed{\text{Energy}\{S_1(t)\} = (\sqrt{E_b})^2 = E_b}$$

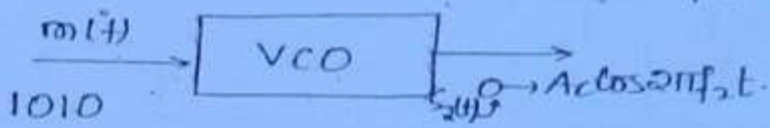
$$\boxed{\text{Energy}\{S_2(t)\} = (-\sqrt{E_b})^2 = E_b}$$

* distance b/w signalling points, $\boxed{d_{12} = 2\sqrt{E_b}}$

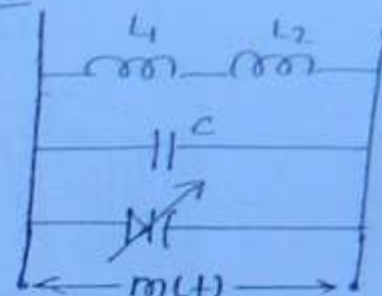
FREQUENCY SHIFT KEYING (FSK):

* In this Binary 1 is represented by high freq^n carrier & Binary 0 by low freq^n carrier.

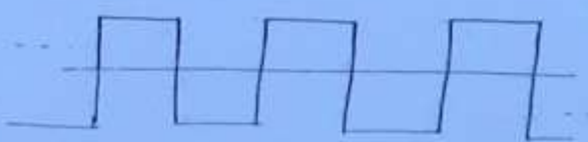
$S_1(t) \to A_c \cos 2\pi f_1 t$



VCO



NRZ ⇒ 1 = +ve
0 = −ve

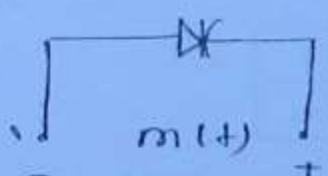


$$\boxed{f_1 = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}}$$

i) Tx of 1:

m(t) is +ve

then



varactor diode connected in Reverse mode.

And, as $C' \propto 1/\omega$

So, in R-B width of depletion layer is high hence C' is less

So f_1 is high.

$$\boxed{\underset{(\text{as } \omega \uparrow)}{RB\uparrow \longrightarrow C'\downarrow} \longrightarrow f_i \uparrow = f_1}$$

ii) Tx of 0:

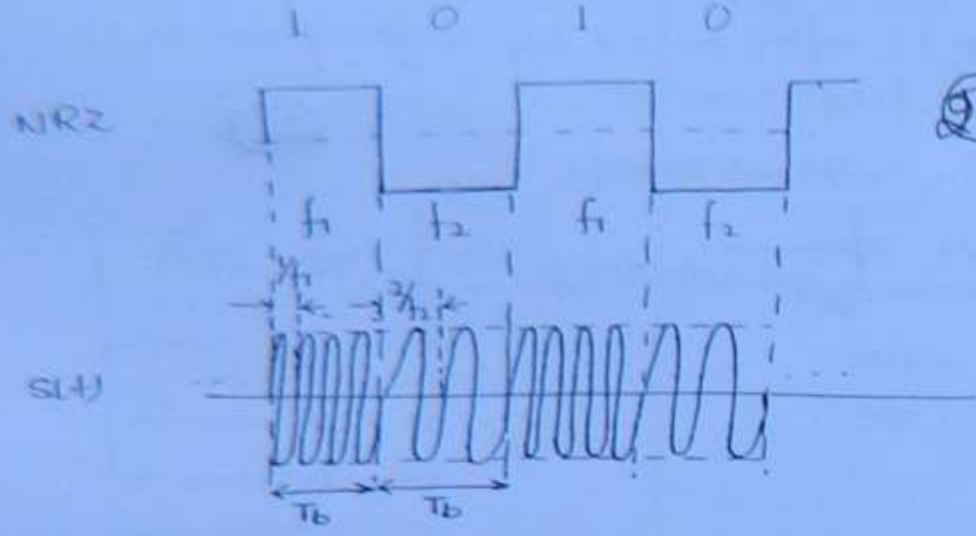
m(t) is −ve

$$\boxed{\underset{(\text{as } \omega \downarrow)}{F\text{-}B \uparrow \longrightarrow C' \uparrow} \longrightarrow f_i \downarrow = f_2}$$

$\Rightarrow \boxed{f_1 >>> f_2}$

As $f_1 > f_2$, then also both the frequencies should be in the Range of MHz.

* Graphical Interpretation :-



2/40

$$T_b = 4/f_1 \quad ; \quad T_b = 2/f_2$$

So, In general,

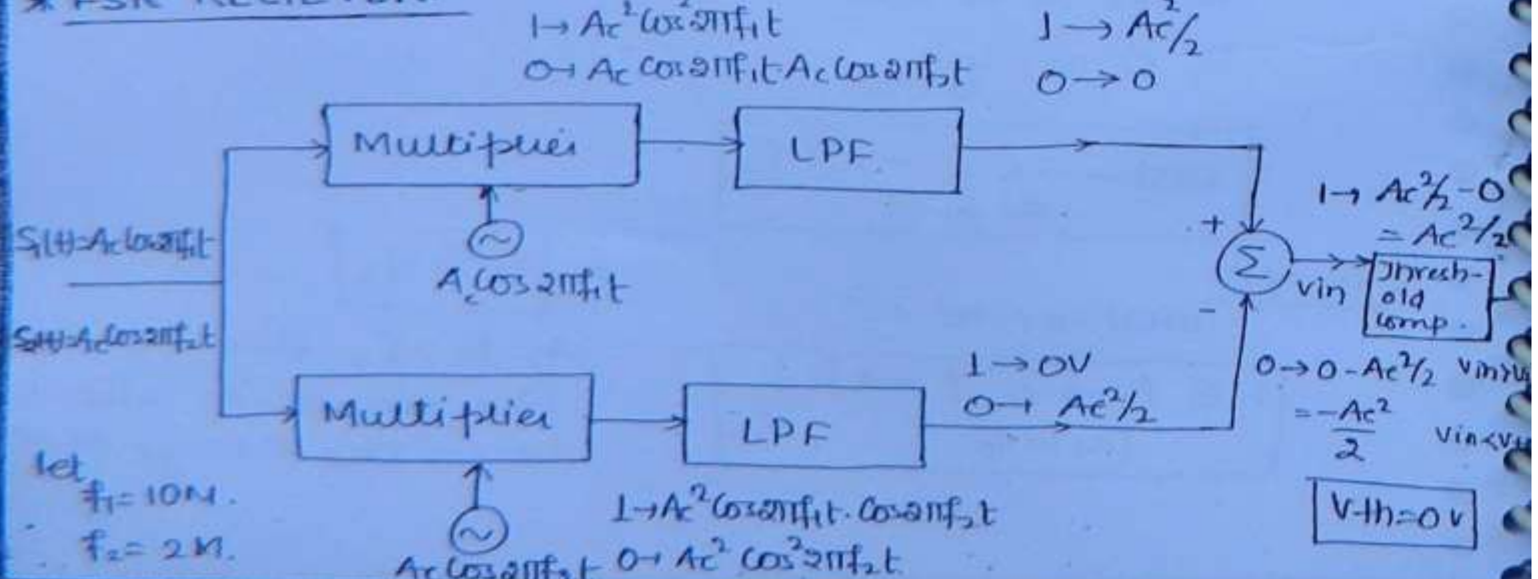
$$T_b = \frac{n_1}{f_1} \quad ; \quad T_b = \frac{n_2}{f_2}$$

$$\boxed{f_1 = \frac{n_1}{T_b}} \quad ; \quad \boxed{f_2 = \frac{n_2}{T_b}}$$

f_1 & f_2 should be Integer multiple of the Bit Rate

ie $\boxed{f_1 = n_1 T_b}$; $\boxed{f_2 = n_2 T_b}$

* FSK RECIEVER :-



Note

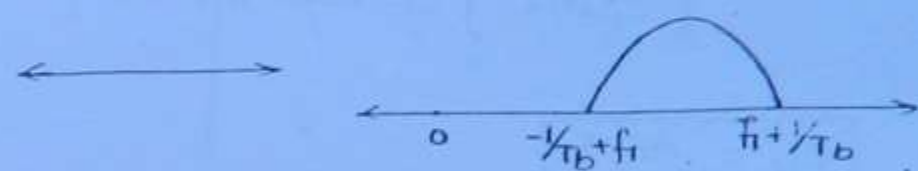
× The demodulation of FSK is affected by QNI

× Transmission B.W:

24

1. Tx of 1:

$S_1(t)$



$\Rightarrow B.W = 2/T_b$

2. Tx of 0:

$S_2(t)$

$\Rightarrow B.W = 2/T_b$

So,

the FSK $B.W = (f_1 + 1/T_b) - (f_2 - 1/T_b)$ {highest +ve freq^n - lowest +ve freq^n}.

$$\boxed{B.W-FSK = f_1 - f_2 + 2R_b}$$

Note:

* FSK needs high Transmission B.W compared to ASK and PSK. (drawback of FSK).

* **Energy per bit:**

* **Tx of 1:**

248

$$E_b = \int_0^{T_b} S_1^2(t)\,dt$$

$$= \int_0^{T_b} (A_c \cos 2\pi f_1 t)^2\,dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2}dt + \int_0^{T_b} \frac{A_c^2}{2} \cancelto{0}{\cos 4\pi f_1 t}\, dt \quad \{\because \text{Complete cycle} = \text{Area} = 0\}$$

$$\boxed{E_b = \frac{A_c^2}{2} T_b}$$

* **Tx of 0:**

$$E_b = \int_0^{T_b} S_2^2(t)\,dt$$

$$= \int_0^{T_b} (A_c \cos 2\pi f_2 t)^2\,dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2}dt + \int_0^{T_b} \frac{A_c^2}{2} \cancelto{0}{\cos 4\pi f_2 t}\, dt \quad \{\because \text{Complete cycle} = \text{Area} = 0\}$$

$$\boxed{E_b = \frac{A_c^2}{2} T_b}$$

Note:

* Transmitter Energy Requirements of PSK and FSK will be the same.

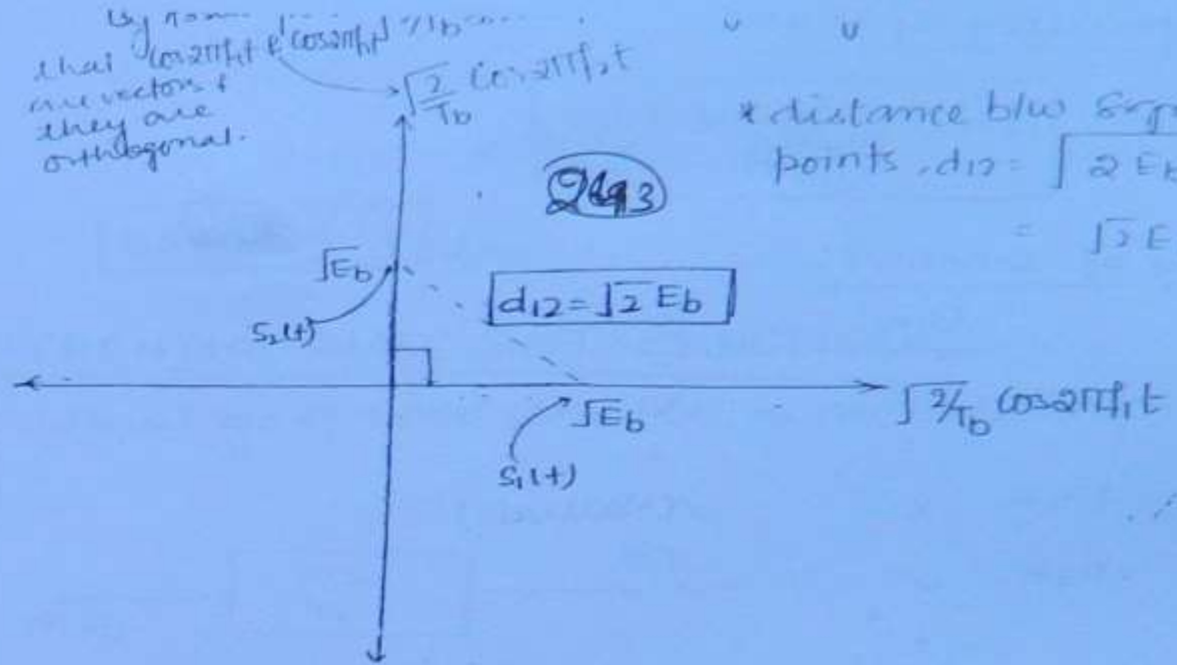
* **CONSTELLATION DIAGRAM:**

$$1 \longrightarrow S_1(t) = A_c \cos 2\pi f_1 t = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t$$

$$0 \longrightarrow S_2(t) = A_c \cos 2\pi f_2 t = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t$$

So,

$$\left.\begin{aligned} S_1(t) &= \sqrt{E_b} \cdot \sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t \\ S_2(t) &= \sqrt{E_b} \cdot \sqrt{2/T_b} \cos 2\pi f_2 t \end{aligned}\right\} \text{In terms of Normalised functions.}$$



* distance b/w signalling points, $d_{12} = \sqrt{2E_b}$

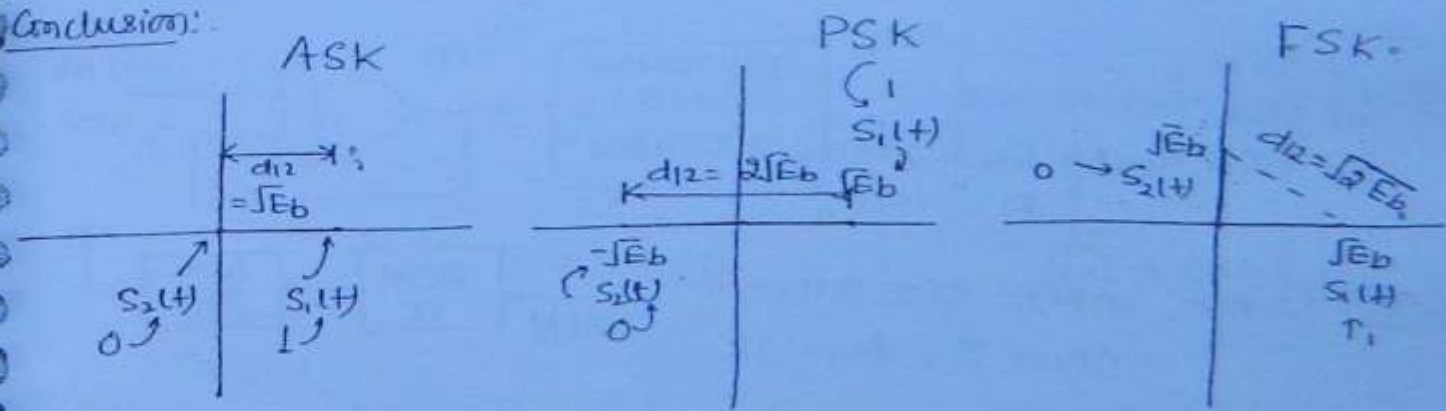
$= \sqrt{2}\, E_b$

We can't locate $\cos 2\pi f_2 t$ as all the axis corresponds to f_1 freq^n.

* $\sqrt{\frac{2}{T_b}} \cos 2\pi f_1 t$ and $\sqrt{\frac{2}{T_b}} \cos 2\pi f_2 t$ are othogonal functions in the interval $(0, T_b)$.

By interpreting these functions as vectors, the phase angle b/w Resulting vectors will be $90°$.

Conclusion:



In a constellation diagram, if the dist. b/w signalling points is less; then P_e will be more, and vice versa.

P_e depends upon the dist b/w signalling pts.

So

$\boxed{P_e : ASK > FSK > PSK}$

Comparison of B.W:

$$\boxed{B.W :- ASK < FSK \;\&\; PSK}$$

276

Usage of schemes:

	B.W	Pe
ASK	✓	×
FSK	×	✓(Moderate)
PSK	✓	✓

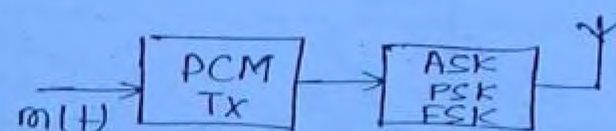
Note:
PSK is much preferred signalling scheme compared to ASK and FSK

Q1 A msg signal of $8\cos 8\pi\times10^3 t$ is given to 10 bit PCM system. The Resulting PCM signal is transmitted through free space, by using Band Pass modulation scheme. Find the Tx signal B.W if modulation scheme is

a) ASK

b) PSK

c) FSK with $F_H = 2MHz$.
$F_L = 1MHz$.



Soln: Given, $m(t) = 8\cos 8\pi\times10^3 t$

$A_m = 8$; $f_m = 4K$.

$n = 10$.

∴ Sampling Rate is not given

So, $f_s = NR = 2f_m = 8K$

So, $R_b = nf_s = 10\times 8K$

$= 80Kbps$.

So, For ASK, $\boxed{B.W = 2R_b = 160K}$; $\boxed{\text{For PSK } B.W = 160K}$

For FSK,

B.W = f_{11} f_1+2R_b

B.W = (2-1)M + 160K

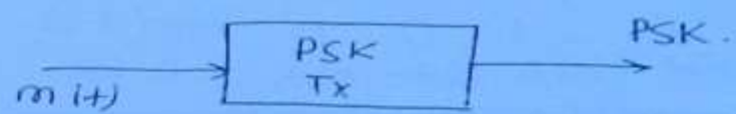
$\boxed{B.W = 1.16M}$ Ans

245

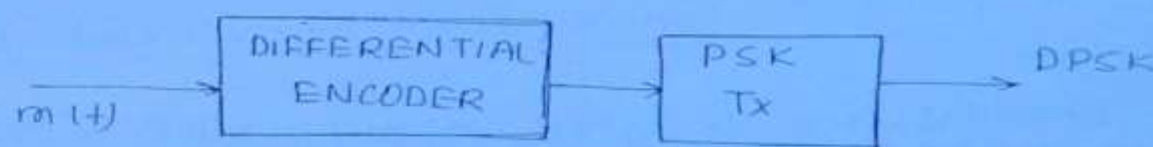
* DIFFERENTIAL PHASE SHIFT KEYING (DPSK):

* The advantage of DPSK over PSK is no QNE.

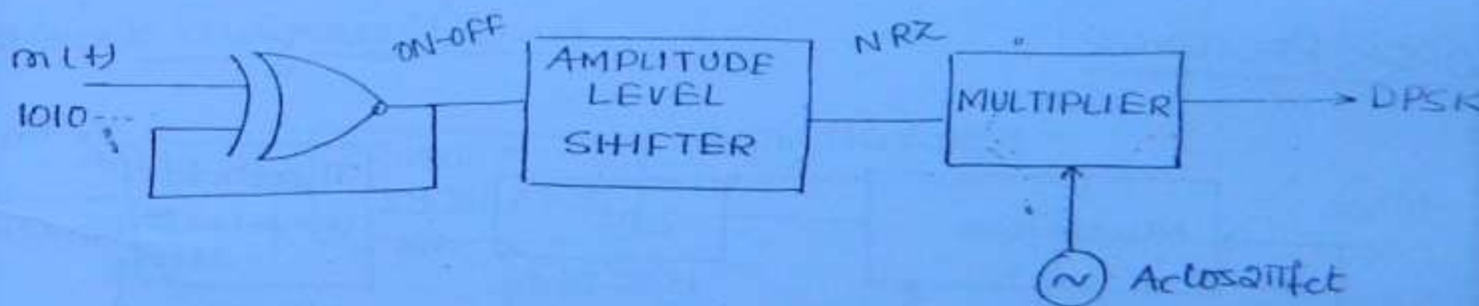
PSK:



DPSK:



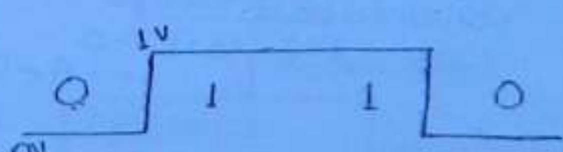
* Internal ckt ary:



m(t) 1 0 1 0

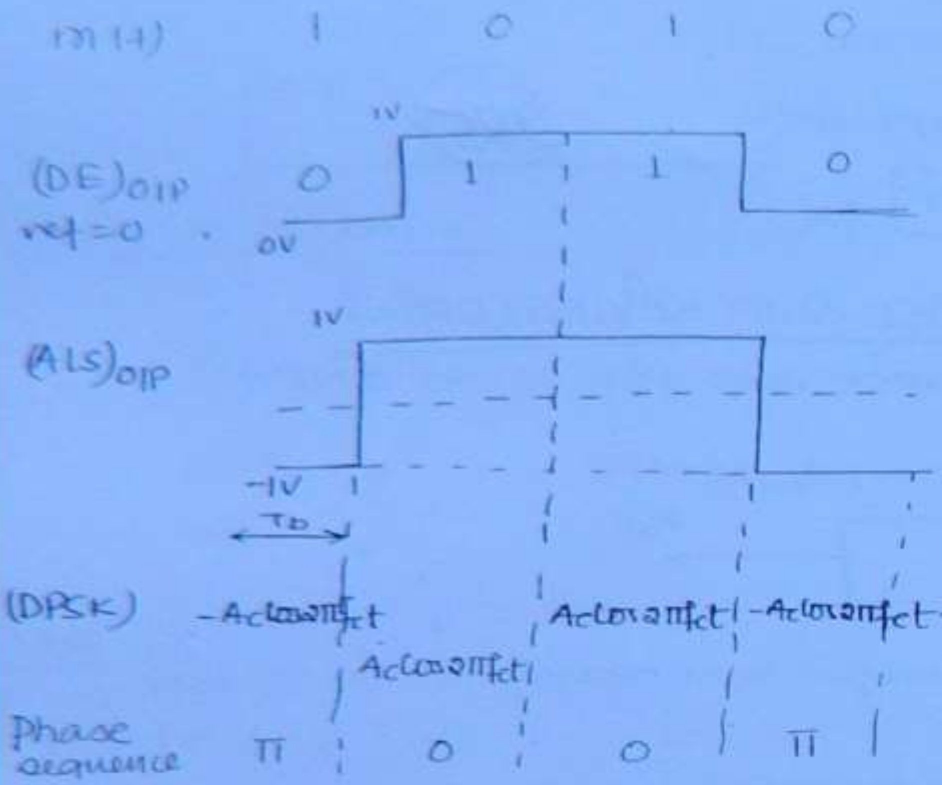
(DE)o/p

1st ref = 0.



ON-OFF form

Note! The o/p of Differential encoder is in ON-OFF form & the input of PSK Tx should be NRZ form. Hence it needs to be converted. It is done by the Amplitude level shifter.

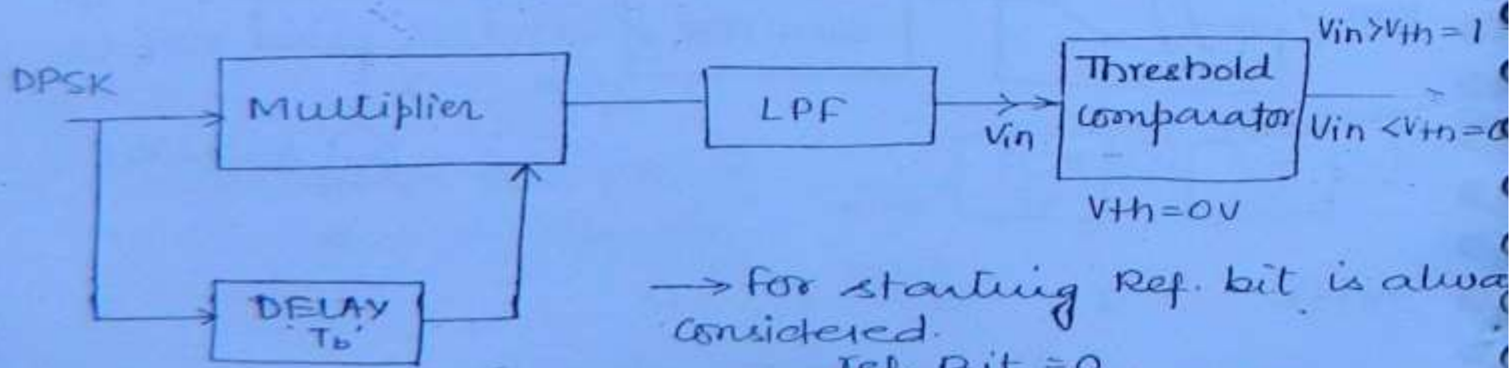


Amplitude level shifter

Note:

m(t) will be given and the ref bit will also be given and the phase sequence of resulting DPSK will be asked.

DPSK RECIEVER:



→ for starting Ref. bit is alwa[ys] considered.

Ref. Bit = 0
= $-A_c \cos 2\pi f_c t$

*Analysis:

DPSK →	$-A_c \cos 2\pi f_c t$	$A_c \cos 2\pi f_c t$	$A_c \cos 2\pi f_c t$	$-A_c \cos 2\pi f_c t$
(Mul)o/p →	$A_c^2 \cos^2 2\pi f_c t$	$-A_c^2 \cos^2 2\pi f_c t$	$A_c^2 \cos 2\pi f_c t$	$-A_c^2 \cos^2 2\pi f_c t$
(LPF)o/p →	$A_c^2/2$	$-A_c^2/2$	$A_c^2/2$	$-A_c^2/2$
Final o/p →	1	0	1	0

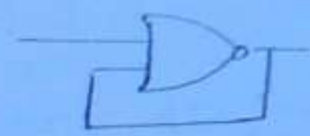
Q) A binary signal [illegible] by the DPSK transmitter, reference bit is 1 find phase sequence of the Resulting DPSK signal

a) $0\pi00$

b) $\pi 0 \pi \pi$

c) $00\pi 0$

d) $\pi\pi 0 \pi$



Soln: $m(t) = 0 \quad 1 \quad 0 \quad 0$

(D·E) o/p $\quad 0 \quad 0 \quad 1 \quad 0$

ref = 1 $\quad -A_c\cos \quad -A_c\cos \quad A_c\cos \quad -A_c\cos$

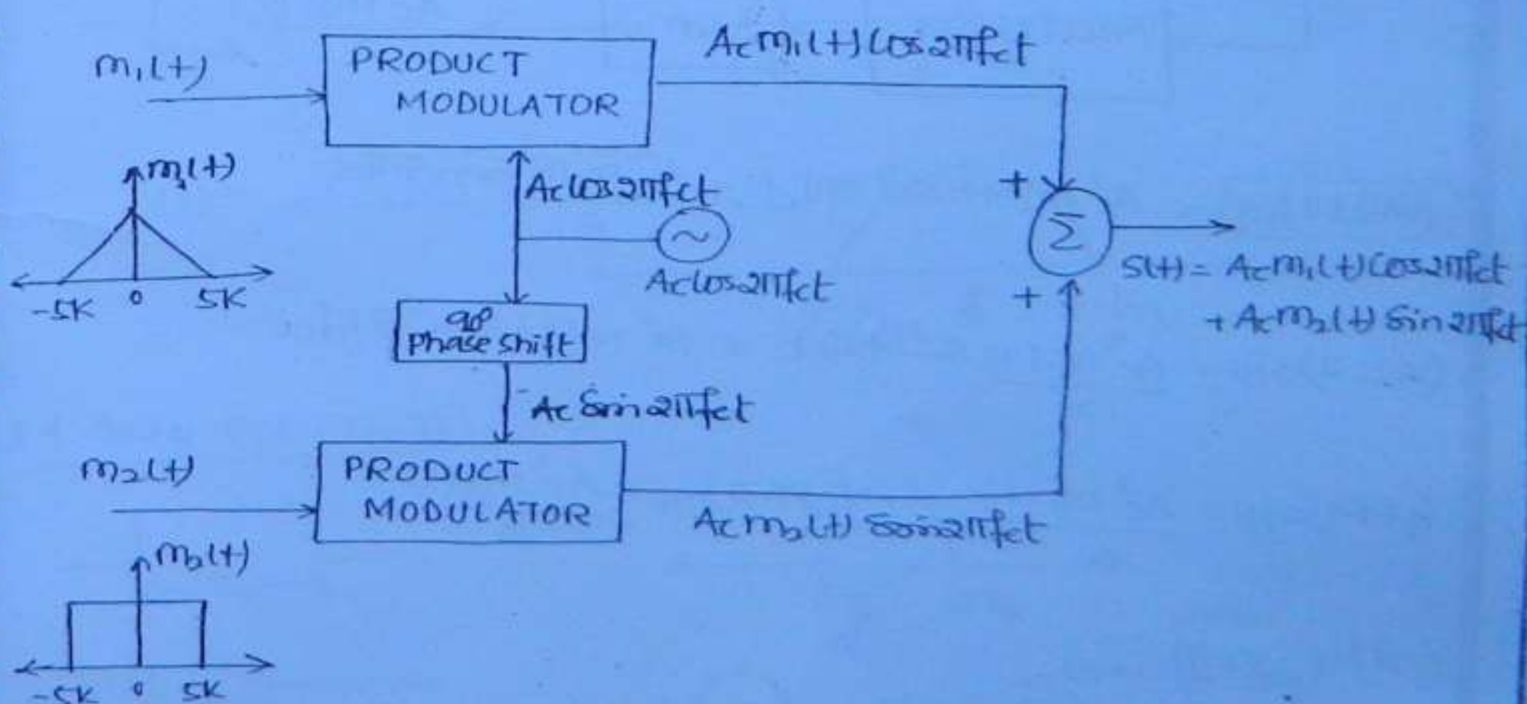
ϕ sequence	π	π	0	π

Ans

* Quadrature Carrier Multiplexing:

* By using this 2 signals will be multiplexed, where the corresponding carriers will have same freqn, and having 90° phase shift b/w them.

* Block diagram (Tx):



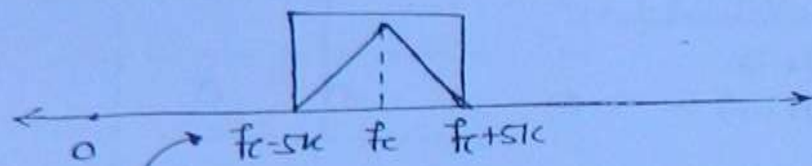
$M_1(t) = A_c m_1(t) \cos 2\pi f_c t \longleftrightarrow \frac{A_c}{2}\left\{ M_1(f-f_c) + M_1(f+f_c) \right\}$.

$M_2(t) = A_c m_2(t) \sin 2\pi f_c t \longleftrightarrow \frac{A_c}{2j}\left\{ M_2(f-f_c) - M_2(f+f_c) \right\}$.

~~284~~ 248

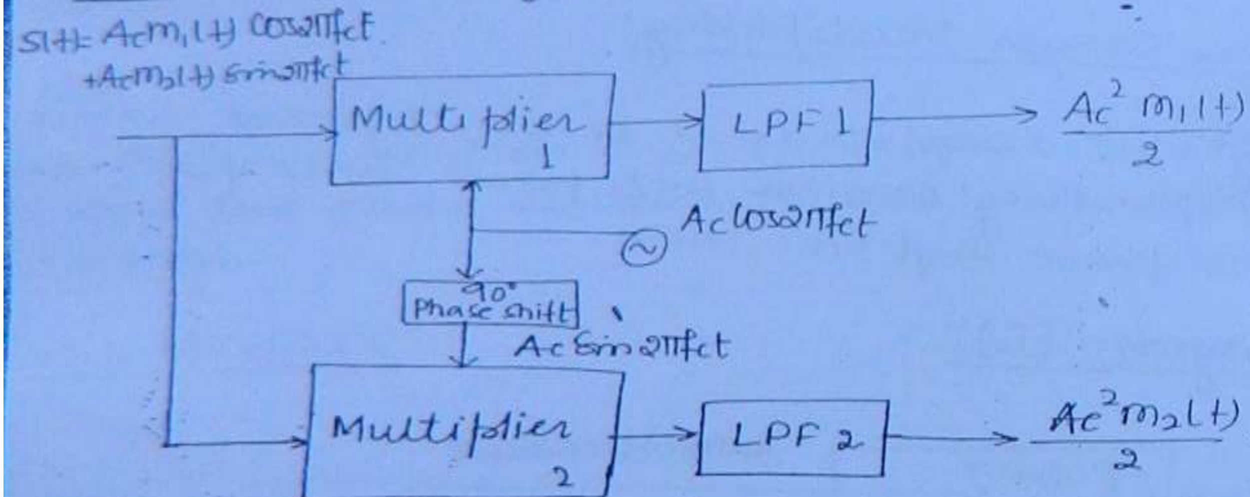
So,

$s(t) \longleftrightarrow$



No interference since carriers are quadrature to each other.

*Reciever Block diagram:



$(\text{Mul } 1)\, o/p = A_c^2 m_1(t) \cos^2 2\pi f_c t + \frac{A_c^2 m_2(t)}{2} \sin 4\pi f_c t$

$(\text{Mul } 2)\, o/p = \frac{A_c^2 m_1(t)}{2} \sin 4\pi f_c t + A_c^2 m_2(t) \sin^2 2\pi f_c t$

$(\text{LPF1})\, o/p = \frac{A_c^2 m_1(t)}{2}$; $(\text{LPF2})\, o/p = \frac{A_c^2 m_2(t)}{2}$

* M-ARRAY SIGNALLING :

* In ASK, PSK & FSK, one bit is transmitted at a time
  ie $N=1$

  No of symbols possible $\Rightarrow M=2 \Rightarrow 0;1$ (229)

* 2 No. of symbols are possible, hence ASK, PSK & FSK are called as Binary signalling schemes or 2 Array signalling schemes.

* ASK $\longrightarrow$ BASK.
  PSK $\longrightarrow$ BPSK.
  FSK $\longrightarrow$ BFSK.

* For 4 Array PSK:

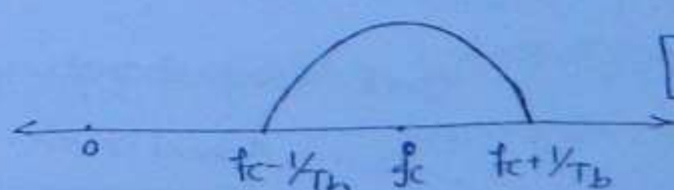
  $M=4$.
  $N=2$. $\{2^{(2)}=(4)\}$.

  Hence 2 bits are transmitted at a time $\boxed{(N=2)}$

  So, no. of symbols possible are $\boxed{M=4}$

*2- Array PSK (N=1):

s(t)



$\boxed{B\cdot W = 2R_b}$

*4 Array PSK (N=2):-

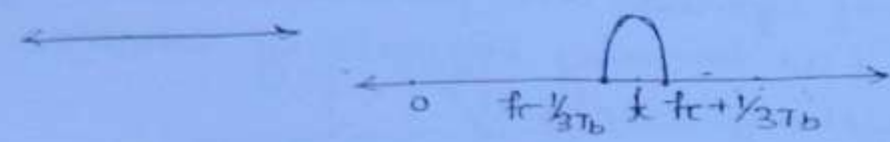
s(t)

$\Rightarrow B\cdot W = \left(f_c+\frac{1}{2T_b}\right)-\left(f_c-\frac{1}{2T_b}\right)$

$= 2\times\frac{1}{2T_b} = \boxed{R_b = BW}$

* 8 Array PSK

s(t)



$$B.W = \frac{2}{3} T_b$$

$$\boxed{B.W = 0.6 T_b}$$

* Conclusion:

* By increasing the No. of bits to be transmitted in specific time instant, then the B.W decreases

$$\boxed{N \uparrow \longrightarrow B.W \downarrow}$$

* If increasing N to very high value the complexity of Tx & Rx increases.

* As the no. of bits to be transmitted in specific time instant increases, transmission B.W required will be decreases. But correspondingly complexity of the system increases.

w.r.t Actual carrier

* $\boxed{\text{Phase shift in M-Array PSK} = \Phi = 2\pi/M}$

a) for $M=2 \Rightarrow \Phi = \pi$ $\Rightarrow 1; S_1(t) = A_c \cos 2\pi f_c t$
$0; S_2(t) = -A_c \cos 2\pi f_c t$ π

b) for $M=4 \Rightarrow \Phi = \pi/2$
4 Array PSK = QPSK. $\Rightarrow$
Quadrature phase shift keying

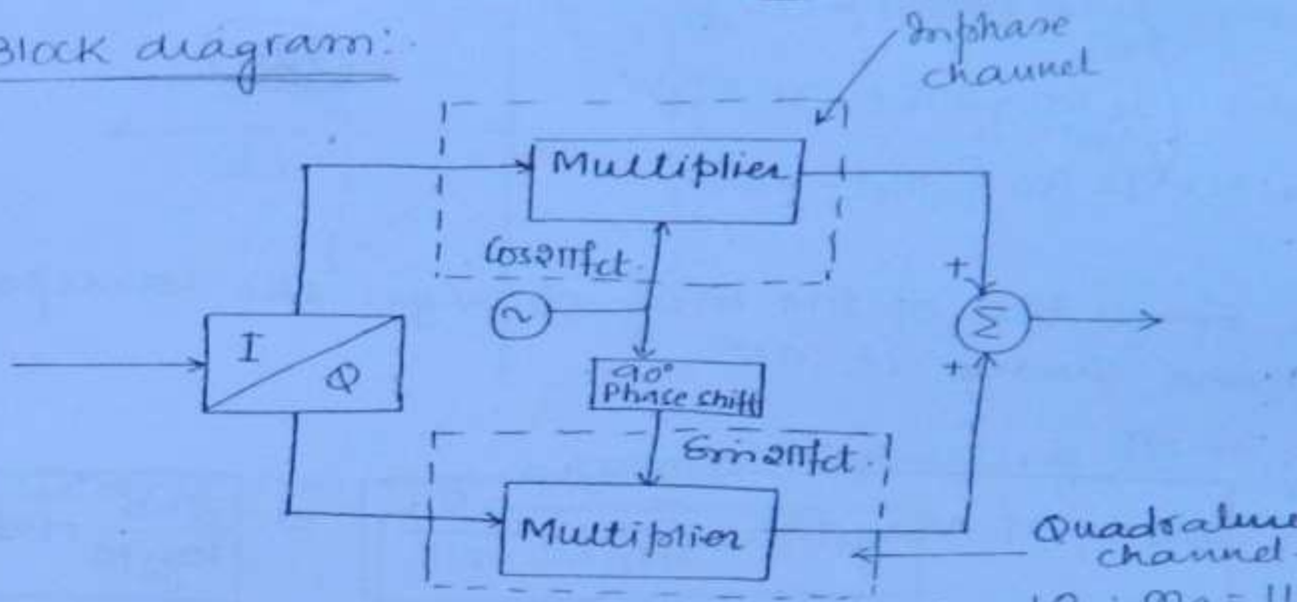
$\rightarrow$ $\boxed{\begin{array}{l} B.W \text{ of BPSK} = 2R_b \\ B.W \text{ of QPSK} = R_b. \end{array}}$

*QPSK Transmitter:

QPSK is 4 Array PSK

257

* Block diagram:

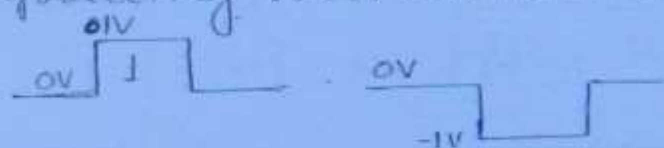


$m_1 = 00$; $m_2 = 01$; $m_3 = 10$; $m_4 = 11$

* NRZ signalling mechanism is used

$1 \rightarrow +ve$

$0 \rightarrow -ve$



$$00 \rightarrow S_1(t) = -\cos 2\pi f_c t - \sin 2\pi f_c t$$

$$01 \rightarrow S_2(t) = -\cos 2\pi f_c t + \sin 2\pi f_c t$$

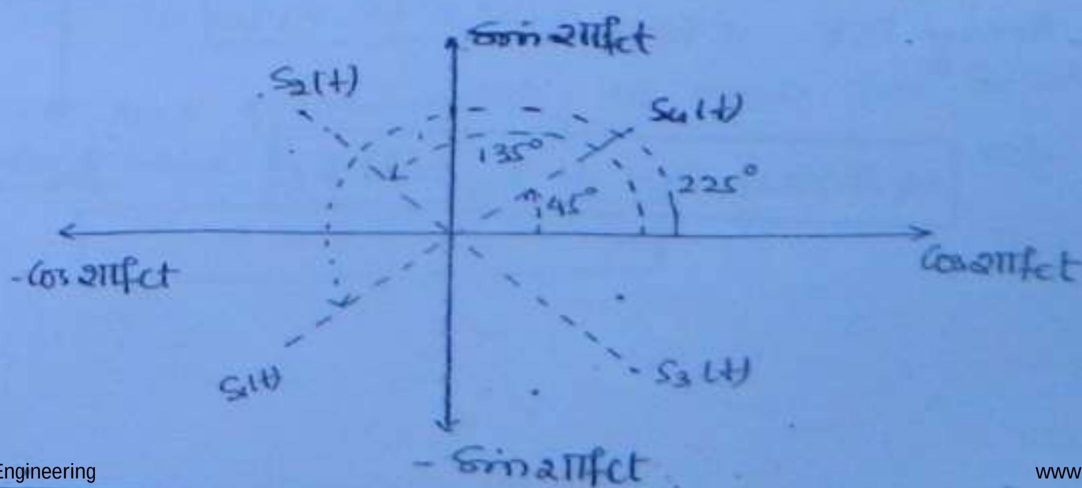
$$10 \rightarrow S_3(t) = \cos 2\pi f_c t - \sin 2\pi f_c t$$

$$11 \rightarrow S_4(t) = \cos 2\pi f_c t + \sin 2\pi f_c t$$

Now, as

$$A\cos 2\pi f_0 t + B\sin 2\pi f_0 t = \sqrt{A^2+B^2}\cos\{2\pi f_0 t + \Phi\}$$

$$\Phi = -\tan^{-1}(B/A).$$



So,

00 $s_1(t) = \sqrt{2}\cos\{2\pi f_c t + [illegible]^\circ\}$

01 $s_2(t) = \sqrt{2}\cos\{2\pi f_c t + 135^\circ\}$

10 $s_3(t) = \sqrt{2}\cos\{2\pi f_c t - 315^\circ\}$

11 $s_4(t) = \sqrt{2}\cos\{2\pi f_c t - 45^\circ\}$.

90° 90° 90° 90°

258

* As single bit of the msg changes the correspondi phase change is 90°.

Note:

$$\boxed{\text{B.W of M-Array PSK} = \frac{2}{NT_b} = \frac{2R_b}{N}} \Rightarrow \boxed{\frac{2R_b}{\log_2 M} = BW}$$

1. For 2 Array PSK, $B.W = 2R_b$. $\{N=1\}$
2. For 4 Array PSK, $B.W = R_b$ $\{N=2\}$

As. $\boxed{M = 2^N}$

only ES } No Gate No PSU

*CONSTELLATION DIAGRAM:

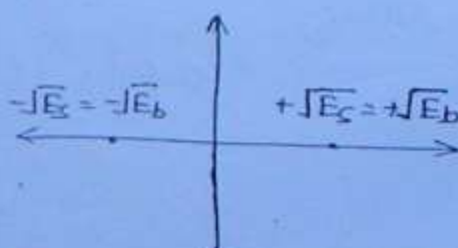
*For M Array PSK; distance of Each of the Signalling pt from the origin $= \sqrt{E_s}$. ← Symbol energy

* ~~For 2 Array PSK ⇒ $\sqrt{E_s} = \sqrt{E_b}$.~~

*For 2 Array PSK ⇒ $E_s = E_b$

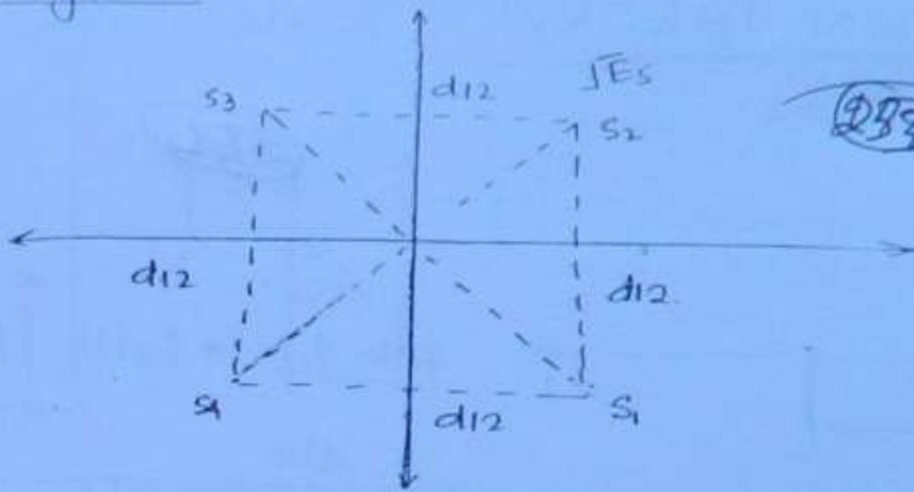
*For 4 Array PSK ⇒ $E_s = 2E_b$.

* So, for $\boxed{\text{M Array PSK} \Rightarrow E_s = N E_b}$



"2-Array PSK"

4 Array PSK:



(258)

* distance b/w two adjacent signalling pts is

$$\boxed{d_{12} = 2\sqrt{E_s}\, \sin \pi/M}$$

* For 2 Array PSK ; $d_{12} = 2\sqrt{E_b} \cdot \sin \pi/2$

$$\boxed{d_{12} = 2\sqrt{E_b}}$$

* For 4 Array PSK; $d_{12} = 2\sqrt{E_s}\, \sin \pi/M$

$$= 2\sqrt{2E_b} \cdot \sin \pi/4$$

$$= 2\sqrt{2E_b} \cdot 1/\sqrt{2}$$

$$\boxed{d_{12} = 2\sqrt{E_b}}$$

Conclusion:

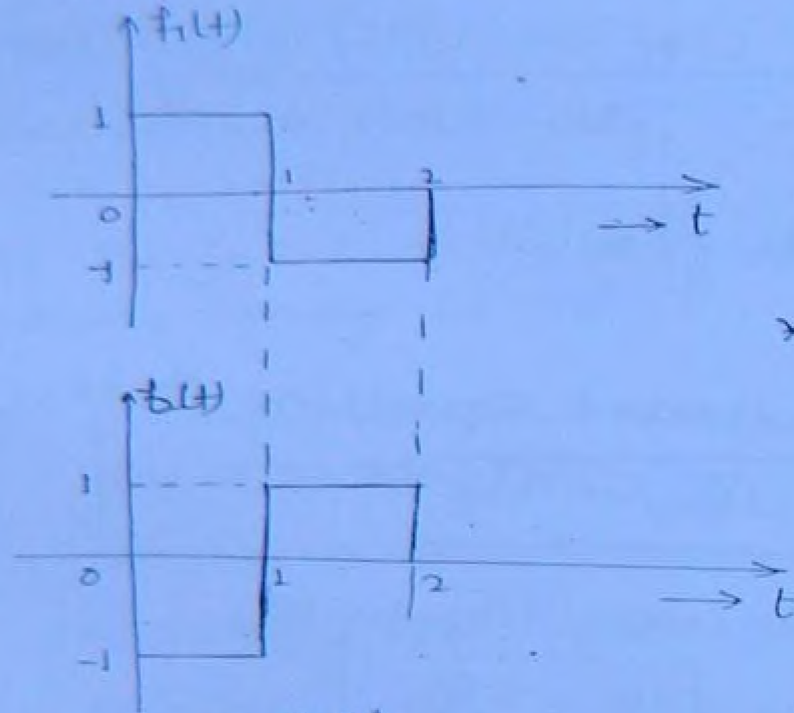
* Since, the distance of adjacent signalling pts is the same. ie $\boxed{d_{12} = 2\sqrt{E_b}}$

Hence, the probability of error for BPSK and QPSK are same

$$\boxed{P_e(BPSK) = P_e(QPSK)}$$

* INFORMATION THEORY:

* Analysis:



As,

$$f(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) * f_2(t$$

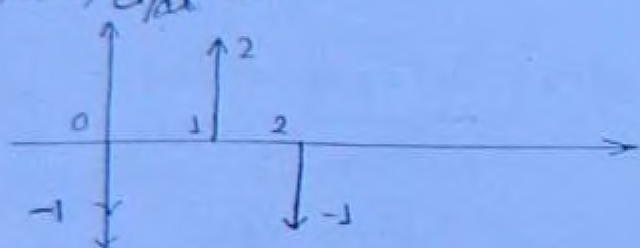
Also,

$$\boxed{\frac{d}{dt}\{f_1(t) * f_2(t)\} = f_1(t) * \left\{\frac{d}{dt} f_2(t)\right\}}$$

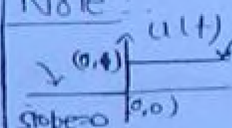
So,

$$f_1(t) * f_2(t) = \int_{-\infty}^{t} f_1(t) * \frac{d}{dt} f_2$$

Now, $d/dt\ f_2(t)$



Note:



$$\frac{d}{dt} u(t) = \frac{\text{change } y}{\text{change } x} = \frac{1}{0} = \infty$$

$$d/dt\ u(t) = \infty,\ t = 0$$
$$= 0;\ t \neq 0$$

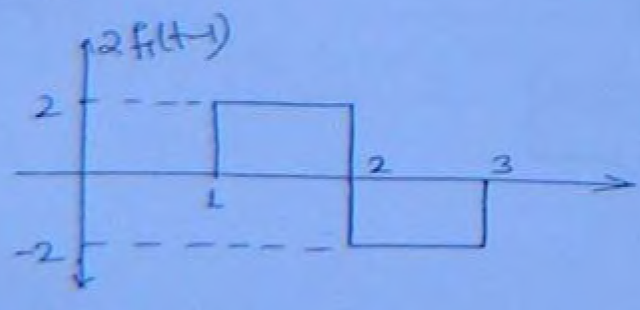
So,

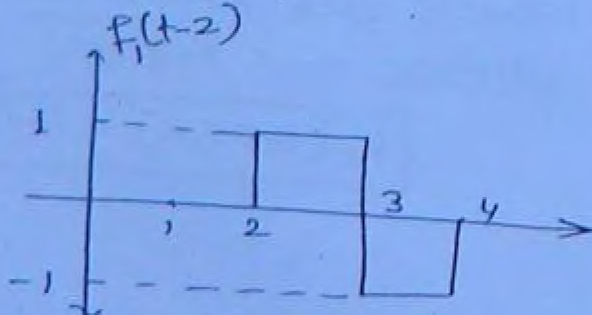
$$\boxed{d/dt\ u(t) = -\delta(t) + 2\delta(t-1) - \delta(t-2)}$$

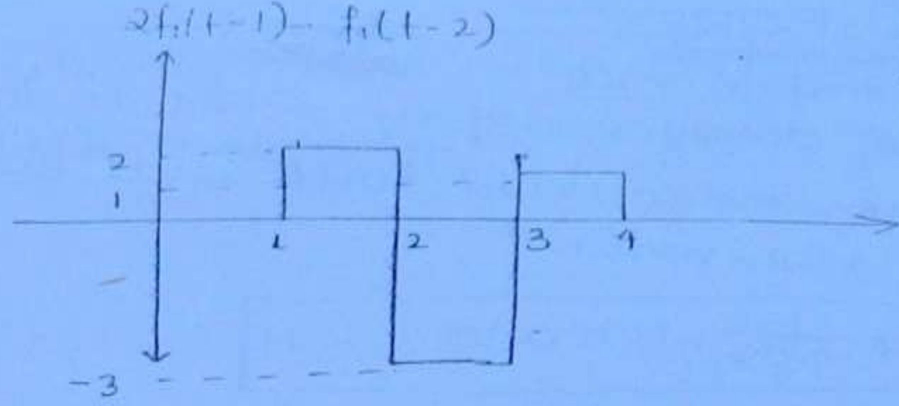
So, $f_1(t) * d/dt\{f_2(t)\} = f_1(t) * \{-\delta(t) + 2\delta(t-1) - \delta(t-2)\}$.

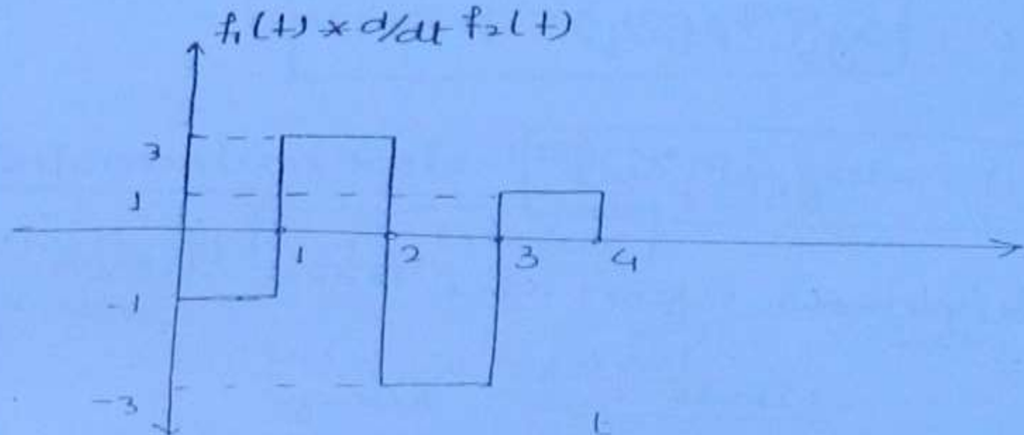
$$\boxed{f_1(t) * \frac{d f_2(t)}{dt} = -f_1(t) + 2f_1(t-1) - f_1(t-2)}$$

Now,







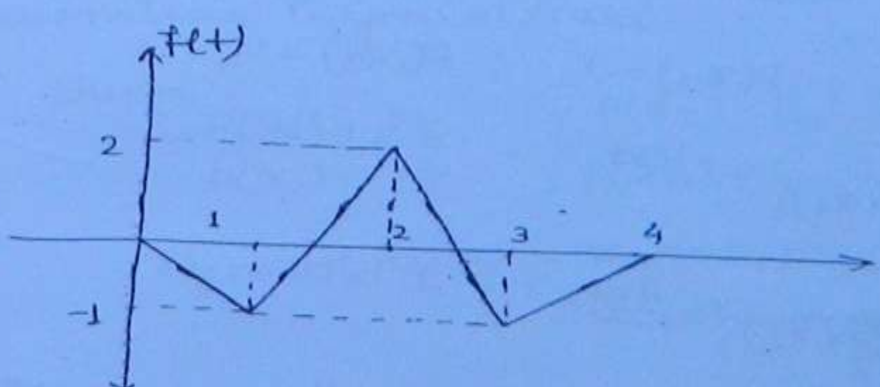


Now, $f(t) = \int_{-\infty}^{t} f_1(t) * d/dt\, f_2(t)$ = Area of the fun

1) At $t = 0.1 \Rightarrow$ Area $= -0.1$

At $t = 0.2 \Rightarrow$ Area $= -0.2$

$t = 1 \Rightarrow$ Area $= -1$.



Note:

$$\boxed{\begin{array}{l} f(t) = f_1 * f_2 \\ A_1 \cdot A_2 = A_1 * A_2 \end{array}}$$
$\leftarrow$ Areas.

* INFORMATION THEORY:

(258)

* Information means importance.
* If the probability of occurence of an event is less then the information associated with that event will be more and vice-versa.

$$I\{x_i\} \propto \frac{1}{P\{x_i\}}.$$

$$I\{x_i\} = \log_b(1/P\{x_i\})$$

or $$\boxed{I(x_i) = -\log_b\{P(x_i)\}}$$

* Units of $I\{x_i\}$ depends upon the base chosen ie

b	Units
2	bits
e	nat
10	decit

Q1. A source is generating 3 possible symbols with probabilities of 1/4, 1/2, 1/4. Find the information associated with each of the symbol.

Soln: Given, $P(x_1) = 1/4$; $P(x_2) = 1/2$; $P(x_3) = 1/4$

$$I(x_1) = \frac{1}{\log_2\{P(x_1)\}} = +\log_2 4 = +2 \text{ bits}$$

$$I(x_2) = \frac{1}{\log_2\{P(x_2)\}} = \log_2 2 = 1 \text{ bit}$$

$$I(x_3) = \frac{1}{\log_2\{P(x_3)\}} = \log_2 4 = 2 \text{ bits}.$$

Note:-

The prob. of occurrance of x_2 is high so the information associated with x_2 will be less.

* Average Information or Entropy (H)

* Units of $\boxed{H \text{ is bits/symbol}}$

* Mathematically, it is given as:- 252

$$H = \sum_i I\{x_i\} . P(x_i)$$

$$\boxed{H = \sum_i p(x_i) \log_2 1/P(x_i)}$$

$$\boxed{H = -\sum_i p(x_i) \log_2 P(x_i)}$$

* Information Rate (R) :-

* Units of $\boxed{R \text{ is bits/sec}}$

• Now,

$$R = \frac{\text{bits}}{\text{Symbol}} \times \frac{\text{Symbol}}{\text{sec}}$$

Symbol (r) Rate

So, $\boxed{R = H \times r}$ ⇒ $\boxed{\text{Information Rate} = \text{Symbol Rate} \times \text{Entropy}}$

Q. A source is generating 4 possible symbols with the probabilities of 1/8, 1/8, 1/4, 1/2.
Find Entropy and Information Rate if the source is generating 1 symbol/msec.

Soln: Given, $P(x_1) = 1/8$; $P(x_3) = 1/4$

$P(x_2) = 1/8$; $P(x_4) = 1/2$

Symbol Rate = 1 symb/msec

r = 1000 Symbol/sec

Now,

$$H = \sum_{i=1}^{4} P(x_i) \log_2 1/P(x_i) = \frac{1}{8}\log_2 8 + \frac{1}{8}\log_2 8 + \frac{1}{4}\log_2 4 + \frac{1}{2}\log_2 2$$

$$\boxed{H = 1.75 \text{ bits/symbol}}$$

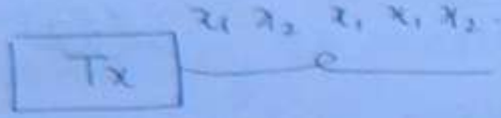
So, $\boxed{R = 1.75 \times 1000 = 1.75 \text{ Kbps}}$ Ans.

note

* Entropy is measure of uncertainty

* Analysis:

1.



2584

Case 1:

$P(x_1) = P(x_2) = 1/2$

$$H = \sum_{i=1}^{2} P(x_i) \log_2 1/P(x_i)$$

$$= \frac{1}{2} \log_2 2 + 1/2 \log_2 2$$

$$\boxed{H = 1 \text{ bits/symbol} = H_{max}}$$

when the prob are equal.

Case 2:

$P(x_1) = 1 ;\ P(x_2) = 0$

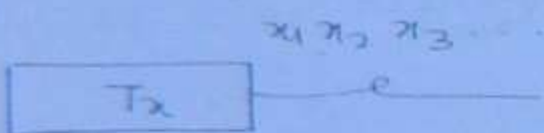
$$H = -\sum_{i=1}^{2} P(x_i) \log_2 P(x_i)$$

$$= 0 + 0$$

$$\boxed{H = 0 \text{ bits/symbol} = H_{min}}$$

when the prob. of one is 1 & other 0.

2.



Case 1:

$P(x_1) = P(x_2) = P(x_3) = 1/3$

$$H = \sum_{i=1}^{3} P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$H_{max} = \frac{1}{3} \log_2 3 + 1/3 \log_2 3 + \frac{1}{3} \log_2 3$$

$$\boxed{H_{max} = \log_2 3 \text{ bits/symbol}}$$

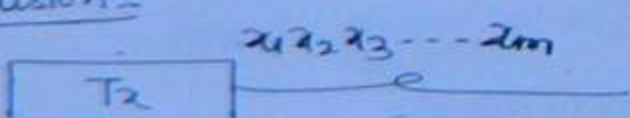
Case 2:

$P(x_1) = 1 ;\ P(x_2) = 0 = P(x_3)$

$$H_{min} = \sum_{i=1}^{3} P(x_i) \log_2 \frac{1}{P(x_i)}$$

$$\boxed{H_{min} = 0 \text{ bits/symbol}}$$

Conclusion:-



$P(x_1) = P(x_2) = P(x_3) = \ldots P(x_m) = 1/M$

$$\boxed{H_{max} = \log_2 M \text{ bits/symbol}} ;\ H_{min} = 0 \text{ bits/symbol}$$

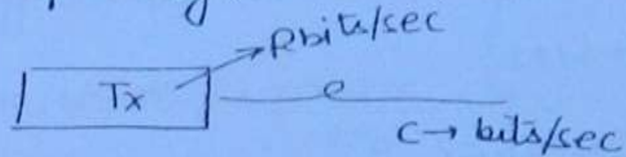
* If all the symbols are having equal prob of occurrence then Entropy will be maxm.

* CHANNEL CAPACITY:

~~185~~ 259

* It specifies the no of bits allowed by the channel in 1 sec

* Channel capacity; C = bits/sec



Hence,

$$\boxed{C \geq R \leftarrow \text{No Information loss}}$$

* SHANON HARTLEY LAW:

It gives the Relation b/w channel capacity (C) and its Bandwidth (B.W)

Mathematically,

Normal S/N (not in dB)

$$\boxed{C = B\log_2(1+S/N)} \qquad (S/N)_{dB} = 10\log_{10}(S/N)$$

where,

C = Channel capacity (bits/sec)

B = Channel B.W (Hz).

S = Signal power expected at channel O/P.

N = Noise power.

$(S/N)_{dB}$	(S/N)
1) 10 dB	10
2) 20 dB	100
3) 15 dB	$10^{1.5}$

Q. For a channel of B.W = 4KHz

(S/N) = 15 dB.

Find the channel capacity

Soln:

As (S/N) = 15 dB

So, $(S/N) = 10^{1.5} = 31.6$

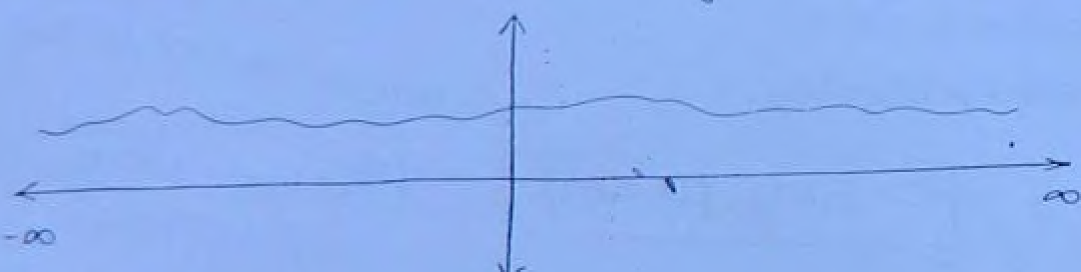
So, $C = B \log_2 \{1 + S/N\}$.

$= 4 \log_2 \{1 + 31.6\}$

$\boxed{C = 20.1 \text{ Kbps}}$ Ans

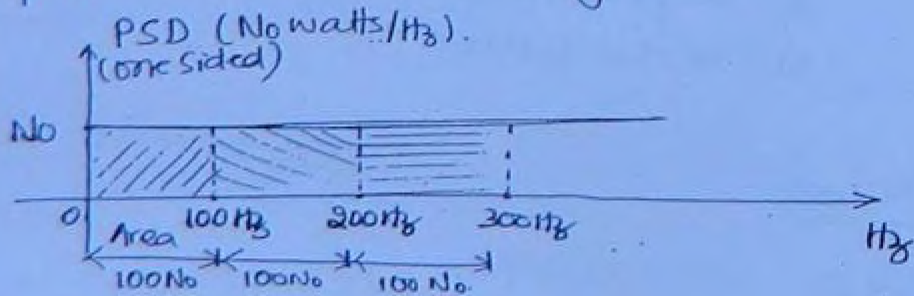
* Capacity of AWGN (additive white Gaussian Noise) channel :-

* white Noise has the frequency spectrum as following.



It covers the all frequency component.

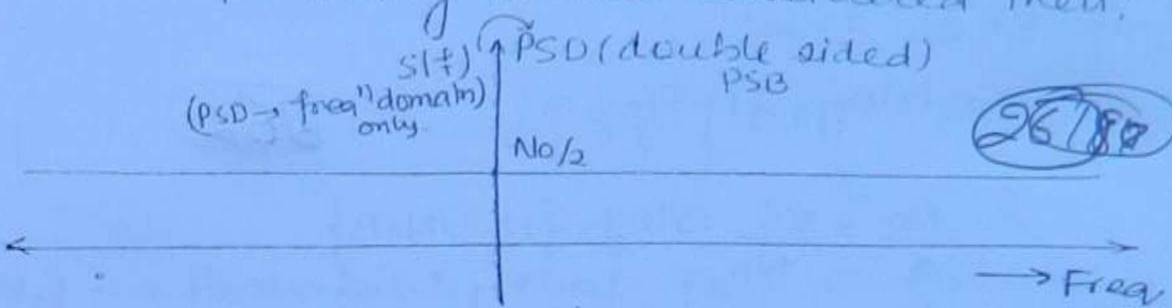
* The PSD of the white Noise is given as:-



* Freqn b/w 0 to 100Hz will be affected by 100No watt of power

* Freqn b/w 100 to 200Hz will be affected by 100No watt of power

* If the -ve frequency is also considered then:



* Regarding white Noise, its power is given as:

$$N(\text{watts}) = \frac{\text{watts}}{\text{Hz}} \times \text{Hz}$$

$$\boxed{N = N_0 \times B} \text{ watts}$$

* Default power spectral density is one sided PSD.

Note:

* Each of the frequency component transmitted through a channel is affected by same amount of white Noise power.

* The channel B.W is given as:

$$\uparrow C = B \uparrow \log_2 \{1 + S/N\}.$$

(linear)

So, for a AWGN channel.

$$\uparrow C = B \uparrow \log \log_2 \{1 + S/N_0 B\}.$$

(non linear)

Conclusion:

* For AWGN channel as $B \rightarrow \infty$.
channel capacity becomes.

$$\boxed{C_\infty = 1.44 \, S/N_0}$$

Proof:

As we know that:

$$C = B\log_2\{1 + S/N_0B\}$$

258

$$C = \frac{N_0}{S} \times \frac{S}{N_0} B\log_2\{1 + S/N_0B\}$$

let $\frac{N_0B}{S} = x$

as $B \to \infty \Rightarrow x \to \infty$.

So,

$$C_\infty = \frac{S}{N_0} \lim_{x \to \infty} x\log_2(1 + 1/x)$$

$$C_\infty = \frac{S}{N_0} \log_2 e$$

V. imp

$$\boxed{C_\infty = 1.44\, S/N_0}$$

Q. For AWGN of having BW 4 KHz, two sided Noise PSD is given by: 10^{-12} watts/Hz. Find the channel capacity required to get signal power of 0.1 mW at the O/P of the channel.

Soln: Given, BW = 4 KHz

$$\frac{N_0}{2} = 10^{-12} \Rightarrow N_0 = 2\times10^{-12}$$

So, $N_0B = 2\times10^{-12} \times 4\times10^3$

$= 8\times10^{-9}$ watts.

Now,

$$C = B\log_2\{1 + S/N_0B\}$$

$$= 4\times10^3 \log_2\left\{1 + \frac{0.1\times10^{-3}}{8\times10^{-9}}\right\}$$

$\boxed{\cancel{C = 54.44\text{ Kbps}}}$ Ans

$\boxed{C = 54.44\text{ Kbps.}}$ Ans

* CHANNEL TRANSITION MATRIX OR CONDITIONAL PROB MATRIX:

$P(0/1) \longrightarrow$ Probability that Txd 1 is Rxd as 0. ~~187~~

$P(1/0) \longrightarrow$ Prob. that Txd 0 is Rxd as 1. 263

So,

$$[P(Y/X)] = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) & \cdots & P(y_n/x_1) \\ P(y_1/x_2) & P(y_2/x_2) & \cdots & P(y_n/x_2) \\ \vdots & & & \\ P(y_1/x_m) & P(y_2/x_m) & & P(y_n/x_m) \end{bmatrix}_{m \times n}$$

So,
$$\boxed{\sum_{j=1}^{n} P(y_j/x_i) = 1 \quad ; \text{for any value of } i}$$

$$P(y_1/x_1) + P(y_2/x_1) + \cdots P(y_n/x_1) = 1$$

* Sum of the elements in each Row of channel x-ition matrix will be equal to 1.

* $P[X] = [P(x_1) \; P(x_2) \; \cdots \; P(x_m)]_{1 \times m}$

Input matrix

$$[P(y)] = [P(y_1) \quad P(y_2) \quad \cdots \quad P(y_n)]_{1\times n}$$

output matrix

So,
$$\boxed{[P(y)]_{1\times n} = [P(x)]_{1\times m} \, [P(y/x)]_{m\times n}}$$

(264)

* Channel Matrix $= [P(x,y)] = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) & \cdots & P(x_1;y_n) \\ P(x_2,y_1) & P(x_2,y_2) & \cdots & P(x_2,y_n) \\ \vdots & & & \\ P(x_m,y_1) & P(x_m,y_2) & \cdots & P(x_m,y_n) \end{bmatrix}_{m\times n}$

Note:

$P(x_1,y_1)$ = Probability that when x_1 is generated & Tx and to be Recieved as y_1.

Now

$$\boxed{[P(x,y)]_{m\times n} = [P(x)]_{m\times m \text{ diagonal}} \times [P(y/x)]_{m\times n}}$$

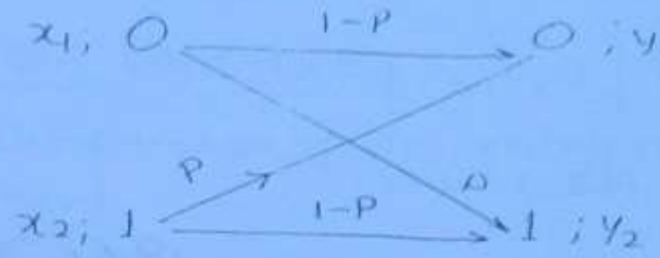
$\therefore [P(x)]_{diagonal} = \begin{bmatrix} P(x_1) & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & P(x_2) & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & P(x_3) & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & & & & & \\ 0 & 0 & \cdots & & & & P(x_m) \end{bmatrix}_{m\times m}$

*Binary Symmetric Channel:

* Tx generating 0 & 1
* Rx recieving 0 & 1
* So, for Binary symmetric channel.

$$\boxed{P(1/0) = P(0/1)}$$

$$\underline{P_{e0}} = \underline{P_{e1}}.$$



$$[P(Y/X)] = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) \\ P(y_1/x_2) & P(y_2/x_2) \end{bmatrix}$$

So $$\boxed{[P(Y/X)] = \begin{Bmatrix} 1-P & P \\ P & 1-P \end{Bmatrix}}$$

*CONDITIONAL ENTROPY:

* It specifies uncertainty about Reciever w.r.t Transmitter.
* Mathematically,

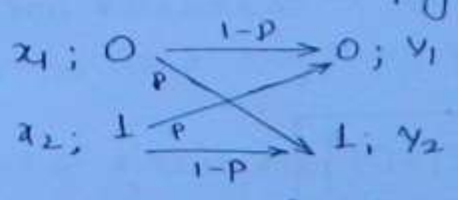
$$\boxed{H(Y/X) = -\sum_{i=1}^{m} \sum_{j=1}^{n} P(x_i, y_j) \log_2 P(y_j/x_i)}$$

let,

1) $P(y_1/x_1) = 0.9$; $P(y_2/x_1) = 0.1$; $P(y_1/x_2) = 1$; $P(y_2/x_2) = 0$.
   $H(Y/X)$ less ← Since $P(Y/X)$ is high
2) $P(y_1/x_1) = 0.4$; $P(y_2/x_1) = 0.6$; $P(y_1/x_2) = 0.5$; $P(y_2/x_2) = 0.5$
   $H(Y/X)$ high ← Since $P(Y/X)$ is low.

* $H(Y/X)$ = height of uncertainty for Rx w.r.t Tx.

Q. Find conditional entropy for Binary symmetric channel.



Soln:

So, $$H(Y/X) = -\sum_{i=1}^{2} \sum_{j=1}^{2} P(x_i, y_j) \cdot \log_2 P(y_j/x_i)$$

Now $$P(Y/X) = \begin{bmatrix} P(y_1/x_1) & P(y_2/x_1) \\ P(y_1/x_2) & P(y_2/x_2) \end{bmatrix} = \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}.$$

Now,

$$P(x,y) = [P(x)]_{diag} \cdot [P(y/x)]$$

let, $P(x_1) = \alpha$; $P(x_2) = 1-\alpha$

266

So, $[P(x)] = [\alpha \quad 1-\alpha]$

$$[P(x)]_{diag} = \begin{bmatrix} \alpha & 0 \\ 0 & 1-\alpha \end{bmatrix}$$

So,

$$P(x,y) = \begin{bmatrix} \alpha & 0 \\ 0 & 1-\alpha \end{bmatrix} \begin{bmatrix} 1-P & P \\ P & 1-P \end{bmatrix}$$

$$P(x,y) = \begin{bmatrix} \alpha(1-P) & \alpha P \\ (1-\alpha)P & (1-\alpha)(1-P) \end{bmatrix} = \begin{bmatrix} P(x_1,y_1) & P(x_1,y_2) \\ P(x_2,y_1) & P(x_2,y_2) \end{bmatrix}$$

Now,

$$H(Y/X) = -\sum_{i=1}^{2}\sum_{j=1}^{2} P(x_i,y_j) \times \log_2 P(y_j/x_i)$$

$$= -\left\{ P(x_1,y_1)\log_2 P(y_1/x_1) + P(x_1,y_2)\log_2 P(y_2/x_1) + P(x_2,y_1)\log_2 P(y_1/x_2) + P(x_2,y_2)\log_2 P(y_2/x_2) \right\}$$

So,

$$H(Y/X) = -\left\{ \alpha(1-P)\log_2(1-P) + \alpha P\log_2 P + (1-\alpha)P\log_2 P + (1-\alpha)(1-P)\log_2(1-P) \right\}$$

$$\boxed{H(Y/X) = -\left\{ P\log_2 P + (1-P)\log_2(1-P) \right\}}$$

So,

$$\boxed{H(Y/X) = P\log_2 1/P + (1-P)\log_2 1/(1-P)}$$

* RANDOM VARIABLES:

> It is the process of Assigning no to the outcome of an experiment.

262

* let, 2 coins are tosed, hence the outcomes are:

{H H, HT, TH, TT} = [S]

All these outcomes are taken under the variable called as sample space variable.

* under some specific condition, the sample space variable is transformed into Random variable.

S	x (Random variable) correspond to no of Heads
H H	2
H T	1
T H	1
T T	0

* when, the Random variable takes the discrete variable then it is called as (discrete Random variable).

* For a variable to be considered as Random variable, the criteria is that the variable should be undeterministic in nature.

Note:

1. A Random variable 'x' is defined as it specifying no. of heads in the exp. of tossing a coin twice.

So,

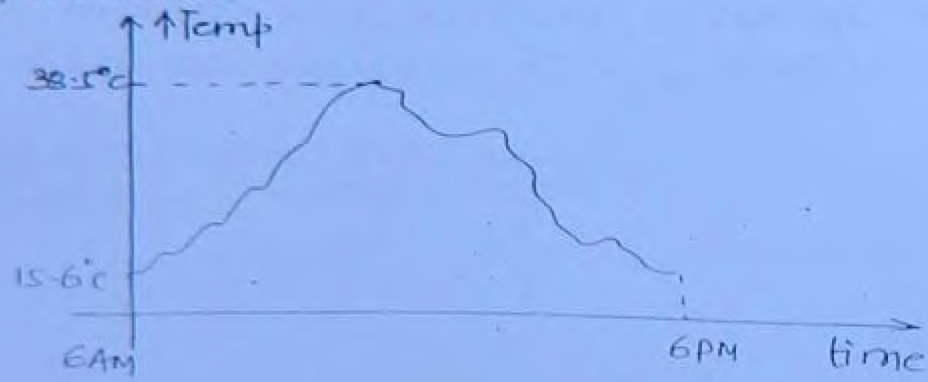
Sample variable {S}	Random variable, x = {x}
H H	2
H T	1
T H	1
T T	0

2. If Random variable takes discrete set of values i.e. it is called as Discrete Random variable.

3. The above is discrete Random variable.

4. If Random variable takes continuous set of values then it is called as continuous Random variable.

5. Random variable which is specifying temp. in a Room from 6 AM to 6 PM corresponds to continuous Random variable.



* PROBABILITY MASS FUNCTION:

It specifies probability of a Random variable taking each of its possible values.

Q. Plot Probability Mass function for a Random variable which is specifying no. of heads in the expt. of Tossing a coin twice

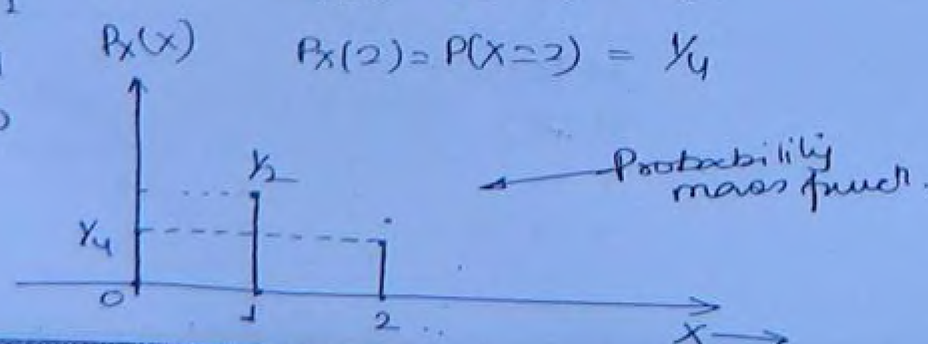
Soln: $P_x(x_i) = P(X = x_i)$.

S	$x = \{x_i\}$
H H	2
H T	1
T H	1
T T	0

So, $P_x(0) = P(X=0) = 1/4$

$P_x(1) = P(X=1) = 1/2$

$P_x(2) = P(X=2) = 1/4$



Properties of [illegible]

1) $0 \leq P_x(x_i) \leq 1$

289

2) $\sum_i P_x(x_i) = 1$

Note:

Prob. Mass func^n (PMF) is used to specify discrete Random variable.

*CUMULATIVE PROBABILITY DISTRIBUTION FUNCTION (CDF):-

* Standard notation is given as $F_x(x) = P(X \leq x)$
* It specifies Probability of Random variable (X) taking the values upto 'x'.

Q. Construct CDF for the above discrete Random variable.

Sol^n:

S	X
H H	2
H T	1
T H	1
T T	0

Now,

$X = \{x\}$ =	0	1	2
$P_x(x)$ =	1/4	1/2	1/4

Now, $F_x(-1) = P(X \leq -1) = 0$.

$F_x(0) = P(X \leq 0) = 1/4$.

$F_x(0.5) = P(X \leq 0.5) = 1/4$.

$F_x(1) = P(X \leq 1) = 1/4 + 1/2 = 3/4 \cdot \{P_x(0) + P_x(1)\}$.

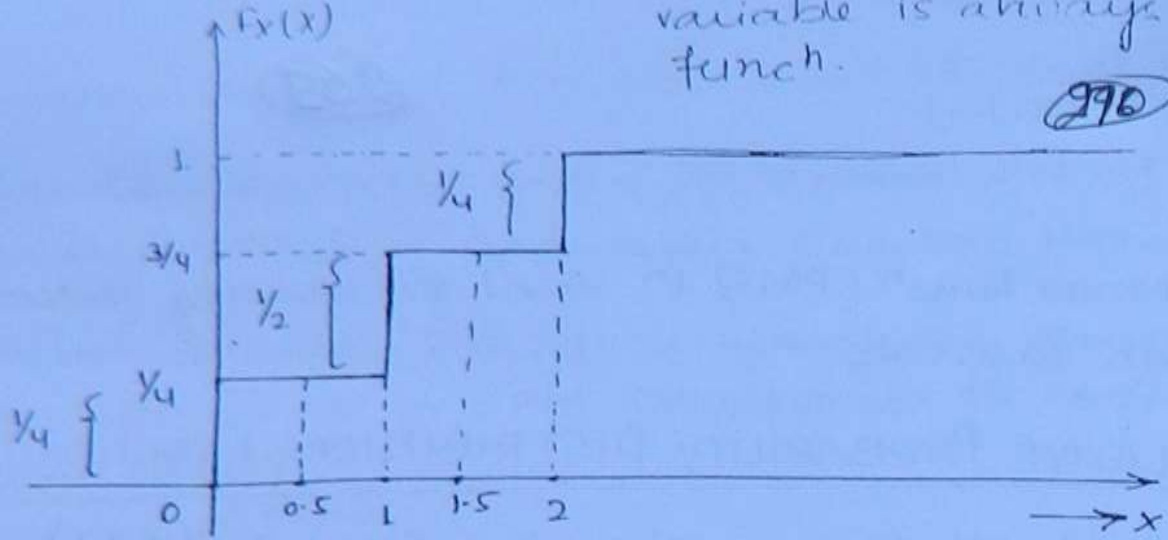
$F_x(1.9) = P(X \leq 1.9) = 3/4$.

$F_x(2) = P(X \leq 2) = 1$.

$F_x(10) = P(X \leq 10) = 1$.

so the plot is given as.

Note: CDF of discrete Random variable is always staircase func^n.



$P(X=1) = \frac{1}{2}$ {Jump offered.}

$P(X=2) = \frac{1}{4}$ {Jump offered.}

Also,

$$\boxed{F_X(x) = \frac{1}{4}u(x) + \frac{1}{2}u(x-1) + \frac{1}{4}u(x-2)}$$

Note:

$F_X(x)$ of a discrete Random variable will be a staircase function.

Q
$X = \{x_i\}$ -1 0 1 2

$P_X(x_i)$ 0.3 $0.2K$ 0.4 0.1

i) $K = ?$

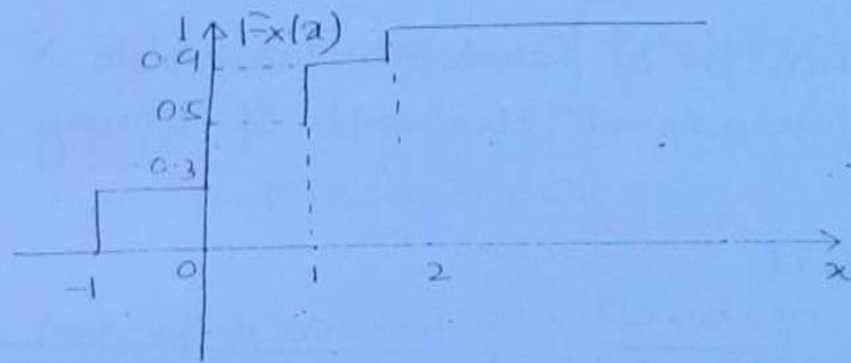
ii) Plot $F_X(x)$

Solⁿ: As $\sum_{i=} P_X(x_i) = 1$

$$0.3 + 0.2K + 0.4 + 0.1 = 1$$

$$\boxed{K = 1} \text{ Ans}$$

ii)



* Properties of $F_X(x)$:

i) $F_X(-\infty) = P(X \le -\infty) = 0.$

ii) $F_X(\infty) = P(X \le \infty) = 1$

iii) $P(X \le x) = F_X(x).$

iv) $P(x_1 \le X \le x_2) = F_X(x_2) - F_X(x_1)$
$\qquad P(X \le x_2) - P(X \le x_1).$

v) $P(X > x) = 1 - F_X(x).$

*** * PROBABILITY DENSITY FUNCTION (PDF):

* Denoted by $f_X(x)$.
* It is generally used to specify continuous Random variable.

* The Relation b/w PDF and distribution function is given as:

$$f_x(x) = \frac{d}{dx} F_x(x)$$

$$\boxed{F_x(x) = \int_{-\infty}^{x} f_x(x)\,dx = P(X \le x)}$$

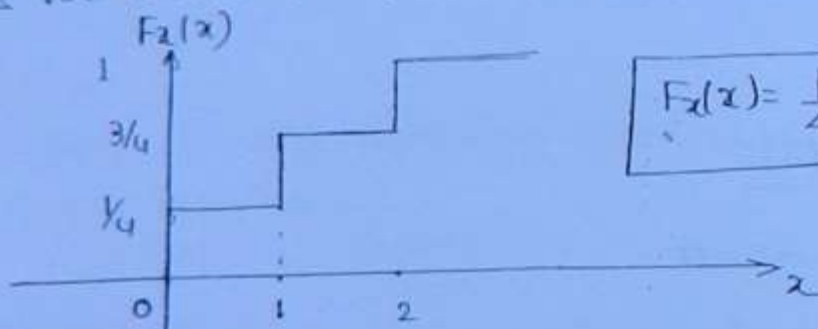
Now,

$$\boxed{P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} f_x(x)\,dx}$$

$$\boxed{P(X \ge x) = \int_{x}^{\infty} f_x(x)\,dx.}$$

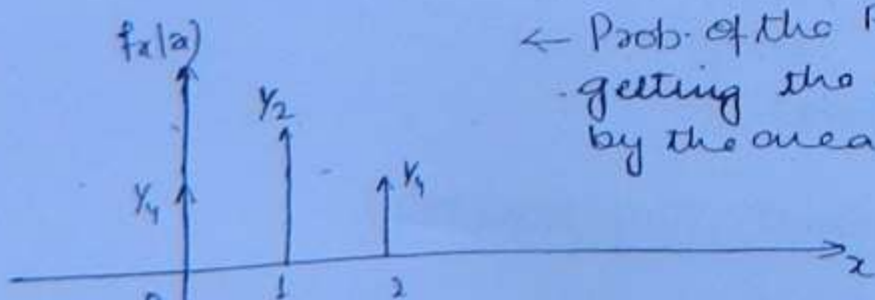
Q. Plot density function for a Random variable X which is specifying no. of heads in the expt of tossing a coin twice.

Soln: As we know that,



$$\boxed{F_x(x) = \frac{1}{4}u(x) + \frac{1}{2}u(x-1) + \frac{1}{4}u(x-2)}$$

So, $f_x(x) = \frac{d}{dx} F_x(x)$

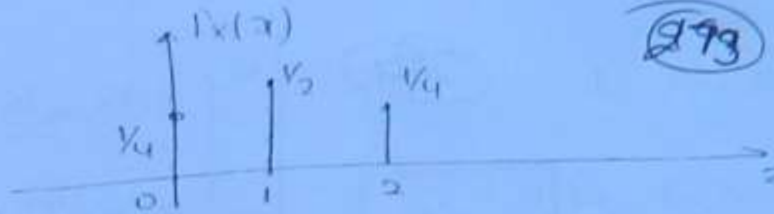


← Prob. of the Random variable getting the prob. can be given by the area of the PDF.

So, $$\boxed{f_x(x) = \frac{1}{4}\delta(x) + \frac{1}{2}\delta(x-1) + \frac{1}{4}\delta(x-2)}$$

Conclusion: Density function of discrete Random variable will be in terms of impulse function.

The Prob. Mass function is given as



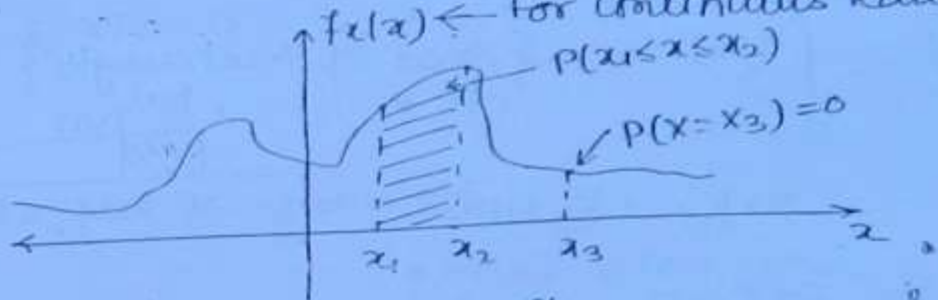
249

$P(X=0) = 1/4$

$P(X=1) = 1/2$

$P(X=2) = 1/4$

Note:



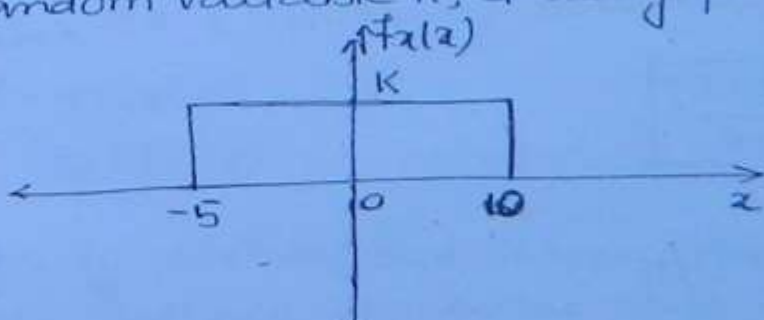
$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f_X(x)\,dx = P(x_1 < X \le x_2)$$

$$P(X = x_3) = 0$$

* The Prob. of a continuous Random variable taking a specific single value will be zero.

Now, $\boxed{P(-\infty \le X \le \infty) = \int_{-\infty}^{\infty} f_X(x)\,dx = 1}$

Q. for a Random variable X; density func^n was given below



a) find K value.

b) $P(-5 \le X \le 10)$.

c) $P(-5 \le X \le 5)$.

d) Plot $f_X(x)$.

Sol^n: It corresponds to the continuous Random variable.

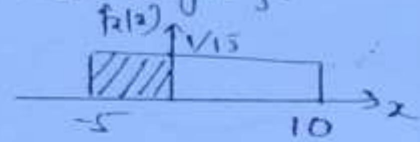
Now as $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$. (2 ab)

So, $15K = 1 \Rightarrow \boxed{K = 1/15}$

Also, $\int_{-5}^{10} K\,dx = Kx\Big|_{-5}^{10} = 1$

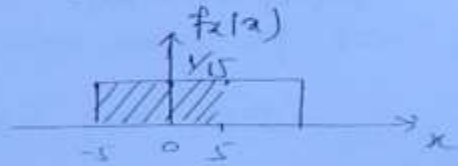
$15 \cdot K = 1 \Rightarrow K = 1/15$

b) $P(-5 \le X \le 0) = \int_{-5}^{0} f_x(x)\,dx$ {Area of Rectangle}.



$= 5 \times 1/15 = 1/3$ units

c) $P(-5 \le x \le 5) = \int_{-5}^{5} f_x(x)\,dx = 10 \times 1/15 = 2/3$ units



d) $F_x(x) = \int_{-\infty}^{x} f_x(x)\,dx$,

$= \int_{-5}^{x} 1/15\,dx = \frac{1}{15} \times x\Big|_{-5}^{x}$

$$\boxed{F_x(x) = \frac{(x+5)}{15}}$$

Now, $F_X(x) = (x+5)/15$

So, $F_X(-10) = 0$ $\{\because F_x(x) = P(X \le x)\}$.

So, $F_x(x) = \int_{-\infty}^{-10} f_x(x)\,dx = 0$.

$F_x(15) = \int_{-\infty}^{15} f_x(x)\,dx = 1$

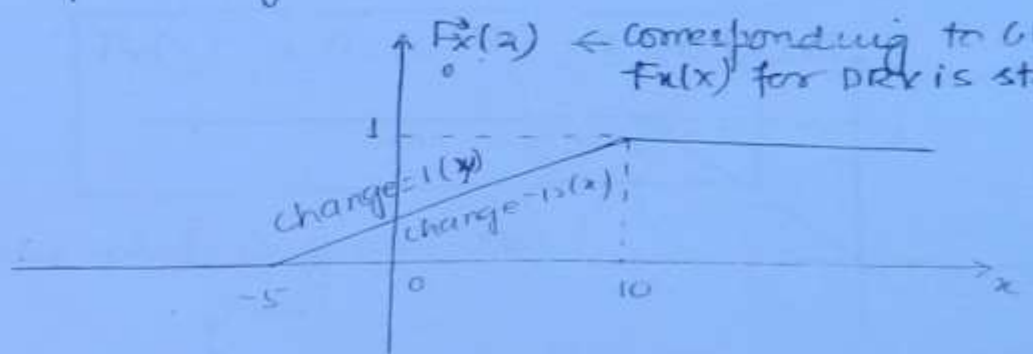
Conclusion:-

$$F_x(x) = \frac{x+5}{15} \; ; \; -5 \le x \le 10$$

$$F_x(x) = 0 \; ; \; x < -5$$

$$ 1 \; ; \; x > 10$$

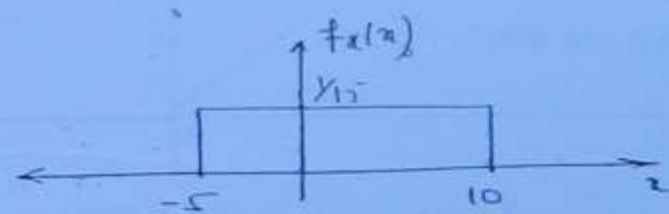
So, the plot is given as:-



Now

$$\frac{d}{dx} F_x(x) = f_x(x) \quad \left\{ \because \frac{d}{dx} = \text{slope} \right\}.$$

So,

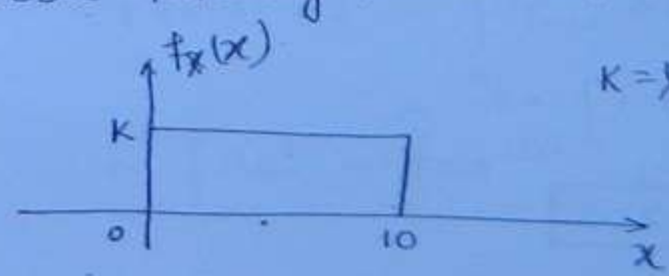


Q2. A Continuous R.V x; uniformly distributed in the interval 0 to 10. Plot, a) $f_X(x)$.

b) $F_X(x)$

Soln: uniformly distributed means that the Prob. of variable taking values at diff. instant is equal.

So,



$K = 1/10$; Since Area = 1.

So, $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

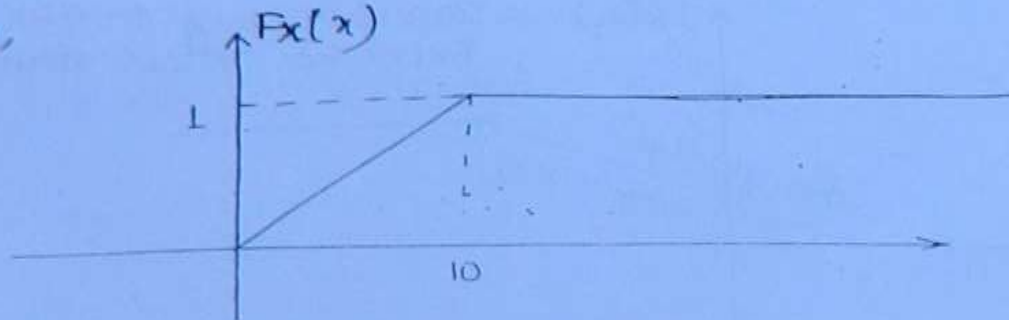
$$= \int_{0}^{x} f_X(x)\,dx$$

$$= \int_{0}^{x} \frac{1}{10}\,dx$$

$$\boxed{F_X(x) = x/10}$$

$$F_X(x) = 0 \;;\; x < 0$$
$$= x/10 \;;\; 0 \le x \le 10$$
$$= 1 \;;\; x > 10.$$

So,



Q. For a CRV; given

$$f_X(x) = ae^{-bx} \;;\; x \geq 0.$$

i) Find Relation b/w a & b.

ii) Plot $f_X(x)$.

Soln: As, $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$$\int_{0}^{\infty} f_X(x)\,dx = 1 = \int_{0}^{\infty} ae^{-bx}\,dx = 1$$

$$= -\frac{a}{b} e^{-bx}\Big|_{0}^{\infty} = 1$$

$$= -\frac{a}{b}\{0-1\} = 1$$

$$\Rightarrow \boxed{a = b}$$

So, $f_X(x) = ae^{-ax} \; ; \; x \geq$

Now, $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$

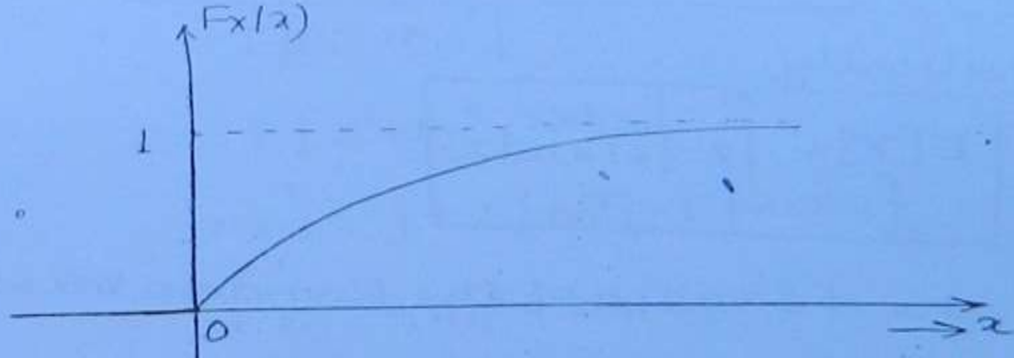
$$= \int_{0}^{x} ae^{-ax}\,dx$$

$$= -\frac{a}{a} e^{-ax}\Big|_{0}^{x}$$

$$= -1\{(e^{-ax}-1)\}$$

$$\boxed{\begin{aligned} F_X(x) &= (1-e^{-ax}) \; ; \; x \geq 0 \\ &\quad 0 \quad ; \; x < 0 \end{aligned}}$$

So, the plot is given as:



Q. For a CRV, X. Given that
$$f_X(x) = ae^{-b|x|}$$
Find the Relation b/w a & b.

Sol^n:- Given that,
$$f_X(x) = ae^{-b|x|}$$

as, $\int_{-\infty}^{\infty} f_X(x) = 1 = \int_{-\infty}^{0} ae^{-b(-x)} + \int_{0}^{\infty} ae^{-b\cdot x} = 1$

$$= \int_{-\infty}^{0} ae^{bx} + \int_{0}^{\infty} ae^{-bx} = 1$$

$$\frac{a}{b} e^{+bx} \Big|_{-\infty}^{0} - \frac{a}{b} e^{-bx} \Big|_{0}^{\infty} = 1$$

$$= \frac{a}{b}\{1-0\} - \frac{a}{b}\{0-1\} = 1$$

$$= a/b + a/b = 1$$

$$\boxed{2a = b}$$ Ans

So, $$\boxed{F_X(x) = ae^{-2a|x|}}$$

Statistical Averages of Random variable:

a) MEAN:

$$\boxed{\text{Mean } [X] = \text{Expectation, } E[X] = \bar{X} = m_1}$$

Mathematically,

$$\boxed{E[X] = \int_{-\infty}^{\infty} x \cdot f_X(x)\, dx}$$

Mean is the d.c value of the Random variable.

) MEAN SQUARE VALUE (MSQ):

$$\boxed{msq[X] = E[X^2] = \overline{X^2} = m_2}$$

Mathematically,

$$\boxed{E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx}$$

※ It gives the total power of the Random variable.

c) VARIANCE (σ^2)

as

$E[K] = K$

$$\boxed{E[K] = \int_{-\infty}^{\infty} K \cdot f_X(x)\,dx = K}$$

and, $E[Kx] = \int_{-\infty}^{\infty} K \cdot x\, f_X(x)\,dx$

$$\boxed{E[KX] = K\,E[X]}$$

And, $\boxed{E[X_1 + X_2] = E[X_1] + E[X_2]}$

So, the variance is defined as:

$$\sigma^2 = E\left[(X - \bar{X})^2\right]$$

$$= E\left[(X - m_1)^2\right]$$

$$= E\left[X^2 + m_1^2 - 2Xm_1\right]$$

$$= E[X^2] + E[m_1^2] - E[2Xm_1]$$

$$= m_2 + m_1^2 - 2m_1 E[X]$$

$$= m_2 + m_1^2 - 2m_1^2$$

$$\boxed{\sigma^2 = m_2 - m_1^2}$$

So, σ^2 = total power - d.c power

$\boxed{\sigma^2 = \text{A.C power of Random variable}}$

d) STANDARD DEVIATION:

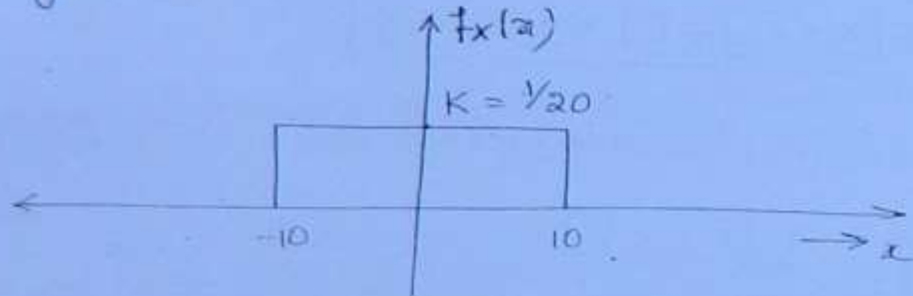
As

σ^2 = variance

So $\boxed{\begin{array}{l}\sigma = \sqrt{\text{variance}} \\ \sigma = \text{AC component of Random variable.}\end{array}}$

Q. A continuous Random variable is uniformly distributed in the interval (-10,10). Find all of its statistical averages.

Soln:-



a) Mean, $m_1 = \int_{-\infty}^{\infty} x \, f_X(x) dx$

$= \int_{-10}^{10} x \cdot \frac{1}{20} dx$

$= \frac{1}{20} \int_{-10}^{10} x dx = \frac{1}{20} \times \frac{x^2}{2} \Big|_{-10}^{10}$

$\frac{1}{40} \times 0$

$\boxed{m_1 = 0}$ Ans

Conclusion:-

If the density function is symmetric about the vertical Axis passing through the origin, then the Mean value of the ~~function~~ R.V is 0.

b) mean square value, $m_2 = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$

$$= \int_{-10}^{10} x^2 \, \tfrac{1}{20}\, dx$$

$$= \frac{x^3}{3}\Big|_{-10}^{10} \times \tfrac{1}{20}$$

$$= \frac{1}{60} \times 2000$$

$$\boxed{m_2 = 33.33} \text{ Ans}$$

c) variance, $\sigma^2 = m_2 - m_1^2$

$$= 33.33 - 0$$

$$\boxed{\sigma^2 = 33.33} \text{ Ans}$$

$\left\{ \because \sigma^2 = m_2 - m_1^2 \right.$

$\because m_1 = 0$

so, $\boxed{\sigma^2 = m_2}$ ***

d) Standard deviation $= \sqrt{\text{variance}}$

$$= \sqrt{33.33}$$

$$\boxed{\sigma = 5.77} \text{ Ans}$$

Q2. For a CRV, X; given

$$f_X(x) = \lambda e^{-\lambda x}; \; x \geq 0$$

find all of its statistical averages.

Soln: i) Mean $= \int_{-\infty}^{\infty} x f_X(x)\, dx =$

$$= \int_0^{\infty} x \cdot \lambda e^{-\lambda x} \cdot dx$$

$$= \lambda \int_0^{\infty} \underbrace{x}_{u} \cdot \underbrace{e^{-\lambda x}}_{v}\, dx \quad \left\{ \because \int uv\, dx = u \int v\, dx - \int u' \int v \right.$$

$$= \lambda \left\{ x \cdot \frac{e^{-\lambda x}}{-\lambda}\Big|_0^{\infty} - \int_0^{\infty} 1 \cdot \frac{e^{-\lambda x}}{-\lambda}\, dx \right.$$

$$= \lambda \left\{ (0-0) + \frac{1}{\lambda^2}\left[e^{-\infty} - e^{0} \right] \right\} \Rightarrow \boxed{m_1 = 1/\lambda}$$

~~288~~ 262

2) Mean Square value,

$$msq = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$$

$$\boxed{msq = 2/\lambda^2}$$ Ans

3) $\sigma^2 = m_2 - m_1^2$

$$= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$\boxed{\sigma^2 = 1/\lambda^2}$$ Ans

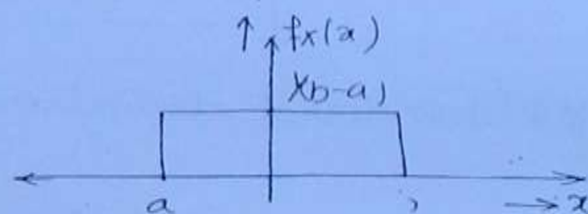
4) Standard deviation $= \sqrt{\sigma^2}$

$$\boxed{S.D = 1/\lambda}$$ Ans

* UNIFORM PROBABILITY DENSITY FUNCTION:

It is defined as:

$$f_X(x) = \frac{1}{(b-a)} \; ; \quad a \le x \le b$$



Q. A CRV; is possesing uniform density function specified above. Find all of its statistical averages.

Sol^n: As, we know that,

$$Mean = \int_{-\infty}^{\infty} x f_X(x)\,dx$$

$$= \int_a^b x \cdot 1/(b-a)\,dx$$

$$= \frac{1}{b-a}\int_a^b x\,dx$$

$$= \frac{1}{(b-a)} \left.\frac{x^2}{2}\right|_a^b$$

$$= \frac{1}{2(b-a)}\left[b^2 - a^2\right] \Rightarrow \boxed{m_1 = \frac{b+a}{2}}$$

2) Mean Square Value, msq $= \int_{-\infty}^{\infty} x^2 f_X(x)\,dx$

283

$$= \int_a^b x^2 \frac{1}{(b-a)}\,dx$$

$$= \frac{1}{3(b-a)}\left[x^3\right]_a^b$$

$$= \frac{b^3-a^3}{3(b-a)}$$

$$\boxed{msq = \frac{b^2+a^2+ab}{3}}$$

3) Now, variance, $\sigma^2 = m_2 - m_1^2$

$$= \frac{b^2+a^2+ab}{3} - \frac{(a+b)^2}{4}$$

$$\boxed{\sigma^2 = \frac{(a-b)^2}{12}}$$

4) Standard deviation $= \sqrt{\sigma}$

$$\boxed{S.D = \frac{(a-b)}{2\sqrt{3}}}$$

* GAUSSIAN DENSITY FUNCTION:

It is given as:

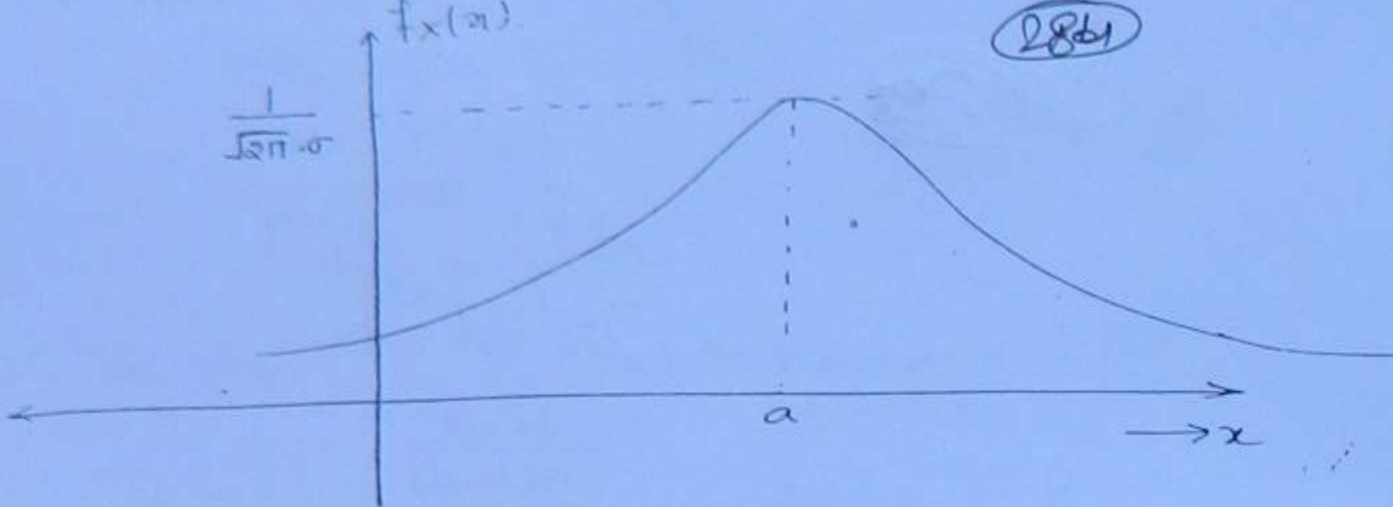
$$\boxed{f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}}}$$

For, $x = a$

$f_X(x)$ will be max^m.

Hence the plot is given as:

(28b1)



a) Mean - $m_1 = \int_{-\infty}^{\infty} x \cdot f_X(x) dx$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-a)^2}{2\sigma^2}} dx$$

After Analysis,

$$\boxed{m_1 = a}$$

b) Mean Square value, $msq = \int_{-\infty}^{\infty} x^2 f_X(x) dx$

$$\boxed{msq = \sigma^2 + a^2}$$

c) Variance, $\sigma^2 = m_2 - m_1^2$

$$= \sigma^2 + a^2 - a^2$$

$$\boxed{\text{Variance} = \sigma^2}$$

d) Standard. deviation

$$\boxed{SD = \sigma}$$

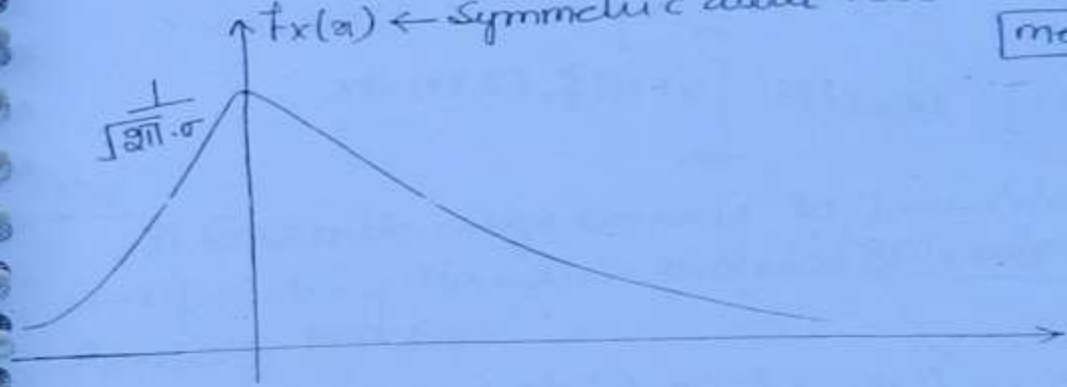
* Analysis:

i) If mean = 0 = $m_1 = 0$

So, $a = 0$

Then, $f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-x^2/2\sigma^2}$ | max at $\underline{x=0}$

So, the plot is given as:



Note:

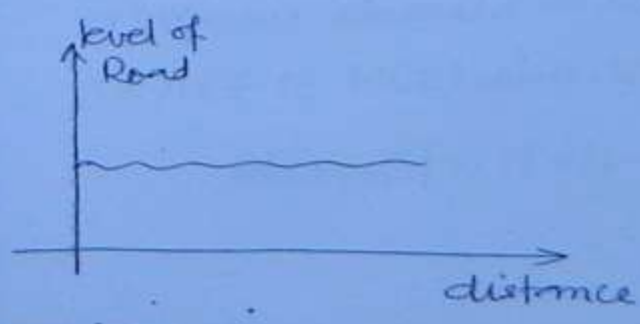
* white Noise possess Gaussian density function, so it is also called as the Gaussian Noise.

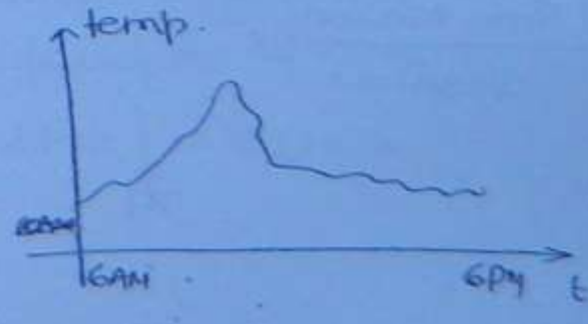
only ES subj numerical

* RANDOM PROCESS:-

* Random variable as a function of time is called as the RANDOM PROCESS.

RANDOM VARIABLE	RANDOM PROCESS
1. denoted as 'x'	1. denoted as $x(t)$.
2. $F_X(x) = P(X \leq x)$	2. $F_X(x,t) = P(x(t) \leq x)$
3. $f_X(x) = \frac{d}{dx} F_X(x)$	3. $f_X(x,t) = \frac{d}{dx} F_X(x,t)$.





*Statistical Averages of Random Process:

288

1) ENSEMBLE AVERAGES:-

The Averages calculated on a group of Similar Random processes are called as Ensemble Avgs.

*depends on density function.

**a) Ensemble mean:

Given as:

$$E[x(t)] = m_1(t) = \int_{-\infty}^{\infty} x(t)\, f_x(x,t)\, dx$$

b) Mean Square Value, msq:

Given as:

$$m_2(t) = \int_{-\infty}^{\infty} x^2(t)\, f_x(x,t)\, dx$$

c) Auto Correlation Function:

Given as:

*** $$\boxed{R(\tau) = E[x(t)\, x(t-\tau)]}$$

2) TIME AVERAGES:

* The Statistical averages computed on a Random process on a time basis is called as Time Averages

*** Time Averages are Independent of density function.

a) Time Mean:

Given as:

$$\langle x(t) \rangle = \int_{t_1}^{t_2} x(t)\, dt$$

b) Mean square value, msq:

Given as:

$$\langle x^2(t) \rangle = \int_{t_1}^{t_2} x^2(t)\,dt$$

c) Auto Correlation Function:

Given as:

$$\langle x(t)x(t-\tau) \rangle = \int_{t_1}^{t_2} x(t)x(t-\tau)\,dt$$

Note:

If Ensemble avgs equals to Time Averages, then the corresponding Random process is said to be "ERGODIC RANDOM PROCESS".

* If only means are same, then it is said to be ERGODIC IN MEAN/MSQ/AUTOCORRELATION.

* STRICT SENCE STATIONARY RANDOM PROCESS:

* If Prob. density function of Random Process is Independent of time, then it is said to be "SSSRP"

So, $\boxed{f_x(x,t) = f_x(x, t+\Delta t)}$

* WIDE SENSE STATIONARY RANDOM PROCESS:

* A Random process is said to be WSSRP, if it satisfies the following

a) Mean should be constant, Independent of t

b) ACF ie $R(\tau)$ should be function of only τ.

$$R(\tau) = E[x(t)\,x(t-\tau)].$$

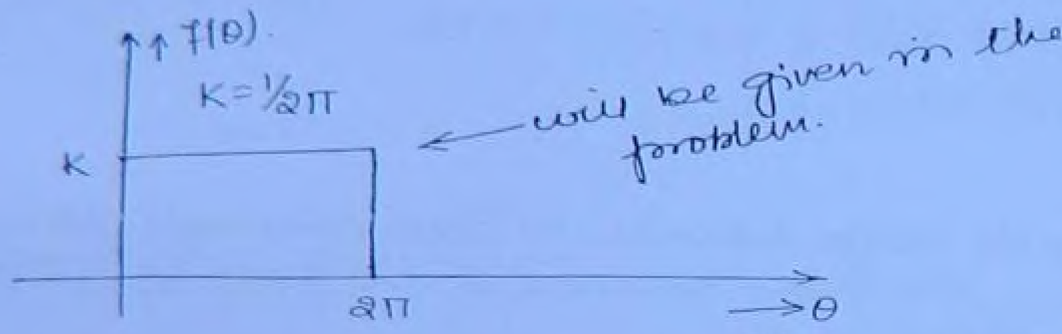
Q A Random process is given by

$$x(t) = A\cos\{\omega_0 t + \theta\}$$

where A & ω_0 are constants and θ is Random variable; which is uniformly distributed in the interval $(0, 2\pi)$.

Find whether the given RP is WSS or not?

Soln:



Now, mean of Funcn: $m_1(t) = \int_{-\infty}^{\infty} x(t) \cdot f(\theta)\, d\theta$

$$= \int_0^{2\pi} A\cos\{\omega_0 t + \theta\} \cdot \tfrac{1}{2\pi}\, d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} A\cos\{\omega_0 t + \theta\}\, d\theta$$

∵ θ is Random variable, all other parameters are considered as considered including t

$$m_1(t) = \frac{1}{2\pi}\int_0^{2\pi} A\cos\omega_0 t \cos\theta\, d\theta - \frac{1}{2\pi}\int_0^{2\pi} A\sin\omega_0 t \sin\theta\, d\theta$$

$$= \frac{A\cos\omega_0 t}{2\pi}\int_0^{2\pi} \overset{0}{\cancel{\cos\theta\, d\theta}} - \frac{A\sin\omega_0 t}{2\pi}\int_0^{2\pi} \overset{0}{\cancel{\sin\theta\, d\theta}}$$

So, $\boxed{m_1(t) = 0 = \text{Constant}}$

Also, ACF $= R(\tau) = E[x(t)x(t-\tau)]$

$$= E[A\cos(\omega_0 t + \theta) \cdot A\cos\{\omega_0(t-\tau) + \theta\}]$$

$$= E[A\cos(\omega_0 t + \theta) \cdot A\cos(\omega_0 t - \omega_0\tau + \theta)]$$

$$R(\tau) = E\left[\frac{A^2}{2}\cos(2\omega_0 t - \omega_0\tau + 2\theta) + \frac{A^2}{2}\cos\omega_0\tau\right]$$

$$R(\tau) = E\left[\frac{A^2}{2}\cos(2\omega_0 t - \omega_0\tau + 2\theta)\right] + E\left[A^2/_2 \cos\omega_0\tau\right]$$

Now As $E[K] = K$

So, $E[A^2/_2 \cos\omega_0\tau] = A^2/_2 \cos\omega_0\tau$ {∵ Integration done w.r.t θ}.

Now,

$$E\left[A^2/_2 \cos(2\omega_0 t - \omega_0\tau + 2\theta)\right] = \int_0^{2\pi} \frac{A^2}{2}\cos(\underbrace{2\omega_0 t - \omega_0\tau}_{A} + \underbrace{2\theta}_{B}) \cdot 1/_{2\pi}\, d\theta$$

$$\Rightarrow \boxed{E\left[A^2/_2 \cos(2\omega_0 t - \omega_0\tau + 2\theta)\right] = 0}$$

So,

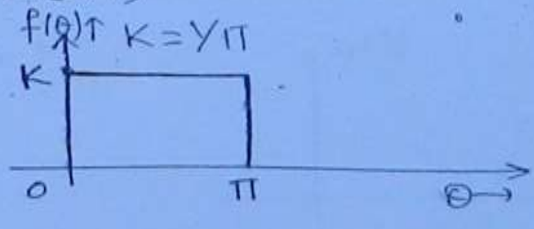
$$\boxed{ACF = R(\tau) = \frac{A^2}{2}\cos\omega_0\tau}$$

Hence, it is function of only τ.

Hence, the given function is WSSRP Ans.

Q. Repeat Above if θ is uniformly distributed in the interval $(0, \pi)$.

Solⁿ:-



Now mean of funcⁿ = $m_1(t) = \int_{-\infty}^{\infty} x(t) f(\theta) d\theta$

$$= \int_0^{\pi} 1/_{\pi}\, A\cos(\omega_0 t + \theta)\, d\theta$$

$$= \frac{A}{\pi}\int_0^{\pi} \cos(\omega_0 t + \theta) d\theta$$

$$m_1(t) = \frac{A}{\pi}\int_0^{\pi}\left[\cos\omega_0 t\cos\theta - \sin\omega_0 t\sin\theta\right]d\theta$$

$$= \frac{A}{\pi}\cos\omega_0 t\int_0^{\pi}\cos\theta\, d\theta - \frac{A\,\cancel{\cos\omega_0 t}\,\sin\omega_0 t}{\pi}\int_0^{\pi}\sin\theta\, d\theta$$

$$= \frac{A}{\pi}\cos\omega_0 t\sin\theta\Big|_0^{\pi} - \frac{A\,\cancel{\cos\omega_0 t}\,\sin\omega_0 t}{\pi}\times -\cos\theta\Big|_0^{\pi}$$

$$= \frac{A}{\pi}\cos\omega_0 t\times(0-0) + \frac{A\,\cancel{\cos\omega_0 t}\,\sin\omega_0 t}{\pi}(\cos\pi - \cos 0)$$

$$= \frac{A\,\cancel{\cos\omega_0 t}\,\sin\omega_0 t}{\pi}(-1-1)$$

$$\boxed{m_1(t) = \frac{-2A\sin\omega_0 t}{\pi}}$$

$m_1(t)$ is not constant, but is a function of t

Hence given RP is not WSS RP.

* CONVOLUTION AND CORRELATION

1. Convolution is used to find the Response of the system
2. Correlation is used to find the Similarity b/w the signals.

Now,
Mathematically, convolution is given as:

$$\boxed{x_1(t) * x_2(t) = \int_{-\infty}^{\infty} x_1(\tau)\, x_2(t-\tau)\, d\tau}$$

* Cross-Correlation means correlation of 2 functions.

Hence
X-Correlation is mathematically given as.

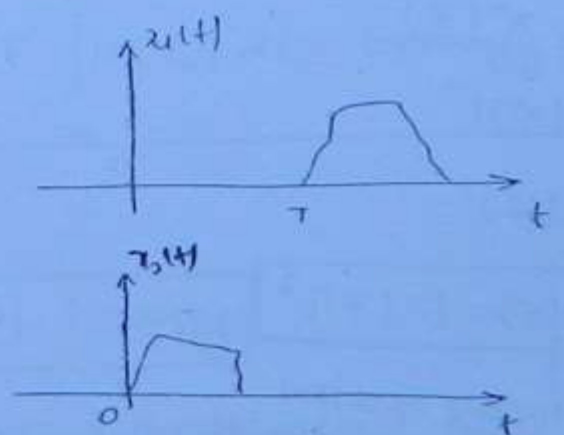
$$\boxed{R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)\, x_2(t-\tau)\, dt}$$

Now, For the 2 Signals to be equal, we have:

$$\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t)\, dt = X(t)$$

If the value of $X(t) = 0$, hence, the Signals are atmost dissimilar.

But let



The value of $\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t)$ is Zero in this case, but some Similarity is present Hence we delay the 2nd Signal by τ and for the value of τ for which max^m area is overlapped, it is said to be the similar value. ie

$$R_{12} = \int_{-\infty}^{\infty} x_1(t)\, x_2(t-\underset{\text{Searching/Scanning parameter}}{\tau})\, d\tau$$

Now
$$R_{12}(\tau) = \int_{-\infty}^{\infty} x_1(t)\, x_2(t-\tau)\, d\tau$$

$$\boxed{R_{12}(\tau) = x_1(t) * x_2(-t) \Big/ t \to \tau}$$

* Auto correlation of a signal means finding the similarity of a signal with its shifted version.

Hence,
Mathematically,
$$ACF[x(t)] = R(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, d\tau$$

So, $\boxed{R(\tau) = x(t) * x(-t)}$

So, by Fourier xform we get:

$$\boxed{R(\tau) \longleftrightarrow X(f) \cdot X(-f)}$$

Note:
Now, if $x(t)$ is Real, then
$$\boxed{X^*(f) = X(-f)}$$

So, $R(\tau) \longleftrightarrow X(f) \cdot X^*(f)$

$R(\tau) \longleftrightarrow |X(f)|^2$

$R(\tau) \longleftrightarrow S(f)$

where, $\boxed{S(f) = |X(f)|^2}$.

Energy spectral density of $x(t)$.

So,
Conclusion:

$$\boxed{\text{Fourier xform of ACF} = \text{ESD}}$$

And, $\boxed{\text{IFT}\left[S(f)\right] = R(\tau)}$

293

Now, $$\int_{-\infty}^{\infty} S(f) e^{j2\pi f\tau} df = R(\tau)$$

put $\tau = 0$.

So, $$\boxed{\int_{-\infty}^{\infty} S(f)\, df = R(0) \quad ---(1)}$$ ← Auto correlation function at origin will give the Area of ESD.

Note:

* $R(\tau)$ is $\max^m$ at $\tau = 0$.
* and as the value of τ is increasing, the similarity is decreasing and $R(\tau)$ is decreasing

Now,
$$R(\tau) = \int_{-\infty}^{\infty} x(t)\, x(t-\tau)\, dt$$

at $\tau = 0$

$$\boxed{R(0) = \int_{-\infty}^{\infty} x^2(t)\, dt = \text{Energy}\left[x(t)\right]} \quad ---(2)$$

So, from (1) & (2) we get:

$$\boxed{\int_{-\infty}^{\infty} S(f)\, df = \int_{-\infty}^{\infty} |X(f)|^2 df = \text{Energy}\left[x(t)\right]}$$ ← PARSEVAL'S THEOREM.

Conclusion:

$$\boxed{\text{Area}\left[\text{ESD}\right] = \text{Energy}}$$

The above discussion is valid only for the Energy Signal. If the Signal is power signal, we have to generalise the discussion by Average Auto-correlation function.

* AVG AUTO CORRELATION FUNCTION :-

289

let
Periodic signal by $x_T(t)$.

* All periodic signal are Power signals; but Reverse is not true.

NOW,
$$R(\tau) = \int_{-\infty}^{\infty} x_T(t) \cdot x_T(t-\tau)\,dt$$

at $\tau = 0$

$$\boxed{R(0) = \int_{-\infty}^{\infty} x_T^2(t)\,dt = \underset{\text{Signal}}{\text{Energy of}} = \infty}$$ ← It fails as value of ACF $= \infty$.

* Autocorrelation function is max^m at $\tau = 0$, but the value should be some finite value

* But for above case, at $\tau = 0$, $R(0) = \infty$.
Hence, the formula for ACF of periodic signal fails for the analysis of Power/Periodic Signals

* ACF shouldn't be ∞. so, it is failed for Power Signals

* For Power Signals, avg auto correlation func^n will be defined.

So,
$$\tilde{R}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_T(t)\, x_T(t-\tau)\,dt \Rightarrow \lim_{T\to\infty} \frac{1}{T}\left[x_T(t) * x_T(-t)\right]$$

$$\boxed{\tilde{R}(0) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x_T^2(t)\,dt = \text{Power of } x_T(t)} \quad \text{---(3)}$$

taking F.T on both sides weget :-

$$\tilde{R}(\tau) \Leftrightarrow \lim_{T\to\infty} \frac{1}{T} |X_T(f)|^2$$

$$\tilde{R}(\tau) \longleftrightarrow \lim_{T\to\infty} \frac{1}{T} \underbrace{S_T(f)}_{\text{ESD.}}$$

$\because$ Energy of Power Signal over entire range $= \infty$.
But for 1 period, E = finite value

So, $\tilde{R}(\tau) \longleftrightarrow S(f)$

Now,
$$IFT[S(f)] = \tilde{R}(\tau)$$

So,
$$\int_{-\infty}^{\infty} S(f) e^{j2\pi f\tau} df = \tilde{R}(\tau)$$

$\tau \to 0$

$$\boxed{\int_{-\infty}^{\infty} S(f)\, df = \tilde{R}(0)} \quad \text{---- (4)}$$

Conclusion:

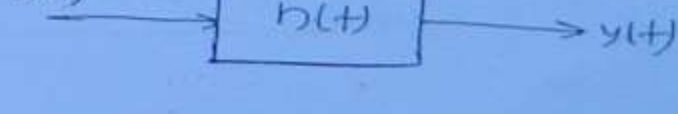
So, from eqn (3) & (4) we get:-

xxx $\boxed{\text{Area}[PSD] = \text{Power}}$

Note:

white Noise corresponds to power signal; as it occupies the spectrum from $-\infty$ to $+\infty$

Note:



$$Y(f) = H(f) \cdot X(f)$$

$$|Y(f)|^2 = |H(f)|^2 |X(f)|^2$$

$$\boxed{(ESD)_{o/p} = |H(f)|^2 \cdot (ESD)_{i/p}}$$

Also,

$$|Y(f)|^2 = |H(f)|^2 |X(f)|^2$$

$$\frac{|Y(f)|^2}{T} = \frac{|H(f)|^2 \, |X(f)|^2}{T}$$

$$\lim_{T\to\infty} \frac{|Y(f)|^2}{T} = |H(f)|^2 \cdot \lim_{T\to\infty} \frac{|X(f)|^2}{T}$$

$$\boxed{(PSD)_{o/p} = |H(f)|^2 (PSD)_{i/p}}$$

Q1. Given,

$x(t) = A\cos\omega_0 t$

Find
i) ACF
ii) PSD
iii) Power

Sol^n: $\because$ $x(t)$ is periodic func^n.
Hence, Avg. ACF has to be calculated.

So,

a) $R(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\,x(t-\tau)\,dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A\cos\omega_0 t\, A\cos(\omega_0 t - \omega_0\tau)\,dt$

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cancel{\frac{A^2}{2}\cos(2\omega_0 t - \omega_0\tau)}^{\,0} + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \cos\omega_0\tau\,dt\,\frac{A^2}{2}$

$\because$ Periodic

$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\cos\omega_0\tau\,dt = \lim_{T\to\infty} \frac{1}{T}\cdot A^2\cos\omega_0\tau \int_{-T/2}^{T/2} dt$

$= \lim_{T\to\infty} \frac{1}{\cancel{T}}\cdot A^2\cos\omega_0\tau\cdot \cancel{T}$

$\boxed{R(\tau) = A^2\cos\omega_0\tau}$ Ans

*** Note 1.

$\left.\begin{array}{c} A\cos(\omega_0 t + \Phi) \\ \text{or} \\ A\sin(\omega_0 t + \Phi) \end{array}\right\} \longrightarrow R(\tau) = \frac{A^2}{2}\cos\omega_0\tau$

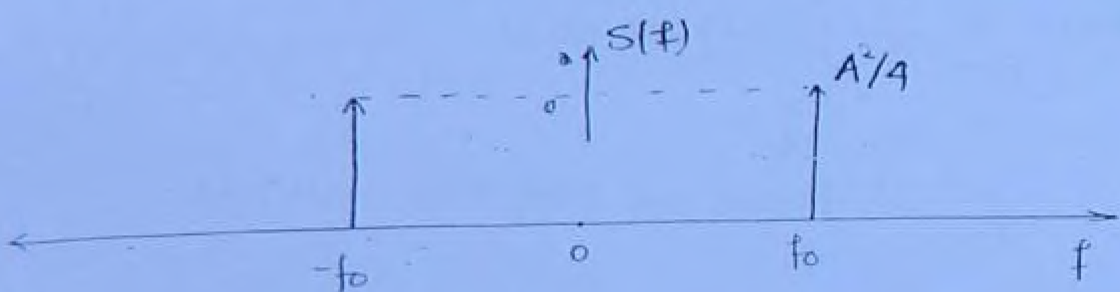
b) $S(f) = FT[R(\tau)]$

$$= FT\left[\frac{A^2}{2}\cos 2\pi f_0 \tau\right]$$

$$= \frac{A^2}{2}\left[\frac{\delta(f+f_0)+\delta(f-f_0)}{2}\right]$$

So, $$\boxed{S(f) = \frac{A^2}{4}\left\{\delta(f+f_0) + \delta(f-f_0)\right\}}$$ Ans

Plot :



c) Power = Area [S(f)]

$$= \frac{A^2}{4} + \frac{A^2}{4}$$

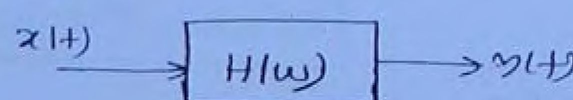
So $$\boxed{\text{Power} = A^2/2}$$ Ans

Also, $\text{Power} = R(\tau)\big|_{\tau=0}$

$$= \frac{A^2}{2}\cos 2\pi f_0 \tau\Big|_{\tau=0}$$

$$\boxed{\text{Power} = R(0) = A^2/2}$$ Ans

Q2. A signal of $x(t) = e^{-2t}u(t)$, is passed through a system given by $H(\omega) = \frac{1}{(j\omega+4)}$. Find Energy spectral density of o/p of the system?



Sol^n: $(ESD)_{o/p} = (ESD)_{i/p} \cdot |H(\omega)|^2$

Now, $$|Y(\omega)|^2 = |X(\omega)|^2 \cdot |H(\omega)|^2$$

Now, $$X(\omega) = \frac{1}{(j\omega+2)} \quad \left\{\because e^{-at}u(t) \longleftrightarrow \frac{1}{s+a}\right\}$$

$$|X(\omega)| = \frac{1}{\sqrt{\omega^2+4}} \Rightarrow |X(\omega)|^2 = \frac{1}{(\omega^2+4)}$$

$$|H(\omega)| = \frac{1}{\sqrt{\omega^2+16}}$$

$$|H(\omega)|^2 = \frac{1}{\omega^2+16}$$

299

So, $$\boxed{|Y(\omega)|^2 = \frac{1}{(\omega^2+4)(\omega^2+16)}}$$ ← ESD of O/P.

Ans

ES Q3. A Random variable, of having Auto correlation function

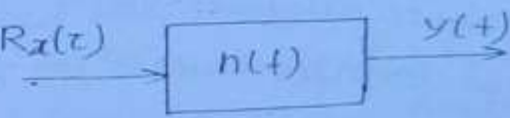
$$R(\tau) = e^{-\sigma|\tau|}$$

is passed through a system, whose Impulse Response is given by $h(t) = \frac{\mu}{2} e^{-\mu t} u(t)$.

Find PSD of the O/P Signal?

Soln: Given, $R(\tau) = e^{-\sigma|\tau|}$

$h(t) = \mu/_2 \, e^{-\mu t} u(t)$



Now as

$$(PSD)_{O/P} = (PSD)_{I/P} \cdot |H(\omega)|^2$$

$(PSD)_{I/P}$ = F.T of $R_x(\tau)$

$$= FT\left[e^{-\sigma|\tau|}\right] \quad \left\{ e^{-a|t|} \Leftrightarrow \frac{2a}{a^2+\omega^2} \right\}.$$

$$\boxed{(PSD)_{I/P} = \frac{2\sigma}{\sigma^2+\omega^2}}.$$

Now, $$H(\omega) \longleftrightarrow \frac{\mu}{2}\left\{\frac{1}{\mu+j\omega}\right\}.$$

$$|H(\omega)|^2 \longrightarrow \frac{\mu^2}{4} \times \frac{1}{(\mu^2+\omega^2)}.$$

So,

$$(PSD)_{O/P} = \frac{2\sigma}{(\sigma^2+\omega^2)} \times \frac{\mu^2}{4} \times \frac{1}{(\mu^2+\omega^2)}$$

$$\boxed{(PSD)_{O/P} = \frac{\sigma\mu^2}{2(\sigma^2+\omega^2)(\mu^2+\omega^2)}}$$

* NOISE IN ANALOG COMMUNICATION (300)

* Noise may be classified as:
  1) Internal Noise. (within the system) –
     - → Shot Noise
     - → *** Thermal Noise
     - → Flicker Noise etc
  2) External Noise (external sources).
     - → Automobile Noise.
     - → Atmospheric Noise.
     - → Solar Noise. etc.

* Thermal Noise is also called as:
  a) White Noise
  b) Gaussian Noise.
  b) Johnson Noise.

* Thermal Noise:

* Due to Thermal Agitation, atoms in the electrical components will gain energy, moves in Random passion and collide with each other; heat will be generated. This corresponds to Thermal Noise.

* Each of the frequency component transmitted through a commn system will be affected by Thermal Noise, so is also called as "WHITE NOISE".

* Thermal Noise Power is given mathematically as:

$$\boxed{\underbrace{N}_{\text{watts}} = \underbrace{KT}_{\text{watts/Hz}}\underbrace{B}_{\text{Hz}}\ \text{watt}}$$

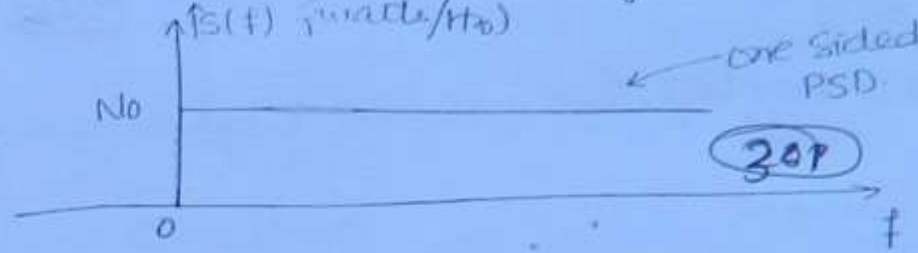
where K = Boltzman const = 1.38×10^{-23} Joules/Kelvin

T = Absolute Temp °K.

B = Bandwidth.

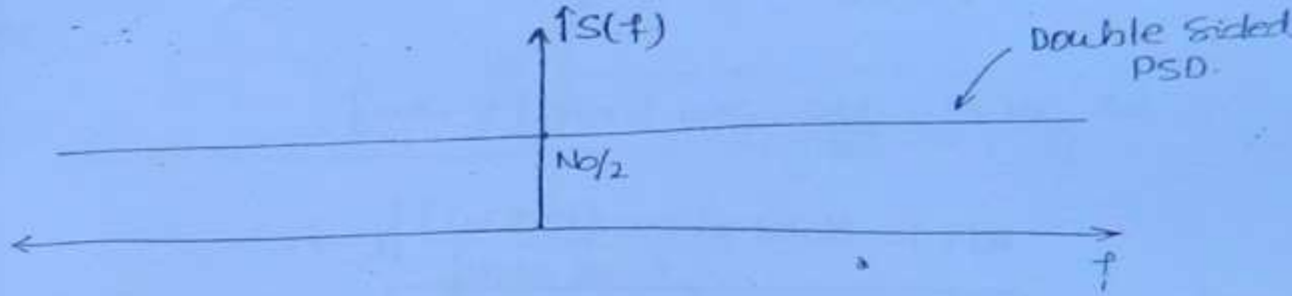
* $\boxed{KT = N_0 = \text{Power spectral density}}$ (const).

* The plot of PSD is a frequency func which is given as



309

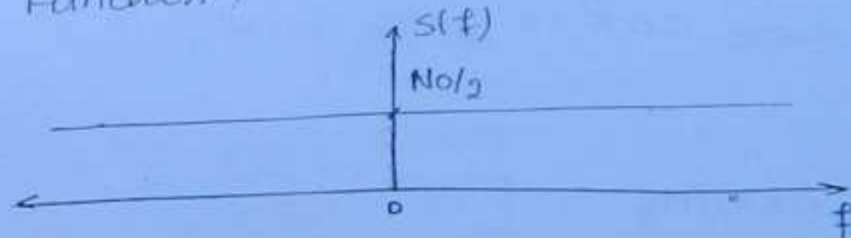
{Area of PSD is given as Power.

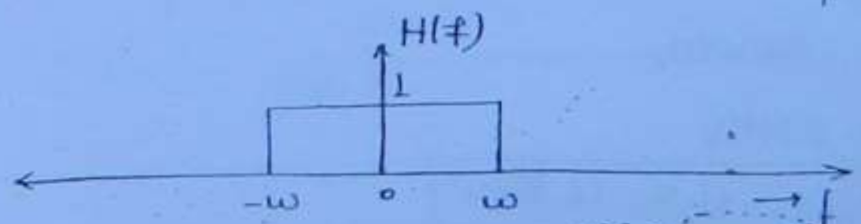


Previous Papers:-

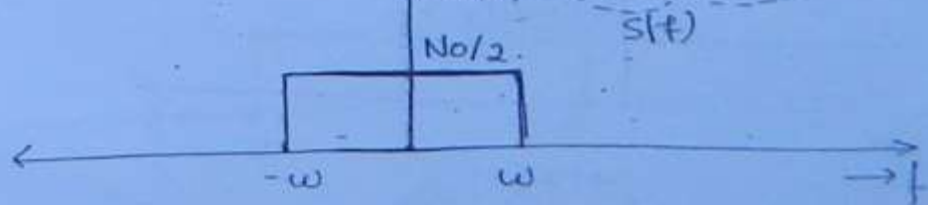
Q1. A white Noise of having 2 sided PSD $N_0/2$ watts/Hz is passed through a LPF whose cut-off frequency (f_c) is ω Hz. Find O/P white Noise power and corresponding Auto correlation function?

Sol^n:





$$S_o(f) = \underbrace{S_i(f)}_{S(f)}\,|H(f)|^2 \rightarrow \text{Same as } H(f) \quad \text{(1)}$$



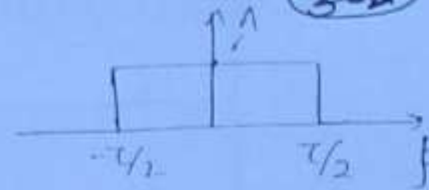
So, O/P Noise power N_0 = Area of $S_o(f)$.

$= 2\omega \times N_0/2$

$$\boxed{\text{O/P Noise power} = N_0\omega \text{ watts}}$$

Autocorrelation func" - $R(\tau) = IFT[S_o(f)]$ 308

Now,

$A\tau \operatorname{Sinc}(t\tau) \longleftrightarrow$ 

comparing we get

$$\tau = 2W \; ; \; A = N_o/2$$

So. $R(\tau) = \frac{N_o}{2} \cdot 2W \operatorname{Sinc}[t \cdot 2W]$

$$R(\tau) = N_oW \operatorname{Sinc}[t \cdot 2W] \Big|_{t \to \tau}$$

$$\boxed{R(\tau) = N_oW \operatorname{Sinc}[2W\tau]} \text{ Ans}$$

Now, $\boxed{R(0) = N_oW = \text{Power}}$ Ans

Q2. A white Noise of having 2 sided PSD 4 Kw/Hz is passed through LPF whose C.O.F is 2KHz. Find O/P white Noise power?

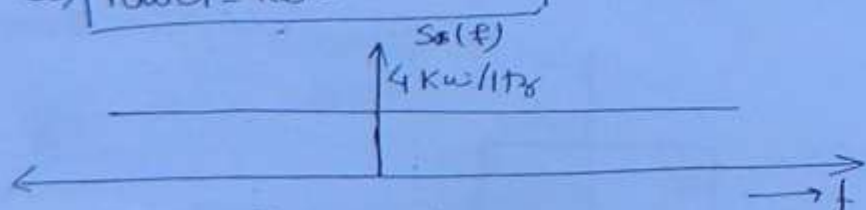
Soln: Given,

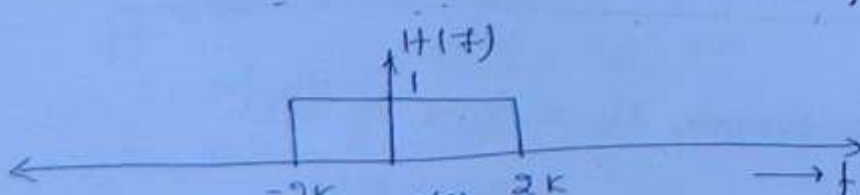
$$N_o/2 = 4\,Kw/Hz$$

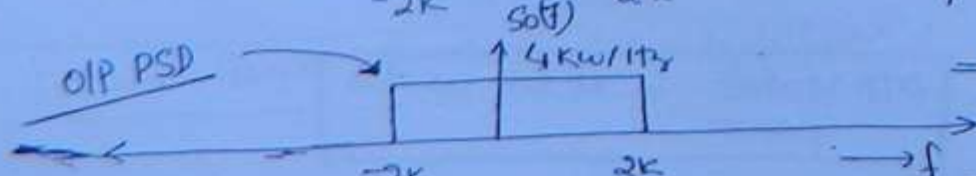
$$N_o = 8\,Kw/Hz$$

$$B = 2\,KHz$$

So, $\boxed{\text{Power} = N_oB = 16\,Kw}$







$\Rightarrow N_o = \text{Area}[S_o(f)]$

$= 4\frac{KW}{Hz} \times 2K$

$\boxed{N_o = 16\,KW}$

Q3 white Noise of having PSD ... is passed through a syst. is specified by

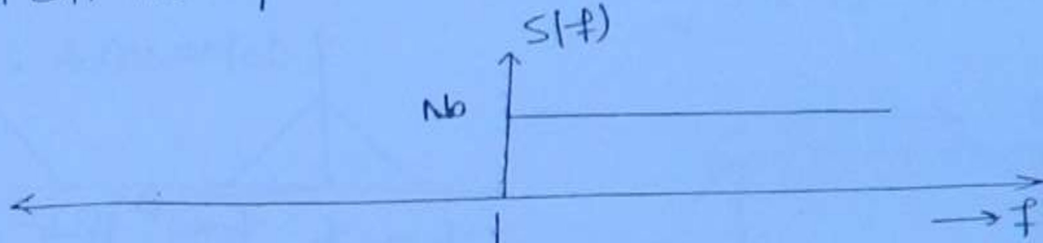
$$H(f) = 2e^{-jf}$$

309

The Resulting is passed through LPF whose Co(f) is B Hz.

Find O/P Noise power.

Sol:



$\downarrow$

$\boxed{H(f) = 2e^{-jf}}$

$\downarrow$

PSD at O/P of the System $\longrightarrow$ $S(f)\cdot|H(f)|^2 = S_o(f)$

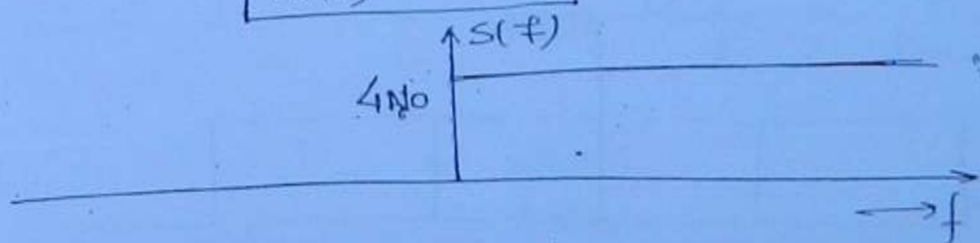
$$H(f) = 2e^{-jf}$$

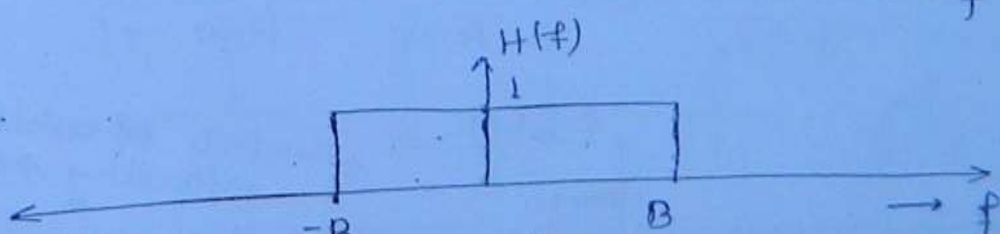
$$|H(f)| = 2\,\underbrace{|e^{-jf}|}_{\to 1}$$

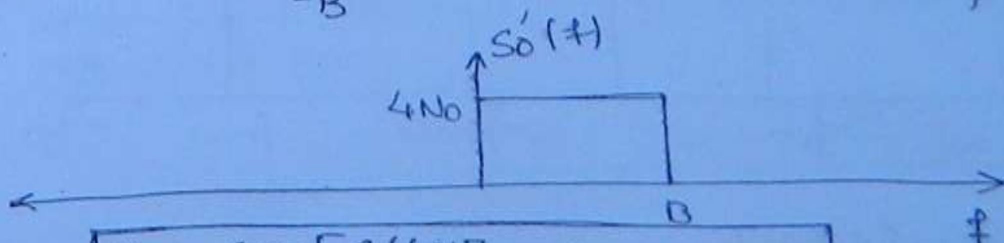
$$|H(f)|^2 = 4.$$

So, $S_o(f) = 4\cdot S(f)$

$\boxed{S_o(f) = 4N_o.}$



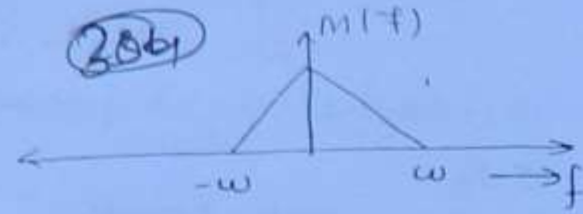




$\boxed{N_o = \text{Area}[S_o'(f)] = 4N_oB \text{ watts}}$

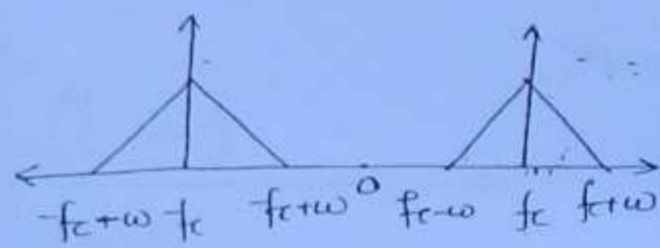
* Analysis (effect of Noise on AM, DSB, SSB)

Assume $m(t) \longleftrightarrow$ 36b



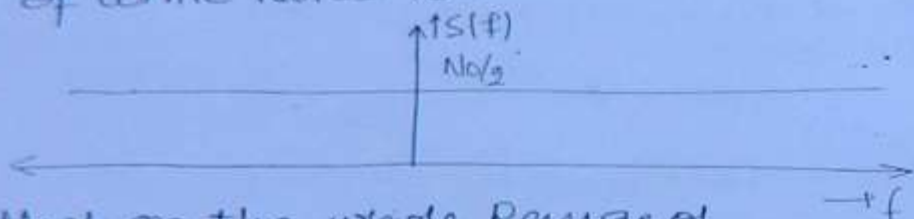
$c(t) = A_c \cos 2\pi f_c t$

$S_{AM}(t) \longleftrightarrow$



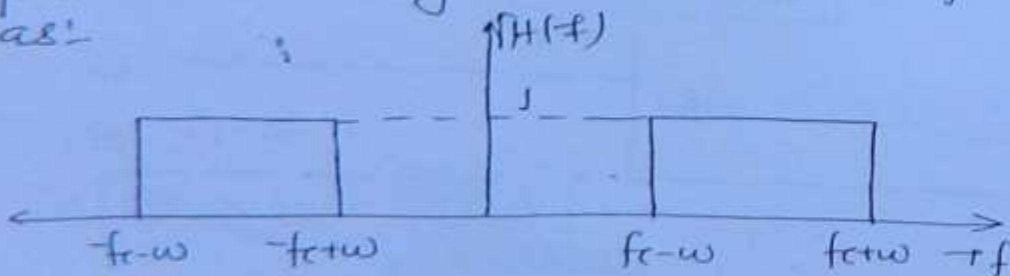
* EFFECT OF WHITE NOISE ON AM & DSB:

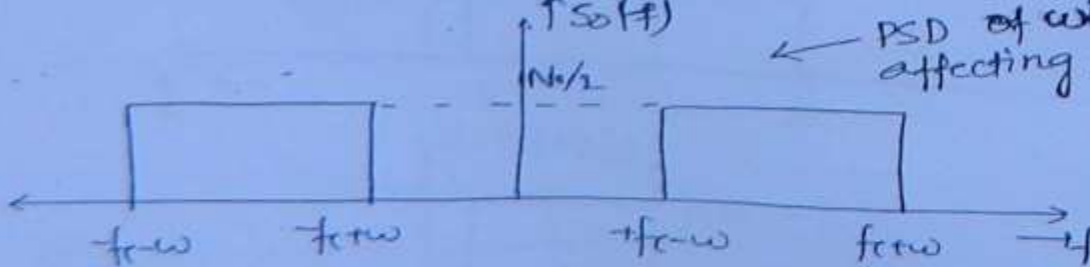
The PSD of white Noise is:



It has effect on the whole Range of frequency

* To visualize the effect on the freq^n components of AM & DSB we pass this through a BPF. So the spectrum is given as:





PSD of white Noise affecting AM & DSB.

So, $N = \text{Area}[S_o(f)]$

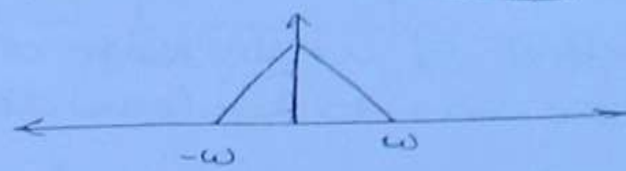
$= 2\omega \times N_0/2 \times 2$ {∵ Two Sided}

So, $\boxed{\text{Noise power, } N = 2N_0 W \text{ watts}}$

*EFFECT OF WHITE NOISE ON SSB:

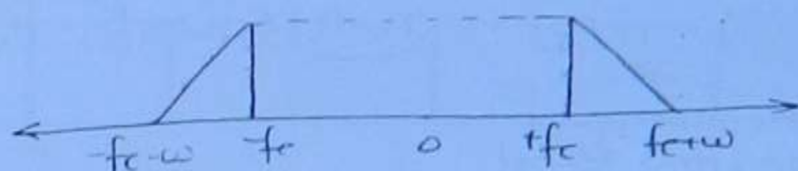
(305)

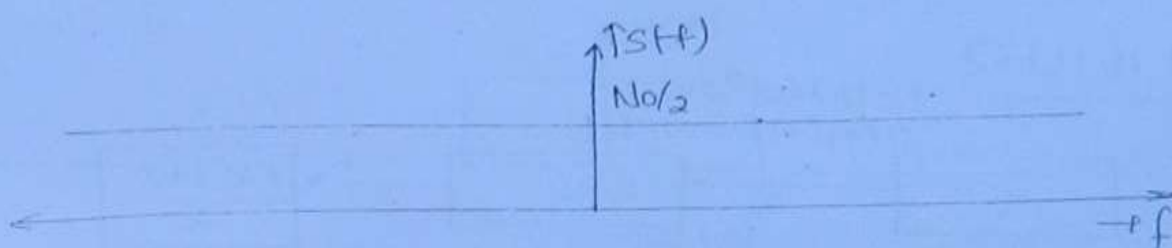
let $m(t) \longleftrightarrow$

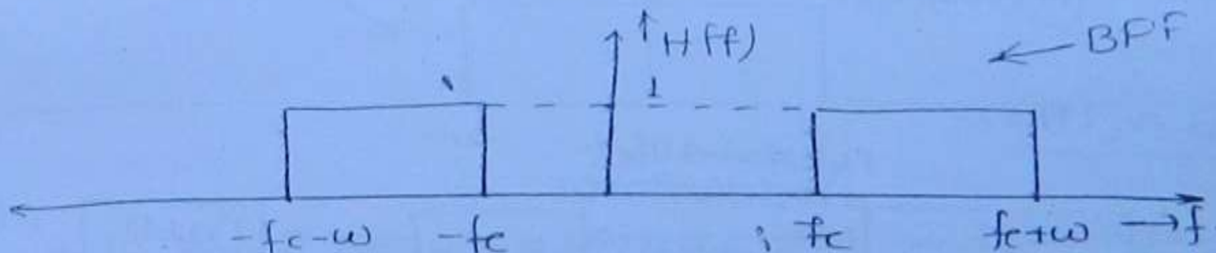


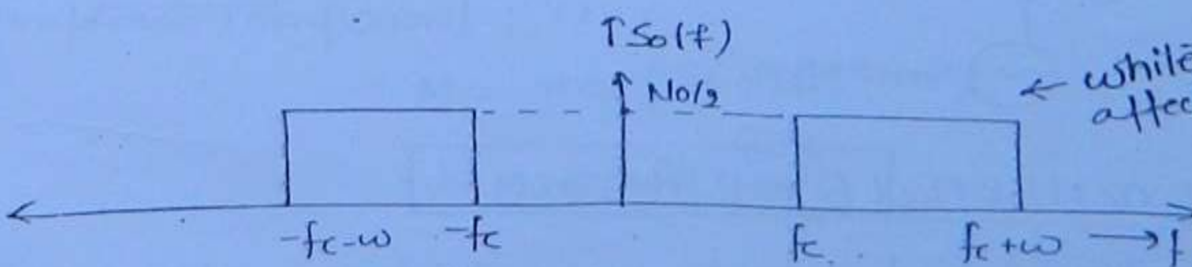
$c(t) = A_c \cos 2\pi f_c t$

So,
$S_{SSB}(t) \longleftrightarrow$









← while Noise power affecting SSB.

So, white Noise power affecting SSB $= N = W \times N_0/2 + W \times N_0/2$

$\boxed{N = N_0 W \text{ watts}}$

* NARROW BAND NOISE.

(338)

* When white Noise is passed through BPF, the Resulting is said to be Narrow Band Noise.

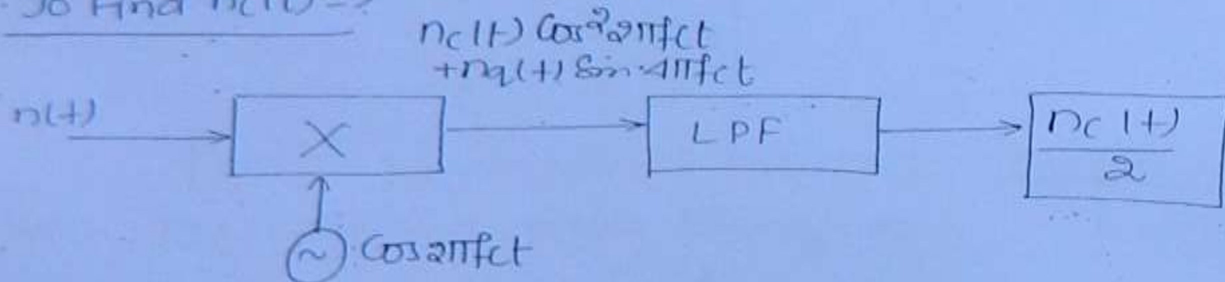
* To find effect of white Noise on AM, DSB and SSB, Narrow Band Noise has to be considered.

$$\boxed{n(t) = IFT\,[S_0(f)]}$$

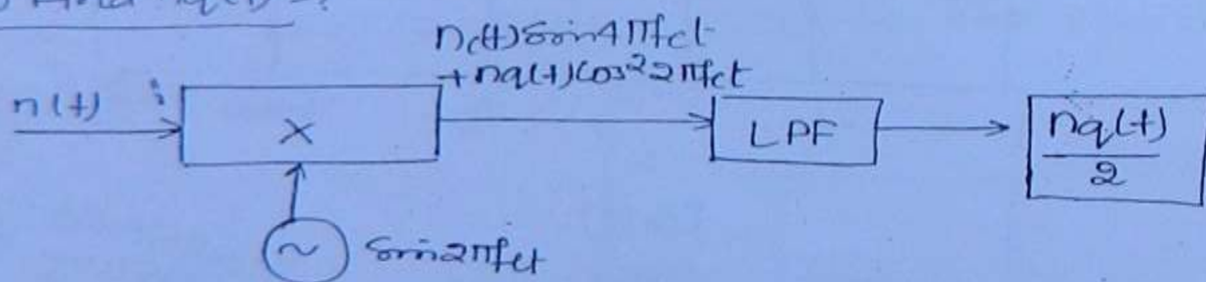
* Analysis:

$$\underset{\text{time domain Narrow Band Noise}}{n(t)} = \underbrace{n_c(t)}_{\text{In phase component}} \cos 2\pi f_c t + \underbrace{n_q(t)}_{\text{quadrature component}} \sin 2\pi f_c t$$

1. To find $n_c(t) = ?$



2. To find $n_q(t) = ?$



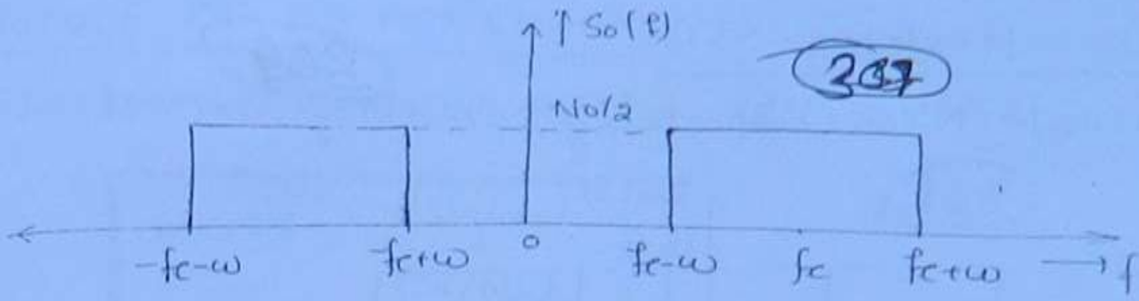
* Effect of $n_c(t)$ & $n_q(t)$ on AM, DSB: (PSD)

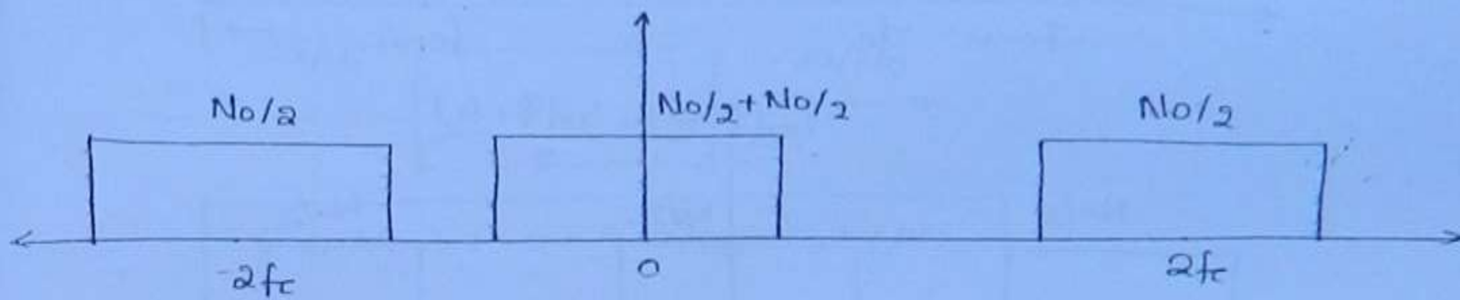
1) by $n_c(t)$:-

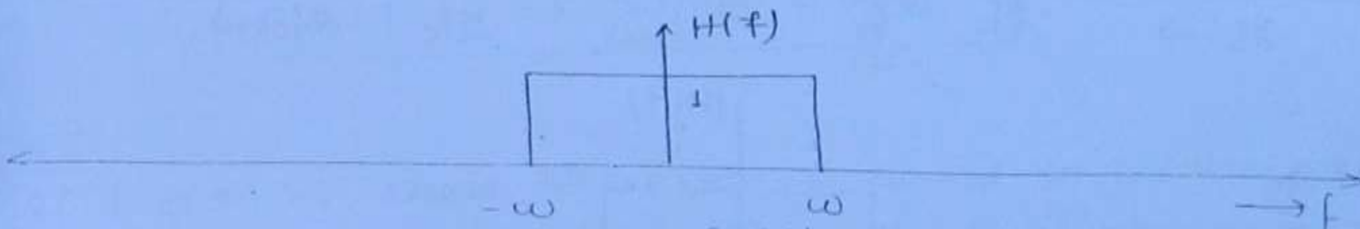
$$n(t) \longleftrightarrow S_0(f)$$

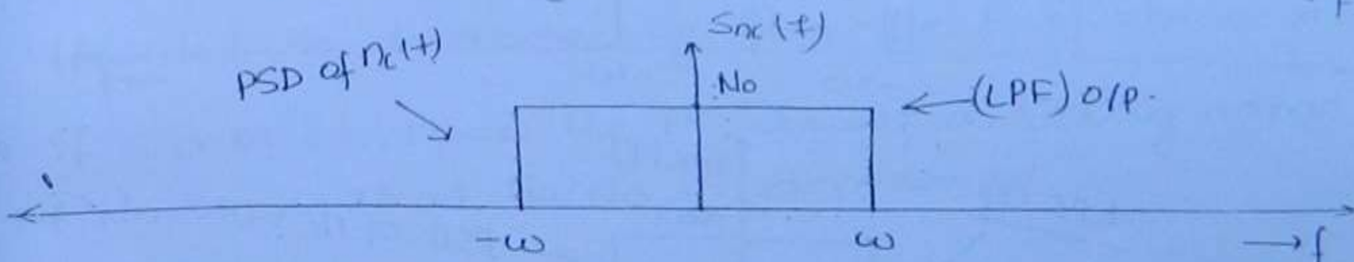
when multiplied by $\cos 2\pi f_c t$, the $S_0(f)$ is shifted left & right by amount f_c

$$S_0\,(O/P)_{mul} \longrightarrow \frac{S_0(f+f_c) + S_0(f-f_c)}{2}.$$









So,
while Noise power affecting AM and DSB due to its inphase component ; N

$$N = \text{Area}[S_{nc}(f)]$$

$$\boxed{N = 2N_0W \text{ watts}}$$

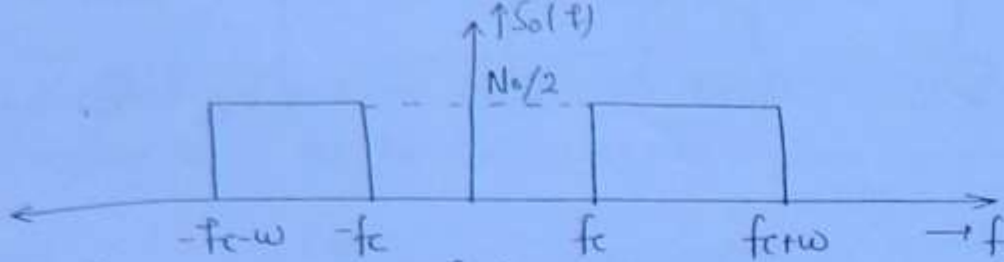
* Due to the whole component of white Noise is also

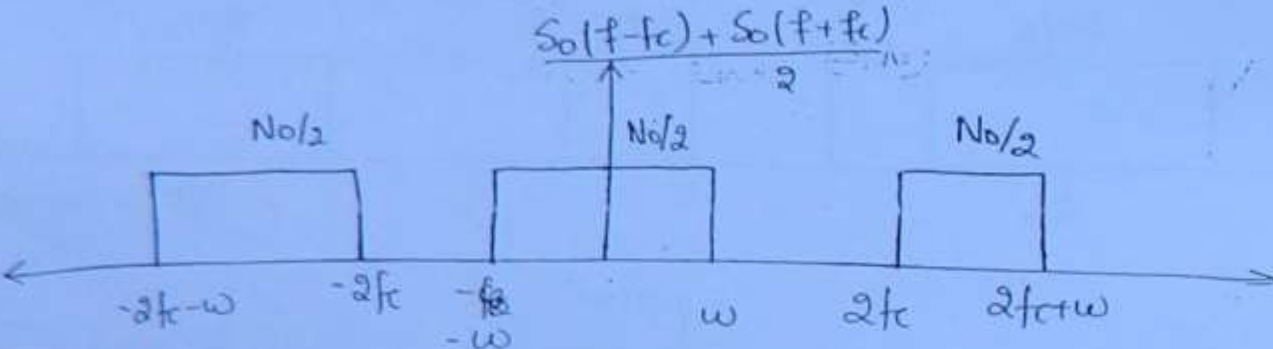
$$N = 2N_0W.$$

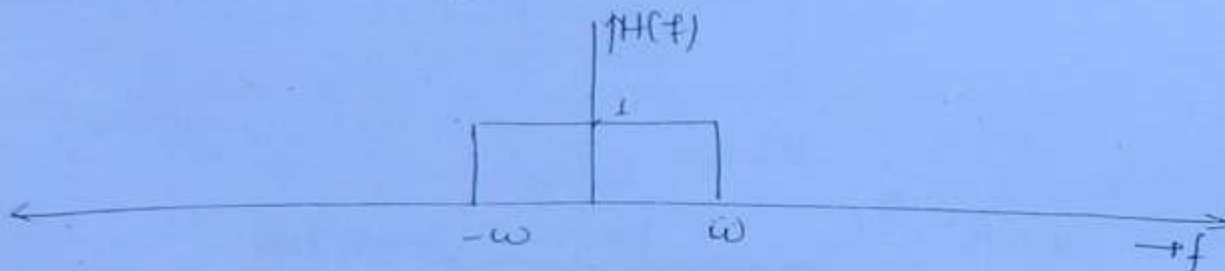
Conclusion:
The effect of white Noise on AM and DSB is only due to its inphase components, so that the effect of quadrature component will be null (ie 0).

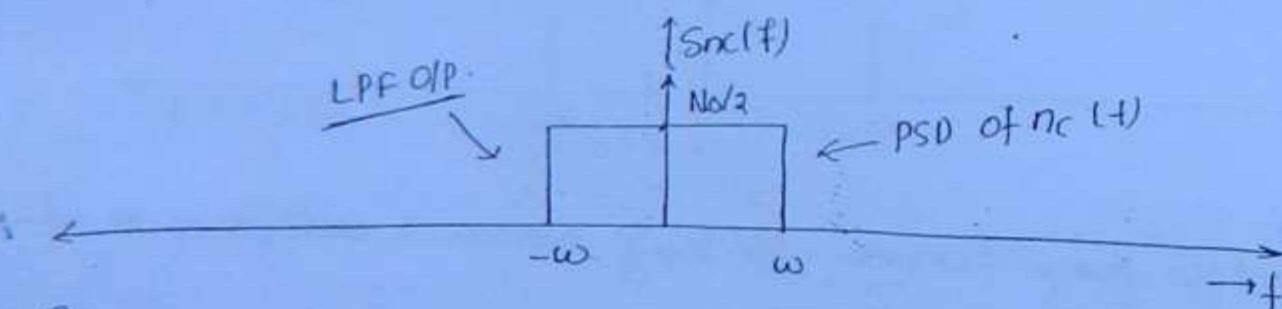
* PSD of n(t) affecting SSB:

* PSD of white Noise affecting SSB is given as









So,

Power = Area $[S_{nc}(f)]$

$\boxed{\text{Power} = N_0 W}$

* Due to Actual white Noise, $\boxed{\text{Power} = N_0 W}$

Conclusion:

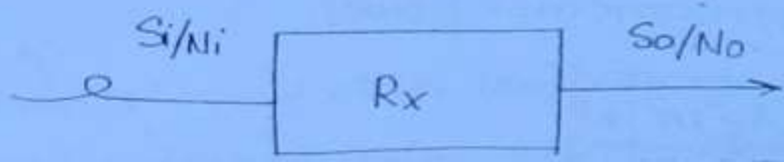
The affect of Quadrature Component of white Noise on SSB will be NULL.

* FIGURE OF MERIT (F.O.M)

Mathematically F.O.M is given as:

303

$$\boxed{F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}}$$



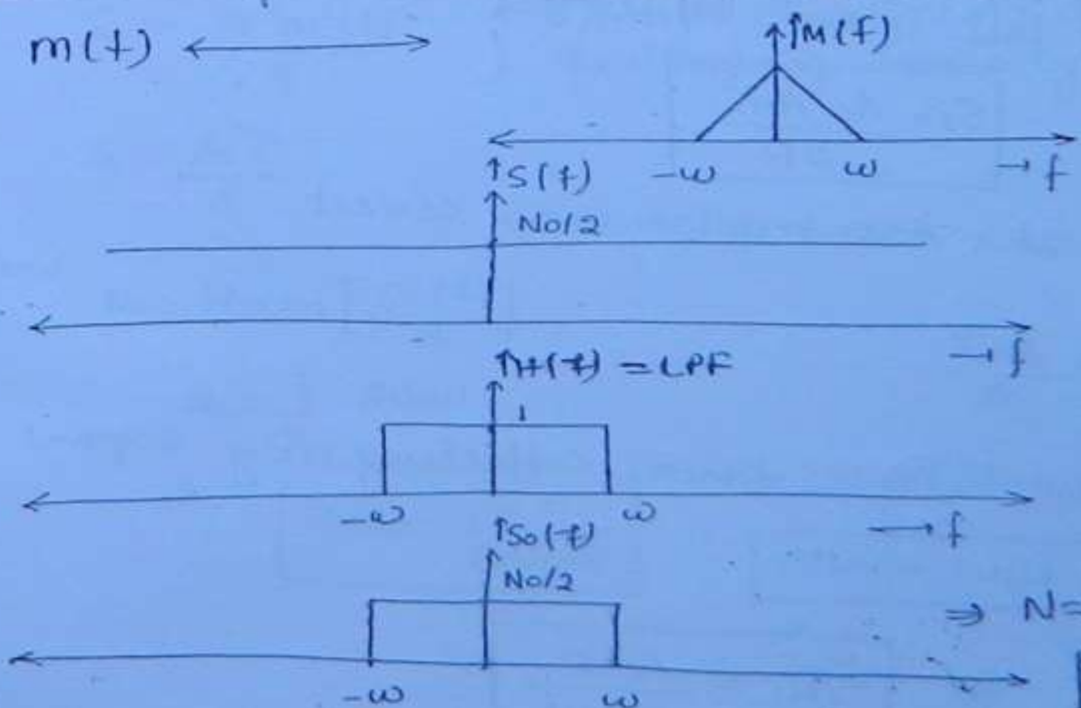
$$\boxed{\begin{array}{l} F.O.M > 1 \Rightarrow S_o/N_o > S_i/N_i \\ F.O.M < 1 \Rightarrow S_o/N_o < S_i/N_i \end{array}}$$

Note:

1. If F.O.M > 1, then Reciever is said to be very much efficient in decreasing the effect of Noise alone.
2. If F.O.M < 1, then Rx is itself adding some amont of Noise, so that So/No is decreased.

* WHITE NOISE POWER AFFECTING MSG SIGNAL:

$m(t) \longleftrightarrow$



$\Rightarrow N = 2\omega \times N_o/2$

$\boxed{N = N_o\omega \text{ watts}}$

* FIGURE OF MERIT OF DSB RX:

As,
$S_{DSB}(t) = A_c m(t) \cos 2\pi f_c t$

Power of signal, $S_i = \frac{A_c^2 m^2(t)}{2R}$

as m(t) is changing, Hence, it gives the Instantantaneous power.

So,
$$S_i = \frac{A_c^2 m^2(t)}{2}$$

let, $m^2(t)$ = Intantaneous power of $m(t) = P$

So, $S_i = \frac{A_c^2 P}{2}$ ...... (A)

Now, let $m(t) = A_m \cos 2\pi f_m t$

So, $S_{DSB} = A_c A_m \cos 2\pi f_m t \cos 2\pi f_c t$

So, $\text{Power (DSB)} = \frac{A_c^2 A_m^2}{4R}$

Now, P of $m(t) = A_m^2 / 2R$

& put in eqn (A) we get:

$$\boxed{S_i = \frac{A_c^2 A_m^2}{2R}}$$

Hence, the Assumption was correct

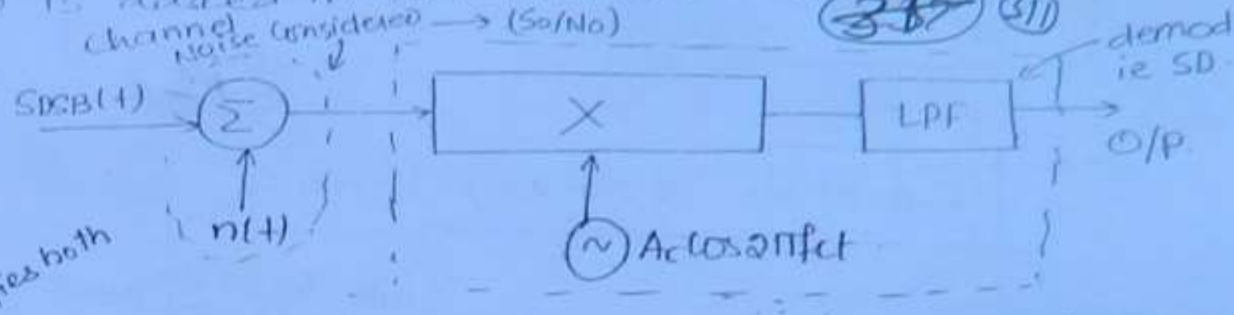
Now,
$$S_i = \frac{A_c^2 P}{2}$$

let, N_i = white noise power affecting msg signal

So, $\boxed{N_i = N_0 W \text{ watts.}}$

So, $\boxed{S_i/N_i = \frac{A_c^2 P}{2 N_0 W}}$

* $S_{DSB}(t)$ is transmitted through channel and some Noise is added in the channel



amplifies both S & N.

* The ampl^r and other components donot affect the (S/N) but only demodulator affect the (S/N). Hence, it is taken into consideration only

So,

$$(\text{mul})\,O/P = \{s(t) + n(t)\}\cos 2\pi f_c t$$

$$= \{A_c m(t)\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t + n_q(t)\sin 2\pi f_c t\}\cdot\cos 2\pi f_c t$$

And,

$$(\text{LPF})\,O/P = \underbrace{\frac{A_c m(t)}{2}}_{\text{Signal}} + \underbrace{\frac{n_c(t)}{2}}_{\text{Noise}}$$

So,

$$S_0 = \text{Power}\left[\frac{A_c m(t)}{2}\right]$$

$$S_0 = \frac{A_c^2 m^2(t)}{4} \qquad \left\{\because A_c m(t) \text{ is either A.C or D.C is not specified. So, Squaring gives Power}\right\}$$

$$S_0 = \frac{A_c^2 P}{4}$$

And,

$$N_0 = \text{Power}\left[\frac{n_c(t)}{2}\right]$$

$$N_0 = \frac{1}{4}\cdot 2N_0W$$

So,

$$\boxed{S_0/N_0 = \frac{A_c^2 P}{8N_0W}}$$

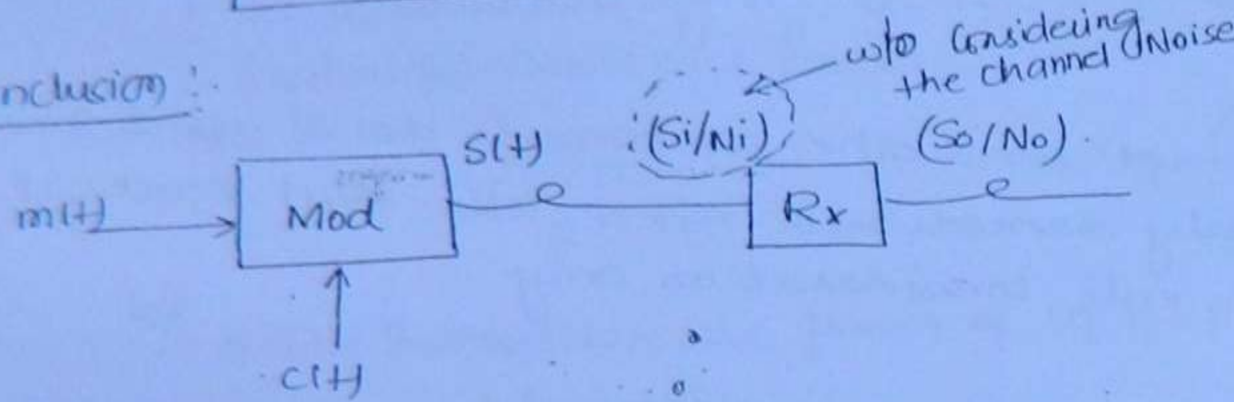
Now,

$$F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}$$

(8.12)

$$\boxed{F.O.M = 1 \Rightarrow (S_o/N_o) = (S_i/N_i)}$$

Conclusion:



* (S_i/N_i) is calculated by considering the effect of Noise on the msg signal. But (S_i/N_i) is to be calculated at the I/P of the Rx by considering the effect of Noise on the modulated signal.
But $(S_o/N_o) = (S_i/N_i)$; hence it may be concluded that the demodulator is eliminating the effect of channel Noise.

* Synchronous detector is working efficiently in nullyfying the white Noise affecting DSB signal in the channel.

* F.O.M of SSB Rx:

General exp of SSB is given as:

$$s(t) = \frac{A_c m(t)}{2} \cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2} \sin 2\pi f_c t$$

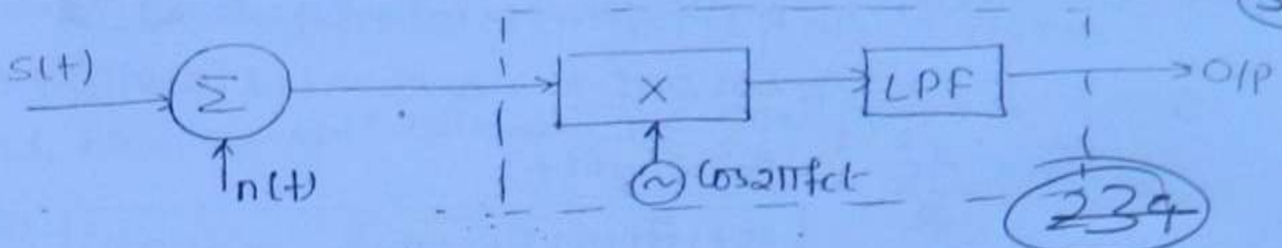
$$S_i = \frac{S_{DSB}}{2} \left\{ \text{half of DSB power} \right\}.$$

$$S_i = \frac{A_c^2 P}{4}$$

Ni: Noise power affecting msg signal

$\boxed{N_i = N_0 \omega \text{ watts}}$ $\Rightarrow$ $\boxed{S_i/N_i = A_c^2 P / 4N_0\omega}$

313



$$(\text{mul})\,o/p = \{s(t) + n(t)\}\cos 2\pi f_c t$$

$$= \left\{ \frac{A_c m(t)}{2}\cos 2\pi f_c t \mp \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t + n_c(t)\cos 2\pi f_c t + n_q(t)\sin 2\pi f_c t \right\}\cos 2\pi f_c t$$

$$(\text{LPF})\,o/p = \underbrace{\frac{A_c m(t)}{4}}_{\text{signal}} + \underbrace{\frac{n_c(t)}{2}}_{\text{Noise}}$$

Now,

$$S_o = \text{Power}\left\{\frac{A_c m(t)}{4}\right\}$$

$$S_o = \frac{A_c^2 m^2(t)}{16}$$

$$\boxed{S_o = \frac{A_c^2 P}{16}}$$

$$N_o = \text{Power}\left\{\frac{N_c(t)}{2}\right\}$$

$$= \frac{1}{4} n_c^2(t)$$

$$N_o = \frac{1}{4} \times N_0\omega$$

$$\boxed{N_o = N_0\omega/4}$$

So,

$$\boxed{\left(\frac{S_o}{N_o}\right) = \frac{A_c^2 P}{4N_0\omega}}$$

$$\therefore \boxed{\left(\frac{S_o}{N_o}\right) = \left(S_i/N_i\right)}$$

So, $\boxed{F.O.M = 1}$

* F.O.M OF AM Rx:

General exp of AM signal is given as: (314)

$$S_{AM}(t) = A_c\{1+K_a m(t)\}\cos 2\pi f_c t$$
$$= A_c\cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t$$

So,

$$S_i = \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 m^2(t)}{2}$$

$$S_i = \frac{A_c^2}{2} + \frac{A_c^2 K_a^2 P}{2}$$

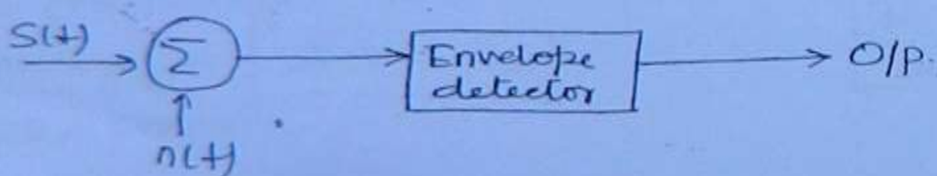
$$\boxed{S_i = \frac{A_c^2}{2}(1+K_a^2 P)}$$

N_i = Noise power affecting msg

$$\boxed{N_i = N_0 W \text{ watts}}$$

So,

$$\boxed{(S_i/N_i) = \frac{A_c^2}{2N_0 W}(1+K_a^2 P)}$$



Now, $(ED)_{I/P} = \{S(t) + n(t)\}$

$$= \{A_c\cos 2\pi f_c t + A_c K_a m(t)\cos 2\pi f_c t + n_c(t)\cos 2\pi f_c t + n_q(t)\sin 2\pi f_c t\}.$$

Now, as

$$A\cos 2\pi f_0 t + B\sin 2\pi f_0 t \xrightarrow{E/D\ O/P} \sqrt{A^2+B^2}$$

$$(ED)_{I/P} = \underbrace{\{A_c + A_c K_a m(t) + n_c(t)\}}_{A}\cos 2\pi f_c t + \underbrace{\{n_q(t)\}}_{B}\sin 2\pi f_c t$$

So,

$$(ED)O/P = \sqrt{\{A_c + A_c K_a m(t) + n_c(t)\}^2 + \{n_q(t)\}^2}$$

(3.15)

As, we know that the effect of quadrature component is 0.

And, the ampl^r blocks D.C components also

So,

$$\boxed{(ED)O/P = \underbrace{A_c K_a m(t)}_{\text{Signal}} + \underbrace{n_c(t)}_{\text{Noise}}}$$

Now,

$$S_o = \text{Power}\{A_c K_a m(t)\} = A_c^2 K_a^2 m^2(t) = A_c^2 K_a^2 P$$

$$N_o = \text{Power}\{n_c(t)\} = 2N_o\omega$$

So,

$$\boxed{\frac{S_o}{N_o} = \frac{A_c^2 K_a^2 P}{2N_o\omega}}$$

Then,

$$F.O.M = \frac{(S_o/N_o)}{(S_i/N_i)}$$

$$= \frac{A_c^2 K_a^2 P \times 2N_o\omega}{2N_o\omega \times A_c^2(1+K_a^2 P)}$$

$$\boxed{F.O.M = \frac{K_a^2 P}{(1+K_a^2 P)}}$$

let,

$m(t) = A_m \cos 2\pi f_m t$

then Power$\{m(t)\}$ = $P = A_m^2/2$

putting in above we get:-

$$F.O.M = \frac{\frac{K_a^2 A_m^2}{2}}{\left(1 + K_a^2 \frac{A_m^2}{2}\right)}$$

$$F.O.M = \frac{(K_a A_m)^2}{\{2+(K_a A_m)^2\}}$$

8/6a

$$\boxed{F.O.M = \frac{\mu^2}{2+\mu^2}}$$

* Also, $\boxed{\eta = \frac{\mu^2}{2+\mu^2} = FOM}$

Now,

$\mu = 0.5 \Rightarrow \eta = 0.11$

$\mu = 0.707 \Rightarrow \eta = 0.2$

$\mu = 1 \Rightarrow \eta = 1/3 = 0.33.$

Conclusion!

FOM ↑ as μ ↑

$$\boxed{(FOM)_{max} = 1/3 \text{ for } \mu = 1}$$

So,

$$\boxed{\left(\frac{S_o}{N_o}\right) = \frac{1}{3}\left(\frac{S_i}{N_i}\right)}$$

Note!

1. The performance of Envelope detector against channel Noise is Poor.

No Imp

* F.O.M OF FM Reciever!

The F.O.M of FM Recievers is given by:.

$$\boxed{F.O.M = \frac{3K_f^2 P}{\omega^2}}$$

where, K_f = freq^n sensitivity of FM mod

P = Power of m(t)

ω = msg B.W

let $m(t) = A_m \cos 2\pi f_m t$

$$P = \frac{A_m^2}{2} \quad \& \quad W = f_m$$

~~245~~ 317

So, $$F.O.M = \frac{3 K_f^2 \cdot A_m^2/2}{f_m^2}$$

$$F.O.M = \frac{3}{2}\left\{\frac{K_f A_m}{f_m}\right\}^2$$

$$= \frac{3}{2}\left\{\frac{\Delta f}{f_m}\right\}^2$$

$$\boxed{F.O.M = \frac{3}{2}\beta^2}$$

* For NBFM:

For NBFM ; $\beta \leq 1$ (small)

$\beta_{max} = 1$

So, $\boxed{F.O.M = 3/2 = 1.5}$

* let $\beta = 0.5 = 1/2$

$$\boxed{F.O.M = \frac{3}{2} \times \frac{1}{4} = 3/8 = 0.3}$$

Conclusion: FOM of NBFM is small.

* For WBFM:

For WBFM ; $\beta > 1$ (high) ~~{βmin = 4}~~

$$FOM = \frac{3}{2} \cdot \beta^2$$

As $\beta \uparrow$ FOM $\uparrow$

but $\beta \uparrow \rightarrow \uparrow B.W = 2(\beta\uparrow + 1) f_m$

Conclusion: So, Generally, the value of $\boxed{\beta \text{ is Restricted to } 10}$

$\boxed{\beta = 10 \quad , FOM = 150}$

WBFM is preferred over NBFM because of its high FOM.

Q1 For an FM, given

$(S/N)_{o/p} = 30\,dB \; ; \; (S/N)_{i/p} = 20\,dB$ (318)

Find the value of β.

Soln: $(S/N)_o = 30\,dB \Rightarrow 10\log_{10}(S/N)_o = 30$

$(S/N)_{o/p} = 10^3 = 1000$

$(S/N)_i = 20\,dB \Rightarrow 10\log_{10}(S/N)_i = 20$

$(S/N)_i = 100$

So, $\dfrac{(S/N)_o}{(S/N)_i} = 10 = \frac{3}{2}\beta^2$

$\beta = \sqrt{\dfrac{20}{3}}$

$\boxed{\beta = 2.58}$

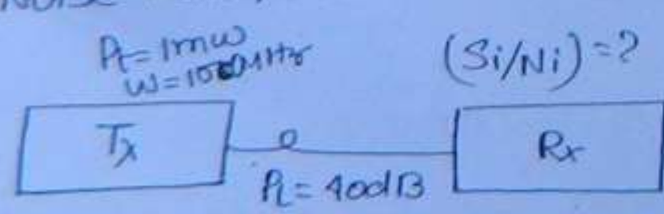
Q2 A Video Signal of having BW of 10MHz, power of 1mW is transmitted through a channel. Power loss in the channel is given by 40dB. Noise PSD is given by 10^{-20} watts/Hz. Find S/N at the i/p of the Reciever.

Soln: Given, B.W = 10MHz.

Power = 1mW.

Power loss in Channel = 40dB.

Noise PSD ; $N_o = 10^{-20}$ watts/Hz



As we know that,

$$N_i = N_0(\omega)$$

$$= 10^{-20} \times 100 \times 10^{6}$$

$$N_i = 10^{-12} \text{ watt}$$

~~245~~ 319

If there is no path loss $\Rightarrow P_t = S_i$

But Path loss = 40dB (P_L).

So, $S_i = (P_t - P_L)$

$$(S_i)_{dB} = (P_t)_{dB} - (P_L)_{dB}$$

$$10\log_{10}(S_i) = 10\log_{10}(P_t) - 10\log_{10}(P_L)$$

$$\Rightarrow \cancel{10\log_{10}}(S_i) = \cancel{10\log}(P_t/P_L)$$

$$S_i = (P_t/P_L)$$

$$S_i = \frac{1\times10^{-3}}{(10^{4})}$$

$$S_i = 10^{-7} \text{ watts}$$

then, $(S_i/N_i) = \frac{(10^{-7})}{(10^{-12})}$

$$(S_i/N_i) = 10^{5}$$

$$\boxed{\left(\frac{S_i}{N_i}\right)_{dB} = 50\text{dB}}$$

Q3 Audio Signal Band limited to 15KHz is transmitted through a channel after modulation. Power Loss in the channel is given by 50dB. 2 sided Noise PSD is 10^{-10} watts/Hz. Find transmitted power required to get $(S/N)_{o/p}$ of 40 dB if the modulation scheme used is:-

a) DSB.

b) AM with $\mu = 1$.

c) FM with $\beta = 5$.

Sol^n Given

$B(\omega), \omega = 15 KHz$

$P_l = 50 dB$

$\frac{N_0}{2} = 10^{-10}$ watts/Hz

(246) (320)

$(S/N)_{O/P} = 40 dB$

a) For DSB

$(S/N)_{O/P} = (S/N)_{I/P}$

So, $(S_i/N_i) = \left(\frac{S_i}{N_0 \omega}\right) = 40 dB$

$$\frac{S_i}{(2\times10^{-10}\times15\times10^3)} = 10000$$

$S_i = 0.03$

NOW, $S_i = \frac{P_t}{P_L} = 0.03$

So, $P_t = (0.03 \times P_L)$

$= 0.03 \times 1 \times 10^5$

$\boxed{P_t = 3 KW}$ Ans

b) For AM :-

$$F.O.M = \frac{\mu^2}{2+\mu^2} = \frac{1}{3}$$

So, $\left(\frac{S}{N}\right)_{O/P} = \frac{1}{3}(S_i/N_i)$

So, $\left(\frac{S_i}{N_i}\right) = 3 \times \cancel{40 dB}\, 10^4$

$S_i = 3\times10^4 \times 2\times10^{-10} \times 15\times10^3$

$\frac{P_t}{P_L} = 0.09$

$P_t = 0.07 \times 10^{4.5}$

$\boxed{P_t = 9KW}$ Ans

) For FM:

329

$FOM = \frac{3}{2}\beta^2$

for $\beta = 5$

$FOM = 37.5$

So, $(S_o/N_o) = 37.5 \times (S_i/N_i)$

$(S_i/N_i) = \frac{10^4}{37.5}$

$S_i = \frac{10^4}{37.5} \times 2 \times 10^{-10} \times 15 \times 10^3$

$\frac{P_t}{P_L} = \frac{1}{1250}$

$\boxed{P_t = 80W}$ Ans

*NOISE IN DIGITAL COMMUNICATION:

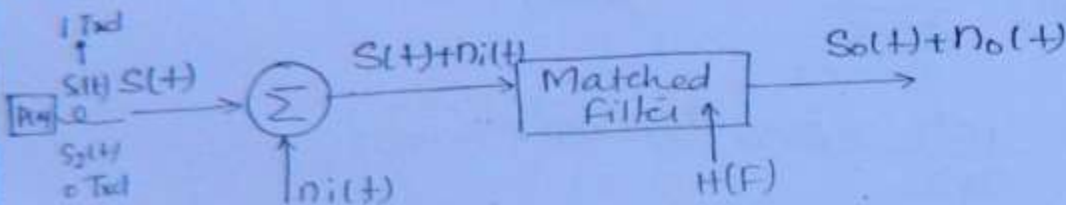
*MATCHED FILTER:

*Importance is there of Matched Filter is to reduce the Prob. of Error

Note:

*Matched Filter is used in digital Rx before threshold comparator

*It increases S/N↑ ratio so that Pe↓ will be decreased.



$$(SNR)_{O/P} = \frac{|S_o(t)|^2}{N_o} \quad ; N_o = \text{O/P Noise power.}$$

*Matched Filter increases the SNR, so we have to calculate the characteristic of MF ie H(F)

* Let,

$n_i(t)$ = white Noise possessing Gaussian density func with '0' mean, and having 2 sided PSD of $N_o/2$ ω/Hz

Now, $S_o(t) = S(t) * h(t)$

where, $S(t)$ = Input Signal Power

$h(t)$ = Impulse Response of MF.

Taking FT we get:-

$$S_o(F) = S(f) \cdot H(f).$$

$$\& \; S_o(t) = IFT\{S_o(F)\}$$

$$= \int_{-\infty}^{\infty} S_o(f) \cdot e^{j2\pi ft} df$$

$$\boxed{s_o(t) = \int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi ft}\,df}$$

Now,
N_o : O/P of Noise power

$$(\text{Noise PSD})_{O/P} = (\text{Noise PSD})_{I/P} \cdot |H(f)|^2$$

$$S_o(f) = \frac{N_o}{2} \cdot |H(f)|^2$$

So. O/P Noise power $\{N_o\}$ = Area$[S_o(f)]$.

$$N_o = \int_{-\infty}^{\infty} S_o(f)\,df$$

$$N_o = \int_{-\infty}^{\infty} \frac{N_o}{2}|H(f)|^2\,df$$

* (SNR) at a specific time instant of $t = T$ is given by:-

Then
$$\boxed{(SNR)_{O/P} = \frac{|s_o(T)|^2}{N_o}}$$

So,
$$\left(\frac{S}{N}\right)_o = \frac{\left|\int_{-\infty}^{\infty} S(f)H(f)e^{j2\pi fT}\,df\right|^2}{N_o}$$

* (S/N) depends upon H(f). Hence for H(f) value the $(S/N)_o$ reaches max^m has to be calculated

Now, according to schwartz's inequality

$$\left(\int_{-\infty}^{\infty} S(f)H(f)\right) \quad \left|\int_{-\infty}^{\infty} x_1(f)\,x_2(f)\,df\right|^2 \le \int_{-\infty}^{\infty} |x_1(f)|^2 df \cdot \int_{-\infty}^{\infty} |x_2(f)|^2 df$$

So, applying above to $(S/N)_o$ we get:-

$$\left|\int_{-\infty}^{\infty} \underbrace{S(f)}_{x_1}\,\underbrace{H(f)e^{j2\pi fT}}_{x_2}\,df\right|^2 \le \int_{-\infty}^{\infty} |S(f)|^2 df$$

[illegible]

$$\left|\int_{-\infty}^{\infty} \underbrace{H(f)}_{x_1}\ \underbrace{S(f)\,e^{j2\pi fT}}_{x_2}\,df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |S(f)\,e^{j2\pi fT}|^2 df$$

$$\left|\int_{-\infty}^{\infty} H(f)\,S(f)\,e^{j2\pi fT}\,df\right|^2 \le \int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |S(f)|^2 df$$

$\left\{\because |e^{j2\pi fT}|^2 = 1\right\}$

Provided, $\boxed{H(f) = S^*(f)\,e^{-j2\pi fT}}$

the equality Relation holds good.

Now,

$$\left(\frac{S}{N}\right)_0 = \frac{\left|\int_{-\infty}^{\infty} S(f)\,H(f)\,e^{j2\pi fT}\,df\right|^2}{\int_{-\infty}^{\infty} \frac{N_0}{2}\,|H(f)|^2 df}$$

$$\left(\frac{S}{N}\right)_0 \le \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \cdot \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}\int_{-\infty}^{\infty} |H(f)|^2 df}$$

provided, $H(f) = S^*(f)\,e^{-j2\pi fT}$, the equality Relation holds good.

So,

$$\left(\frac{S}{N}\right)_{0\,max} = \frac{\cancel{\int_{-\infty}^{\infty} |H(f)|^2 df} \cdot \int_{-\infty}^{\infty} |S(f)|^2 df}{\frac{N_0}{2}\cancel{\int_{-\infty}^{\infty} |H(f)|^2 df}}$$

ESD of $S(t)$

$$\left(\frac{S}{N}\right)_{0\,max} = \int_{-\infty}^{\infty} |S(f)|^2 df \Big/ (N_0/2)$$

$$\left(\frac{S}{N}\right)_{max} = E/(N_0/2)$$ (325)

where, E = Energy of s(t)

So, $$\boxed{\left(\frac{S}{N}\right)_{max} = (2E/N_0)}$$

$$\left(\frac{S}{N}\right) = E/(N_0/2)$$

S → mapped to Energy of Input Signal

N → mapped to Input PSD of Input $n_i(t)$

Note:
For max (S/N) is corresponding to the Ratio of Input Signal Energy and Input Noise PSD.

Imp ***

* Impulse Response of Matched Filter.

As, $h(t) = IFT[H(f)]$

$$= \int_{-\infty}^{\infty} H(f) e^{+j2\pi ft} df$$

$$= \int_{-\infty}^{\infty} S^*(f) e^{-j2\pi fT} \cdot e^{+j2\pi ft} df$$

Now, if s(t) is Real, then $S^*(f) = S(-f)$.

So,
$$h(t) = \int_{-\infty}^{\infty} S(-f) e^{-j2\pi fT} \cdot e^{j2\pi ft} df$$

let $-f \rightarrow f$

$$h(t) = \int_{\infty}^{-\infty} S(f) e^{j2\pi fT} \cdot e^{-j2\pi ft} (-df)$$

$$\because -\int_a^b = \int_b^a$$

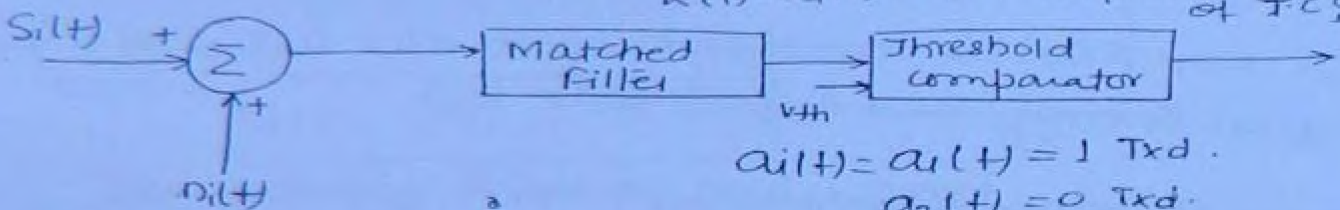
So, $$h(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi fT} \cdot e^{-j2\pi ft} df$$

$$h(t) = \int_{-\infty} S(f)\, e^{j2\pi f(T-t)}\, df$$

So, $\boxed{h(t) = s(T-t)}$ ***

* Probability Error of digital Signalling schemes:

$z(t) = a_i(t) + n_o(t)$ {$z \rightarrow$ input voltage expelled as input of T.C}



$a_i(t) = a_1(t) = 1$ Txd.

$a_2(t) = 0$ Txd.

$S_i(t) = S_1(t) \rightarrow$ binary '1' was Txd.

$S_2(t) \rightarrow$ binary '0' was Txd

* Assume, $n_i(t)$ corresponds to white Noise of having 2 sided PSD $N_0/2$, and possessing Gaussian density func^n with 0 mean.

Case 1:
Assume no signal component was transmitted by the Tx. $(a_i(t) = 0)$.

So, $z(t) = n_o(t)$

$z = n_o$.

So, $E[z] = E[n_o]$.

$E[z] = 0$ {$\because$ mean of n_i is 0, hence mean of n_o is also 0.}.

So, $f(z) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-(z-a)^2/2\sigma^2}$ {$\because$ Gaussian density func^n}.

$\sigma^2 \leftarrow$ variance of z = AC power.

Now, $z = n_o$

variance$[z]$ = variance$[n_o]$

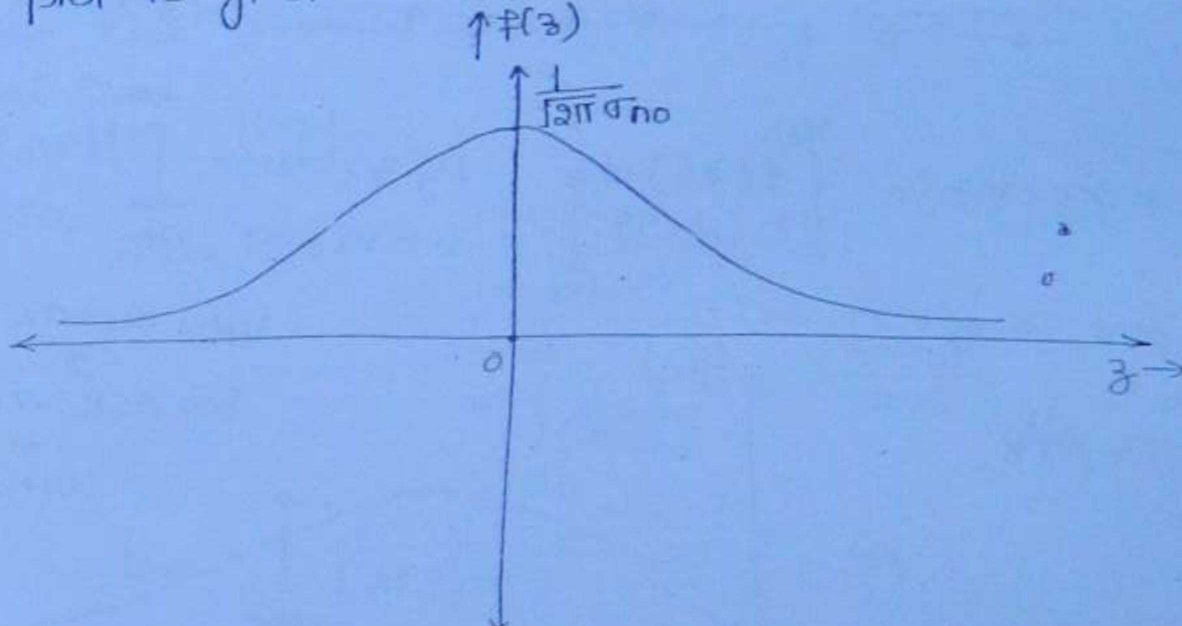
A-C power(z) = AC power(n_o).

So, let $\sigma = \sigma_{no}$ = variance of noise

$$f(z) = \frac{1}{\sqrt{2\pi\sigma_{no}^2}} \, e^{-\frac{(z-a)^2}{2\sigma_{no}^2}}$$

So, $$f(z) = \frac{1}{\sqrt{2\pi\sigma_{no}^2}} \cdot e^{-z^2/2\sigma_{no}^2} \quad \left\{ \because \text{mean} = 0,\ a = 0 \right\}$$

The plot is given as:-



* As the Noise is white Noise, so the strength of Noise is very small. So,

P(no small) = high.

P(no large) = low.

$$\left\{ \because P(X \le x) = \int_{-\infty}^{x} F_X(x)\,dx. \right.$$

Case 2:-

Assume binary 1 was transmitted.

So, $z = a_1(t) + n_0(t)$

$z = a_1 + n_0$

$E[z] = E[a_1 + n_0] = E[a_1] + \cancel{E[n_0]}^{\,0}$.

$E[z] = E[a_1] = a_1$

$E[z] = a_1$

The density function is given by:-

$$f(z/1) = \frac{1}{\sqrt{2\pi\sigma_{n0}^2}} \cdot e^{-\frac{(z-a_1)^2}{2\sigma_{n0}^2}}$$

{∵ A.C power and a_1 is dc term so A.C power of $a_1 = 0$, A.C power of $n_0 = \sigma^2_{n0}$ }

Now,

Pe_1 = Prob. of Rx 0 when 1 was Txd. $= P\{z < V_{th}\}$

Pe_0 = " 1 " 0 " $= P\{z > V_{th}\}$

So,

$$P\{z < V_{th}\} = \int_{-\infty}^{V_{th}} f(z/1)\, dz \;;\; P\{z > V_{th}\} = \int_{V_{th}}^{\infty} f(z/0)\, dz.$$

* The plot is given as:-

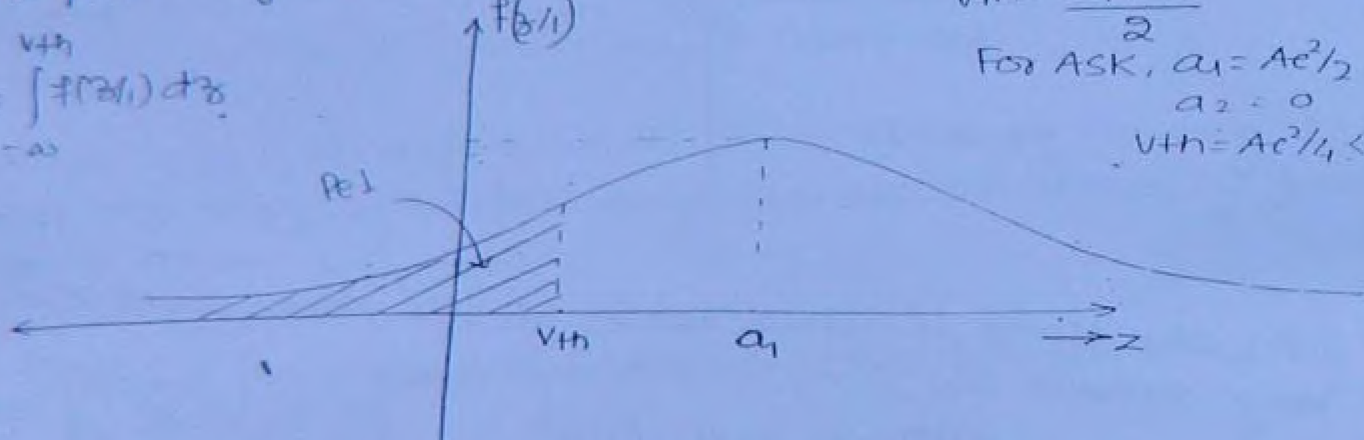
$$V_{th} = \frac{a_1 + a_2}{2}$$

For ASK, $a_1 = A_c^2/2$

$a_2 = 0$

$V_{th} = A_c^2/4 < a_1$

$$Pe_1 = \int_{-\infty}^{V_{th}} f(z/1)\, dz$$



Case 3: when Binary '0' was transmitted.

$$z = a_2 + n_0.$$

$$E[z] = E[a_2] + 0.$$

$$E[z] = a_2$$

So, $$f(z/0) = \frac{1}{\sqrt{2\pi\sigma_{n0}^2}}\, e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}}$$

$V_{th} = A_c^2/4 > a_2 = 0.$

* Plot is:-

$$Pe_0 = \int_{V_{th}}^{\infty} f(z/0)\, dz$$

Note:

1 when Binary 1 was transmitted, no error occurs if $z > V_{th}$

then $\boxed{P_{e1} = P(z < V_{th})}$ (329)

2. when Binary 0 was transmitted, no error occurs if $z < V_{th}$

$$\boxed{P_{e0} = P(z > V_{th})}.$$

* Assume the channel was Binary symmetric channel so that, $\boxed{P_{e1} = P_{e0}}$

So, $$P_{e0} = P(z > V_{th}) = \int_{V_{th}}^{\infty} f(z/0)\, dz.$$

$$= \int_{V_{th}}^{\infty} \frac{1}{\sqrt{2\pi\sigma_{n0}^2}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}} \cdot dz$$

$$P_{e0} = \int_{V_{th}}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_{n0}} \cdot e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}} \cdot dz$$

$$P_{e0} = \frac{1}{\sqrt{2\pi\sigma_{n0}^2}} \int_{+V_{th}}^{\infty} e^{-\frac{(z-a_2)^2}{2\sigma_{n0}^2}} \cdot dz$$

Now as error funcⁿ is given as:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-y^2/2}\, dy.$$

Now, let, $\frac{z-a_2}{\sigma_{n0}} = y$

$z - a_2 = \sigma_{n0}\, y.$

$dz = \sigma_{n0} \cdot dy$

$z = 0 \Rightarrow y = \infty.$

$z = V_{th} \Rightarrow y = \left(\frac{V_{th} - a_2}{\sigma_{n0}}\right)$

$$= \frac{\frac{a_1+a_2}{2} - a_2}{\sigma_{n0}}$$

$$= \frac{(a_1-a_2)/2}{\sigma_{n0}}$$

$$\left\{ y = (a_1-a_2)/2\sigma_{n0} \right\}$$

So $P_{e0} = P_{e1} = P_e = \frac{1}{\sqrt{2\pi}\,\cancel{\sigma_{n0}}} \int_{\frac{a_1-a_2}{2\sigma_{n0}}} e^{-y^2/2}\, \cancel{\sigma_{n0}}\, dy$ 330

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{\frac{a_1-a_2}{2\sigma_{n0}}}^{\infty} e^{-y^2/2}\, dy$$

$$P_e = Q\left\{ \frac{a_1-a_2}{2\sigma_{n0}} \right\} \quad \left[\because Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2}\, dy \right].$$

× The Plot of $Q(x)$ is given as:-

Now, $P_e = Q\left\{ \sqrt{\frac{(a_1-a_2)^2}{4\sigma_{n0}^2}} \right\}$

× Analysis:-

$s_1(t)$, $s_2(t)$, $n_i(t)$ → M·F $\{h(t)\}$ → $a_1(t)$, $a_2(t)$, $n_0(t)$

Now $\frac{(a_1-a_2)^2}{\sigma_{n0}^2} = \frac{\text{diff Signal Power}}{\text{total Noise power}}$

$\because \sigma^2 = m_2 - \cancel{m_1^2}^{\,0}$ (mean)
(total power)

× $S_i(t)$, $n_i(t)$ → M·P → $S_0(t)$, $n_0(t)$

$$\left(\frac{S}{N}\right)_0 = \frac{|S_0(t)|^2}{N_0} = E/(N_0/2)$$

when, $h(t) = S(T-t)$.

Now $\frac{(a_1-a_2)^2}{\sigma_{no}^2}$ $\rightarrow$ $E_d/(N_o/2)$; E_d = diff signal energy
$h(t) = s(T-t)$ ie energy $\{S_1(t) - S_2(t)\}$

Now, $s(t) = S_1(t) - S_2(t)$

331

So, $P_{emin} = Q\left[\sqrt{\frac{1}{4} \cdot \frac{E_d}{N_o/2}}\right]$

$$\boxed{P_{emin} = Q\left[\sqrt{\frac{E_d}{2N_o}}\right]}$$

* From M.F op^n:-

$\frac{a_1 - a_2}{\sigma_{no}^2}$ is maximised to $(E_d/N_o/2)$ so that P_e will be minimised

* P_e of ON-OFF SIGNALLING SCHEMES:

$1 \rightarrow S_1(t) = A_c$

$0 \rightarrow S_2(t) = 0V$

So, $P_e = Q\left[\sqrt{\frac{E_d}{2N_o}}\right]$

$$E_d = \text{Energy}\left[S_1(t) - S_2(t)\right]$$

$$= \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$$

$$= \int_0^{T_b} (A_c - 0)^2 dt$$

$$\boxed{E_d = A_c^2 T_b}$$

So, $\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_o}}\right]} = P_e = Q[x_1]$

* P_e of NRZ signalling scheme

$1 \longrightarrow S_1(t) = A_c$

$0 \longrightarrow S_2(t) = -A_c$

332

$$E_d = \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$$

$$= \int_0^{T_b} (2A_c)^2 dt$$

$$\boxed{E_d = 4A_c^2 T_b}$$

So, $P_e = Q\left[\sqrt{\frac{4A_c^2 T_b}{2N_0}}\right]$

$$\boxed{P_e = Q\left[\sqrt{\frac{2A_c^2 T_b}{N_0}}\right]} \Rightarrow P_e = Q[X_2]$$

* As $\boxed{x\uparrow \rightarrow Q[x]\downarrow}$

Conclusion:

As $X_2 > X_1$

So, $\boxed{P_e(\text{NRZ}) < P_e(\text{ON-OFF})}$

* P_e OF ASK:

$1 \longrightarrow S_1(t) = A_c \cos 2\pi f_c t$

$0 \longrightarrow S_2(t) = 0$

So, $E_d = \int_0^{T_b} \{S_1(t) - S_2(t)\}^2 dt$

$$= \int_0^{T_b} \{A_c \cos 2\pi f_c t\}^2 dt$$

$$= \int_0^{T_b} \frac{A_c^2}{2} dt + \frac{A_c^2}{2} \int_0^{T_b} \cancelto{0}{\cos 4\pi f_c t \, dt}$$ { $\because f_c$ is integer multiple of complete cycles }

$$E_d = \frac{A_c^2}{2} T_b$$

So,

$$P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$$

(333)

$$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]}$$

*P_e OF PSK:

$1 \rightarrow s_1(t) = A_c \cos 2\pi f_c t$

$0 \rightarrow s_2(t) = -A_c \cos 2\pi f_c t$

So, $E_d = \int_0^{T_b} \{s_1(t) - s_2(t)\}^2 dt$

$= \int_0^{T_b} \{2A_c \cos 2\pi f_c t\}^2 dt$

$= \int_0^{T_b} 4A_c^2 \cos^2 2\pi f_c t \cdot dt$

$= \frac{4A_c^2}{2} T_b + \frac{4A_c^2}{2} \int_0^{T_b} \cancel{\cos 4\pi f_c t}^{\,0} \, dt$

$(E_d = 2A_c^2 T_b)$

So,

$$P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$$

$$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]}$$

*P_e OF FSK :

$1 \rightarrow s_1(t) = A_c \cos 2\pi f_1 t$ $(f_1 > f_2)$.

$0 \rightarrow s_2(t) = A_c \cos 2\pi f_2 t$

$E_d = \int_0^{T_b} \{s_1(t) - s_2(t)\}^2 dt$

$$E_d = \int_0^{T_b} (A_c \cos 2\pi f_1 t - A_c \cos 2\pi f_2 t)^2 dt$$

334

$$= \int_0^{T_b} A_c^2 \cos^2 2\pi f_1 t \, dt + \int_0^{T_b} A_c^2 \cos^2 2\pi f_2 t \, dt - 2 \int_0^{T_b} A_c^2 \cos 2\pi f_1 t \cos 2\pi f_2 t \, dt$$

$$= \frac{A_c^2}{2} \int_0^{T_b} dt + \frac{A_c^2}{2} \int_0^{T_b} \underbrace{\cos 4\pi f_1 t \, dt}_{\to 0} + \frac{A_c^2}{2} \int_0^{T_b} dt + \frac{A_c^2}{2} \int_0^{T_b} \underbrace{\cos 4\pi f_2 t \, dt}_{\to 0}$$

$$- 2 \frac{A_c^2}{2} \int_0^{T_b} \{ \underbrace{\cos 2\pi (f_1 + f_2) t}_{\to 0} + \cos 2\pi (f_1 - f_2) t \} dt$$

$$E_d = \frac{A_c^2 T_b}{2}$$

So, $P_e = Q\left[\sqrt{\frac{E_d}{2N_0}}\right]$

$P_e = Q\left[\sqrt{T_b A_c^2 / 4N_0}\right]$

$\boxed{P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]}$

Conclusion:

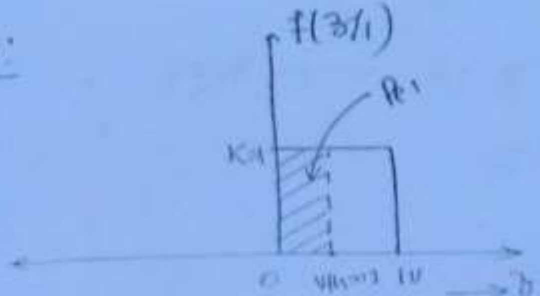
Signalling scheme	P_e
ASK	$Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]$
FPSK	$Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]$
PSK	$Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]$

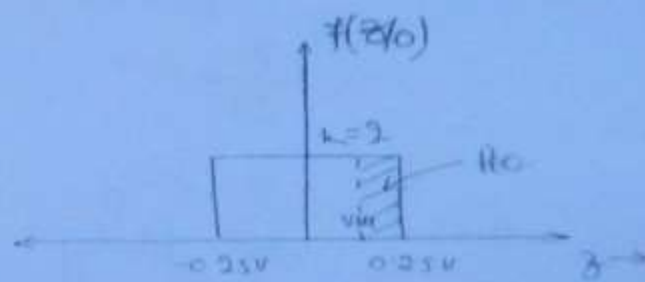
So, $\boxed{P_e(PSK) < P_e(FSK) < P_e(ASK)}$

335

Q1 A Binary Tx is transmitting 2 possible binary symbols specified by 0 & 1. When Binary 1 was transmitted, the signal voltage at the I/P of Threshold Comparator will be in b/w 0V & 1V with equal probability. When Binary '0' was transmitted signal voltage lies b/w −0.25V & 0.25V with equal probability. Threshold voltage is given by 0.2V. Find avg. Pe?

Sol^n:





Now, $Pe_{avg} = \frac{Pe_1 + Pe_0}{2}$

∵ Channel is not BSC:

$$Pe_1 = P(z < V_{th}) = \int_{-\infty}^{V_{th}} f(z/1)\,dz$$

$$= \int_0^{0.2} 1 \cdot dz = 0.2$$

Now,

$$Pe_0 = P(z > V_{th}) = \int_{V_{th}}^{\infty} f(z/0)\,dz$$

$$= \int_{0.2}^{0.25} 2\,dz = 0.1$$

So, $Pe_{avg} = \frac{Pe_0 + Pe_1}{2} = \frac{0.1 + 0.2}{2}$

$$\boxed{Pe_{avg} = 0.15}$$

Complementary Error Function; erfc(x):

Mathematically given as:

(38)

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2}\, dy$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-z^2/2}\, dz \; ; \; z = \text{dummy variable.}$$

let $\frac{z}{\sqrt{2}} = y$

$z = \sqrt{2}\, y$

$dz = \sqrt{2}\, dy$

$z = x \Rightarrow y = x/\sqrt{2}$

$z = \infty \Rightarrow y = \infty$

So,

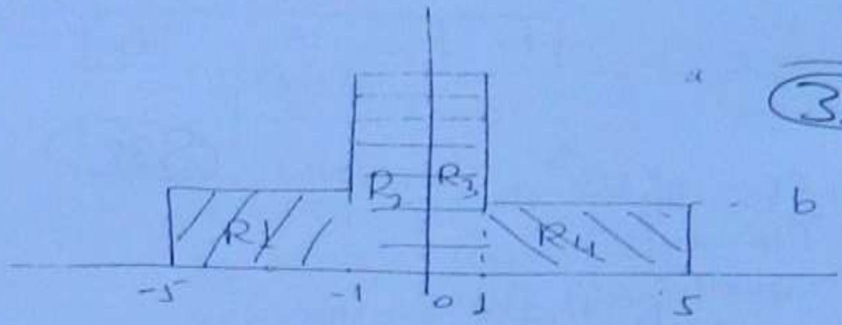
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-y^2} \cdot \sqrt{2}\, dy$$

$$Q(x) = \frac{1 \times 1 \times 2}{2\sqrt{\pi}} \int_{x/\sqrt{2}}^{\infty} e^{-y^2}\, dy$$

$$\boxed{Q(x) = \frac{1}{2} \operatorname{erfc}\left(x/\sqrt{2}\right)}$$

And, $\boxed{\operatorname{erfc}[x] = \frac{e^{-x^2}}{x\sqrt{\pi}}}$

(332)



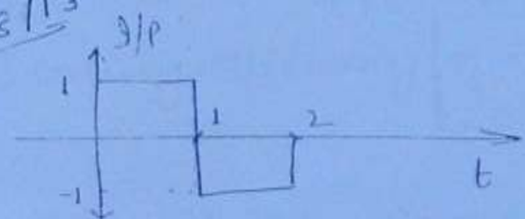
$Prob(R_1) = 1/4 = Area(R_1)$

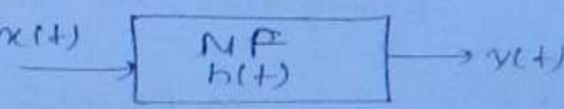
$\Rightarrow 4b = 1/4 \Rightarrow \boxed{b = 1/16}$

$Area(R_2) = Prob(R_2) = 1/4$

$\boxed{a = 1/4}$

Pg-28/13





$y(t) = x(t) * h(t)$

$h(t) = s(T-t)$

when T is not given, then
take T = ~~at~~ Total duration of given Signal = 2

$T = 2$

So, $h(t) = s(2-t)$

So, $y(t) = x(t) * s(2-t)$

$s(t) \longrightarrow s(2-t)$

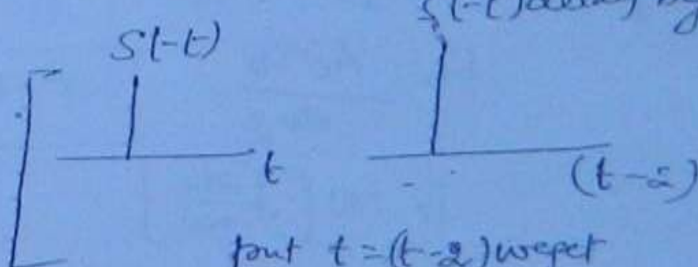
time Reversal

$\downarrow$

$s(-t)$

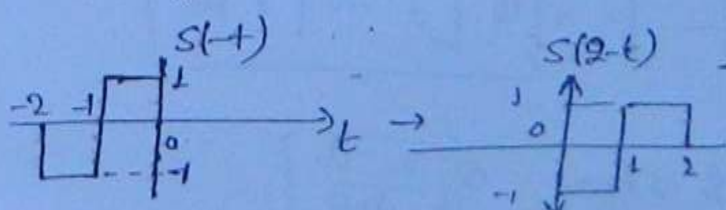
$\downarrow$ delay by 2 unit

$s(-(t-2)) = s(-t+2)$



put $t = (t-2)$ w.r.t

$s(-t+2)$



$\boxed{So,\ y(t) = x(t) * s(2-t)}$

Page 31/2 1st model P_e

Given. $P_t = 1KW$

$P_L = 60dB$.

Tx — C — Rx, $P_L = 60dB$

338

$$S_i = \frac{P_t}{P_L} = \frac{1\times10^3}{10^6} = 0.1mW$$

$N = 10^{-4}$ watt.

$$P_e \text{ of ASK} = Q\left[\sqrt{\frac{A_c^2 T_b}{4N_0}}\right]$$

∴ No data is given regarding above.

And o/p of MF is matched to output energy & PSD of noise

$$S/N = E/(N_0/2)$$

Now,

Energy/bit, $E_b = \frac{A_c^2 T_b}{2}$. & $P_e = Q\left[\sqrt{E_b/2N_0}\right]$.

So, $E_b \rightarrow S$

$\frac{N_0}{2} \rightarrow N$

So, $\boxed{P_e = Q\left[\sqrt{S/2 \cdot 2N}\right]}$; Before matched filter operation

(2) For PSK

$$P_e = Q\left[\sqrt{\frac{A_c^2 T_b}{N_0}}\right]$$

$$E_b = \frac{A_c^2 T_b}{2}$$

$$P_e = Q\left[\sqrt{\frac{2E_b}{N_0}}\right]$$

$$= Q\left[\sqrt{\frac{2S}{2N}}\right] = \boxed{Q\left[\sqrt{S/N}\right] : P_e}$$

P_e 2nd model

(3) $R_b = 2.5\times10^6$ bits/sec

$\frac{N_0}{2} = 10^{-20}$ watt/Hz

$A_c = 1\mu Volt$

P_e of FSK $= Q\left[\sqrt{\frac{A_c^2 T_b}{2N_0}}\right]$

$$= Q\left[\sqrt{\frac{1\times10^{-12}\times}{2.5\times10^6\times 2\times2\times10^{-20}}}\right]$$

As $Q[x] = \frac{1}{2}\,erfc\left[x/\sqrt{2}\right]$

× SOURCE CODING THEOREMS

Tx —— $x_1 x_2 x_3$

339

$x_1 = 001 \rightarrow$ Code length $= 3$

$x_2 = 0010 \rightarrow$ code length $= 4$

$x_3 = 0011 \rightarrow$ Code length $= 4$

× how efficiently is the code working is calculated by the coding efficiency.

× Avg. code length $= L =$ bits/symbol.
Mathematically given as:-

$$L = \sum_i n_i P(x_i).$$

× So, coding efficiency, $\boxed{\eta = \frac{L_{min}}{L}}$

× According to the Source coding Theorem $\Rightarrow \boxed{L \geq H}$

$$\boxed{\% \eta \uparrow \quad H / L \downarrow}$$

× If avg. code length is small, then it is called to be as that the coding efficiency is high.

$$\boxed{H = -\sum_i P(x_i) \log P\{x_i\}}$$

$\underline{Q}$:-

[illegible]
possible symbol with prob. of 0.3, 0.25, 0.2, 0.12, 0.08, 0. [illegible]
Find coding efficiency.

(340)

Soln:

Step 1 → Arrange all prob. in decreasing order

0.3
0.25
0.2
0.12
0.08
0.05

Step 2 → divide the whole prob in such that the 2 sets have equiprobable probabilities (i.e almost close prob).

0.55	0.3	0
	0.25	0
0.45	0.2	1
	0.12	1
	0.08	1
	0.05	1

Note: Assign 0 to above set and 1 to the lower set or vice-versa, but only one has to be followed.

Step 3: Again divide the 2 set with the same process as in 2nd step

	0.3	0	0			→ $n_1 = 2$
	0.25	0	1			→ $n_2 = 2$
0.2	0.2	1	0			→ $n_3 = 2$
0.25	0.12	1	1	0		→ $n_4 = 3$
	0.08	1	1	1	0	→ $n_5 = 4$
	0.05	1	1	1	1	→ $n_6 = 4$

Step 4: Repeat 3

Step 5:

$$L = \sum_{i=1}^{6} n_i \, p(x_i)$$

$$L = (2\times0.3 + 2\times0.25 + 2\times0.2 + 3\times0.12 + 4\times0.08 + 4\times0.05)$$

$$\boxed{L = 2.38 \text{ bits/symbol}}$$

341

Step 6:

$$H = -\sum_{i=1}^{6} p(x_i)\log\{p(x_i)\}.$$

$$\boxed{H = 2.36 \text{ bits/symbol}}$$

So,

$$\% \eta = H/L$$

$$\% \eta = \frac{2.36}{2.38} \times 100$$

$$\boxed{\% \eta = 99.9\%}$$ Ans

Q2. Construct HOFFMAN coding for the above problem

Soln: Step 1: Arrange in decreasing order

0.3
0.25
0.2
0.12
0.08
0.05

Step 2: Sum the last 2 prob.

ie $0.08 + 0.05 = 0.13$

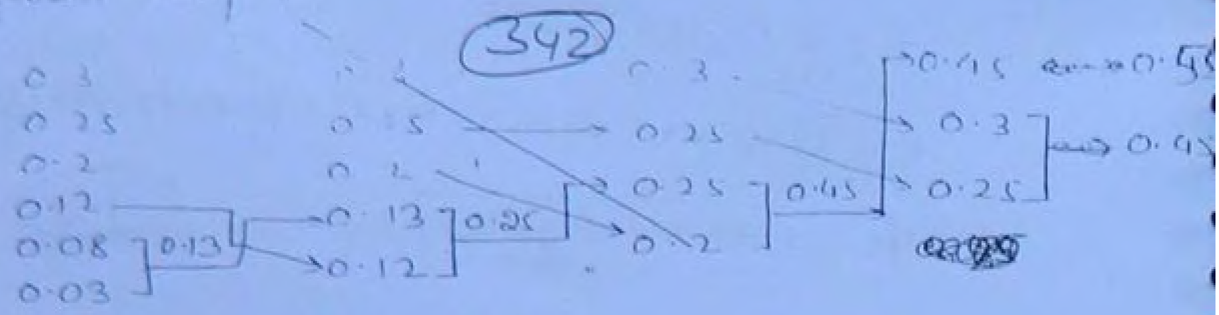
0.3
0.25
0.2
0.12
$\left.\begin{matrix}0.08\\0.05\end{matrix}\right]$ — 0.13

Step 3: by taking step 2 in considera-tion again arrange Prob in decreasing order

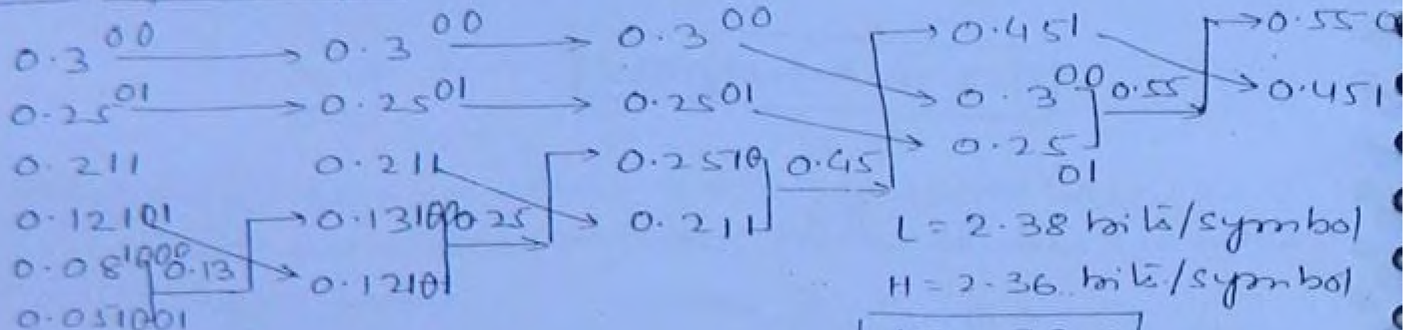
$$\begin{array}{ll} 0.3 & 0.3 \\ 0.25 & 0.25 \\ 0.2 & 0.2 \\ 0.12 & 0.13 \\ 0.08 & 0.12 \\ 0.05 & \end{array}$$

$\left.\begin{matrix}0.13\\0.12\end{matrix}\right]$ — 0.25

342



Step 4 Repeat Step 3



L = 2.38 bits/symbol

H = 2.36 bits/symbol

$\boxed{\% \eta = 99\%}$

Step 5: Associate '0' to all prob corresponding to 0.55 & 1 to prob corresponding to 0.45

Step 6: Move Backward and repeat above.

p34/1

1. i) $H = -\sum_{i=1}^{8} P(a_i) \log \{P(a_i)\}$.

$= 1.96$ bits/symbol.

ii) Prob. of occur of '0' $= (3 \times 1/2 + 2 \times 1/4 + 2 \times 1/8 + \cdots) \times 1/3$

Prob. of occuring of '1' $= 0.2$

iii) $\eta = \frac{H}{H_{max}} = \frac{1.96}{\log_2 8}$ | ∵ no coding technique was given, so $L = H_{max}$

$\eta = \frac{1.96}{3}$

$\eta = 65.3\%$.

iv) by Shannon fano, ...

v) $\eta = 100\%$

Q2.

x_1	0.5	0		$n_1 = 1$
x_2	0.4	1	0	$n_2 = 2$
x_3	0.1	1	1	$n_3 = 2$

So, $L = 1 \times 0.5 + 2 \times 0.4 + 2 \times 0.1$

$= 0.5 + 0.8 + 0.1$

$= 1.4$ bits/symbol

$\eta = 90.7\%$

× 2nd order extension code:

		Pe
$x_1 x_1$	$\rightarrow$	0.25
$x_1 x_2$	$\rightarrow$	0.2
$x_1 x_3$	$\rightarrow$	0.05
$x_2 x_1$	$\rightarrow$	0.2
$x_2 x_2$	$\rightarrow$	0.16
$x_2 x_3$	$\rightarrow$	0.04
$x_3 x_1$	$\rightarrow$	0.05
$x_3 x_2$	$\rightarrow$	0.04
$x_3 x_3$	$\rightarrow$	0.01

$\xrightarrow{}$ S.F coding.

Q. Given $z = x + y$, where x and y are Random variables having density function in the form of Rectangular pulse. Density func^n of z will be:

a) Rectangular pulse

b) Triangular pulse

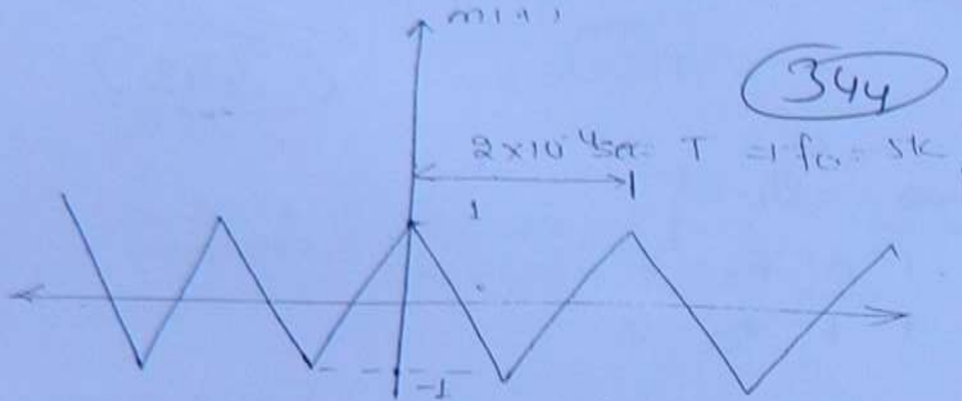
c) Gaussian Pulse

d) None

$\because f(z) = f(x) * f(y)$

$f(z) =$

344



$$m(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn2\pi f_0 t}$$

upto 3rd Harmonic freq^n.

$m(t) = 3f_0, 2f_0, 1f_0 \ldots$ } {∵ multi tone modulation}

$f_{max} = 15KHz \; \{3f_0\}$

Now,

$K_f = 2\pi \times 10^5 \quad ; K_p = 5\pi$

The units are not mentioned & π terms are involved hence

$\omega_i = \omega_c + K_f m(t)$

but all the analysis was done for

$f_i = f_c + K_f m(t)$

So, $\dfrac{\omega_i}{2\pi} = \dfrac{\omega_c}{2\pi} + \dfrac{K_f m(t)}{2\pi}$

$\therefore K_f = \dfrac{2\pi \times 10^5}{2\pi} \quad ; K_p = 5\pi/2\pi$

$= 1 \times 10^5 \quad\quad K_p = 5/2$

$B_{FM} = 2(\Delta f + f_m)$

$= 2\{\dfrac{K_f A_m}{\cancel{f_m}} + f_m\}$

$= 2\{1 \times 10^5 \times 1 + 15 \times 10^3\}$

$= 230K$

For PM, if the msg. is not sinusoidal the formula of B.W & Power changes.

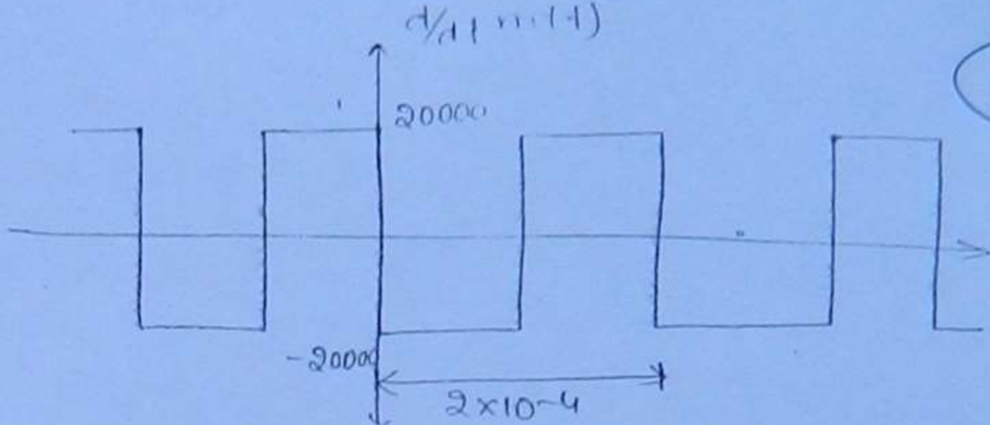
So,

PM of $m(t)$ = FM of $\frac{d}{dt} m(t)$.

B.W of PM of $m(t)$ = B.W of FM of $\frac{d}{dt} m(t)$

Now, $d/dt\ m(t) = \text{slope of } m(t) = \pm \frac{2}{10^{-4}} = \pm 20{,}000$

345



So, $B.W = 2\{\Delta f + f_{max}\}$

$= 2\{K_f A_m' + f_{max}\}$

& B.W of PM is to be calculated by FM $d/dt\ m(t)$, then K_f replaced by K_p

$B.W = 2\{K_p A_m' + f_{max}\}$

$= 2\{5/2 \times \cancel{20000}^{10000} + 15000\}$

$= 2 \times 55k$

$BW = 130k$

) $B.W \uparrow = 2(\Delta f \uparrow^{2} + f_m)$.

$B.W \uparrow = 2(\Delta\phi \uparrow^{2} + 1) f_m$

conclusion ...

346

PAM:

Generation → AND gate

demod → LPF (Integrator)

PWM:

Generation → Monostable M.V

demod. → LPF (Integrator).

PPM:

Generation → PWM → $\boxed{\frac{d}{dt}}$ → $\boxed{\text{clipper}}$ → PPM

demod → PPM → $\boxed{\text{Bistable M.V}}$ → PW.M.

* Granular Noise power = $\frac{\Delta^2}{3}$; Δ = Step Size

* % of modulation for FM = $\frac{\Delta f}{\Delta f_{max}}$; $\Delta f_{max} = 75K$ standard.

* For PAM → Roll off factor (α)

$$\boxed{B.W = \frac{R_b}{2}(1+\alpha)}$$